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P D Killworth, K Döös & S R Thompson

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ABSTRACT

This review outlines our current knowledge of the effects of resolution on ocean general circulation models used for climate studies. Comparisons are made between the marginally eddy-resolving models currently in use.

Poleward heat flux is the quantity which it is most vital to obtain accurately in order that coupled atmosphere-ocean models will function correctly. This quantity has consistent values over a wide range of resolutions, although its longitudinal distribution may vary. Other quantities, such as eddy energy, depend heavily on resolution and only show signs of asymptoting to some 'correct' value for extremely fine resolution.

Grid resolution should be balanced between the horizontal and vertical, in the sense that the parameter Nh/fL should be of order unity, where N is the buoyancy frequency, f the Coriolis parameter, and h and L are the vertical and horizontal spacings respectively. A simple numerical representation of the Eady problem shows that erroneous instabilities can occur if this quantity differs noticeably from unity.

Coarse resolution models must use parameterization schemes to emulate the mesoscale effects that the model cannot resolve. A review of the extant parameterizations is given. A promising line of attack may involve working with a modified velocity, which includes the along-isopycnal eddy transport of tracer quantities.

KEYWORDS

FRAM
NUMERICAL MODELLING
OCCAM
OCEAN MODEL

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1. INTRODUCTION: MODEL REQUIREMENTS FOR CLIMATE AND ITS SIMULATION

Climate modellers face great difficulties when they seek to reproduce today's, or predict the future's, climate. The climate system involves atmosphere, ocean, ice, land surface, biology and chemistry. The level of understanding of these different regimes varies, with the atmosphere relatively well understood, and the ocean rather more poorly so. Coupling these different systems together makes requirements on our understanding of them. From the point of view of a specialist in any of these areas, the ideal climate model would consist of a set of modules. Each module would be fed by, and feed, other modules; its internal workings would not need to be known by the other modules, provided merely that it returned the necessary output for the remainder of the model. Thus, for an atmospheric specialist, an ocean model that returned (say) the surface heat and moisture flux as a function of position and time would be adequate - how it obtained these values would be unimportant. Of course, an oceanographer might regard an atmospheric model in a similar light.

What is needed, then, is an ocean model that behaves in as similar a fashion to reality *as far as the atmosphere above it is concerned.* It is unlikely to be feasible in the near future that ocean models used in climate prediction will be able to reproduce the ocean circulation with the level of detail which would satisfy an ocean modeller (but cf. the comparison of heat fluxes between Southern Ocean observations by de Szoeke and Levine, 1981 and FRAM fluxes by Thompson, 1993). Many processes will have to be subsumed, ignored, or parameterised in some fashion. Whatever the representation, though, the result must feed into the other modules 'correctly'.

But the definition of 'correctly' is not clear. If only the surface fluxes are a necessary output from an ocean model, to what accuracy and resolution are they required? For example, an ocean model which resolves the western boundary layers of the ocean, such as the Gulf Stream, is already probably providing significantly more finely resolved surface information than the atmospheric model above it. Does this matter for the atmospheric model? If these fluxes are now averaged in some fashion, can the inherent misplacement of the fluxes by the averaging, and the errors that the averaging induces, cause difficulties for the atmospheric model? Thinking again of the Gulf Stream example, would a misplacement of the large heat loss from ocean to atmosphere north of the Gulf Stream have a global, rather than a local, effect? Fine resolution ocean models suggest that the thermohaline conveyor belt is a rather more chaotic structure of different interbasin pathways, with water particles initially very close having trajectories which diverge dramatically (Döös, 1994a). What effect does the inevitable loss of detail in the thermohaline circulation have on its representation in climate models?

Time resolution is also important. The natural time scale for all ocean processes apart from barotropic motions - which only have a small effect on surface fluxes - is long compared with typical atmospheric time scales. Thus one might imagine that ocean time scales will be well resolved by any model and not give any difficulties. However, the ocean has a very large heat content, and so erroneous heat *storage* can significantly accelerate or delay atmospheric response to an ocean anomaly.

In this paper we review what is known about the representation of the ocean using varying spatial resolution in numerical models. In Section 2 we examine the types of behaviour exhibited in observations and by ocean models which are too small to be resolved by climate models. Section 3 examines numerical requirements for ocean models, Section 4 examines what is known about parameterisations, and Section 5 discusses the existing comparisons of ocean models at various resolutions. We conclude with some recommendations.

2. SUBGRIDSCALE OCEAN PHENOMENA

In this section we briefly discuss ocean phenomena which are too small to be resolved by climate models, and what is known about their parameterisation.

2.1 Eddies, rings and meddies

The most important oceanic feature omitted by climate models is probably the ubiquitous mesoscale eddy field. Eddies occur on scales of order a few internal deformation radii, and so vary in size between about 100 km in the subtropics to perhaps 30 km in subpolar regions (estimates of deformation radii are given, eg., by Houry *et al.*, 1987). A survey of our knowledge of eddies is given by Robinson (1982). Recent satellite data (Shum *et al.*, 1990) have permitted a global description of surface eddy energy (subject to certain stringent assumptions about either eddy isotropy or time scales of variability depending on the method used). Despite internal differences between estimates, these studies have demonstrated that eddy kinetic energy varies by orders of magnitude between intense, eddy-active regions such as western boundary currents, and the quiet regions typically found in the eastern subtropical gyres (cf. Fig. 1). It can be quite hard to isolate and follow a particular eddy in the 'soup' which is typical of the mid-ocean.
Eddies are believed to be formed by several processes, including local baroclinic/barotropic instability, wind forcing, and shedding by strong currents over topography. The first of these processes must be associated with a readjustment of the density field, because the potential energy serves as an important source for the instability. This would be predominantly poleward on average, but active local currents such as the Gulf Stream are not oriented East-West and so their instabilities can act to move heat and salt in a non-poleward direction.

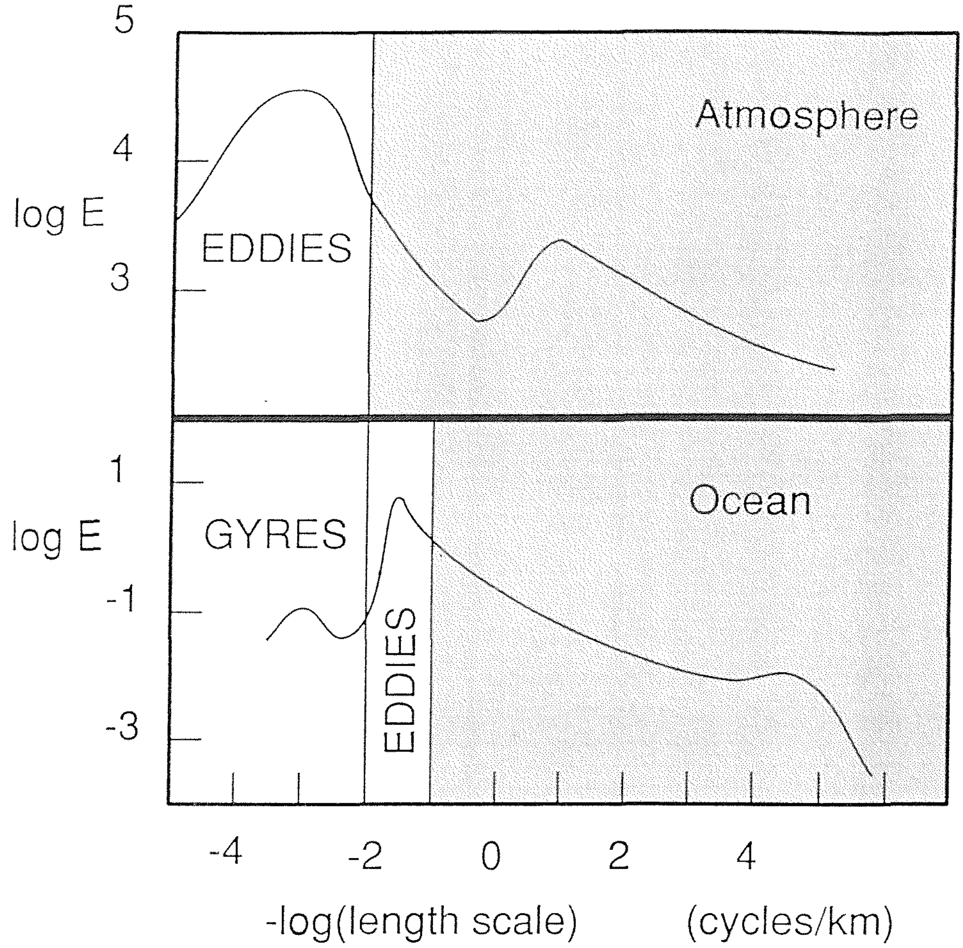

Figure 1: Kinetic energy spectrum as a function of horizontal wave number in the atmosphere and ocean. The ordinate is in units of m^2s^{-1}.

Gulf Stream eddies, usually termed rings, can be quite long-lived (Olson, 1991), which implies that they may be unusually efficient in moving heat, salt and momentum through the ocean. Their average hydrographic structure is fairly well known, and consists of a 500 to 700m thick layer of thermally anomalous water. They are about 60 km in radius, fitting Killworth's (1983a) theory of minimum eddy radius. Warm-core rings have shorter lifetimes than cold, largely because they are routed by the slope topography north of the Gulf Stream and tend to re-enter the Stream south and west of their formation region. Were it not for the surface heat loss undergone by warm-core rings, they would not thus transfer any properties meridionally. Cold rings, on the other hand, can migrate long distances and hence form efficient transport mechanisms. The Agulhas rings in the FRAM model were estimated to transport 0.2 PW equatorward in the S. Atlantic (Webb *et al.*, 1991).

Smaller eddies, termed 'submesoscale coherent vortices' by McWilliams (1985), are also beginning to be found in large numbers (there is evidence that their signal in early CTD work was considered anomalous and discarded). Typical among these are the 'meddies', or Mediterranean eddies. These consist of lenses about 60 km diameter containing, at depths of about 600 to 1500 m, about one month's outflow of warm, salty Mediterranean outflow. They are found all over the subtropical N. Atlantic. The best studied is Meddy Sharon (Armi *et al.*, 1989) which was tracked for two years as it moved slowly southwards, gradually decaying, until finally it disappeared. Because of the large heat and salt anomalies present in Meddies (4°C, 0.6 psu), they can conceivably transport large amounts of heat equatorward if there are enough of them present. Their number is unknown (they are after all small, and hence hard to detect) but estimates by Armi *et al.* (1989) and others suggests at least tens of Meddies present at any one time. With a typical anomalous heat content of a Meddy of around 3×10^{19} J and a frequency of tens per year moving southwards, this only yields a few hundredths of a PW, compared with the larger values for Agulhas rings.

Eddy effects are typically parameterised in numerical models by isotropic Laplacian or biharmonic diffusion and viscosity terms acting on the mean flow, mainly for numerical reasons to be discussed below. (But see Gent and McWilliams, 1990 for some discussion.) This is based - rather loosely - on assumptions of isotropic stirring by small-scale motions, which replaces $\overline{v'T'}$ terms by a downgradient diffusion term acting on the mean flow. It is difficult to see how these assumptions can hold in general from the discussion above. Ring formation is primarily a non-isotropic phenomenon; Agulhas heat transport has a well-defined direction and magnitude associated with it which bears little resemblance to gradients of the mean flow around it; and the Meddy transport, whatever its size, is of uniform direction but opposite to the mean gradient. How values of the coefficients are arrived at will also be discussed below.

2.2 Fronts

Fronts form in the ocean for many reasons, of which large-scale convergence is a frequent candidate (although, of the three major Antarctic fronts, one is in a region of convergence, one in divergence, and one in a region of neither). They are narrow (of width of order a deformation radius or two), often contain strong along-front velocities, and are highly meandering in nature. Fronts are important because of the strong ageostrophic vertical motions present in them (cf Pollard and Regier, 1992), often of order 20 m/day. These motions can expose large amounts of subsurface water to atmospheric effects, and the resulting air-sea fluxes of heat and salt can be considerable.

It has proven extremely difficult to mount suitable observation regimes to determine frontal dynamics because of the meandering (the Fasinex cruise is a good example here), and other than phenomenologically we know little about the dynamics.

Process models of fronts abound (e.g. Welander, 1981). Most frontal situations can be shown to be generically unstable, at least for cut-down models such as one- or two-layer systems (Killworth and Paldor, 1987; Benilov, 1993). Three-dimensional numerical frontal models have been successfully run (James, 1987) but it is probably true to say that these, in company with most high-resolution numerical process studies, have still to advance our understanding of how to parameterise the problem correctly.

2.3 Other narrow currents

Frontal features are not the only areas of strong narrow currents (although such currents may well have fronts associated with them). Western boundary currents are also not well resolved by climate models - indeed, many climate models do not contain recognizable western boundary currents at all. The resolution of such currents is important for heat transport. In the simplest (one- and a half-layer) case, the net (geostrophic) mass transport by a narrow current is proportional to the difference in h^2 at each side of the current, where h is the fluid depth. In such a case, this transport is also proportional to the heat transport. So the problem of obtaining the correct transport reduces to that of getting the correct layer depths on each side of the boundary current. That in the fluid interior is presumably adequately represented by Sverdrup-like dynamics which are well resolved by coarse climate models. However, the depth on the landward side of the western boundary current is determined, even in this simple case, by ageostrophic dynamics which are not resolved. Accordingly, the poleward heat flux will be wrongly estimated by the model.

Narrow currents are also involved in deep inter-basin exchanges through sills, which carry much of the thermohaline circulation. Observations (cf. Saunders, 1990) suggest that eddy activity is strong in such regions. It is typical that sills largely disappear from coarse resolution models both because of the smoothing that is required to remove two-grid-point noise from the topography and because the sill topography is frequently too narrow to represent. Indeed, Wadley (1994) found difficulty in emulating flow through the Vema channel in FRAM: even at that model's fine resolution, the channel had disappeared!

Theory (cf. Killworth, 1992) shows that even sills which are wide compared with a deformation radius must inevitably have narrow, ageostrophic flows associated with them. Again, these cannot be properly represented by coarse resolution models. There are no estimates of how much error poor resolution causes, although Wadley (1994) has shown that an underestimate of the observed flux by up to a factor of 20 is possible even in the high resolution (by climate standards) FRAM model.

2.4 Topographic representation

Until recently, the ubiquity of the GFDL model for ocean climate studies meant that topography was always represented as a collection of steps. The appearance of isopycnic and sigma-coordinate models, which formally maintain an accurate representation of the topography, has given modellers potential improvements in this field. However, coarse resolution of topography may simply be inadequate on dynamical grounds. Consistent estimates of the flux of the Antarctic Circumpolar Current in coarse resolution models give values about 100 - 120 Sv, which are well in line with observations (cf. Nowlin and Klinck, 1986). Marginally eddy resolving models, on the other hand, give fluxes of order 190 Sv. (Semtner and Chervin, 1992; Webb *et al.* 1991). The difference is due partly to topographical variations, and partly to interpolatory errors between widely spaced moorings (Grose, 1992), but some of it lies in the poor treatment of topographic form drag by coarse-resolution models.

2.5 Deep convection

Oceanic deep convection forms the closure of the thermohaline circulation. It occurs in two forms: slope convection, and open-ocean convection (cf. Killworth 1983b for a detailed review). Slope convection involves the movement of dense water, formed typically by intense winter cooling on a wide continental shelf, downwards and laterally along a neighbouring continental slope in a dense and narrow turbulent plume. This plume mixes with its environment as it descends the slope, and eventually leaves the slope somewhere near a depth at which it has the same density as its environment. Open-ocean convection involves the formation of narrow 'chimneys' containing one or more vertically convecting cells. These cells are driven by surface cooling, and surface losses of energy can exceed 500 W m^{-2}. Laboratory experiments (Maxworthy and Narimousa, 1991) and numerical experiments (e.g. Jones and Marshall, 1993), together with direct oceanic measurements (Schott and Leaman, 1991) confirm that strong vertical motions (5 cm s^{-1}) act to move water particles rapidly over the top 2000 m of ocean.

In both cases the relevant words describing the mixing regions are 'narrow' and 'cooling'. Horizontal length scales are less than, and in many cases much less than, a local deformation radius. The cooling occurs during autumn, winter and perhaps early spring. No basin-scale or larger numerical models have yet been able to resolve such length scales, and many models do not have seasonal surface forcing (or if they do, the forcing is climatological and often lacks the extreme events which cause deep convection). As a result, representation of deep convection in climate models spreads over many gridpoints and is far weaker than is observed. Most models represent vertical mixing either by an infinitely rapid mixing when there is static instability, or by an increase of several orders of magnitude in the vertical diffusion coefficients when stability drops below some threshold value.

It is unclear what effect this weakness in the vertical mixing has on the models' representation of the thermohaline circulation. The localised forcing in the real ocean acts like a distributed source of heat and salt, and observations suggest that this is rapidly smeared out by local turbulence and mixing into water masses occupying many deformation radii in the horizontal. Is the much broader representation of the process by coarse resolution models adequate or not? A promising technique for open-ocean convection is due to Alves (in preparation), who has sought to embed a classical ensemble of vertical plumes within a grid box of a coarse resolution model. A critical test of this approach will be to compare it with the simple vertical averaging used in numerical models, which recent scaling arguments (at least for convection into unstratified fluid columns) have suggested are adequate (Marshall, personal communication).

2.6 Ice

At high latitudes, ice plays a crucial role in maintaining the thermohaline circulation. In times of freezing, it rejects dense saline water into the ocean beneath and so can trigger deep convection. It acts as a barrier between atmosphere and ocean, thus modifying the momentum, heat and salt uptake by the ocean. It transports large volumes of fresh water from formation sites to export areas. Aagaard and Carmack (1989) have estimated that one-third of the fresh water budget of the Arctic is due to ice export in the East Greenland Current, for example.

Ice thermodynamics is relatively well understood (the majority of models use a version of Semtner's 1974 thermodynamic model). It is ice dynamics that still presents problems. In regions of divergence, simple ice drift formulations based on wind speed and direction work well (Wamser and Martinson 1993), but in convergence regions ice ridging and other processes occur which are not well understood. The internal ice dynamics appears to possess many similar features to ocean dynamics, despite the difference in rheologies: for instance, ice can possess a potential vorticity; eddying at the edge of ice streams (the so-called marginal ice zone) is frequent, and occurs on the same scales as the ocean. Since much of the ice-driven ocean motions also occur in the marginal ice zone, including much deep convection, it is clear that ideally models should resolve the ice scales as well as the ocean scales. Unfortunately, there is evidence (Gray and Killworth 1993) that the standard ice model in use, due to Hibler (1979) possesses instabilities in divergent regions which are masked only by the use of artificially high Laplacian and higher diffusivities; it is estimated that a reduction of grid spacing to eddy scales would induce this instability. Newer rheologies (e.g., Flato and Hibler, 1992) do not suffer from this problem.

3. NUMERICAL CONSIDERATIONS

3.1 What prevents a numerical model from functioning?

We noted above that all ocean models require artificial treatment of some sort to ensure that they actually function, ie., that they do not blow up. For example, an attempt to run a gridpoint model without artificial viscosity and diffusion will fail. Such coefficients (and forms) are chosen primarily on the basis of numerical stability and only secondarily for physical reasons, if at all. We first examine the numerical restrictions on these coefficients (since, if these are not satisfied the model will fail, and physical requirements become irrelevant).

A list of 'standard' numerical requirements is given by Killworth, Smith and Gill (1985). These include the restriction that no numerical process may spread across a gridpoint in a time step, which results in the Courant-Friedrichs-Lewy criterion for wave propagation and advection, and diffusive and viscous restrictions on timesteps.

Other physical requirements usually enter. For example, there is a need to resolve the western boundary layer adequately (its width varies as the one-third power of lateral viscosity). This places an *upper* bound on grid spacing, which can be in conflict with the *lower* bound set by timestep restrictions above. Thus the method of choosing the horizontal viscosity Ah deserves more detailed explanation. The condition of no-slip at the lateral walls requires that a viscous boundary layer be resolved at the side walls. If this boundary layer is not resolved the error excites a computational mode that extends out into the interior, resulting in a checkerboard pattern in the stream function (Bryan *et al.* 1975).

The viscous boundary layer width according to Munk (1950) is

$$\Delta < L_m = \frac{\pi}{\sqrt{3}}\left(\frac{A_h}{\beta}\right)^{1/3} \tag{1}$$

where A_h is the horizontal friction and β the northward gradient of the Coriolis parameter. Therefore a given grid spacing Δ requires that

$$A_h > \beta\left(\frac{\sqrt{3\Delta}}{\pi}\right)^3 \tag{2}$$

Another criterion arises through the inaccuracy of the finite-difference model at the smallest scales that can be resolved. Even in a completely stable computation a computational mode can be excited when centred differencing is used if the Reynolds number is not less than a certain critical value. The criterion derived by Chen (1971) is

$$Re_{grid} = \frac{U\Delta}{A_h} < 2 \tag{3}$$

where U is a typical velocity. These two requirements are illustrated in Fig. 2. The solid line represents the Munk layer thickness criterion and the dashed the Reynolds number criterion. U = 0.1 m/s and the latitude is chosen to be the equator, where the most severe conditions are. Values below the lines may permit the growth of a computational mode. Neither Semtner and Chervin's (1992) model nor FRAM satisfy these criteria. The lack of checkerboard pattern close to the western boundary suggests therefore that the non-linear advection terms act in a similar way to the horizontal viscosity, to widen the western boundary. See Pedlosky (1979) for more details. Covey (1992) showed that the western boundary currents improved qualitatively as resolution increases.

Other related restrictions have been discovered. Weaver and Sarachik (1990) found artificial circulatory cells at the equator in their vertically coarsely resolved model. These were shown to be due entirely to the poor vertical resolution, and were caused by a failure of the model to adequately represent the predominantly vertical advective-diffusive balance $wT_z = \kappa T_{zz}$. Here z is the vertical coordinate, w the vertical velocity, T the temperature, and κ the vertical diffusivity. The analytical solution of this problem is an exponential with depth scale κ / w When the cell Peclet number $(w \Delta z / \kappa)$ exceeds 2, this solution is not adequately resolved in the same manner as the Reynolds number above, and the numerical solution becomes a decaying oscillatory function of depth, changing sign at each gridpoint.

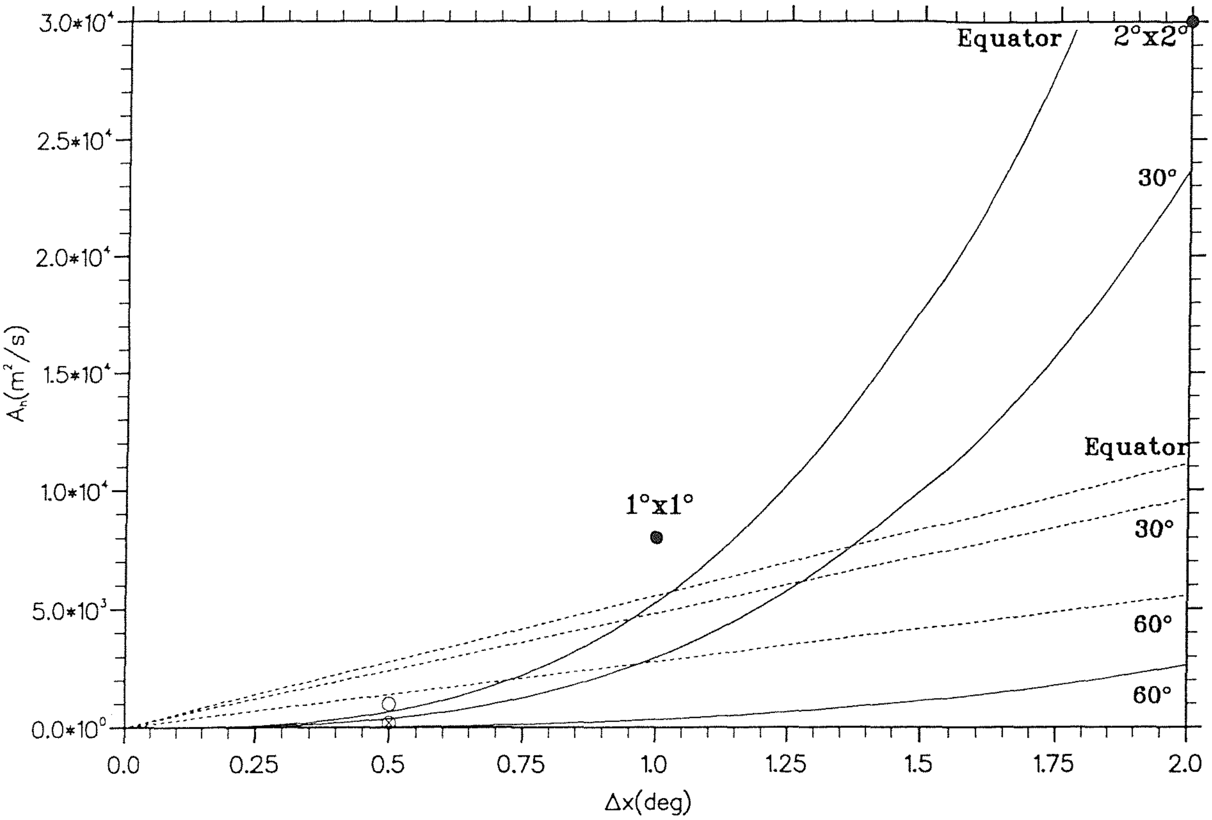

Figure 2: The Munk layer thickness criterion (solid lines) and the Reynolds number criterion (dashed lines) as a function of resolution for the equator, 30° and 60°. U = 0.1 m/s. The dots illustrate parameters chosen by Covey (1992) for his 2° and 1° runs. The ○ represents Semtner and Chervin (1992) and the ⊗ FRAM. Values below the lines may permit the growth of a computational mode.

Killworth (1989) had independently discovered this effect in an 8-level ocean code. This model was run with a prescribed surface temperature ranging between 4 and 20°. Its steady state solution had bottom temperatures less than -1°, but no oscillations were detectable in the solution. He found that vertical overturning had occurred in the solution induced by the natural movement towards oscillations in the vertical. This had induced uniform values of temperature (and salinity in general, although this had not been included in the model), and acted as an erroneous source of cooling at depth.

Analysis of the rarely published vertical velocity fields in models does indicate that cell Peclet numbers frequently exceed 2, either near-surface (because of down- or upwelling induced by the nearness of a vertical wall) or at depth (because of large values of Δz). When no apparent oscillatory behaviour is visible, we must deduce that horizontal advective or diffusive terms are also present in the term balance and so prevent a clearly erroneous solution: but the resulting solution can still be in error.

The influence of different numerical advection schemes on model circulation has been investigated by Gerdes *et al.* (1991). A centred difference scheme, an upstream scheme and a flux-corrected transport scheme were compared. It was found that the properties relevant for climate response experiments, such as meridional heat transport and thermocline thickness depend not only on explicit mixing parameters, but also on implicit parameters hidden in the numerical scheme. They pointed out that models with higher vertical resolution are less sensitive to implicit mixing. The second order accuracy of the central difference scheme is an advantage, and, as the computational expense increases only linearly with vertical resolution, this approach is recommended for long integrations.

3.2 Do models perform correctly?

Even if models give a plausible solution, they may still be seriously wrong, as a previous example showed. A series of papers (Davey *et al.* 1983, Hsieh *et al.* 1983, Wajsowicz 1986) examined the numerical wave properties of a finite-difference ocean model, in the spirit of the atmospheric analyses of, e.g., Mesinger (1974). Their findings were confirmed by direct numerical integrations of ocean codes by Cherniawsky and Mysak (1989) at varying resolution. These findings were that the speed of a numerical Kelvin wave is almost exactly proportional to the grid resolution used (provided that the deformation radius - the natural length scale of the wave - is not properly resolved, of course), and is independent of the eddy viscosity used. Numerical planetary (Rossby) wave speeds depend only weakly on resolution, but are too heavily damped if the wave is not resolved by the grid used. Since both waves are natural components of oceanic response to forcing, this suggests that grids need to be adequate to give not too distorted a response; Cherniawsky and Mysak (1989) suggest grids less than 1° in size.

Drijfhout (1992), performing an intercomparison between various model types, noted that formulations which did not conserve potential vorticity led to poor eddy simulations, with unpleasant noise in extrema and corresponding poor behaviour in integrated quantities such as heat flux. Most published ocean models do not show highly differentiated fields such as *w* or potential vorticity simply because they do display a high degree of noise. Yet invertibility principles (Hoskins *et al.*. 1985) suggest that modellers should first seek to reproduce such fields correctly, and then less differentiated fields, which require a degree of integration to acquire them, will automatically be smooth.

3.3 What are the best numerical choices?

Circulation models have frequently been used as test beds for nonlinear simulations which can have no analytical results, and it is normally assumed that the model is performing adequately. For example, Haidvogel and Holland (1978) ran a two-layer quasigeostrophic model of an eddy-resolving ocean basin. They then used an analytical-numerical method to solve some parallel shear flow problems numerically for the corresponding linear instability theory. (The work was extended by Holland and Haidvogel, 1978). Both studies looked only at system parameters (beta parameter, width of channel etc.) and not at the numerical resolution of the model.

An early test of numerical resolution was made by Simmons and Hoskins (1976) who solved the same (atmospheric) instability problem with varying horizontal and vertical resolution, using a sigma-coordinate primitive equation model. They used 2, 5, 8 and 16 vertical levels, and concluded that '... inadequate vertical resolution can result in excessive growth rate' and '... caution must be exercised in attaching physical significance to the short-wave spectra of such model'.

The main difficulty for ocean model resolution is the small size of the eddies. The length scale for mesoscale motion is set by the deformation radius, so a theoretical measure of eddy-resolution is the ratio between the deformation radius and the grid size. The barotropic deformation radius is of the order of 1000 km in the ocean and therefore easily resolved. On the other hand, the baroclinic deformation radius is rarely resolved in GCMs, even in the high resolution ones. The first baroclinic deformation radius versus the grid size is plotted in Fig. 3. The ratio (deformation radius/Δx) must be greater than one if the deformation radius is to be resolved at all. The figure shows that the deformation radius becomes very small at high latitudes, where the stratification is weak. The horizontal resolution for the polar seas would therefore have to be of the order of 0.01°. This would make it more than a million times more expensive than the Semtner and Chervin (1992) 0.5° resolution run (keeping the ratio of horizontal/vertical resolution constant).

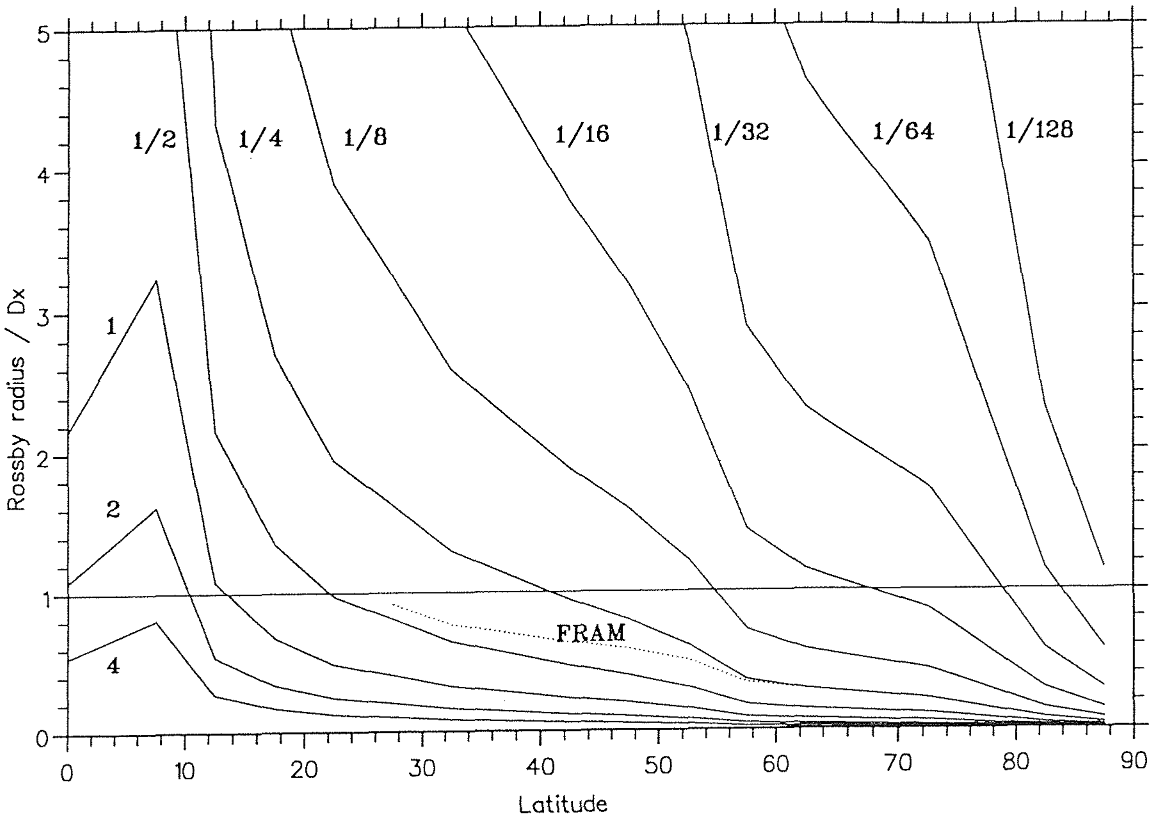

Figure 3: Deformation radius as a function of latitude (from Houry *et al.*,1987).

Models will probably therefore not be able to resolve the deformation radius in the foreseeable future. The question remains as to what extent this affects the solution. Furthermore, does poor resolution affect only the local dynamics? Can important processes such as bottom water formation be adequately parameterised using coarse resolution?

One of the few tests of the accuracy of a primitive equation ocean general circulation model was made by Wood (1988). He tested his channel model to see if it could reproduce the analytical-numerical results of Killworth, Paldor and Stern (1984) for two-layer fluid instability. With a grid spacing of one-quarter of a deformation radius, and 8-10 points above the interface, he obtained the analytical results to within 10%. From Wood's point of view, this validated his model (the majority of his paper looked at nonlinear effects). For the present discussion however, it demonstrates that 10% errors can be obtained even from an apparently very well-resolved model.

So how should values of grid spacing, etc., be selected? Fox-Rabinowitz and Lindzen (1993) suggested that the resolutions chosen should be balanced, in the sense that fine horizontal and coarse vertical resolution, or vice versa, should be avoided. Their study built on the earlier work of Lindzen and Fox-Rabinowitz (1989). Both studies found that models need $\Delta x \sim (N / f)\Delta z$, where $\Delta x$is a typical horizontal spacing, and N and f are a typical buoyancy frequency and Coriolis parameter respectively. (We assume here that Δy is of the same order as Δx which may not be the case for, e.g., equatorial models.) Such a scaling would be expected from the natural quasigeostrophic balance $NH / fL \sim 1$ where H,L are depth and height scales respectively. For typical values of N / f between 10 and 100, and 200m vertical resolution, we find that a horizontal resolution of 2-20 km is required! Thus the resolution of most ocean models can be considered poor. Fox-Rabinowitz and Lindzen suggest filtering if adequate resolution can not be satisfied. Experience from ocean models at high latitude, when filtering has been employed as an option, is that the filtering - essentially an operation on a scalar - is ineffective on fluxes, which are products of scalars, and other techniques should be employed. Adequate resolution is but one!

The need for adequate resolution was addressed by Vallis and Hua (1988), who looked at how model resolution affected the spectral distribution of explicit eddy viscosity effects. They found that omitting the subgrid scales in coarse resolution runs seriously affects the inverse energy cascade to smaller wavenumber, and hence produces errors in the largest scales of flow, which one would assume a priori were well resolved.

A related topic was examined by Barnier *et al.*, (1991), who ran a 2-gyre, 6 layer quasigeostrophic EGCM with 20 and 10 km resolution. They found much better behaviour at higher resolution, although the model Gulf Stream perhaps extended too far into the mid-Atlantic. To get consistent resolution, they found that three-dimensional scales (in terms of a pseudowavenumber $K^2 = k^2 + 1 / a^2$, where k is horizontal wavenumber and a the deformation radius for the mode of interest) should be within a factor of 7 of the spectral peak to be resolved.

What this means is that for a baroclinic instability peak at around $\left(1/\sqrt{2k_1}\right)$,where $k_1 = 1/a$, we must resolve $K_{\max}^2 = (49/2)k_1^2$. Converting this means that, at the extreme of a purely horizontal structure $k^2 = (49/2)k_1^2$ and for the opposite extreme of a purely vertical structure we must resolve vertical scales $k_n = \sqrt{49/2k_1} \sim 5k_1$. Thus Barnier *et al.*'s overall conclusion was that resolution must be distinctly finer than one would estimate; higher order vertical modes play important parasitic roles in baroclinic instability.

We can estimate this directly, by performing the equivalent of the Kelvin and Rossby wave analysis discussed above for the simplest instability problem - the Eady problem. The exact, and approximate numerical, solutions are derived in the Appendix. We find broad agreement with the studies discussed above, in that good vertical resolution (at least 10 points in the vertical) and good horizontal resolution ($k\Delta x < 0.5$, for any wavenumber *k* which needs to be resolved) is required if the resulting waves are not to be grossly misrepresented. Indeed, stable Eady solutions - which analytically have two wave solutions propagating in opposite directions - become *unstable* if they are badly resolved, with growth rates of the same order as the correct frequency. Such erroneous instabilities may be among those features of coarse resolution models which need to be controlled by artificial damping terms.

4. PARAMETERISATION OF MESOSCALE EFFECTS

Little is known as to the best method for parameterising mesoscale effects, and much of what appears in the literature is remarkably ad hoc. We shall examine here what *is* known about parameterisations.

4.1 Choice of the diffusion and viscosity

Holland (1971) made a study of tracer distributions simulated by the GFDL model. The meridional circulation consisted of a small region of sinking at high latitudes and upwelling at lower latitudes. In contrast to the traditional view that the upwelling is broadly spread over the oceanic interior, it was mostly confined to the western boundary current region in the model. The reason was thought to lie in the particular choice of the horizontal coefficients of momentum and heat diffusion: $A_m = 5 \times 10^4\ m^2s^{-1}$ and $A_h = 5 \times 10^3\ m^2s^{-1}$. When the horizontal coefficient of heat diffusion is too large relative to the horizontal coefficient of momentum diffusion, the temperature contrast across the western boundary current, required by the thermal wind relation, can be maintained only by a vertical upwelling of cold water adjacent to the boundary. The three dimensional circulation conspires to pump much of the water sinking to abyssal depths in high latitudes back to the surface in the boundary layer, leaving little to feed the interior. Note that the presence or lack of interior upwelling affects thermocline maintenance. The lesson of this work is that the large scale circulation is so sensitive to the choice of diffusion parameters that very different regimes of flow are possible even with the same physics and boundary conditions. Following these results, almost all subsequent model studies made use of large horizontal viscosities and as small heat diffusivities as possible on a given grid, to create a flow regime resembling the classical one. However, we still do not know if the boundary upwelling found in the models is realistic.

Olbers (1989) uses various different forms of the beta-spiral method to estimate the diffusion coefficients. The estimated coefficients show patterns which, by and large, confirm our concept of mixing activity in the ocean: larger values occur in areas of strong currents and eddy activity. The peak values, however, appear fairly large: in the ACC estimates for diapycnal coefficients are as high as $10^{-3}\ m^2s^{-1}$.

The vertical viscosity A_v has traditionally being chosen as a constant. A parametrisation of the turbulent mixing processes as a function of the Richardson number [$Ri = N^2/(dU/dz)^2$] was introduced by Pacanowski and Philander (1981). They set the vertical viscosity to

$$A_v = \frac{A_o}{(1+\alpha Ri)^n} + A_b \tag{4}$$

and the vertical diffusion:

$$K_v = \frac{A_o}{(1+\alpha Ri)^n} + K_b \tag{5}$$

which led in their study to a more realistic simulation of the response of the equatorial oceans to different wind stress patterns. The parametrisation was tested on the Indian Ocean where the monsoon reverses the equatorial currents every year. In the case of eastward winds, results agree well with observations. For westward winds it is of crucial importance that the heat flux into the ocean is taken into account, so that it can stabilise the upper layers and reduce the intensity of the mixing, especially in the east. Smith and Hess (1993) compared this parameterisation with a second-moment closure model (Mellor and Yamada, 1982). SMC models, as they are presently formulated, are meant to account for small scale turbulence; in the ocean; this translates into scales of the order of tens of meters. Larger scales should in principle, be accounted for by fine resolution or by sub-grid scale parameterisation. The comparison of the two parameterisations showed that the Pacanowski and Philander model was mainly driven by changes in the stratification rather than shear-generated instabilities, and the position and the width of the mixing transition zone between high and low mixing values was found to be sensitive to the parameters of the model. In the second-moment closure model the master length scale limit effectively determines the threshold of the mixing zone, while the inclusion of storage, advection, and diffusion terms in the turbulent kinetic energy equation affects both position and extent of the transition zone. The study also showed that there is no simple functional relationship between the gradient Richarson number and the intensivity of mixing in the second-moment closure schemes.

Blanke and Delecluse (1993) made another comparison between a Richardson number-dependent parameterisation of the mixing coefficients and a turbulent closure model for the tropical Atlantic Ocean. Obvious contrasts between the two experiments on the sea surface temperature and on the dynamics indicated that turbulent vertical diffusion plays a major role in the surface processes simulated by the model. Comparisons with observations showed that the turbulent closure scheme improves the representation of sea surface temperature, the vertical mixed-layer structure, the equatorial meridional cell, as well as the equatorial undercurrent, which becomes more energetic.

Greatbatch and Lamb (1990) let the vertical eddy viscosity depend on the buoyancy frequency N and the Coriolis parameter f

$$A_v = \frac{Af^2}{N^2} \tag{6}$$

This leads to isopycnal mixing of f/q, where q is the abbreviated form of the potential vorticity, provided that A is independent of the vertical coordinate. If additionally, A is also independent of the latitude, then on a beta-plane, this implies homogenisation of q within closed q-contours on isopycnal surfaces. This conclusion extends to spherical geometry if $A_v = af^2\left(\beta N^2\right)$. A sensitivity study of a similar parameterisation was carried out by Cummins *et al.* (1990). A case with constant diffusivity was compared with a case in which the diffusivity is inversely proportional to N. The stability-dependent parameterisation of vertical diffusivity yields a poleward heat flux similar to that of a small, constant diffusivity. However this parameterisation increases the mean temperature in the deep ocean by about 0.8°C and the strength of the meridional circulation by 40%. In addition, the stability dependent diffusivity is found to increase the stratification in the deep ocean. The experiment suggested that it may be possible to calibrate the rate of deep-water formation in GCMs, without affecting the poleward heat transport, by varying the magnitude of the vertical diffusivity below the thermocline.

The above parameterisations have been for mixing along fixed depths. However, it is believed that the mixing in the ocean takes place along isopycnal surfaces. One solution to this is to abandon the z-coordinate system in favour of a density coordinate system. Another solution is to introduce a more complex mixing tensor which senses the local isopycnal slope and conducts diffusion in that direction. This technique has been described by Redi (1982) and Cox (1987). The theory has been extended by Gent and McWilliams (1990) and Gent *et al.* (1993).

5. MODEL-MODEL AND MODEL-OBSERVATION COMPARISONS

This section consists of a brief review the state of the art of ocean numerical modelling, and look at what has been achieved in some of the many global and limited area, high and low resolution integrations to date. We also consider the shortcomings of the various models, and compare the model output with observations.

5.1 Global high resolution models

Perhaps the most advanced in the field of high resolution global ocean models is the work of Semtner and Chervin (1992), although the model is not strictly global, as it extends only as far north as 65°N. They have used the best available computer of the day to carry out a 32.5 year run of a 0.5°, 20 level model.

The model reproduces the mean and eddy characteristic of mid-latitude gyre circulations, western boundary currents, zonal equatorial flows, and the ACC. There is some indication of eddy intensification of the mean flow of the ACC and of separated boundary jets. Global thermohaline circulation of North Atlantic Deep Water (NADW) occurs in deep western boundary currents linked by the ACC, with zonal jets playing an important role near the equator (being associated with enhanced equatorial upwelling which may arise out of the Pacanowski and Philander vertical mixing scheme).

Eddies transport heat and salt down the gradients and along the mean flow in strong current regions. The complexity of these processes suggests that they may not be easy to represent by parameterisations. Eddy transports are particularly important for warming the Pacific upwelling branch of the thermohaline circulation and for transporting salt across the equator into the North Pacific. Mid-latitude instabilities are too weak compared with observations. Heat transport is northward between 28 and 47°S, perhaps due to insufficient eddy heat transport in the ACC (particularly near the Brazil Falklands Current Confluence and the Agulhas retroflection) which in turn could be due to insufficient resolution. During the integration a change from Laplacian to biharmonic closure was made and the two regimes compared. The time-mean flows were narrowed and strengthened in the latter case. This is seen as a being a problem for climate simulations where one might hope to represent time-mean flows with the correct size and strength and be able to use accelerated timestepping because of quasi-steadiness. Semtner and Chervin also comment that negative viscosity effects of eddies may require proper parameterisation in regions of intense currents.

Of course, such an integration has an extremely high computational cost. Semtner and Chervin quote a figure of 500 machine hours per model decade for a 0.5 degree model with 20 levels in the vertical. (Accelerated spinup strategies do exist, but these are of little use for eddy-resolving computations since they alter the physics of the system.) Semtner and Chervin also point out that, if mesoscale eddies are to be properly represented, then grid spacing should be about 0.5° in the tropics, but as small as 1/8° in polar regions (cf. the discussion in the previous section). Going to a 1/8° grid requires about 100 times as much computational power. Also, the extended integrations needed for climate studies lead to another factor of 10. They suggest that development of massively parallel processor machines may allow such integrations to be carried out in the next few years.

5.2 Limited domain eddy resolving models

Because of the computational impossibility of running eddy-resolving global ocean models at present, many groups have chosen to limit the region of the globe covered by their models. Obviously, this approach is not suitable for coupled atmosphere-ocean climate models, but serves as an indicator of what can be achieved, given the computer resources, in an ocean model.

Recent examples of high resolution limited domain ocean models include FRAM (Fine Resolution Antarctic Model), AIM (Atlantic Isopycnic Model), and the model of the North Atlantic developed by the CME (Community Modelling Effort) group. Brief descriptions of these models and discussion of their degree of realism follow.

FRAM is based on the model of Semtner (1974) and Cox (1984), covering the region south of 24°S at a resolution of 1/4° in latitude and 1/2° in longitude with 32 levels in the vertical. This gives a resolution of 27 km in both directions at 60°S. A more detailed description of the model initialisation and the main properties of the early stages of integration is given by Webb *et al.* (1991).

The heat, salt and mass transport across several zonal sections have been studied by Saunders and Thompson (1993). They used model output from the end of the spinup period (after 6 years of integration), this being the time at which the state of the model most resembled the climatological analysis fields of Levitus (1982). The results were found to be generally realistic, although the production and export of abyssal water is underestimated. Also, mesoscale processes were not found to play a major role in transporting heat (across the sections considered) except at 60°S. Further work on the heat transport in the model by Thompson (1993) reveals that, in the later, prognostic phase of the integration, eddies appear to play an important role in transporting heat across the Antarctic Polar Front. Output from the model is compared with the results of De Szoeke and Levine (1981), who employed a novel approach to avoid the uncertainties which arise when using hydrographic data to estimate heat transport. The results from FRAM support the conclusions of De Szoeke and Levine, that the mean geostrophic motions transport little heat across the Polar Front, although use of a zonal section renders the mean flow more important (with heat transported by north-south meanders of the ACC). Insufficient resolution leads to an underestimate of poleward heat transport in the model, but the subgrid scale parameterisation of eddy heat transport goes some way to bringing the model results into closer agreement with the observations.

Model/observation comparisons concerning oceanic variability have also been made. Stevens and Killworth (1992) compared energy levels in FRAM with FGGE drifters and long term current meter moorings, finding reasonable agreement, although the magnitudes were smaller in the model, particularly in lower energy regions. Quartly and Srokosz (1993) compared the model with Geosat data in the Agulhas retroflection region (just off the southern tip of Africa). They found that the magnitude and position of the high variability regions are in reasonable agreement, although they did not find realistic seasonal variations in the model in this region. They also commented that inadequacies in the representation of topography in the model cause the Agulhas current to retroflect in the wrong place. If this is the case then the possibility arises that small details in the topography can affect the large-scale features of the circulation. It has been proposed by Gordon (1985) that the southern end of the Atlantic Conveyor Belt (Broecker 1991) is completed by upwelling and a return flow from the Pacific and Indian Oceans via the Agulhas retroflection region. Presumably, errors in representing the retroflection will affect the transfer of heat between the Oceans, and thus the amount of heat made available to the Conveyor Belt via this route.

Snaith (1992) also compared FRAM with Geosat data, and found that both the model and satellite data showed the strong influence of bottom topography on current location and on the formation of instabilities, even in deep water. The smoothing of the topography in the model, which was applied for reasons of numerical stability, reduced slope gradients and made narrow deep fractures broader and shallower. This affects the distribution and intensity of variability.

Another high resolution integration based on the Cox-Bryan code is that of the WOCE Community Modelling Effort (CME). They have developed a North and Equatorial Atlantic model with a resolution of 1/3° x 0.4° and 1/6° x 0.2° (Böning and Budich 1992). The model qualitatively reproduces the observed current structures and also simulates the seasonal variation of the Florida Current, and the O(100 days) fluctuations of the Antilles Current and the deep western boundary current.

The kinetic energy has often been used as a measure of how well the eddies are resolved. The increase of horizontal resolution increases the kinetic energy in both oceanic as well as atmospheric general circulation models (Eliasen and Laursen 1990, Böning and Budich 1992, Schmitz and Thompson 1993). Böning and Budich (1992) and Böning (1989) found an increase in the eddy kinetic energy of up to a factor of two, depending on location, with less change in the better resolved - in terms of deformation radii - subtropics than in subpolar regions. Eliasen and Laursen (1990) found that the eddy momentum transport increased significantly with increased horizontal resolution in an atmospheric GCM, but they could emulate this using a strongly scale-dependent linear diffusion. Is there an upper limit of required resolution? Schmitz and Thompson (1993) used a two layer primitive equation model to simulate the Gulf Stream. They found that the kinetic energy increased by a factor of two when increasing the resolution from 0.2° to 0.1°, and estimated this to be realistic compared to observations. A further change to 0.05° resolution had little change.

The previous examples of ocean models have been of the Semtner-Cox-Bryan type. An alternative to this type of fixed level model uses an isopycnic formulation. One such model is AIM, a high resolution integration of which is still under way. In a parallel project both Cox (GFDL) and isopycnic (Bleck) models have been integrated for 30 years from rest, with the same forcing and, as far as possible, the same parameterisations. Experiments with the Cox code have demonstrated strong sensitivity of overturning to the bathymetry between the North Atlantic and the GIN Sea. Both models exhibit long-term weakening trends in meridional heat transport up to year 30 (by which stage they appear to be near equilibrium), probably due to closed lateral boundaries; the Cox model peaking with 0.47 PW , the Bleck model with 0.70 PW , both at around 25°N. Both models have basinwide annual mean cooling, the Bleck model more so (perhaps because it produces high latitude water masses more strongly , thus converting light (warm) to dense (cold) water more vigorously than the Cox model. In general the Haney surface restoration term is smaller in the Bleck model implying that the circulation is more consistent with observations (personal communication, the AIM team).

5.3 Global low resolution models

So how well do low resolution models perform? In many cases, surprisingly well. We noted that an important aspect of the models is their simulation of the meridional heat transport. This has turned out to be surprisingly insensitive (in the northern hemisphere) to the resolution even for eddy-resolving resolutions. Comparing an eddy resolving model to a lower resolution integration, Bryan (1986, 1991) found that an enhanced mean flow carrying surface water poleward and deep water equatorward almost exactly balances the eddy transport in the subtropical gyre. Particles in eddies in the model move along isopycnal surfaces. In the real ocean, heat transfer may occur along such surfaces without buoyancy transport because temperature gradients exist along the surfaces, and can be quite large at high latitudes. There may be a coupling between eddies and the large scale circulation through large-scale wind stirring providing energy for baroclinic instability. It is not yet clear how significant this eddy-mean flow compensation is for climate models. Covey (1992) carried out a sensitivity study of four horizontal resolutions with a GCM ranging from 1/2 x 1/2° to 4 x 4°. In the southern hemisphere the heat transport increased with decreasing resolution, almost entirely due to the sub-grid diffusion. This suggests that it may be possible to reproduce the total meridional heat transport in higher resolution models by careful selection of the diffusion.

The general circulation does not generally change qualitatively with resolution. Bryan (1987) showed, however, that the size of the vertical heat diffusion changed the deep circulation in the ocean. The meridional overturning increased from 8 Sv to 30 Sv as the vertical heat diffusivity was increased from 10^{-5} to 25 x 10^{-5}, exhibiting a 1/3 power dependence. Döös (1994b) and Döös and Webb (1994) showed however that the meridional overturning cells are not necessarily associated with diapycnal motion. They also showed that the meridional cells that outcrop at the surface are driven entirely by the northward Ekman transport and are therefore insensitive to the resolution. The shape of the deep part of this cells can however change with resolution and parameterised diffusion. Simulated transport through Drake Passage (Covey, 1992) intensifies with increasing resolution and appears to converge toward a value of 200 Sv, which should be compared with the observed 120 Sv.

Among the most recent studies of low resolution global ocean models are those of England (1992, 1993). England (1993) uses a hierarchy of coarse resolution global ocean models. He finds that the model representation of Antarctic Intermediate Water (AAIW) depends on the level of horizontal diffusion, and that use of a isopycnal mixing scheme improves the situation. After some experimentation, he created a model which reproduces the vertical distribution of deep and intermediate water masses quite accurately. It is noteworthy that he needed to use a wintertime salt flux adjustment to represent brine rejection over and above that already catered for within the model.

England (1992) looked at formation of AAIW and Antarctic Bottom Water (ABW) in coarse resolution models (4.5° latitude by 3.75° longitude, 12 levels) and found that the salinity of the shelf water in the Weddell and Ross Seas is critical. He suggested that the sea-ice component of climate models is critical for capturing high salinity shelf water and bottom water formation. Also the bathymetry of Drake Passage is shown to determine the shape and strength of an intense meridional overturning cell in the Southern Ocean which affects the formation and spreading of AAIW.

England *et al.* (1992) compared the Manabe and Stouffer ocean circulation (from a coupled model) with a Sverdrup model. The Sverdrup theory captures the major features found in the model except in regions where deep water is formed (ie where the flow is thermohaline). Compared with observations, the model has an unrealistically intense halocline around Antarctica, prohibiting formation of bottom water in the Weddell and Ross Seas. This also means that AAIW sinks too far (as there is no bottom water to stop it), making the overall bottom water too fresh and warm. NADW appears in one of the climate states but it does not penetrate deep enough. This is because it is too fresh, due to deep overturning near the equator rather than thermohaline overturning of Weddell Sea NADW.

Rhines (1993) points out that the vertical velocity in low resolution models is sensitive, with unrealistic equatorial jets and massive upwelling at western boundaries. The former can be due to numerical diffusion and the latter due to the 'Veronis effect' (Veronis, 1975) in which strong diapycnal mixing and vertical motion are the consequence of large Cartesian horizontal diffusion acting across steeply tilted density surfaces. He goes on to say that, as a result of some experiments where Michael Cox was increasing the vertical resolution in his 3D model to look at topographic effects on the deep circulation, Cox found that increasing the number of small topographic features seemed to cause unrealistic flows. Although this has not been followed up rigorously to date, it suggests that continuing towards ever increasing resolution might not necessarily be the best way forward for realistic modelling of the ocean circulation.

5.4 Other model studies of interest

One problem with setting up models which include the North Pole is that of the convergence of the meridians and the associated timestepping difficulties. Semtner and Chervin avoided this problem by ending their model domain at 65°N, whilst recognizing that this could seriously affect the realism of their results by ignoring the regions of North Atlantic Deep Water formation. Another approach is to use Fourier filtering techniques, or, following the technique of the OPA group (Delecluse 1993), to construct a system of co-ordinates in which the singular point is moved into a continental area. The OCCAM project proposes to tackle the problem using a two-model approach in which the North Atlantic and Arctic Ocean are treated separately from the rest of the global oceans, on a rotated grid which has its meridians converging over land. While this approach is similar to that of the OPA, it has the advantage of allowing the standard finite difference formulations to be used.

The OPA, a 'medium' resolution global model (1 to 2° resolution) has been developed at LODYC (Delecluse, 1993). The model appears to produce too much equatorial upwelling due to inadequacies in the mixing scheme (a problem with many models). The Pacanowski and Philander scheme used in this models is not able to generate strong enough mixing in the surface layers and the simulated mixed layer is always too shallow. Because the shear is very strong in the surface layers the impact of the wind forcing is concentrated in a very thin surface layer. Gaspar *et al.* (1990) developed a more sophisticated formulation. The scheme uses a prognostic equation for TKE and an estimate of characteristic mixing length. This parameterisation was tested in a tropical version of the OPA code. The Ekman drift became deeper and slower, reducing by a factor of 2 the equatorial divergence. Also the Equatorial Undercurrent was much better represented.

Hirst and Godfrey (1993) have made four model runs examining the effect of Indonesian throughflow (their model resolution is 1.59 x 2.81°, 12 levels). The throughflow warms the Indian ocean and cools the Pacific. It has a large local effect on SST in the Agulhas region, Leeuwin current, Tasman Sea, Equatorial Pacific and 2 bands in the mid-latitude South Pacific. Subsurface changes are more widespread, but are insulated from the surface. The additional heat is released upon encounter with upwelling or convection and may be far from the source of the perturbation. The throughflow leads to an extra 0.3PW (12%) zonally integrated poleward heat transport.

6. SUMMARY

Much of the preceding report suggests that even higher resolution than is currently being used in stand alone ocean models is required to adequately model important features of the ocean circulation.

- Vallis and Hua (1988) found that omitting subgrid scales produces errors in the large scale flows.
- Barnier *et al* (1991) showed how representing baroclinic instability requires finer resolution than one would estimate. Analysis of the Eady problem reveals that at least 10 points in the vertical and $k\Delta x \approx 0.5$ (where k is the wavenumber) are necessary to avoid serious misrepresentation of linear waves.
- Semtner and Chervin (1992) suggest that a resolution of 1/8° is needed to model the mesoscale eddy field in polar regions. We provide evidence that distinctly higher resolution is needed.
- Some features of the models change with increasing resolution, for example, the kinetic energy in both ocean and atmosphere GCMs, and the transport in the ACC both increase.
- There is evidence that eddies do play a role in meridional heat transport.

The kind of resolution implied by the above is obviously impossible for climate studies (indeed it is still a long way away for global ocean models). Thus it becomes important to understand the effects of not resolving certain scales. Furthermore, given that there are processes that it will be impossible to represent adequately, the possibility arises that increasing resolution beyond a certain point will not significantly improve the model.

- Increasing eddy heat transport can be compensated for by the mean flow, thus increasing the resolution may not affect the total meridional heat transport in the model ocean. Covey (1992) suggests that 4° is not adequate for climate simulations, but diminishing returns set in below 1°.

- Going to much higher resolutions will highlight new problems, such as poor representation of interaction with topography.

- Horizontal and vertical resolution should be balanced (Fox-Rabinowitz and Lindzen 1993).

Further questions :

- Which are the most suitable parameterisations and how can they be improved upon?

- What scales and processes are important for the atmosphere?

- How are important processes represented in models of different resolution?

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APPENDIX: THE NUMERICAL EADY PROBLEM

The Eady problem is well discussed in the literature (cf. Gill, 1982), and serves both as a canonical baroclinic instability problem, as well as a useful guide to the behaviour of linear disturbances in the presence of mean shears and density gradients. It therefore also serves as an extension of the discussions by Davey *et al.* (1983), Hsieh *et al.* (1983), and Wajsowicz (1986) on the numerics of linear wave modes.

We consider an f-plane ocean in the quasigeostrophic regime, bounded by two rigid surfaces at $z = \pm H$. The background buoyancy frequency is independent of depth, and there is a mean flow $\bar{u} = \bar{u}_z z$, together with a corresponding northward density gradient in thermal wind balance. Small perturbations of the form $\psi(z)$ exp $(ikx\text{-}iwt)$ are sought. For simplicity, no y-variation is permitted. Analytically, conservation of potential vorticity (here zero) becomes

$$\varphi_{xx} + \frac{f^2}{N^2}\varphi_{zz} = -k^2\varphi + \frac{f^2}{N^2}\varphi_{zz} = 0 \tag{A1}$$

The requirement of zero vertical velocity at top and bottom gives

$$\left(\bar{u} - \frac{\omega}{k}\right)\varphi_z = \bar{u}_z\,\varphi, \quad z = \pm H \tag{A2}$$

Standard algebra then gives the nondimensional eigenfrequencies $\hat{\omega} = \left(N\omega / f\,\bar{u}_z\right)$ as

$$\hat{\omega}^2 = (\alpha - \tanh\alpha)\,(\alpha - \coth\alpha) \tag{A3}$$

where α is the nondimensional wavenumber

$$\alpha = \frac{NkH}{f} \tag{A4}$$

For $\alpha < 1.2$, $\hat{\omega}$ is pure imaginary, corresponding to linear growth. The maximum growth rate occurs at $\alpha = 0.804$, where $\hat{\omega} = 0.31$. For short waves ($\alpha > 1.2$) $\hat{\omega}$ is real, and there are two equal and opposite roots for the two wave speeds $c = \hat{\omega}\,k^{-1}$. Clearly α of order unity corresponds to wavelengths of order the local deformation radius.

Numerically the problem is cast onto a grid of (x,z) points, and standard centred differences are taken. Using normal difference notation, (A1) becomes

$$\delta_x^2 \varphi + \frac{f^2}{N^2} \delta_z^2 \varphi = 0 \tag{A5}$$

$$\bar{u} \delta_x \overline{(\delta_z \varphi)}^x - i\omega \delta_z \varphi = \bar{u}_z \delta_x \overline{(\varphi)}^x, z = \pm H \tag{A6}$$

Here the quantity $\delta_z \varphi$ must be evaluated by one-sided differences. Again seeking sinusoidal solutions in x, we have

$$\delta_x^2 \to \frac{2(\cos k \Delta x - 1)}{\Delta x^2}, \quad \delta_x \overline{(\ )}^x \to \frac{i \sin k \Delta x}{\Delta x} \tag{A7}$$

As a result we can remove the x-variation, leaving the quantity

$$F = k \Delta x \tag{A8}$$

as a measure of the horizontal resolution of the wave. The vertical resolution can be measured by dividing the vertical into (M-1) layers (i.e., numbering gridpoints downward 1, 2, ..., M). Denoting the gridpoint number by l, (A5) and (A6) become

$$\varphi_{l-1} + \varphi_l \left[-2 + 2\alpha^2 \sigma^2 \left(\frac{\cos F - 1}{F^2} \right) \right] + \varphi_{l+1} = 0, \quad l = 2, \ldots, M-1 \tag{A9}$$

$$\left(-i\hat{\omega} + \alpha \frac{i \sin F}{F} \right) \left(\varphi_1 - \varphi_2 \right) = \alpha\sigma \frac{i \sin F}{F} \varphi_1 \tag{A10}$$

$$\left(-i\hat{\omega} - \alpha \frac{i \sin F}{F} \right) \left(\varphi_{M-1} - \varphi_M \right) = \alpha\sigma \frac{i \sin F}{F} \varphi_M \tag{A11}$$

where $\sigma = 2/(M - 1)$ is a measure of the vertical resolution.

This is an eigenvalue problem of the form $Ax = \lambda Bx$ where A, B are matrices and λ is the eigenvalue. Standard subroutines are readily available for its solution. We shall restrict our attention to three representative cases.

Case 1: $\alpha = 0.804$, maximum instability.

Fig. (A1) shows contours of the ratio $\left(\hat{\omega} / \hat{\omega}_{analytic}\right)$ and $\hat{\omega}$ itself, for varying values of F between 0 (good resolution) and π (bad resolution, corresponding to a $\pm$ pattern). Provided M exceeds about 10, solutions of the eigenfrequency do not change, although the eigenfunction may do so. The dependence on F is more stringent. If F exceeds about 0.5, estimates of growth rate are in error.

It may be argued that coarse-resolution climate models would not resolve - and hence would not represent - wavelengths of order a deformation radius. However, putting

$$\Delta x = 100\,km, \quad N = 10^{-2,-3}\,s^{-1}, \quad H = 4\,km, \quad f = 10^{-4}\,s-1$$

where the two values for N probably represent extremes, gives

$$\alpha \leq 12,\ 1.2 \text{ respectively.}$$

Clearly in the former case the Nyquist cutoff at $\alpha = \pi$ would apply. A still coarser model (400 km resolution) would change the numbers to 3, 0.3 respectively. So in most of these cases the wavelength of maximum growth would be resolved, albeit poorly. Fig. (A1) then shows that such a wave would be very badly represented.

Case 2: $\alpha = 1.5$, stable waves.

Fig. (A2) shows the equivalent results for a stable, shorter wave. A rather surprising result is found. Badly resolved waves - basically those with $F > 1.5$ for good vertical resolution, or $M < 7$ for good horizontal resolution - turn out to be unstable. This area is indicated by negative values on Fig. (A2b), although these really refer to growth rates and not frequencies. At such wavelengths, requirements on resolution in both directions are distinctly more stringent than in Case 1. If these are not satisfied, erroneous growth rates will be observed which will need eddy diffusion or viscosity terms to remove them and give good numerical behaviour.

Case 3: $\alpha = 0.2$, unstable long waves.

Fig. (A3) shows a final set of curves, which are very similar to those in Fig. (A1). Since these are for longer waves, it becomes easier still for coarse resolution models to represent these waves - and again to find growth rates off by factors of 2 if the resolution is not good.

We thus conclude from this simple model that adequate representation of simple baroclinic instabilities requires both good vertical resolution (at least 10 points in the vertical) and good horizontal resolution $(k\Delta x < 0.5)$ if the resulting waves are not to be grossly misrepresented - or, indeed, develop unphysical instabilities. Such results are in good agreement with those presented by Hua and Haidvogel (1986) and Barnier *et al.* (1991).

Fig. A1 (a)

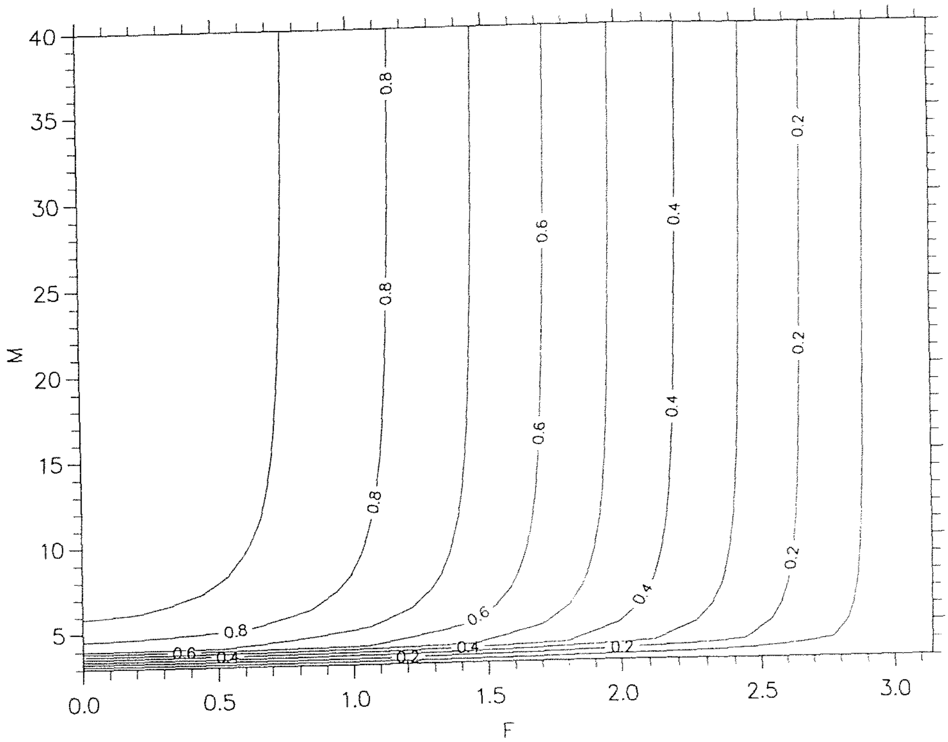

Fig A1 (b)

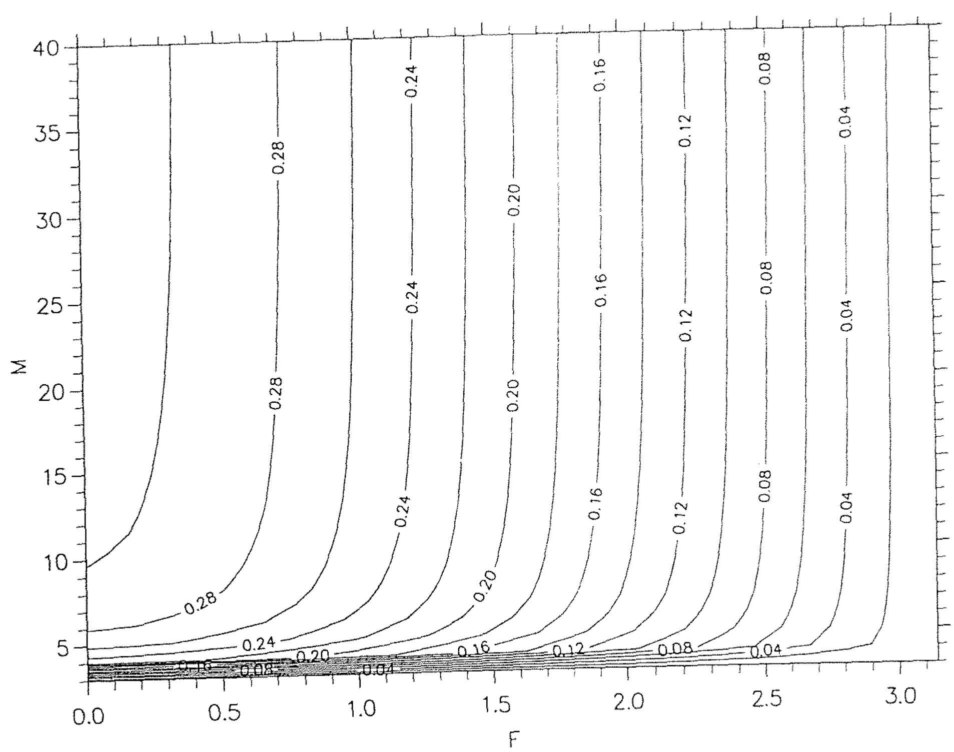

Figure A1: Contours of (a) ratio of computed to analytical growth rate and (b) actual growth rates, plotted against measures of horizontal resolution $F = k\Delta x$ and the number of vertical points M. The value of α is 0.803, ie., that for maximal growth.

Fig. A2 (a)

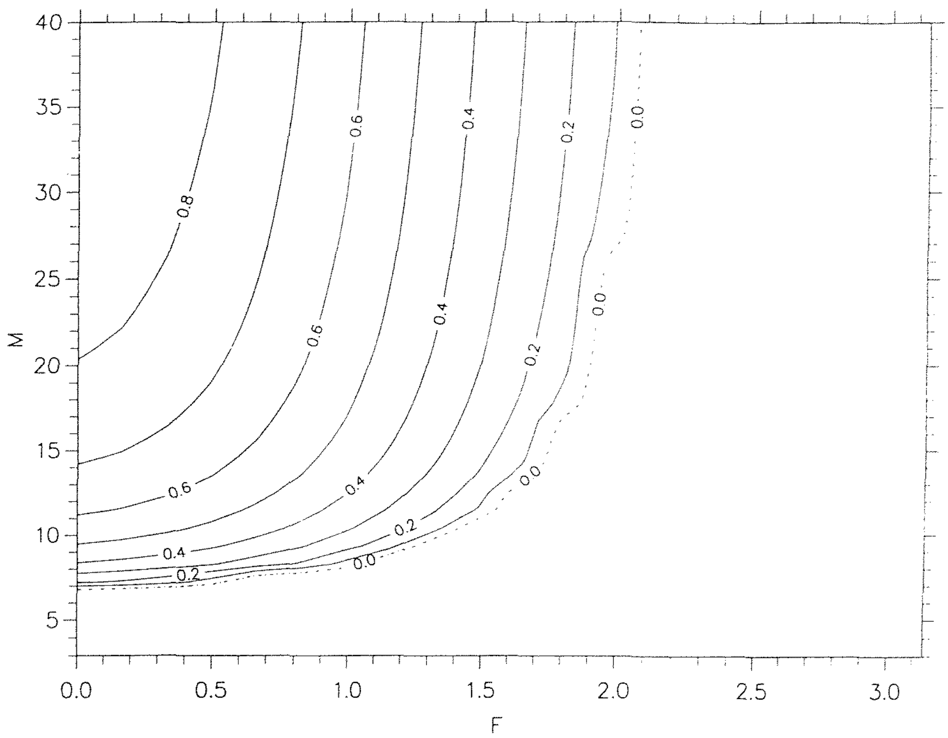

Fig. A2 (b)

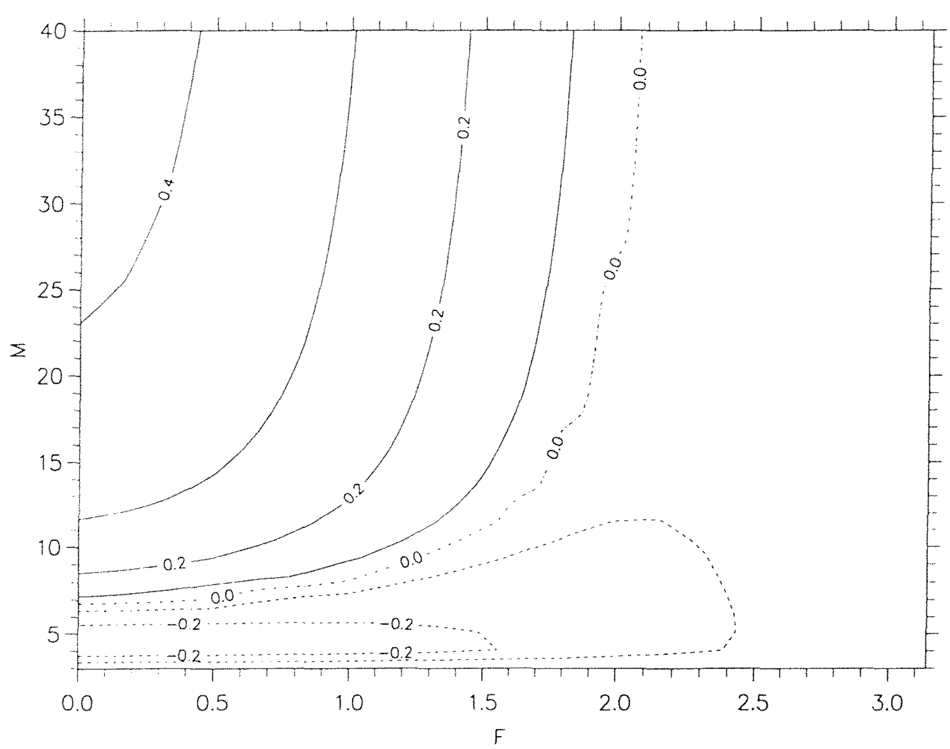

Figure A2: As for Fig. A1 but for $\alpha = 1.5$, an analytically stable case. The area to the right of the dashed curve in Fig. A2a is unstable numerically. The growth rates are shown (negatised for clarity) on Figure A2b.

Fig. A3 (a)

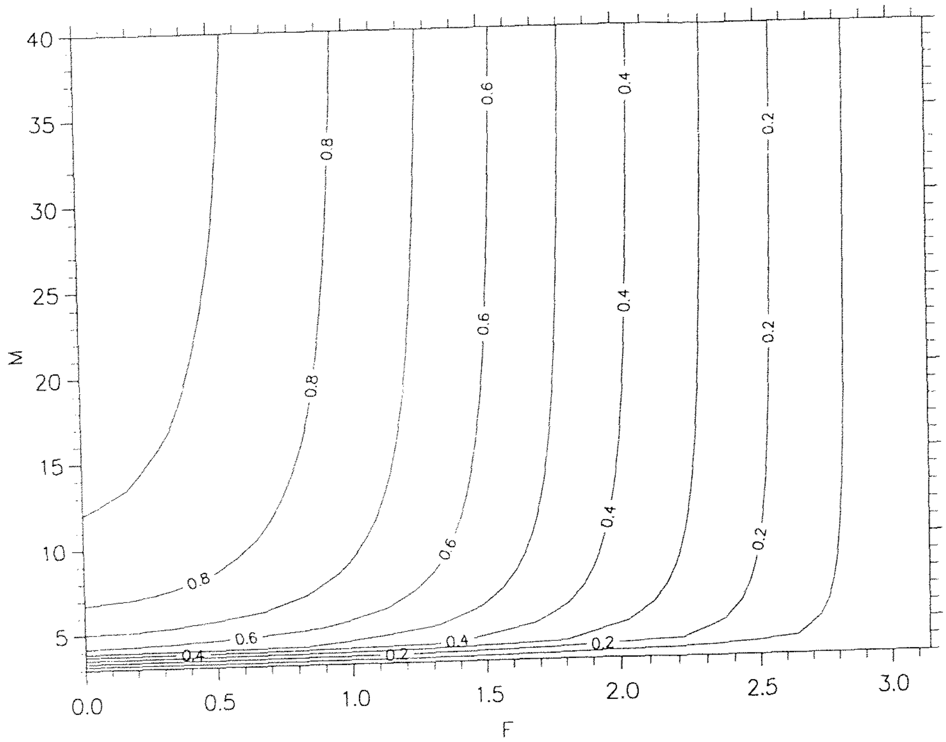

Fig. A3 (b)

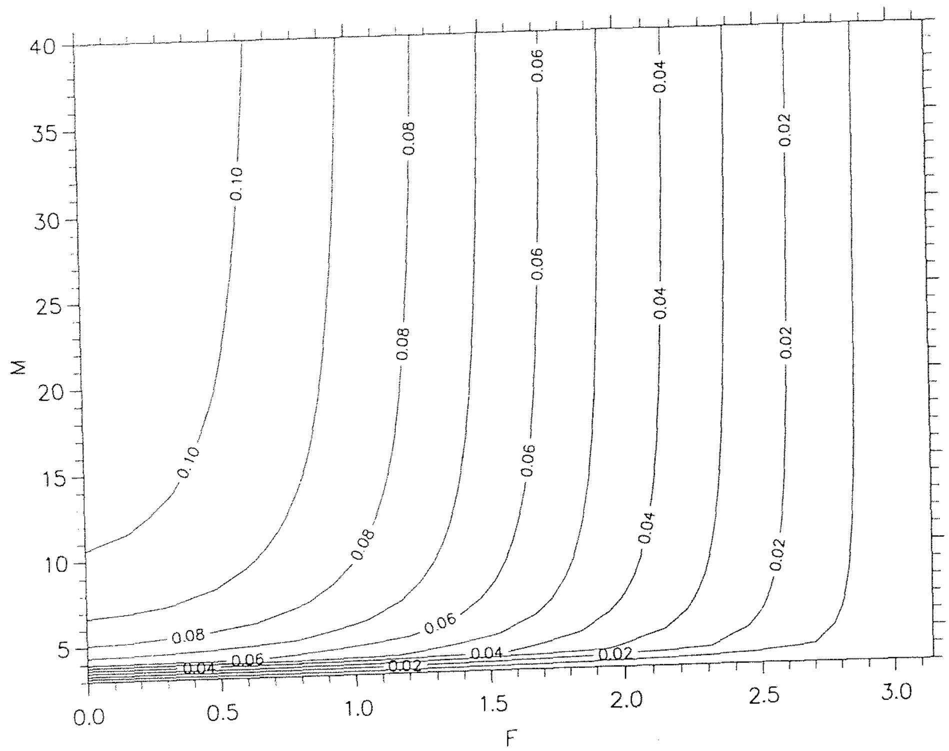

Figure A3: As for Figure A1, but for $\alpha = 0.2$, a more weakly unstable case.

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