NATIONAL INSTITUTE OF OCEANOGRAPHY
WORMLEY, GODALMING, SURREY

Steady Displacements of Moorings Produced by Ocean Currents

by

T. R. BARBER

N.I.O. Internal Report No. A. 54

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1. Introduction

Non-vertical water motion makes a subsurface mooring lean in the direction of the horizontal component of the current thereby displacing each point on the cable both horizontally and downwards from its nominal position vertically above the anchor. Furthermore, these displacements increase both as the current increases and as the cable is ascended. This behaviour of moorings, then, can be understood to have a disturbing influence on current meter records, especially when comparring those obtained from closely spaced meters near the top of the mooring. A brief outline and comparison is first made between three methods for obtaining cable displacements, followed by a closer examination of several typical moorings using the method considered to be the most reliable.

2. Outline of the three methods.

The simplest approach is a single-level model given by Fofonoff (1965). He assumes that (a) the mooring cable of length L remains straight, (b) the nett buoyancy F_B of everything above the anchor acts at the subsurface float, (c) the area A of only the upper third of the mooring is considered when calculating the drag which is taken to act horizontally at the float, and (d) the cable tension just beneath the float is equal to F_B. The resulting Fofonoff (F) expression for the horizontal displacement x of the float from the vertical through the anchor thus becomes

$$x = \frac{\rho C_D A L V^2}{2 F_B}$$

where C_D is the drag coefficient of the float in an horizontal current of magnitude V, and ρ is the sea water density.

A more elaborate calculation is that due to Lampietti and Snyder (1965) who consider the equilibrium of an element of curved cable by making the following assumptions. (a) Both the current and drags are everywhere horizontal, (b) the cable is nearly vertical, and (c) the drag q on and the weight ω of the cable, both per unit length, are constant over the whole cable. The resulting expression for x given by Lampietti and Snyder (L-S) is

$$x = \left(\frac{H}{\omega} + \frac{q T_F}{\omega^2}\right) \ln\left(\frac{T_F}{T_S}\right) - \frac{q L}{\omega}$$

Here H is the drag on the float, and T_F and T_S are the vertical components of the tension at the top and bottom of the cable respectively.

A cable with attached masses and which passes through a current profile which can be taken to be stepped, i.e. a mooring cable, is easily studied with the L-S expression by applying it to the component sections taken in descending order. The section boundaries occur wherever there is a change in conditions which will alter the cable shape. These are (a) a change in cable diameter or density which will alter ω and q, (b) the attachment to the cable of a float or heavy body, e.g. a current meter, and (c) a change in the current speed which will alter q. The value of H for a section is the total horizontal drag on everything above that particular section, and L is now the length of the section and not that of the whole mooring. The horizontal displacement of a section boundary, e.g. at a current meter or at the subsurface float, from the vertical through the anchor is now given by the vector sum of the displacements of the sections below the point in question.

The third method is based on a computer program given by Mihoff (1966) which numerically solves expressions involving the normal and tangential forces acting on the cable. However, the program as given by Mihoff applies to the towing of submerged bodies for which the cable lies in the first quadrant as described by Pode (1951), whereas a mooring cable lies in Pode's second quadrant. Thus, using the same theory as Mihoff, which is given more clearly by Eames (1968), expressions pertaining to the second quadrant were derived. The relevant forces acting on an element AB of cable are shown in fig. 1. Here T is the tension, WT is the weight per unit length of cable, and R is the drag on unit length of cable when it is normal to the current. μ is a number between zero and one, the cable's inclination to the horizontal α on the leeward side is taken to be negative, and the arc length S is measured positively downwards from the float. Eames gives μ to be 1 for a perfectly faired cable, to be 0 for a bluff cable, and the value for cable similar to that used for moorings is given to be in the range 0.02 to 0.05.

If the resultant tangential and normal forces on unit length of cable arising from drag and cable weight are P and Q, respectively, in the directions shown in fig. 1, then the fundamental differential equations governing the equilibrium shape of the cable are

$$P\,dS + dT = 0$$

and

$$Q\,dS + T\,d\alpha = 0$$

Obtaining P and Q from inspection of fig. 1, then

$$dT/dS = \mu R \cos\alpha + WT \sin\alpha$$

and

$$\frac{d\alpha}{dS} = \frac{-\mu R \sin\alpha + (1-\mu) R \sin^2\alpha + WT \cos\alpha}{T}$$

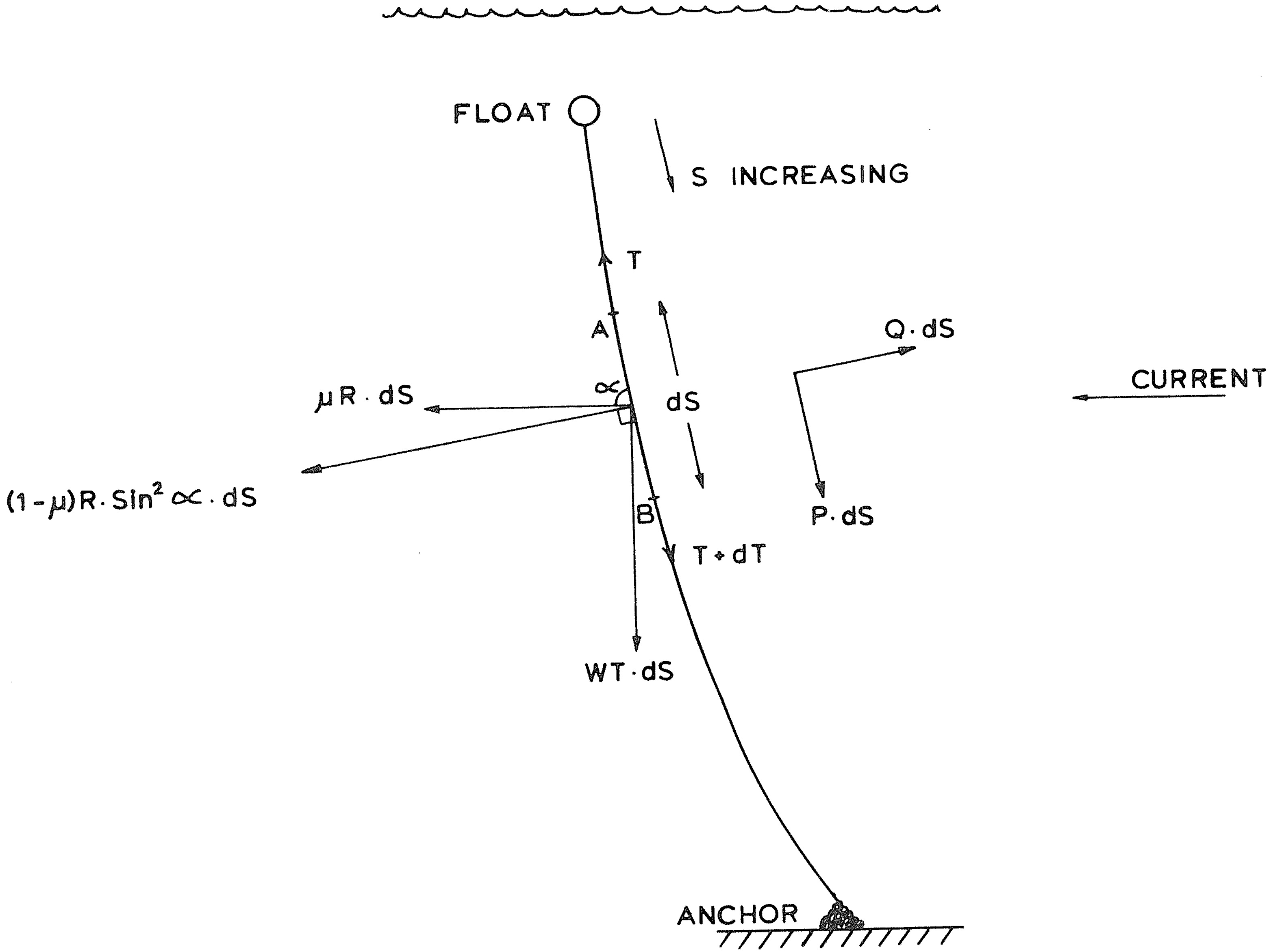


Fig. 1. Forces acting on the element AB of cable.

bearing in mind that α is negative. R here is positive, that is the drag acts outwards on the convex side of the cable, or correspondingly, the current approaches the cable on the concave side. If the cable enters a layer where the current direction is reversed, then the curvature of the cable changes sign. This is allowed for by writing R in the form,

$$R = \frac{1}{2} \rho \cdot CR \cdot D \cdot V|V|$$

where CR is the drag coefficient of the cable which has a diameter D . The current V is taken to be positive past the float and then negative if the current reverses lower down the cable.

Apart from the above mathematical alterations, the Mihoff program was modified drastically as regards format, was expressed mainly in metric units, and was adapted to consider a mooring in many sections. The program has to begin at a point where both the tension and inclination of the cable are known. This point thus is usually at the uppermost end of the cable where these two quantities are obtained via the expressions

$$T = (DRAG^2 + BWT^2)^{\frac{1}{2}}$$

$$\alpha = TAN^{-1}(-BWT/DRAG)$$

where DRAG and BWT are the float drag and nett buoyancy respectively. Hence knowing dT/ds and $d\alpha/ds$ a predictor-corrector method is used for calculating further values of T and α at selected points down the mooring. Additionally, this program, which is called SHAPE, gives the coordinates of these points with respect to the float, unlike the F and $L-S$ expressions which only give horizontal excursions.

SHAPE was further modified to obtain approximate estimates of the influence of dan buoy drag on a mooring. Negatively or neutrally buoyant upper cable, i.e. the cable between the dan buoy and the subsurface float, will assume a curve similar to the curve ACB shown in fig. 2. Because the upward pull of the dan buoy on the upper cable is unknown, the tension and inclination of the upper cable beneath the dan buoy cannot be calculated as at the subsurface float. Hence both dT/ds and $d\alpha/ds$ for the upper cable also remain unknown. The following three assumptions were therefore made.

(a) Because the drag normal to the cable decreases on descending the upper cable, the drags on the upper cable were calculated at the inclination which it would have if it remained straight.

(b) The cable is taken to be neutrally buoyant. This is very nearly true for polypropylene line which is used in practice.

(c) The upward pull of the dan buoy on the upper cable is ignored. This may be a bad assumption in the case of strong currents because then the upper cable does pull the dan buoy

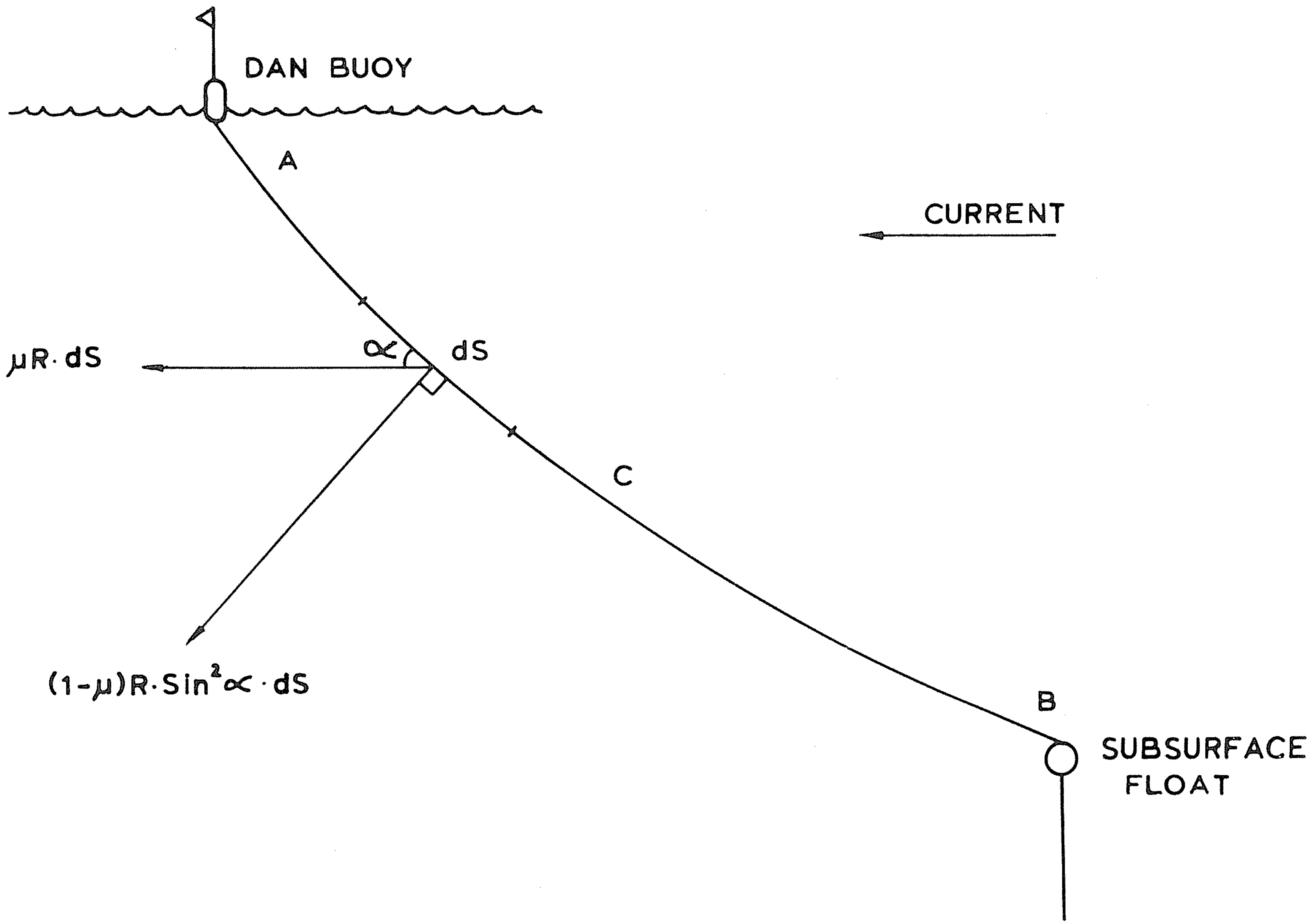


Fig. 2. Schematic configuration of the upper cable.

slightly lower in the water.

Because the equilibrium depth of the subsurface float is not initially known, the first run of the program takes the mooring to be vertical in order to calculate the inclination of the upper cable from the sounding, mooring cable length and the length of the upper cable. A new height (smaller) of the mooring is hence obtained which is fed back to give a new value of the upper cable inclination (steeper) for starting a second run. The program is thus cycled until the fractional change in depth of the subsurface float becomes less than an arbitrarily chosen value. A further run is then made, this time printing the results.

3. Comparison of the F, L-S and SHAPE methods.

The mooring and current profile adopted are shown in fig. 3 together with the displacements given by each of the three methods. The displacements refer to the bottom of each attachment to the cable. The instruments A1-A4 are Bergen current meters, the subsurface float is a 4ft. diameter sphere, 6mm cable is used throughout, and the cable lengths are in metres. All the relevant data needed for these calculations are listed in the appendix.

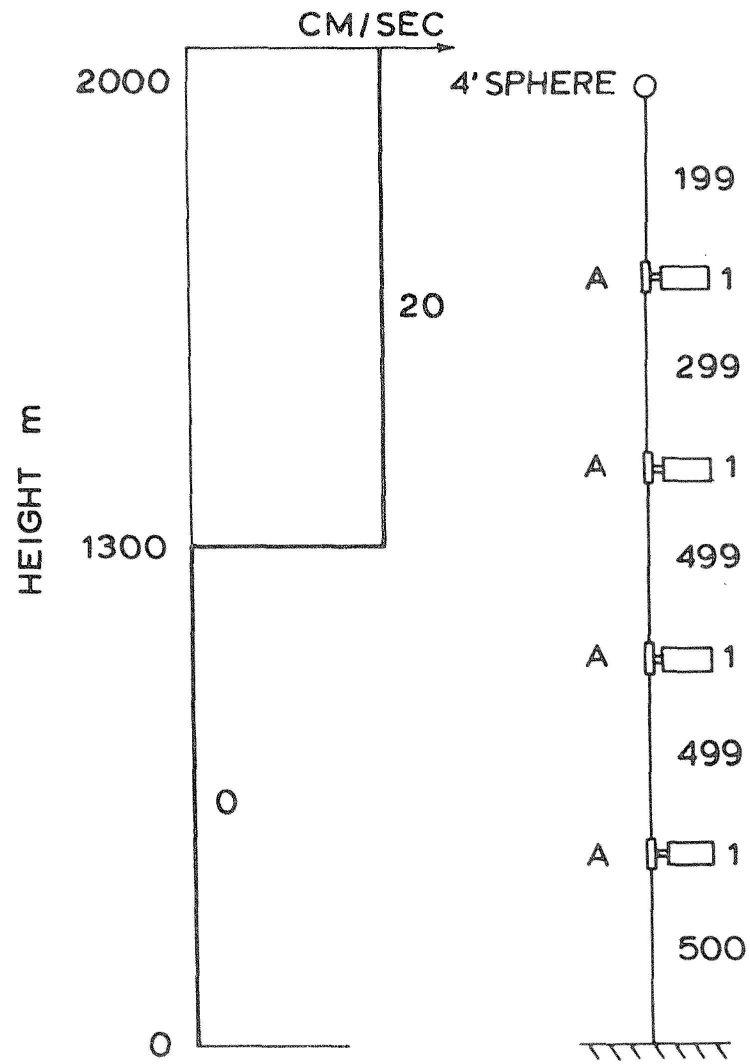
As seen from fig. 3, the L-S and SHAPE results agree closely with each other whereas the F result is markedly less than either of them. However, on applying the F, L-S and SHAPE methods to the NIO mooring 058 subjected to strong currents (~40cm/sec), the horizontal excursions of the cylindrical floats were, respectively, 538m, 261m and 259m. In other words the L-S and SHAPE results are again similar, but the F result is now much larger than either of them. Further L-S and SHAPE calculations for weaker currents (~20cm/sec) on this mooring yielded respective displacements of 97.6 and 91m. Therefore it generally appears that the L-S result for the horizontal excursion of the subsurface float is slightly larger than that given by SHAPE, whereas the F result may be anything from approximately half to twice that of either.

4. SHAPE applied to typical moorings.

In view of the results of section 3, the SHAPE method was chosen to examine the 2000, 4000 and 5000m moorings which are shown schematically in figs. 4, 8 and 12, respectively. These moorings are similar to what are used in practice, and were designed using the general data given in the appendix to fulfil the following conditions:-

(a) the nett weight of the mooring is around 300lbf, and
(b) the safety factors for the cables are at least $2\frac{1}{2}$.

The resulting displacements and corresponding current profiles are given in figs. 5-7, 9-11, and 13-15, together with the inclinations to the horizontal of each end of the main cable. The 2000 and 4000m moorings are considered both with and without a dan buoy, and so values for the soundings have been assumed. These are shown in the relevant figures. However, solutions could not be obtained in the case of the 4000m mooring with a dan buoy in strong currents. Figure 9 shows that the 4' sphere without a dan buoy is depressed by 131.1m to a depth of 231.1m ,



NOMINAL HEIGHT m	HORIZ. DISPL. m			DEPRESS^{N}. SHAPE m
	F	L-S	SHAPE	
2000	22.52	35.17	35.05	0.36
1800		34.41	34.36	0.35
1500		31.66	31.61	0.34
1000		22.85	22.85	0.26
500		12.30	12.31	0.15

Fig. 3. The test mooring and current profile with displacements given by the three methods.

assuming the sounding to be 4100m. Thus the upper cable length was first chosen to be 300m. Unfortunately, the new depth of the float was given to be larger than this value after the first cycle of the main program, and a similar failure resulted after four cycles when the upper cable length had been increased to 600m. This value is much larger than is normally used in practice, and so further calculations were not pursued. Under real conditions, though, the dan buoy exerts a stronger upward pull on the upper cable as it settles lower in the water and hence it may not submerge when only 300m of upper cable are employed.

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Pode, L. 'Tables for Computing the Equil. Config. of a Flexible Cable in a Uniform Stream', David Taylor Model Basin, Report 687 (1951).

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Appendix. General Data.

Most of the values are quoted to more significant figures than are allowable by experimental errors but they are what was used in the input to SHAPE.

Miscellaneous

acceleration due to gravity = 980.665 cm sec^{-2}

1 lb = 453.59 gm

1 lbf= 444819 dyne

The sea is taken to have a density of 1.026 gm/cc, and with a salinity of 35‰ at 10°C the kinematic viscosity ν is 1.356×10^{-2} stokes (see Myers et al. 1969).

The latter quantity is needed when calculating the Reynolds number R for the flow passed a body from the expression

$$R = V d / \nu$$

where V is the current speed and d is a representative linear dimension of the body.

Acoustic release AR

weight in sea water = 40 lbf = 17792760 dyne

dimensions: 3' 8" x 8"

area presented to current = 2044 cm^2

Cables

All the cables are unfaired and the drag parameter μ is taken to be 0.05

(1) Polypropylene: Neutrally buoyant, 5 tonf maximum tension, approximately 2" in circumference, i.e. diameter = 1.62 cm.

(2) Steel: Details are given below. T is the nominal maximum tension which the cable will withstand, and T_s is the maximum tension in order to have a safety factor of at least $2\frac{1}{2}$.

cable dia. mm	T lbf	Ts lbf	wt/unit length	
			lbf/m	dyne/cm
4	2240	896	0.11	489.2
6	4480	1792	0.22	978.5
8	8960	3584	0.4921	2189.0

Command Pinger CP

weight in seawater = 25 lbf = 11120475 dyne

cylindrical case: 4' 5" x 4" diameter

area presented to current = 1394 cm^2

Current Meters

(1) Bergen A

weight in sea water = 40 lbf = 17792760 dyne

cylindrical case: 33cm x 13cm diameter

area presented to current = 430 cm^2

(2) Braincon B

weight in sea water = 40 lbf = 17792760 dyne

cylindrical case: 100cm x 22cm diameter

area presented to current = 2200 cm^2

Dan Buoy

Body taken to be a vertical cylinder, 42.34cm in diameter with

38 cm length submerged

area of body presented to current = 1608.92 cm^2

Bottom pole taken to be 10' long and 3.2cm in diameter, area of pole presented to current = 975.36 cm^2

Hence total area of buoy presented to current = 2584.28 cm^2

Subsurface floats.

(1) Cylindrical.

Cylindrical portion is 6' 2" x 18" diameter with hemispherical ends.

Displacement in sea water = 812 lbf

Mass of cylinder is given as 323 lb

Therefore buoyancy of a cylinder = 489 lbf

Treble cylinder unit:

considered as a cylinder 228.8cm long x 92.8cm diameter. These cylinders tow broadside on to the current, hence area presented to current = 21233 cm^2

buoyancy = 1467 lbf = 652549473 dyne.

(2) Spherical

diameter 4'

area presented to current = 11690 cm^2

buoyancy = 1440 lbf = 640539360 dyne.

<u>Drags</u>

The drags were calculated from the expression

$$\frac{1}{2} \rho C_D A V^2$$

where ρ is the seawater density, C_D is the drag coefficient, A is the area of the body presented to the current, and V is the current speed. C_D was obtained from graphs given by Schlichting (1955) which show C_D plotted as a function of the Reynolds number. The latter was obtained by considering everything except the spherical float as circular cylinders of diameter equal to the smaller of the dimensions given above for each item.

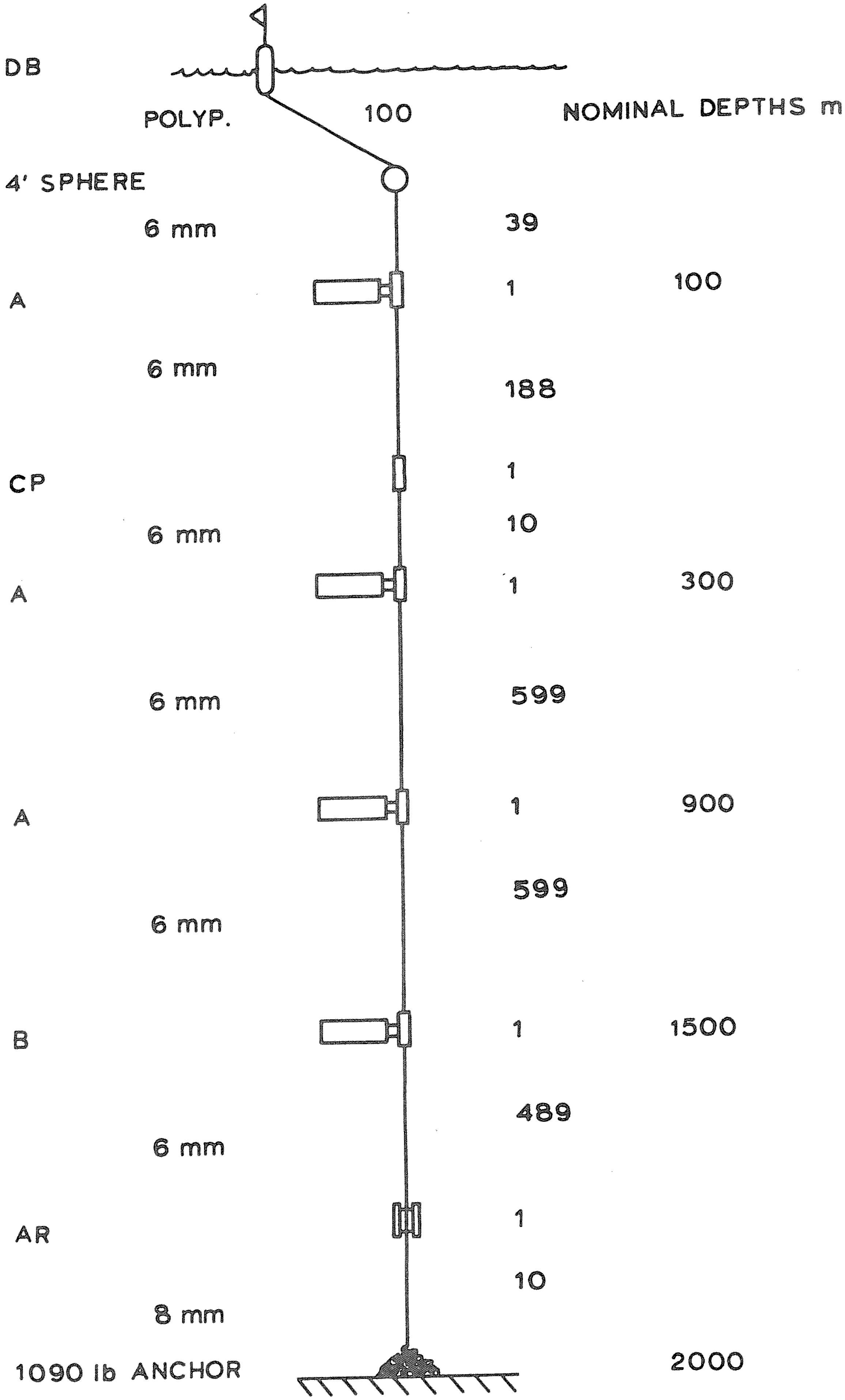
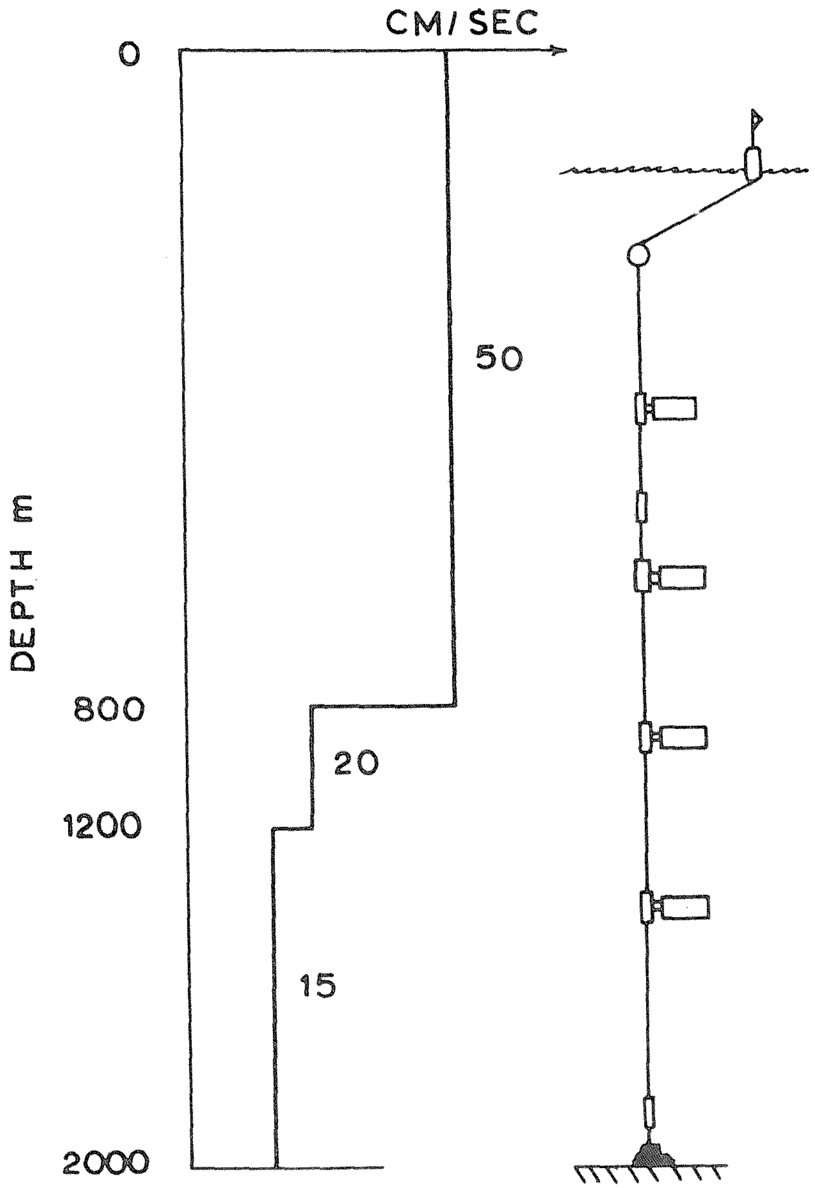


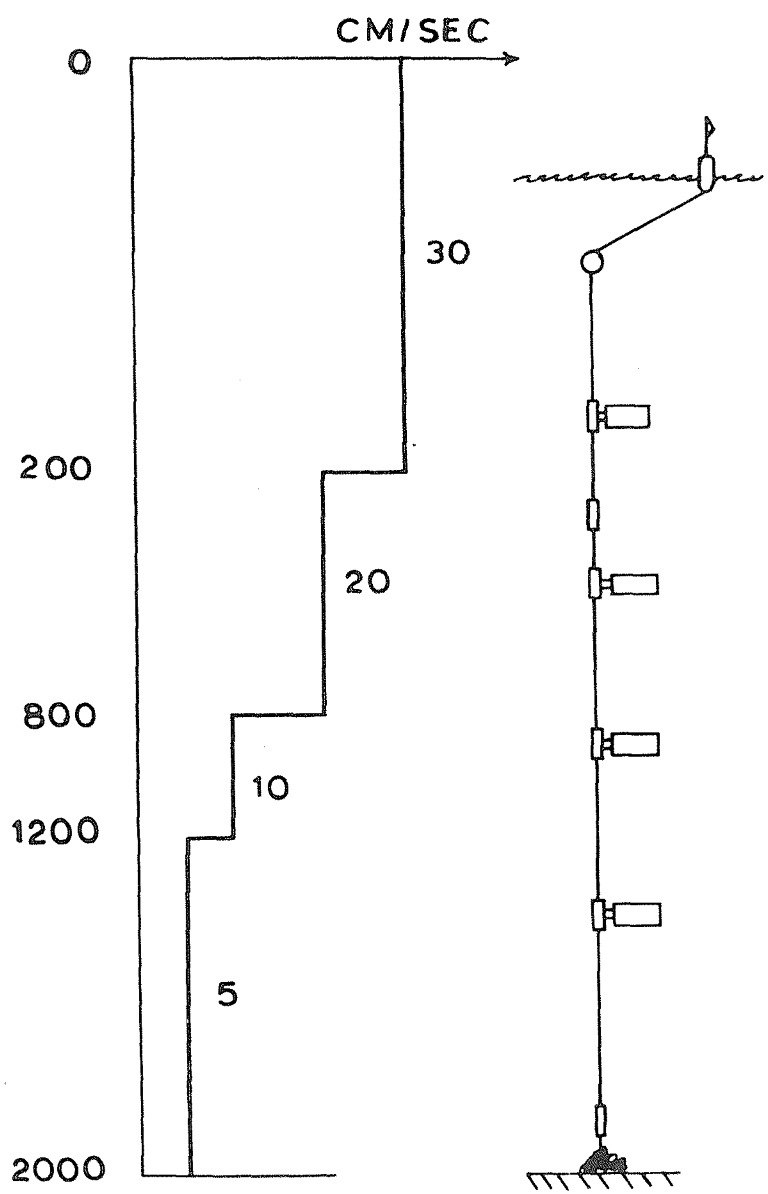
Fig. 4. Schematic diagram of the 2000m mooring. Cable lengths are in metres.



NOMINAL DEPTH m	DEPTH m NO DB	DEPTH m WITH DB	HORIZ. DISPL. m NO DB	HORIZ. DISPL. m WITH DB
0				342.42
60	75.30	85.03	220.67	289.79
100	115.29	125.00	220.50	288.50
300	315.25	324.76	216.42	278.72
900	913.07	920.69	167.49	210.77
1500	1507.47	1511.67	86.00	107.28

INCLIN.°	NO DB	WITH DB
TOP	89.879	88.318
BOTTOM	78.577	75.808

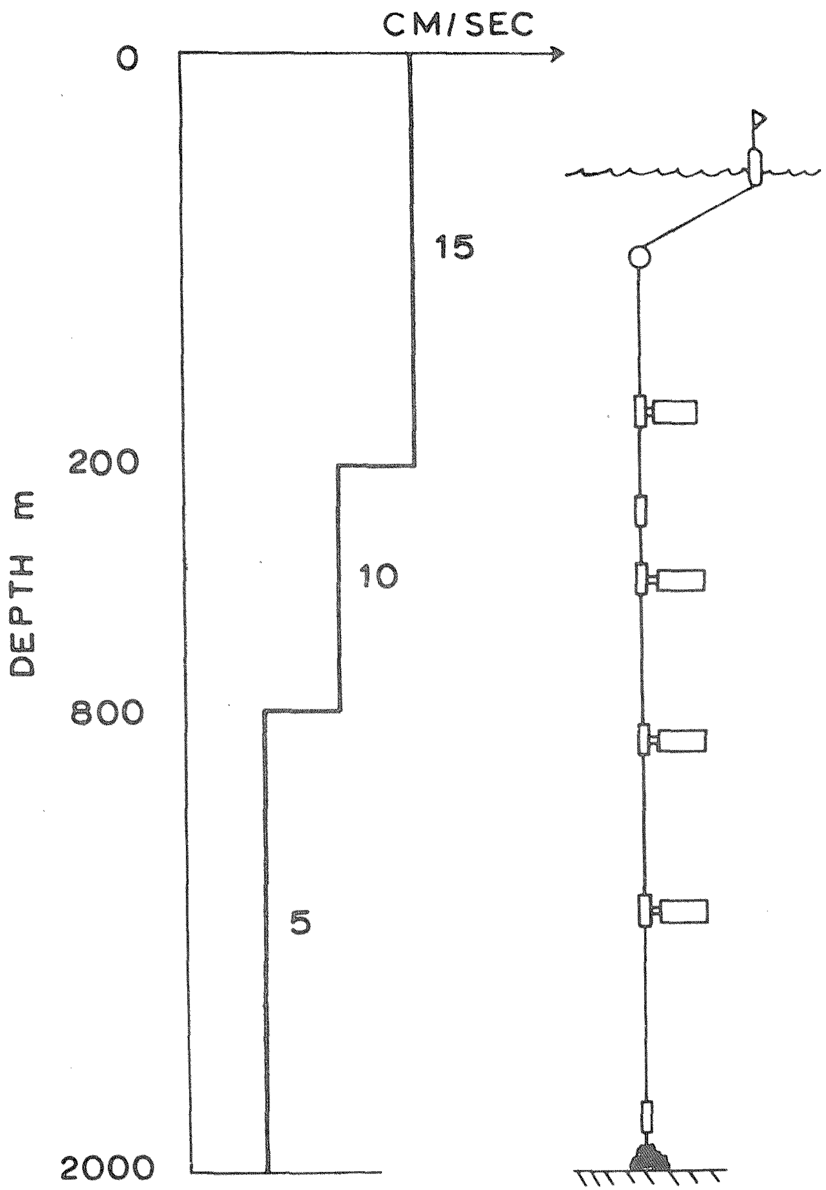
Fig. 5. Displacements of 2000m mooring in STRONG currents, plus inclinations of main cable.



NOMINAL DEPTH m	DEPTH m		HORIZ. DISPL. m	
	NO DB	WITH DB	NO DB	WITH DB
0				146.87
60	60.85	61.33	53.28	67.88
100	100.84	101.32	53.15	67.53
300	300.84	301.30	51.40	64.60
900	900.71	901.07	39.14	48.36
1500	1500.40	1500.60	19.95	24.54

INCLIN.°	NO DB	WITH DB
TOP	89.859	89.540
BOTTOM	87.387	86.797

Fig. 6. Displacements of 2000m mooring in MEDIUM currents, plus inclinations of main cable.



NOMINAL DEPTH m	DEPTH m		HORIZ. DISPL. m	
	NO DB	WITH DB	NO DB	WITH DB
0				99.65
60	60.09	60.12	16.19	19.74
100	100.08	100.11	16.15	19.65
300	300.08	300.11	15.64	18.85
900	900.71	900.10	12.08	14.32
1500	1500.05	1500.06	6.40	7.51

INCLIN. °	NO DB	WITH DB
TOP	89.950	89.872
BOTTOM	89.129	88.986

Fig. 7. Displacements of 200m mooring in WEAK currents, plus inclinations of main cable.

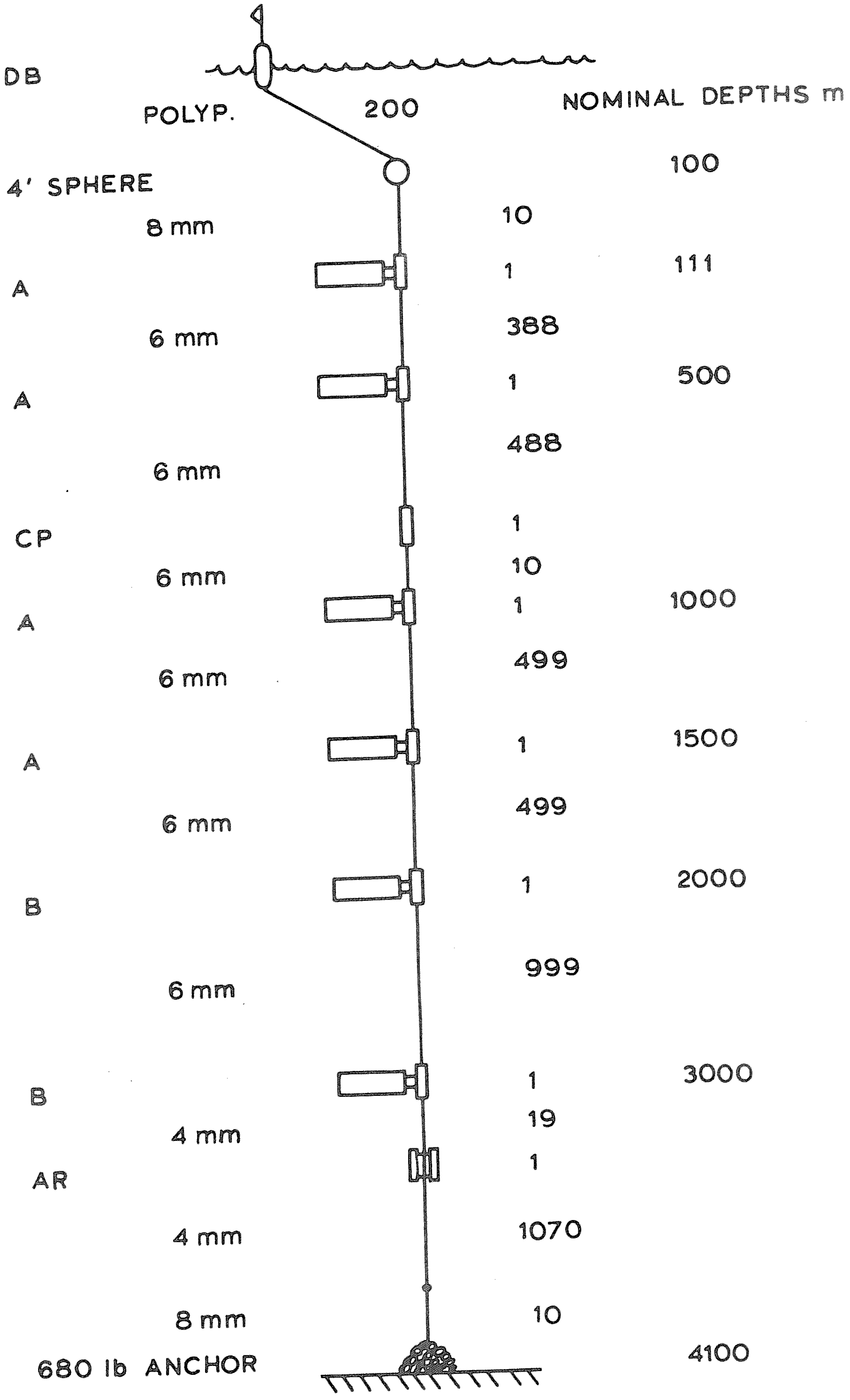
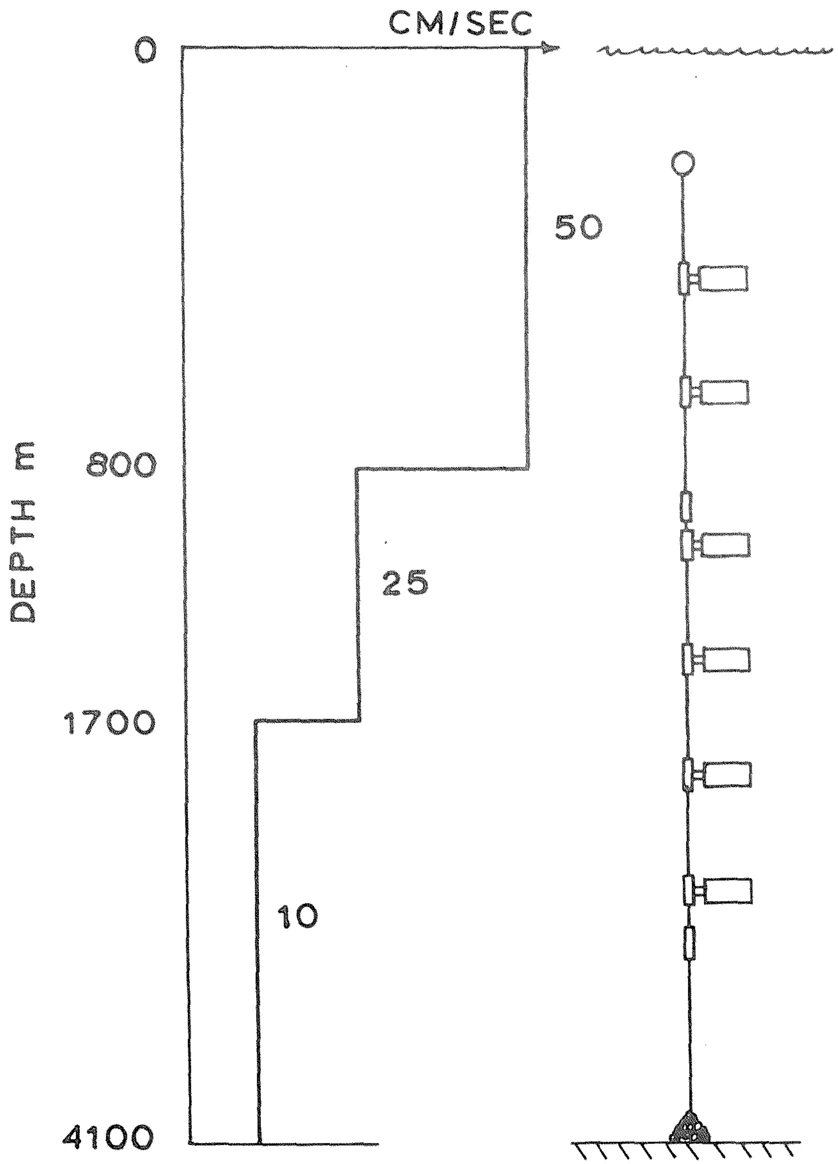


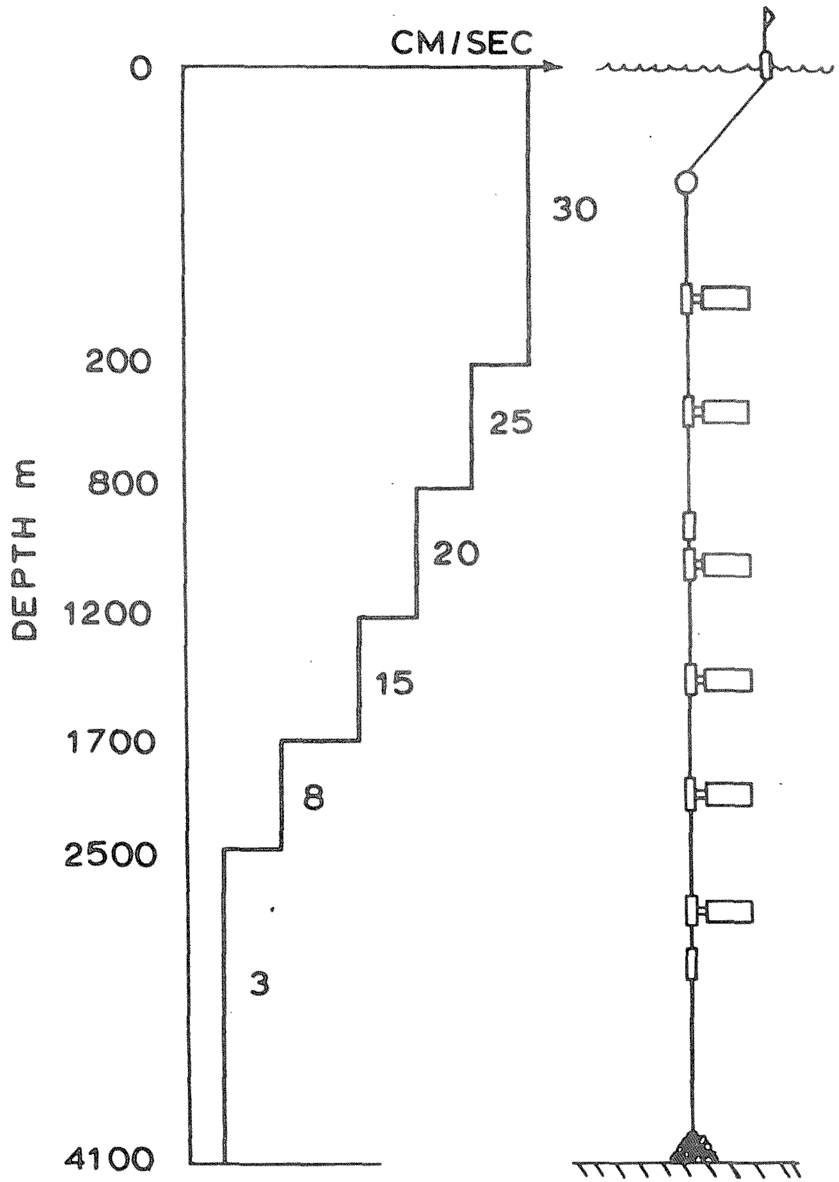
Fig. 8. Schematic diagram of the 4000m mooring.
Cable lengths are in metres.



DEPTH m		HORIZ. DISPL. m
NOMINAL	ACTUAL	
100	231.10	881.19
111	242.09	881.16
500	630.89	869.88
1000	1128.80	825.09
1500	1624.10	756.86
2000	2115.96	667.15
3000	3085.42	423.35

	INCLIN. °
TOP	89.879
BOTTOM	63.518

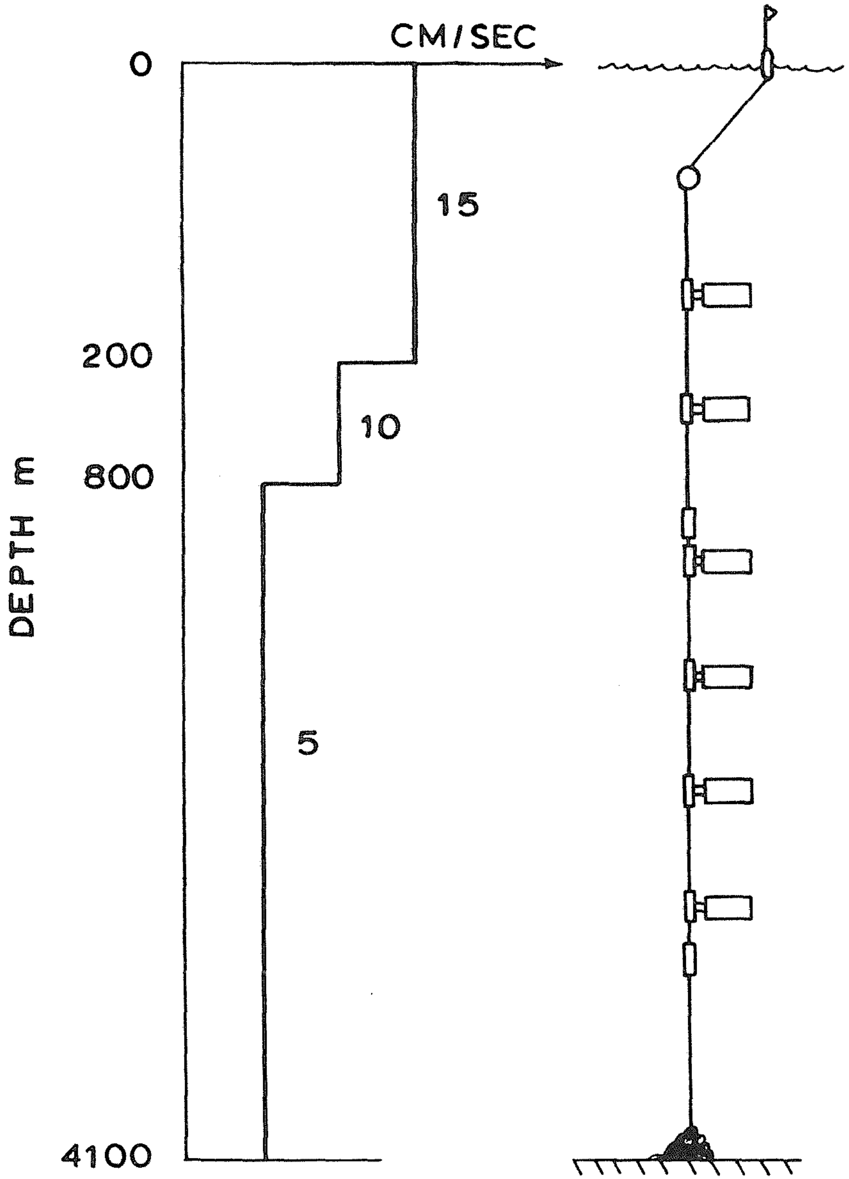
Fig. 9. Displacements of 4000m mooring without a dan buoy in STRONG currents.



NOMINAL DEPTH m	DEPTH m		HORIZ. DISPL. m	
	NO DB	WITH DB	NO DB	WITH DB
0				559.53
100	117.68	126.92	324.06	404.96
111	128.67	137.91	324.03	404.83
500	517.65	526.82	319.72	396.59
1000	1017.44	1026.42	305.62	376.83
1500	1516.87	1525.49	281.87	346.41
2000	2015.84	2023.89	249.76	306.46
3000	3011.69	3017.63	159.34	195.38

INCLIN. °	NO DB	WITH DB
TOP	89.859	89.318
BOTTOM	80.231	77.997

Fig. 10. Displacements of 4000m mooring in MEDIUM currents, plus inclinations of main cable.



NOMINAL DEPTH m	DEPTH m		HORIZ. DISPL. m	
	NO DB	WITH DB	NO DB	WITH DB
0				257.83
100	100.91	101.27	70.89	85.36
111	111.90	112.26	70.88	85.34
500	500.90	501.26	69.77	83.50
1000	1000.89	1001.24	66.69	79.39
1500	1500.87	1501.21	62.23	73.73
2000	2000.84	2001.16	56.32	66.43
3000	3000.67	3000.91	38.00	44.42

INCLIN. °	NO DB	WITH DB
TOP	89.941	89.851
BOTTOM	87.520	87.127

Fig. 11. Displacements of 4000m mooring in WEAK currents, plus inclinations of main cable.

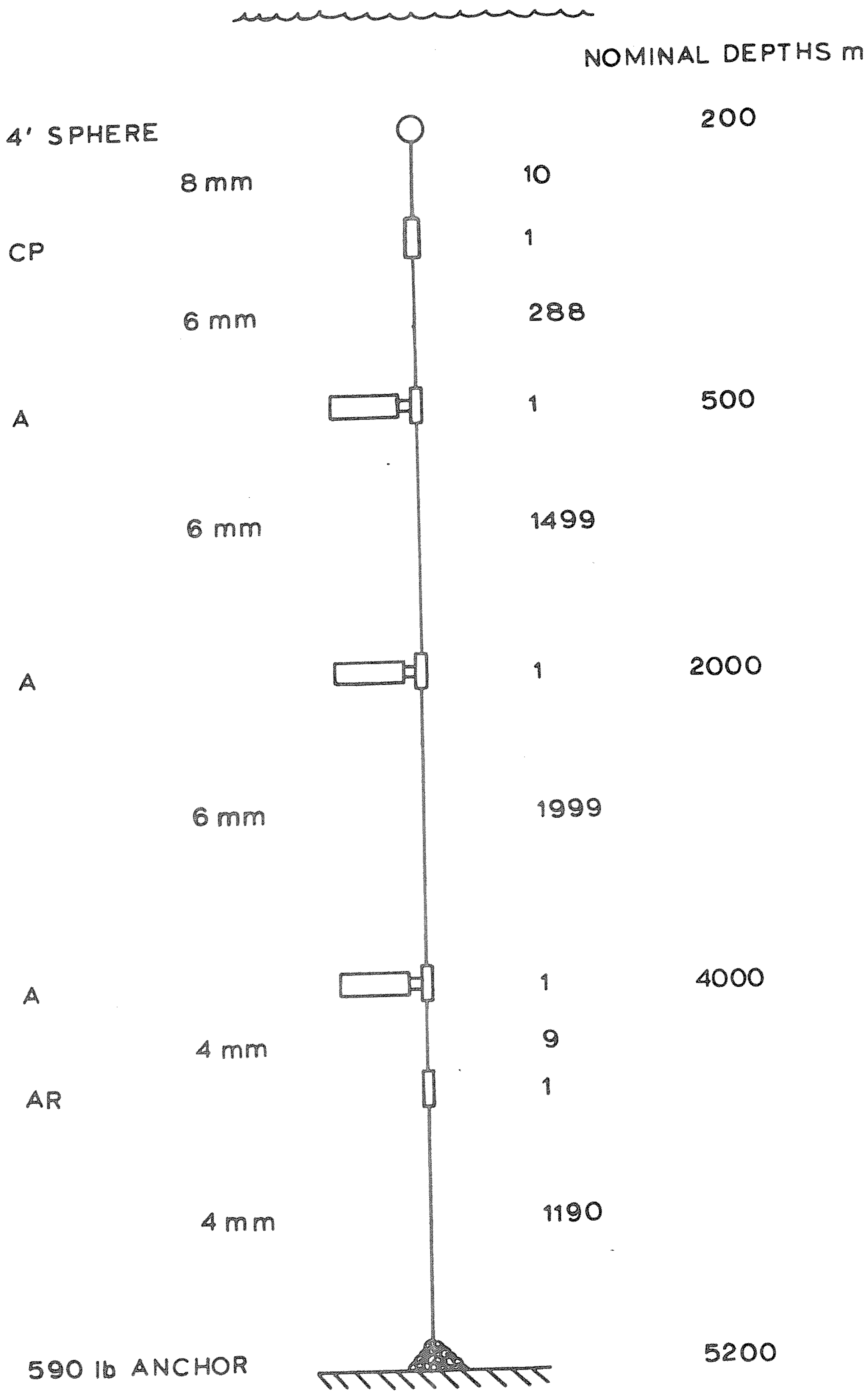
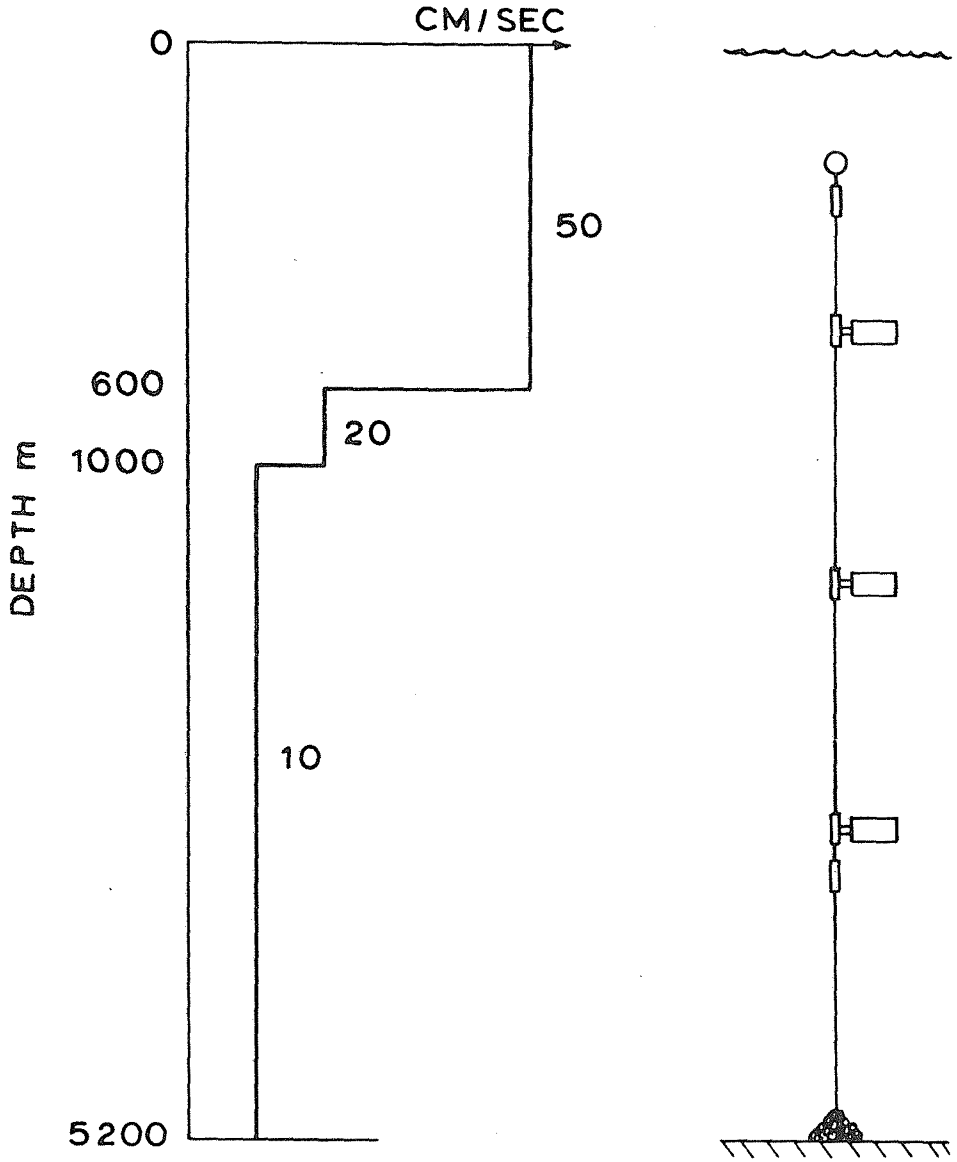


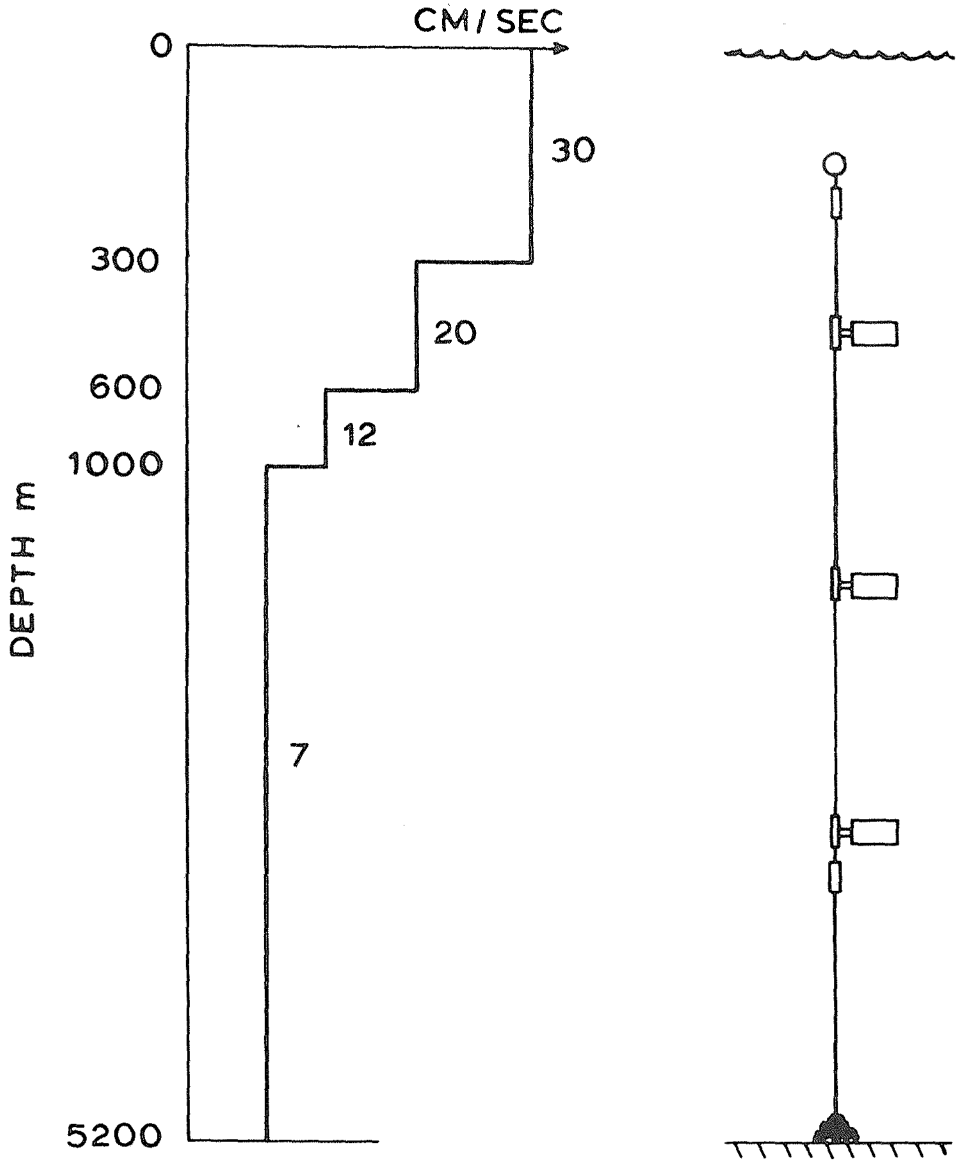
Fig. 12. Schematic diagram of the 5000m mooring. Cable lengths are in metres.



DEPTH m		HORIZ. DISPL. m
NOMINAL	ACTUAL	
200	292·26	804·23
500	592·15	797·04
2000	2087·60	681·79
4000	4064·12	383·94

	INCLIN. °
TOP	89·879
BOTTOM	66·893

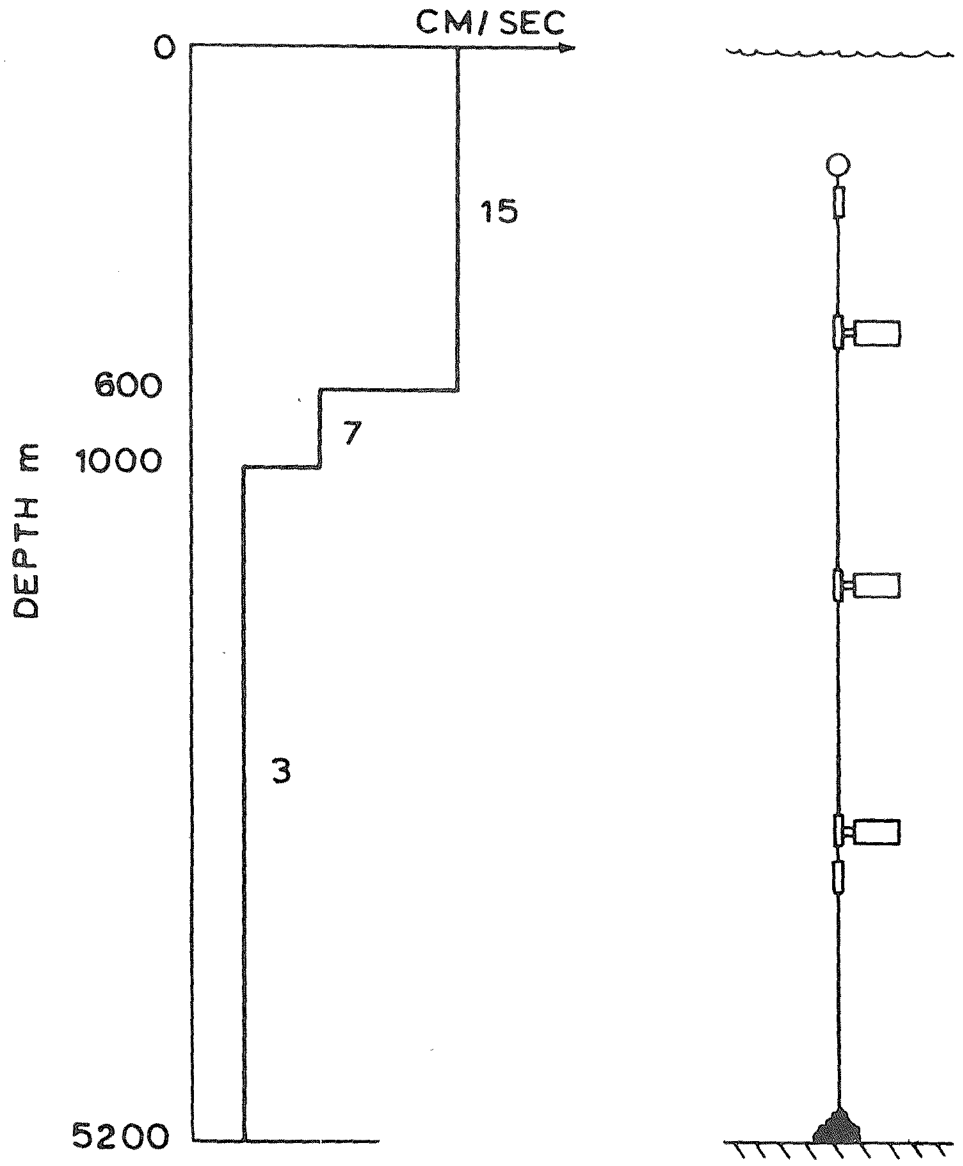
Fig. 13. Displacements and inclinations of 5000m mooring in STRONG currents.



DEPTH m		HORIZ. DISPL. m
NOMINAL	ACTUAL	
200	211.48	275.83
500	511.46	273.12
2000	2011.08	239.80
4000	4008.45	140.80

	INCLIN. °
TOP	89.859
BOTTOM	81.444

Fig. 14. Displacements and inclinations of 5000m mooring in MEDIUM currents.



DEPTH m		HORIZ. DISPL. m
NOMINAL	ACTUAL	
200	201.37	97.92
500	501.36	97.02
2000	2001.30	83.49
4000	4000.97	47.89

	INCLIN. °
TOP	89.950
BOTTOM	87.137

Fig. 15. Displacements and inclinations of 5000m mooring in WEAK currents.