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The absolute calibration of echo-sounders used for the detection of mid-water organisms

by

M. J. TUCKER and A. R. STUBBS

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CONTENTS

ABSTRACT

A brief account is first given of the theory of echo-sounding for mid-water organisms. Four methods for the calibration of echo-sounders are then described.

These are:

1. The use of standard targets
2. The measurement of the electrical characteristics of the transducers
3. Transmission between two identical transducers
4. The use of a calibrated hydrophone.

The results obtained by using these methods to calibrate a real transducer are given.

1. INTRODUCTION

The absolute calibration of echo-sounders used in marine research is most important if results from different machines are to be compared and interpreted in terms of the size or abundance of the organisms producing the echoes.

The authors have set out to produce a guide to the calibration of echo-sounders aimed primarily at those who use the echo-sounder as a tool in their work. They have therefore not used the logarithmic system favoured by acoustic engineers, but have used instead ordinary physical units which they feel will be easier for the non-specialist.

The reader who is not familiar with electric circuits is advised to read only the first two or three paragraphs of section 4.2 describing calibration by measuring the electrical characteristics of the transducer. But though this method seems very involved at first sight, it is in many ways the best and most convenient method of calibration and is reasonably straightforward once the principles have been mastered.

It seems to be difficult to get high accuracy in underwater acoustic measurements. In the calibrations by the methods described in sections 4.1 and 4.2, the authors consider a 20% accuracy to be reasonable. In measuring targets in practice, the uncertainty in the value of 'a' (Figure 1) may also introduce considerable error.

2. SYMBOLS

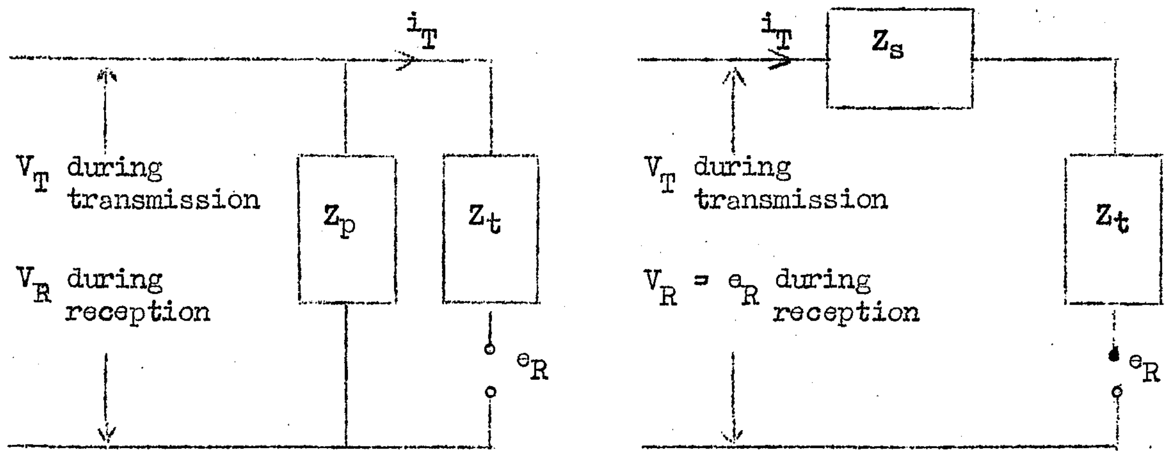
2.1 List of symbols

The dimensions are given in c.g.s. units.

A (sq. cms.)	the area of the active face of the transducer.
a (cms^{-1})	coefficient of absorption of sound in water, i.e. in a plane wave $F_s = F_0\,10^{-as}$.
B (num)	a constant depending on the beam pattern of the transducer. $B = 3.54 \times 10^{-2}$ for a rectangular transducer whose face width is large compared with a wavelength. $B = 3{\cdot}69 \times 10^{-2}$ for a large circular transducer.
c (cms/sec)	the velocity of sound.
E (num)	the efficiency of the transducer, i.e. the ratio of the transmitted acoustic power P_T to the electrical input power (losses in the tuning capacitor or inductor being counted as transducer losses).
e_R (volts)	the voltage across an untuned transducer on open circuit during the reception of an echo.
F (watts/cm^2)	acoustic energy flow.
$f(\phi, \psi)$ (num)	the amplitude of the sound at a bearing ϕ, ψ relative to the amplitude on the axis of the beam at the same range.
i_R (amps)	the current through a transducer during reception when the terminals are short-circuited.
i_T (amps)	the current through an untuned transducer during transmission.
K (cm)	the calibration constant of the system for a single target defined by $\frac{V_R}{V_T} = K\frac{\sqrt{\sigma}}{s^2}\,10^{-as}$
K^1 ($cm^{-3/2}$)	the calibration constant of the system for a diffuse target defined by $\frac{V_R}{V_T} = K^1\frac{\sqrt{m}}{s}\,10^{-as}$
L (cms)	the "ping length", i.e. $\frac{1}{2}T$ x velocity of sound in water.
M (ohm.cm)	calibration constant defined by $\frac{e_R}{i_T} = M\frac{\sqrt{\sigma}}{s^2}\,10^{-as}$
M^1 (ohm $cm^{3/2}$)	calibration constant defined by $\frac{e_R}{i_T} = M^1.\frac{\sqrt{m}}{s}\,10^{-as}$
m (cm^{-1})	the coefficient of volume reverberation. The sum of the acoustic cross-sections of all scatterers in unit volume (averaged over the relevant volume).
N	calibration constant defined by $\frac{i_R}{V_T} = N\frac{\sqrt{\sigma}}{s^2}\,10^{-as}$
N^1	calibration constant defined by $\frac{i_R}{V_T} = N^1\frac{\sqrt{m}}{s}\,10^{-as}$
P_R (watts)	acoustic power intercepted by a transducer during an echo.

P_T (watts)	acoustic power radiated by a transducer during a pulse, totalled over all angles.
p (dynes/sq.cm)	acoustic sound pressure.
s (cms)	the distance of the target from the transducer.
T (secs)	the pulse length.
V_R (volts)	the voltage appearing across the terminals of a transducer which is tuned but otherwise on open circuit, during an echo.
V_T (volts)	the voltage across the terminals of a tuned transducer during transmission.
α (rads)	the width of the acoustic beam.
θ (rads)	the effective solid angle of the transducer defined as follows. If all the acoustic energy emitted by the transducer were uniformly distributed through the solid angle θ , then the intensity within this angle would be the same as the actual intensity on the axis of the transducer. It will be shown in Appendix I that $\theta = \lambda^2/A$ for a large transducer.
λ (cms)	the wavelength of the sound in water at the frequency of the system.
μ (num)	reflectivity of a material (in sea water).
ρ (grms/cm^3)	the density of a material.
σ (sq. cms)	the acoustic cross-section of a target. The target gives the same echo as an object which collects the acoustic energy passing through a cross-section σ perpendicular to the beam and re-radiates it uniformly in all directions. For a large, perfectly reflecting sphere, σ is its actual cross-section.
ϕ, ψ (rads)	bearing of a target relative to the axis of the beam.

2.2 Diagrams showing meanings of electrical symbols



(a) Parallel-tuned (b) Series-tuned

Z_t represents the transducer.

Z_s and Z_p are capacitors for magnetostriction transducers or inductors for piezo electric transducers, and are assumed to tune with the transducer at the frequency of mechanical resonance of the transducer.

Note that $V_R = e_R$ for a series-tuned transducer but not for a parallel-tuned transducer.

3. PRINCIPLES AND FORMULAE GOVERNING THE AMPLITUDE OF THE ECHO

3.1 General

For marine biological work two types of target are important: the individual scatterer, for example, a single fish; and scatterers diffuse in volume, for example, a fish shoal. Scattering from the sea bed or sea surface will not be discussed here.

It will be assumed that either:

(a) the same transducer is used for both transmission and reception, or

(b) the transmitting and receiving transducers are similar.

Assuming the characteristics of the transducer and of the transmission pulse to remain constant, the amplitude of the echo has to be measured and interpreted in terms of the acoustic size of the scatterer or group of scatterers.

The acoustic size of a single target is described in terms of its acoustic or scattering cross-section which is denoted by σ in this report. The echo strength is the same as that from an object which intercepts the energy passing through a cross-section σ perpendicular to the path of the sound and re-radiates it uniformly in all directions. A perfectly reflecting sphere whose diameter is large compared with λ has an acoustic cross-section equal to its physical cross-section. σ is, however, usually smaller than the physical cross-section of the target, but can also be larger if, for example, the target contains a resonant air bubble.

If there are so many scatterers that the echoes superimpose and individuals cannot be distinguished, the average received power is measured and this is the sum of the powers reflected by all the individual scatterers "in view" at any instant. This is interpreted in terms of the average sum of the scattering cross-sections per unit volume, which is known as the coefficient of volume reverberation and is denoted by "m" in this report.

In the first instance a fairly long transmission pulse of constant amplitude is considered, in which case the received voltage can be calculated in terms of comparatively simple parameters of the system. If a short pulse of rapidly varying amplitude, such as that produced by a capacitor-discharge system, is used, the situation is more complicated. This case is discussed in Section 6.

The electrical measurements required are either the voltage across the transducer or the current through it during transmission and either the open-circuit voltage across it, or the short-circuit current through it during reception of an echo (see Section 2). Methods of measurement of these quantities are discussed in Section 5.

It is assumed that the frequency used is the same as the frequency of mechanical resonance of the transducer, since this simplifies calculation. However, for the direct method of calibration using standard targets, so long as the frequencies both remain constant, resonance need not be assumed.

In this report it is assumed that at the range of the target, the angular distribution of energy in the sound beam has reached its long-range pattern. If the largest dimension of the transducer face is w, this means in practice that s must be greater than $2 w^2/\lambda$

3.2 Single target on beam axis

The problem is in two parts. The first is mainly geometrical; that of calculating the power intercepted by the transducer during an echo as a proportion of the total acoustic power (summed over all directions) radiated during transmission. This is given by

$$\frac{P_R}{P_T} = \frac{\sigma A}{4\pi\theta s^4} \quad 10^{-2as} \tag{1}$$

The derivation of this equation is given in Appendix I. The first part of the r.h.s. is due to the spreading-out of the sound in the water, the second due to the absorption.

It can be shown that for a large transducer $\theta = \dfrac{\lambda^2}{A}$

The second problem is to calculate P_R and P_T in terms of electrical quantities which can be measured. An interesting result here is that if the transducer is tuned electrically,

$$\frac{V_R}{V_T} = 2E\sqrt{\frac{P_R}{P_T}} \tag{2}$$

where E is the efficiency with which the transducer converts electrical into acoustic energy.

This formula is exact for a series-tuned magnetostriction or parallel-tuned piezoelectric transducer, but involves a small approximation for a parallel-tuned magnetostriction or a series-tuned piezo electric transducer.

Taking formulae (1) and (2) together

$$\frac{V_R}{V_T} = 2E \frac{1}{s^2} \sqrt{\frac{\sigma A}{4 \pi \theta}} \, 10^{-as}$$
$$= K \frac{\sqrt{\sigma}}{s^2} \quad 10^{-as} \qquad (3)$$

Where K, the calibration constant we wish to determine, equals $\sqrt{\frac{E^2 A}{\pi\theta}}$

Similarly, we can define a constant M such that

$$\frac{e_R}{i_T} = M \frac{\sqrt{\sigma}}{s^2} \quad 10^{-as} \qquad (4)$$

and a constant N such that

$$\frac{i_R}{V_T} = N \frac{\sqrt{\sigma}}{s^2} \quad 10^{-as} \qquad (5)$$

3.3 Large diffuse target

Here $$\frac{P_R}{P_T} = B \frac{mAL}{s^2} \quad 10^{-2as} \qquad (6)$$

The derivation of this equation is given in Appendix I. The constant B depends on the beam-pattern of the transducer, and is $3{\cdot}54 \times 10^{-2}$ for a rectangular transducer whose face dimensions are large compared to a wave-length (in practice, it is probably adequate if the smallest dimension > 2 λ), L (sometimes called the "ping length") is half the length of the pulse in water, and is the length of the volume from which echoes are received at any instant. For a large circular transducer $B = 3{\cdot}69 \times 10^{-2}$.

Taking equations (2) and (6) together

$$\frac{V_R}{V_T} = 2E \frac{1}{s} \sqrt{B\, mAL} \,.\, 10^{-as}$$
$$= K' \frac{\sqrt{m}}{s} \quad 10^{-as} \qquad (7)$$

where $K' = K \sqrt{4 \pi \theta BL}$ (8)

Thus, if K is known, K' can be calculated from known parameters of the system.

Similarly, we can define constants M' and N' by

$$\frac{e_R}{i_T} = M' \frac{\sqrt{m}}{s} \quad 10^{-as} \qquad (9)$$

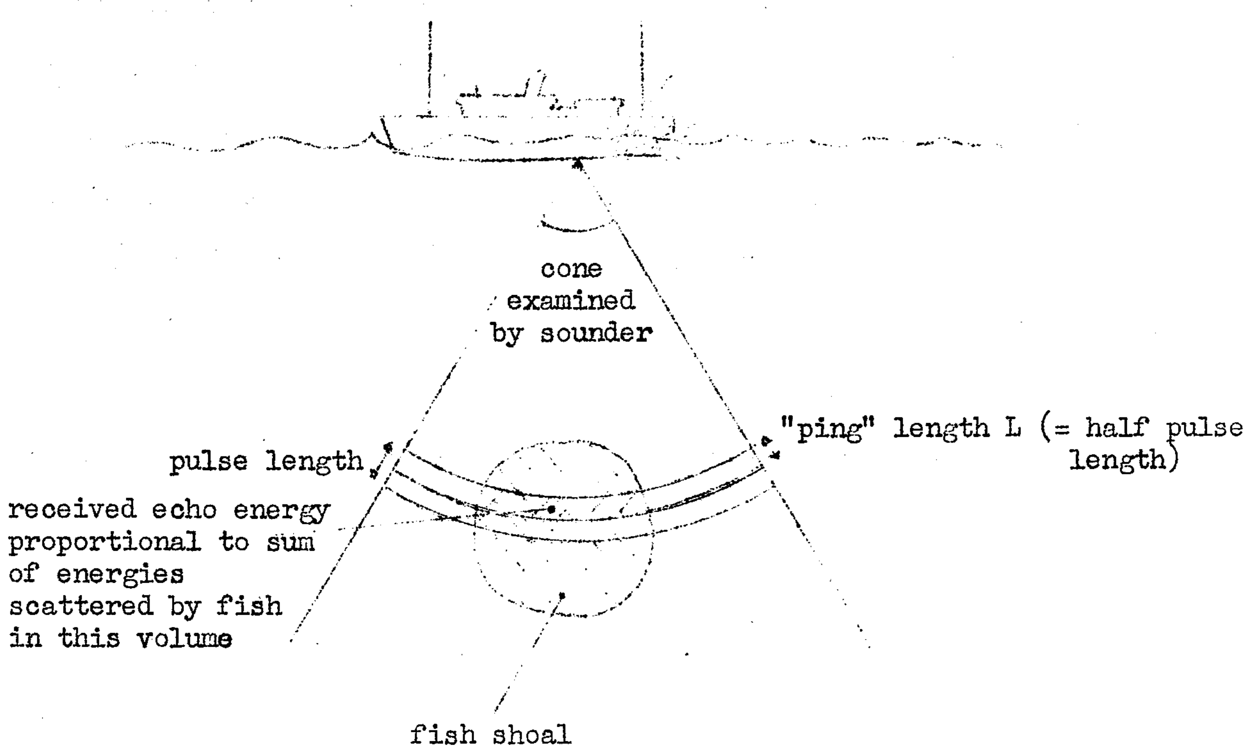
so that $$M' = M \sqrt{4 \pi \theta BL} \qquad (10)$$

and $$\frac{i_R}{V_T} = N' \frac{\sqrt{m}}{s} \; 10^{-as} \tag{11}$$

so that $$N' = N \sqrt{4 \pi \theta \, BL} \tag{12}$$

3.4 Small diffuse target on beam axis

Formulae (1) and (3) apply, but σ is now interpreted as the sum of the scattering cross-sections of all the animals in the "ping-length" L.



3.5 Note on units

It seems likely that in many cases the method of calibration using standard targets will be used by itself. In this case formula (3) may conveniently be used with 's' in fathoms (note that the value of 'a' in terms of fathoms must also be used). When the other methods of calibration are also being used, it is probably easier to work throughout in c.g.s. units, since tables for conversion of fathoms to metres are available on most research ships and there is less chance of error through the use of an inconsistent set of units.

4. METHODS OF CALIBRATION

4.1 Calibration by means of standard targets

This is probably the most satisfactory method of calibration, since it is simple and direct. It involves suspending a standard target, usually either a heavy rigid sphere or a hollow air-filled sphere, on the axis of the acoustic beam of the sounder, and measuring the voltage produced by the echo.

In the theoretical case of a large, perfectly reflecting sphere, geometrical optical methods show that the energy intercepted by the sphere is re-radiated uniformly in all directions. For diffraction effects to be less than 20%, the diameter of the sphere must be greater than 1.1 λ (wavelength in water) and for an accuracy of 10%, the diameter must be greater than 3 λ.

In practice, materials such as iron, steel or brass with reflectivities μ of 0·94, 0·93 and 0·93 respectively, are used. With high reflectivities such as these, the sound penetrating the surface of the sphere echoes around inside it and will escape more or less uniformly in all directions. The precise amount scattered in any direction will depend on interference effects, but these are difficult to calculate and it has to be assumed that the final result is still a re-radiation of the incident sound uniformly in all directions, and that $\sigma = \pi r^2$. The recommended method of suspension is shown in Figure 2.

A hollow, air-filled sphere such as a trawl float may also be used, in which case it is assumed that $\mu = 1$. In this case it is important to ensure that the float really is a sphere: a dent near the top, for example, can produce a major error. Another difficulty in cast floats is that the shape of the inside surface is important but cannot be seen.

Air held in the suspension wire can cause appreciable echoes, and it is therefore advisable to use either piano-wire or monofilament nylon.

Perhaps the greatest difficulty in this method is to ensure that the target is accurately on the axis of the echo-sounder beam. We have found it helpful to move the target horizontally at a given depth until the greatest echo is obtained.

The calculated value for σ and the measured values of the transmitted and received voltages are inserted in formula (3), from which K may then be calculated. If possible, the calibration should be repeated for several values of the depth s, which should be read from the echo-sounder chart since it is the distance from the transducer to the target which is required.

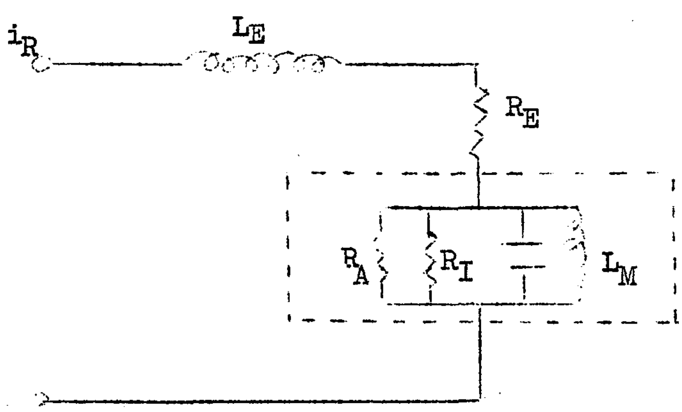
4.2 Calibration by measuring the transducer impedance

4.2.1 Principles

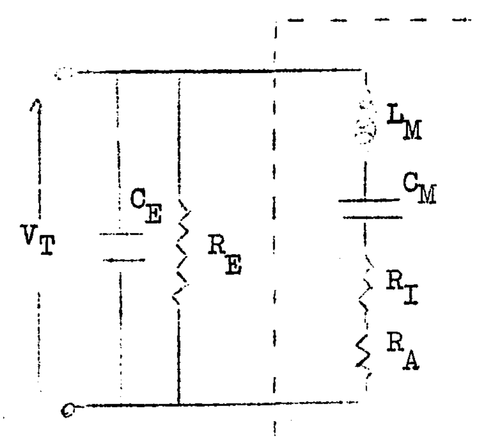
This is the most indirect method of calibration, and involves some rather subtle electrical measurements. It only works when the transducer face is in direct contact with the sea (that is, when it is not mounted in a tank inside the hull) but has the great advantage that an approximate calibration can be made with the ship tied up to a dock, and a reasonably accurate calibration is possible if measurements can also be made with the transducer in air. It therefore does not depend on having good weather at sea.

From electrical measurements it is possible to calculate the amount of power converted into acoustic energy. Using equations (1) and (6), the acoustic power reflected back onto the transducer can then be calculated, and from the electrical measurements it is possible to calculate the voltage this will induce in the transducer.

For the present purpose we may consider the equivalent circuits of transducers to be as shown below.



Magnetostriction transducer



Piezo electric transducer

Here L_E, C_E, R_E represent the purely electrical or "clamped" impedances of the transducers. The circuits inside the dotted boxes represent the effect of the mechanical coupling. C_M and L_M resonate at the frequency of mechanical resonance. R_I represents the internal mechanical losses, and the power dissipated in R_A is radiated as acoustic energy.

It is shown in Appendix I that for a magnetostriction transducer

$$K = 2 \frac{R_X^2}{R_A(R_E + R_X)} \sqrt{\frac{A}{4 \pi \theta}} \qquad (13)$$

$$K^1 = 2 \frac{R_X^2}{R_A (R_E + R_X)} \sqrt{BAL} \qquad (14)$$

$$M = 2 \frac{R_A R_I^2}{(R_A + R_I)^2} \sqrt{\frac{A}{4 \pi \theta}} \qquad (15)$$

$$M^1 = 2 \frac{R_A R_I^2}{(R_A + R_I)^2} \sqrt{BAL} \qquad (16)$$

where $R_X = R_A R_I / (R_A + R_I)$

Note that $\frac{M}{K} = \frac{M^1}{K^1} = R_E + R_X$ which is the effective series resistance of the transducer at resonance.

For a piezo electric transducer

$$K = 2 \frac{R_A R_E}{(R_I + R_A)(R_I + R_A + R_E)} \sqrt{\frac{A}{4 \pi \theta}} \qquad (17)$$

$$K^1 = 2 \frac{R_A R_E}{(R_I + R_A)(R_I + R_A + R_E)} \sqrt{BAL} \qquad (18)$$

$$N = 2 \frac{R_A}{(R_I + R_A)^2} \sqrt{\frac{A}{4 \pi \theta}} \qquad (19)$$

$$N^1 = 2 \frac{R_A}{(R_I + R_A)^2} \sqrt{BAL} \qquad (20)$$

Note that $\frac{N}{K} = \frac{N^1}{K^1} = \frac{1}{R_I + R_A} + \frac{1}{R_E}$ which is the conductance of the transducer at resonance.

4.2.2 Methods of measurement

Note: The measurements cannot be made with the transducer in an ordinary tank since standing-wave patterns are set up. A pond, river or harbour is suitable, since in these the sound energy can spread out and dissipate.

a) Magnetostriction transducer

Let the impedance $Z = R + j X$

If R is plotted against X for various frequencies, a diagram of the form shown in Figure 4(a) should be produced. At a frequency well away from resonance $R = R_E$, and at resonance $R = R_E + R_X$, so that both R_E and R_X can be found from such a diagram.

If now the transducer is taken out of the water, no acoustic power is radiated and R_A is effectively open-circuited so that $R_X = R_I$. Thus, from the impedance diagrams in air and water, R_E, R_A and R_I may be determined.

In practice the diagram is not quite so simple as shown in Figure 4, owing to electrical and mechanical imperfections, but the necessary measurements can still be obtained with adequate accuracy (see Section 7.2 and Figures 5 and 6).

A suitable bridge circuit for measuring the impedance is shown in Figure 3(a). At balance

$$R = \frac{R_2}{R_3} \cdot R_V \qquad\qquad X = \frac{1}{\omega C}$$

The variable capacitor may conveniently be a decade capacitor with a maximum capacitance of about 1 μ F.

b) Piezo electric transducer

In this case the admittance Y is used. ($Y = 1/Z$)

Let $Y = G + j B$

A plot of G against B gives a similar diagram to the one shown for the impedance of a magnetostriction transducer. Away from resonance, $G = \frac{1}{R_E}$, and at resonance, $G = \frac{1}{R_E} + \frac{1}{R_I + R_A}$. If the transducer is taken out of the water, R_A is effectively short circuited and at resonance $G = \frac{1}{R_E} + \frac{1}{R_I}$, so that, as before R_E, R_I and R_A can be found.

A suitable bridge circuit for measuring the admittance is shown in Figure 3(b). At balance

$$G = \frac{R_2}{R_3} \cdot \frac{1}{R_V} \quad \text{and} \quad B = \frac{R_2}{R_3} \cdot \omega C$$

4.3 Use of a second similar transducer

Suppose two identical transducers face one another with a separation 's' and a transmission takes place from one to the other, then

$$\frac{P_R}{P_T} = \frac{A}{\theta s^2} \qquad 10^{-as} \tag{21}$$

Using equations (2) and (3) this gives

$$\frac{V_R}{V_T} = 2 K \sqrt{\pi} \frac{1}{s} \quad 10^{-(as/2)} \tag{22}$$

so that K may be calculated from measurements of V_R, V_T and s. Similarly,

$$\frac{e_R}{i_T} = 2 M \sqrt{\pi} \quad \frac{1}{s} \qquad 10^{-(as/2)} \tag{23}$$

$$\text{and} \qquad \frac{i_R}{V_T} = 2\,N\,\sqrt{\pi}\,\frac{1}{s}\,10^{-(as/2)} \tag{24}$$

This method is in principle simple and direct, but the authors have been unable to obtain consistent results in practice. Their calibrations were carried out horizontally in a harbour and the arrangement was such that multiple path effects should have been negligible. It is thought that the trouble was due to the difficulty in ensuring that each transducer was precisely on the axis of the other.

4.4 Use of a calibrated hydrophone

a) Magnetostriction transducer

If a calibrated hydrophone is available, this may be used to calibrate the transducer.

It will be shown in the appendix that

$$P_T = i_T^2 \left(\frac{R_I}{R_I + R_A}\right)^2 R_A \tag{25}$$

Using this equation, the acoustic energy flow on the axis of the beam of a transducer at a distance s is

$$F = \frac{P_T}{\theta s^2}\,10^{-as} = \frac{i_T^2}{\theta s^2} \cdot \frac{R_A R_I^2}{(R_A + R_I)^2}\,10^{-as} \tag{26}$$

F may be measured with the hydrophone, θ is calculated from the dimensions of the transducer as before, i_T and s may be measured so that the factor $\frac{R_A R_I^2}{(R_A + R_I)^2}$ may be calculated.

This may now be inserted in equations (15) and (16) to give M and M'.

b) Piezo electric transducer

For this case

$$F = \frac{V_T^2}{\theta s^2} \cdot \frac{R_A}{(R_I + R_A)^2} \tag{27}$$

$\frac{R_A}{(R_I + R_A)^2}$ may thus be calculated and inserted in equations (19) and (20) to give N and N'.

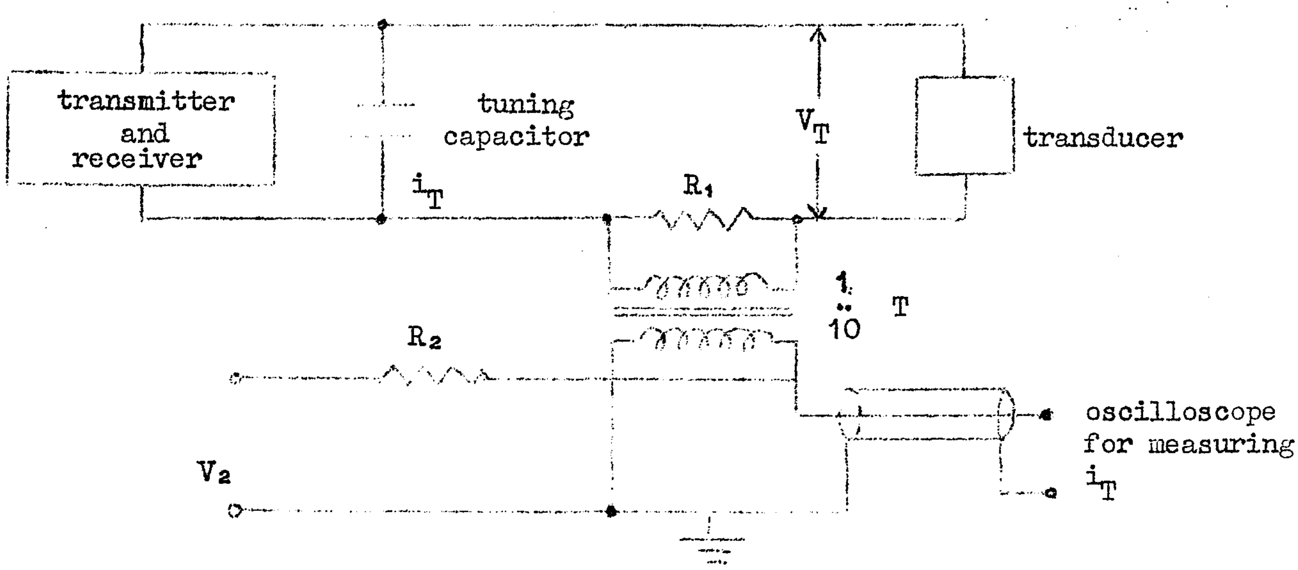
Since hydrophones are usually calibrated in sound pressure p', the following conversion is required (calculated for typical sea conditions)

$$F = 6.4 \times 10^{-13} p^2 \text{ watts/cm}^2 \text{ (if } p \text{ in dynes/cm}^2\text{)} \tag{28}$$

Though apparently a fairly simple method, it involves many instrument calibrations, each of which introduces an extra source of error.

5. METHOD OF MEASURING VOLTAGE AND CURRENTS DURING TRANSMISSION AND RECEPTION

5.1 Magnetostriction transducer (parallel-tuned)



V_T is measured by connecting an oscilloscope directly across the transducer.

i_T is measured by measuring the voltage drop across a low-value resistor R_1 in series with the transducer.

e_R is measured by generating a voltage across R_1 which is adjusted until it is the same as the echo voltage. This voltage is produced by passing a known current through R_1 from V_2. If the transformer ratio is 10:1, this current is 10 V_2/R_2.

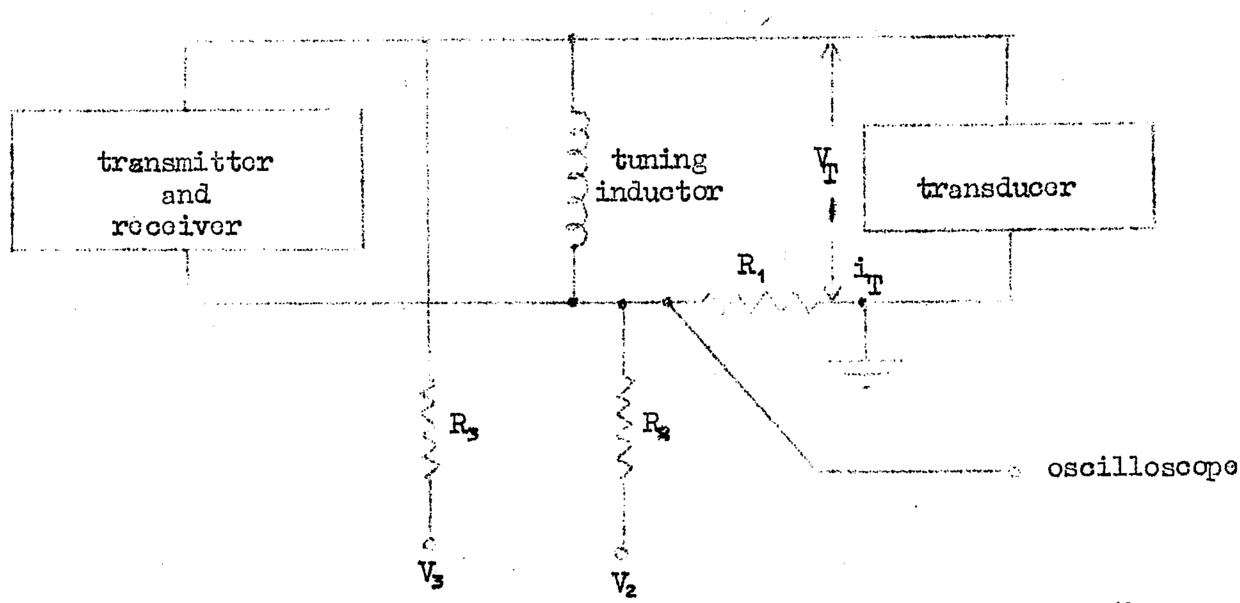
i_R is not required in this case.

For many transducers a value of R_1 of 0.1 ohm is suitable. It is convenient to make this of 24 s.w.g. Eureka wire (or similar type) and to measure it by passing through a metered current of about 0.5 amp from a 1.5 volt dry cell and measuring the voltage drop that this produces using another meter. The primary impedance of T should be about 100 R_1 at the frequency of operation. If R_1 is 0.1 ohm and the transformer is wound on a Ferroxcube core type LA1, a primary winding of 20 turns of 24 s.w.g. enamelled copper wire and a secondary winding of 200 turns of 38 s.w.g. should be suitable for frequencies between 10 kc/s and 50 kc/s.

The authors have found electrical pick-up to be a major problem when measuring V_T and i_T. To test for this, disconnect the live lead <u>only</u> of the C.R.O. as near the transducer as possible when measuring V_T or disconnect one of the connections of T to R when measuring i_T and make sure no voltage is picked up on the C.R.O.

It is essential that the oscillator frequency is the same as the transmitter frequency.

5.2 Piezo electric transducer (parallel-tuned)



Voltages relative to earth

V_T, i_T and e_R may be measured as before, except that in most cases a transformer across R_1 will not be necessary since, owing to the higher impedance of the system, R_1 may be a considerably higher value resistor (typically 10 ohms).

In this case i_R is important, and is measured by injecting a current through R_3 to produce a signal equal in amplitude to the echo signal.

Pick-up should again be checked by removing the appropriate lead at the transducer end and ensuring that the signal disappears.

It is also important to ensure that the oscillator frequency when measuring echoes is the same as the transmitter frequency.

6. IMPULSE DISCHARGE SYSTEMS

Many commercial echo-sounders use a transmission pulse generated by discharging a capacitor through the transducer using mechanical contacts or a gas-filled tube as a switch. Such systems are not so convenient for measurement as the keyed oscillator type, but satisfactory calibration for measuring the size of single targets is possible using either the standard targets, or a second similar transducer. The other two methods cannot be used.

When measuring a diffuse target it is necessary to estimate the "ping length" L. This can probably be done by examining the echo from a single target using an oscilloscope with an expanded time-base (a commercial "fish-lens" type of system may be suitable for this). Ideally, one should square the amplitude of the pulse envelope, measure the area under this curve and divide it by the peak value. However, a reasonably good measure is probably the depth interval between the half-power points.

The frequency of the oscillator used for measuring the echo should be the resonant frequency of the transducer. The precise frequency is not important so long as it is the same for both calibration and measurement.

7. EXAMPLE OF AN ACTUAL CALIBRATION

The transducer calibrated was a single section of the 36 kc/s Sonar fitted to "Discovery II". This has an active face of 6 cm x 17·5 cm, giving beam-widths of 40° x 15° (between half-power points).

The calibrations have been made in terms of the transmission current i_T and the received voltage e_R, since these are the more fundamental measurements. At 36 kc/s $a = 1 \cdot 1 \times 10^{-5}$ cm^{-1}.

7.1 Standard targets

7.1.1 Cast iron sphere 12 cm in diameter

$i_T = 4 \cdot 1$ amps $\quad \sqrt{\sigma} = 6\sqrt{\pi} = 10 \cdot 63$ cms.

s fathoms (measured from recorder chart)	s cm	e_R µV (measured)	$e_R s^2 10^{1 \cdot 1 \times 10^{-5} s}$
22	4,020	125	$2 \cdot 26 \times 10^3$
49	8,960	22	$2 \cdot 22 \times 10^3$
67	12,250	14	$2 \cdot 87 \times 10^3$
63	11,520	14	$2 \cdot 50 \times 10^3$
45	8,230	31	$2 \cdot 50 \times 10^3$
35	6,400	56	$2 \cdot 70 \times 10^3$
		Average	$2 \cdot 52 \times 10^3$

$$M = \frac{e_R s^2 10^{as}}{i_T \sqrt{\sigma}} = \frac{2 \cdot 52 \times 10^3}{4 \cdot 1 \times 10 \cdot 63} = 57 \cdot 8 \text{ ohm cm}$$

7.1.2 Trawl float 19 cm in radius

$\sqrt{\sigma} = 16{\cdot}7$ cm

s fm	s cm	e_R μV	$e_R s^2 10^{1{\cdot}1 \times 10^{-5} s}$
19	3470	250	$3{\cdot}26 \times 10^3$
27	4940	125	$3{\cdot}44 \times 10^3$
		Average	$3{\cdot}35 \times 10^3$

giving M = 49·4 ohm cm.

<u>Note:</u> It was discovered after this calibration had been completed that the transmitter frequency had been just under 1 kc/s above the transducer resonant frequency. The effect of this would be to reduce M as measured here by about 20% relative to the value measured by the second method.

This frequency error arose from using two oscillators with different errors in their dial calibrations. There is a moral here!

7.2 Measurement of the electrical characteristics of the transducer

Figures 5 and 6 show the measured impedance diagrams of the transducer. Note the effect of a secondary mode of resonance which is particularly marked in the first diagram. From these it is estimated that

$$R_I = 22{\cdot}6 \text{ ohms}$$

$$R_A = 9{\cdot}12 \text{ ohms}$$

Inserting these values in equation (15)

$$M = 9{\cdot}26 \sqrt{\frac{A}{4\pi\theta}}$$

$$= 9{\cdot}26 \ \frac{A}{2\lambda} \cdot \frac{1}{\sqrt{\pi}}$$

$$= 66{\cdot}9 \text{ ohm cm}$$

7.3 Use of a second similar transducer

For this calibration and that described in the next section, the authors used a continuous oscillation. On thinking over the results, it seems possible that the use of a pulse might have shown up multiple path effects or other possible sources of error.

The measured values were $i_T = 0{\cdot}2$ amp

$s = 1{,}500$ cm

$e_R = 10^{-2}$ volts

Inserting these values in equation (23) gives M = 21.2 ohm cm.

As has been mentioned earlier, the received voltage fluctuated enormously, probably mainly due to the difficulty of keeping each transducer on the axis of the other. The figure of 10 m V for the received voltage is thus probably considerably low.

7.4 Use of calibrated hydrophone

i_T = 28 milliamps

s = 1.74×10^3 cm

Sound pressure p measured with the hydrophone = 150 dynes/cm^2.

Using equation (28)

$F = 1.44 \times 10^{-18}$ watts/cm^2

Putting these figures into equation (26) gives

$$\frac{R_A R_I^2}{(R_A + R_I)^2} = 8.9 \text{ ohms}$$

Using equation (15) M = 128 ohm cm.

Note: This was not a satisfactory calibration for several reasons, the most important being the difficulty in obtaining even a reasonably steady output from the hydrophone. The reason for this is not known. Other possibilities of error arose from imperfect instrumentation.

APPENDIX I

DERIVATION OF SOME EQUATIONS USED IN THE REPORT

a) Echo from a small target on the beam axis

θ is defined, in effect, by saying that if it is assumed that all the transmitted power P_T is uniformly distributed over a solid angle θ, this will give the correct intensity along the beam axis (but not, of course, the correct intensity in any other direction). Thus, at a range s, P_T is distributed over a shell of area θs^2 and a target of acoustic cross-section σ will intercept a power of $P_T \dfrac{\sigma}{\theta s^2}$

This intercepted power is re-radiated uniformly in all directions, so that the receiving transducer of area A will intercept a proportion $A/4\pi s^2$ of it. This received power is P_R, so that

$$P_R = \frac{A}{4\pi s^2} \cdot \frac{\sigma}{\theta s^2} \cdot P_T = \frac{A\sigma}{4\pi\theta s^4} P_T$$

If the effect of absorption in the water is included this becomes equation (1).

b) Calculation of the effective solid angle θ.

This becomes complicated unless it is assumed that the transducer is large compared to a wave-length. In practice, this means that the smallest dimension of the transducer face must be greater than 2λ.

The sound pressure on the axis of the transducer at a given long range is proportional to the integral of the pressure over the transducer face and is thus proportional to A. The power per unit solid angle on the axis, P_U, is thus proportional to A^2.

If the transducer is large enough for its radiation near the surface to be effectively a plane wave, the total power radiated, P_T, is proportional to A.

Now by definition $\theta = P_T/P_U$ and is therefore proportional to $1/A$.

This argument does not involve the shape of the transducer, so that θ depends only on the area and not on the shape.

Thus calculation of θ for, say, a rectangular transducer, gives a formula valid for all shapes of transducer face.

Consider a rectangular transducer with lengths of side q and r, and a direction which is the intersection of planes making angles ϕ and ψ with the planes through the beam axis and parallel to the sides r and q respectively. Also for convenience we may write

$$y = (\pi r/\lambda) \sin \psi$$

$$z = (\pi q/\lambda) \sin \phi$$

Then the pressure at a point in this direction relative to a point at the same range on the beam axis is

$$f(\phi,\psi) = \frac{\sin y}{y} \cdot \frac{\sin z}{z}$$

(Quoted, for example, in "Ultrasonics" by P. Vigoureux.)

It is now necessary to assume that the dimensions of the transducer are large compared with λ, so that effectively all the energy is radi ted in directions for which ϕ and ψ are small. Thus,

$$P_T \simeq P_u \iint f^2(\phi,\psi)\, d\phi\, d\psi$$

but $\theta = P_T/P_u$, so that

$$\theta \simeq \iint f^2(\phi,\psi) \quad d\phi\, d\psi$$

the integration being carried out over values of ϕ and ψ containing effectively all the radiated energy.

Thus, $$\theta = \iint \frac{\sin^2 y}{y^2} \cdot \frac{\sin^2 z}{z^2} \quad d\phi\, d\psi$$

but $$dy = (\pi r/\lambda) \cos \psi\, d\psi \simeq (\pi r/\lambda)\, d\psi$$
$$dz = (\pi q/\lambda) \cos \phi\, d\phi \simeq (\pi q/\lambda)\, d\phi$$

therefore $$\theta = \frac{\lambda^2}{\pi^2 rq} \iint \frac{\sin^2 y}{y^2} \cdot \frac{\sin^2 z}{z^2}\, dy\, dz$$

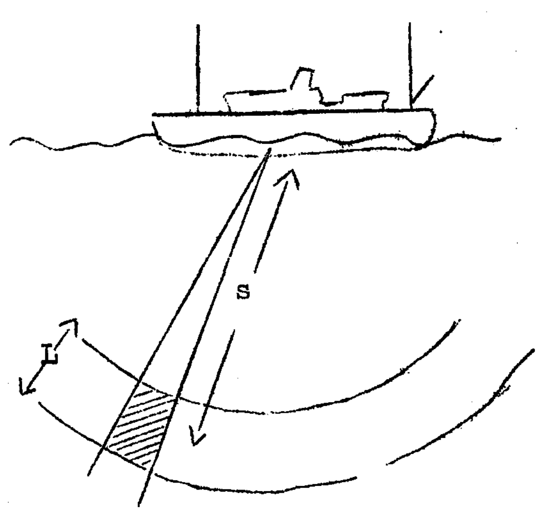
Since $\frac{\sin^2 y}{y^2}$ falls off qu' ckly with increasing y, these integrals may be taken from $-\infty$ to $+\infty$.

From tables of integrals

$$\int_{-\infty}^{+\infty} \frac{\sin^2 y}{y^2}\, dy = \pi$$

therefore $$\theta = \frac{\lambda^2}{\pi^2 rq} \cdot \pi^2$$
$$= \frac{\lambda^2}{A}$$

c) The echo from a large diffuse target



It will again be assumed that the beam is narrow. A beam width of less than 40° between 3db points is probably "narrow" for the purposes of this theory, since, as will be seen, the effects of the outer fringes of the beam are very small.

The power per unit area in the beam at a range s, is

$$\frac{P_T}{\theta s^2} \cdot f^2(\phi,\psi)$$

Remembering that the sensitivity of the transducer to signals received from a direction (ϕ,ψ) is reduced by $f^2(\phi,\psi)$ in terms of power, and using arguments similar to these used in (a) above, the effective power received by the transducer from a target with an acoustic cross-section σ is

$$P_R = P_T \quad \frac{A\sigma}{4\pi\theta s^4} \quad f^4(\phi,\psi)$$

Since ϕ and ψ are small, the solid angle enclosed by increments $\delta\phi$ and $\delta\psi$ is $\delta\phi\delta\psi$. In this solid angle, the volume of water from which echoes are being received at any instant is $Ls^2\delta\phi\delta\psi$. Thus, if on the average there are Q targets of acoustic cross-section σ in unit volume, the power received by the transducer is

$$\delta P_R = P_T \quad \frac{AQ\sigma Ls^2}{4\pi\theta s^4} \quad \delta\phi\delta\psi \; f^4(\phi,\psi)$$

If the scatterers are not all the same size, $Q\sigma$ is replaced by m, the average sum of the acoustic cross-sections of all scatterers in a unit volume, giving

$$\delta P_R = P_T \quad \frac{AmL}{4\pi\theta s^2} \quad f^4(\phi,\psi) \; \delta\phi\delta\psi$$

and the total power received by the transducer is

$$P_R = P_T \quad \frac{mAL}{s^2} \cdot \frac{1}{4\pi\theta} \iint f^4(\phi,\psi) \; \delta\phi\delta\psi$$

If $$B = \frac{1}{4\pi\theta} \iint f^4(\phi,\psi) \; \delta\phi\delta\psi$$

then B is a function of the beam pattern only, and this becomes equation(6)when the exponential absorption term is added. The integration for B can be performed in a similar manner to that performed above in determining θ. Using the fact that

$$\int_{-\infty}^{+\infty} \frac{\sin^4 y}{y^4}\, dy = \frac{2\pi}{3}$$

this gives, for a rectangular transducer whose face dimensions are large compared with λ, that $B = 1/9\pi$ independent of the beam-width.

The authors have not yet been able to solve analytically the case of a circular transducer, but a numerical integration gives $B = 3{\cdot}69 \times 10^{-2}$ when the diameter is large compared with λ.

d) <u>Derivation of calibration constant in terms of the impedances of the transducers</u>

Referring to the diagrams on page 11, for a magnetostriction transducer working at mechanical resonance, during transmission

$$P_T = i_T^2 \left(\frac{R_I}{R_A + R_I} \right)^2 R_A$$

When considering the voltage produced by an echo, it is easiest to consider first a transducer with no internal losses, i.e., R_I is infinite. In this case there will be some load resistance R_L (which includes the resistance of R_E) which completely absorbs the incident acoustic energy P_R. The voltage across R_L will then be $\sqrt{P_R R_L}$. If the transducer is now open-circuited, the incident sound will be entirely reflected and the pressure on the transducer face will double, so that the voltage across the transducer terminals will be $2\sqrt{P_R R_L}$. Inspection of the equivalent circuit on page 11 will show that in these conditions $R_L = R_A$, so that the received echo is equivalent to an e.m.f. in series with R_A of $2\sqrt{P_R R_A}$. *

Thus $$e_R = 2\sqrt{P_R R_A} \; \frac{R_I}{R_A + R_I}$$

and $$\frac{e_R}{i_T} = 2 \frac{R_A R_I^2}{(R_A + R_I)^2} \sqrt{\frac{P_R}{P_T}}$$

combining this with equation (1) gives

$$\frac{e_R}{i_T} = 2 \frac{R_A R_I^2}{(R_A + R_I)^2} \sqrt{\frac{A}{4\pi\theta}} \quad \frac{\sqrt{\sigma}}{s^2} \; 10^{-2as}$$

from which the formula for M given in equation (17) is derived.

Using equation (6) instead of equation (1) gives the value for M'.

If L_E is tuned with a series capacitor, and writing for convenience

$$R_X = \frac{R_A R_I}{R_A + R_I}$$

then $$i_T = V_T / (R_E + R_X)$$

and $$\frac{V_R}{V_T} = 2 \frac{R_X^2}{R_A(R_E + R_X)} \sqrt{\frac{P_R}{P_T}}$$

from which equations (13) and (14) may be derived.

The efficiency is given by

$$E = \frac{P_T}{i_T V_T} = \frac{R_X^2}{R_A(R_E + R_X)}$$

---

* If difficulty is experienced in following this reasoning, the reader is referred to "Ultrasonics" by P. Vigoureux, where the formula is quoted on page 47 in a slightly different form.

so that $$\frac{V_R}{V_T} = 2E\sqrt{\frac{P_R}{P_T}}$$ which is equation (2)

If the circuit is tuned with a parallel capacitor, the above result holds if $\omega^2L^2 \gg R_E + R_X$.

By similar reasoning, for a piezo-electric transducer

$$\frac{i_R}{V_T} = 2\frac{R_A}{(R_I + R_A)^2}\sqrt{\frac{P_R}{P_T}}$$

If the transducer is tuned by a parallel inductor

$$\frac{V_R}{V_T} = 2\frac{R_A\ R_E}{(R_I + R_A)(R_I + R_A + R_E)}\sqrt{\frac{P_R}{P_T}}$$

$$= 2E\sqrt{\frac{P_R}{P_T}} \quad \text{as before.}$$

If the transducer is tuned by a series inductor, this result is true if $\omega^2C^2 \gg \left(\frac{1}{R_E} + \frac{1}{R_I + R_A}\right)^2$

APPENDIX II

USEFUL FORMULAE, CONSTANTS, ETC.

Physical System

1. Velocity of sound c

For echo-sounding, the standard velocity is taken as 4,800 ft/sec. = $1{\cdot}463 \times 10^5$ cm/sec. This corresponds approximately to S = 33‰, T = 5°C and atmospheric pressure.

For fish-detection work it may be more convenient to use $c = 1{\cdot}5 \times 10^5$ cm/sec. which corresponds approximately to S = 33‰, T = 15°C and atmospheric pressure.

$c = \lambda \times$ frequency.

2. Energy, etc.

Energy density $W = \frac{p^2}{\rho c^2}$

If p is in dynes/cm²

ρ in grams/cm³

c in cm/sec

Then W is in erg/cm³

Energy flow $F = cW = \frac{p^2}{\rho c}$

If p, ρ, and c are in c.g.s. units, F is in erg/(sec)(cm²)

or $F = \frac{p^2}{\rho c} \times 10^{-7}$ watt/cm²

$\simeq 6{\cdot}5 \times 10^{-7}\ p^2$ watt/cm² in sea-water.

3. Reflectivity

For normal incidence, reflectivity μ is given by

$$\mu = \frac{\rho_1 c_1 - \rho_2 c_2}{\rho_1 c_1 + \rho_2 c_2}$$

where the suffixes $_1$ and $_2$ refer to the two media.

4. Beam angles

For a rectangular transducer in the plane containing the acoustic axis and the breadth ℓ

Beam angle, axis to half-power direction: $\sin \alpha = 0{\cdot}44\ \lambda/\ell$

axis to first minimum: $\sin \alpha = \lambda/\ell$

For a circular transducer of diameter d

Beam angle axis to half-power direction: $\sin \alpha = \frac{0{\cdot}51\lambda}{d}$

axis to first minimum: $\sin \alpha = \frac{1{\cdot}22\lambda}{d}$

APPENDIX III

SUMMARY OF AN ALTERNATIVE METHOD FOR CALCULATING ECHO-STRENGTH

The theory presented above does not follow the method used by acoustic engineers, which will now be outlined for reference purposes.

If by definition the acoustic intensity $I = p^2$, then the intensity I_0 at a range s on the axis of the transducer is given by

$$I_0 = \frac{I_U}{s^2} \, 10^{-as}$$

Here $I_U = p_U{}^2$ is the intensity at unit distance from the transducer, ignoring attenuation. Thus the output of a transducer may be specified in terms of p_U, and is often specified as dynes/cm² at 1 yard.

The intensity I_R at the receiving hydrophone due to the echo from an object with acoustic cross-section σ is

$$\begin{aligned} I_R &= I_0 \frac{\sigma}{4\pi s^2} \, 10^{-as} \\ &= I_U \frac{\sigma}{4\pi} \quad \frac{1}{s^4} \quad 10^{-2as} \end{aligned}$$

In the logarithmic system, the echo level in decibels relative to 1 dyne/cm²

is

$$\begin{aligned} 10 \log_{10} I_R = 10 \log_{10} I_U &+ 10 \log_{10} \frac{\sigma}{4\pi} \\ &- 40 \log_{10} s - 20 \, as \end{aligned}$$

$10 \log_{10} I_U$ is termed the "Source strength" S decibels

$10 \log_{10} \frac{\sigma}{4\pi}$ is termed the "Target strength" T decibels.

Thus the echo level is $S + T - 40 \log_{10} s - 20 \, as$ decibels relative to 1 dyne/cm² (a is in bels per unit distance; for a in decibels, the last term would be $-2as$).

Note that in the logarithmic system of units <u>all</u> quantities are relative, and the reference must always be quoted. For example,

S may be decibels relative to 1 dyne/cm² at 1 metre

T may be decibels relative to a sphere of 4 metres diameter, or often relative to a sphere 4 cms diameter. For a perfectly reflecting sphere of diameter d, $T = 20 \log_{10} (\frac{d}{4})$.

Failure to quote the reference has in the past occasionally caused confusion.

In the system used in this report, the transmitted energy flow F (i.e. power per unit area) at a range s is given by

$$F = \rho c I_0 = \frac{P_T}{\theta s^2} \, 10^{-as}$$

from which

$$I_U = \frac{P_T}{\theta \rho c}$$

In order to work in terms of voltages or currents across the transducer, it is necessary to know two calibration factors, for example, p_U/V_T and e_R/p_R (where $p_R = \sqrt{I_R}$). In calibration methods 1 and 3, these combine into the empirically determined calibration constant. In method 2 they can be calculated from the electrical measurements and the dimensions of the transducer, and in method 4 the first constant is measured directly, and the second constant may also be measured directly by measuring the same echo on the calibrated hydrophone and the receiving transducer.

When measuring the reverberation coefficient of a diffuse target or calculating the voltage induced in the receiver by water-borne noise, a measure of the directivity of the transducer is required. For this purpose the "Directivity index" D is used, and it can be shown that

$$D = 10 \log_{10} \frac{\theta}{4\pi}$$

For some purposes this system of calculation is convenient. For example, since water-borne noise is usually specified in r.m.s. pressure, it enables the signal-to-noise ratio to be easily calculated.

The authors feel that the system used in this report is more suited to the present purpose, in that it shows the fundamental factors involved more clearly.

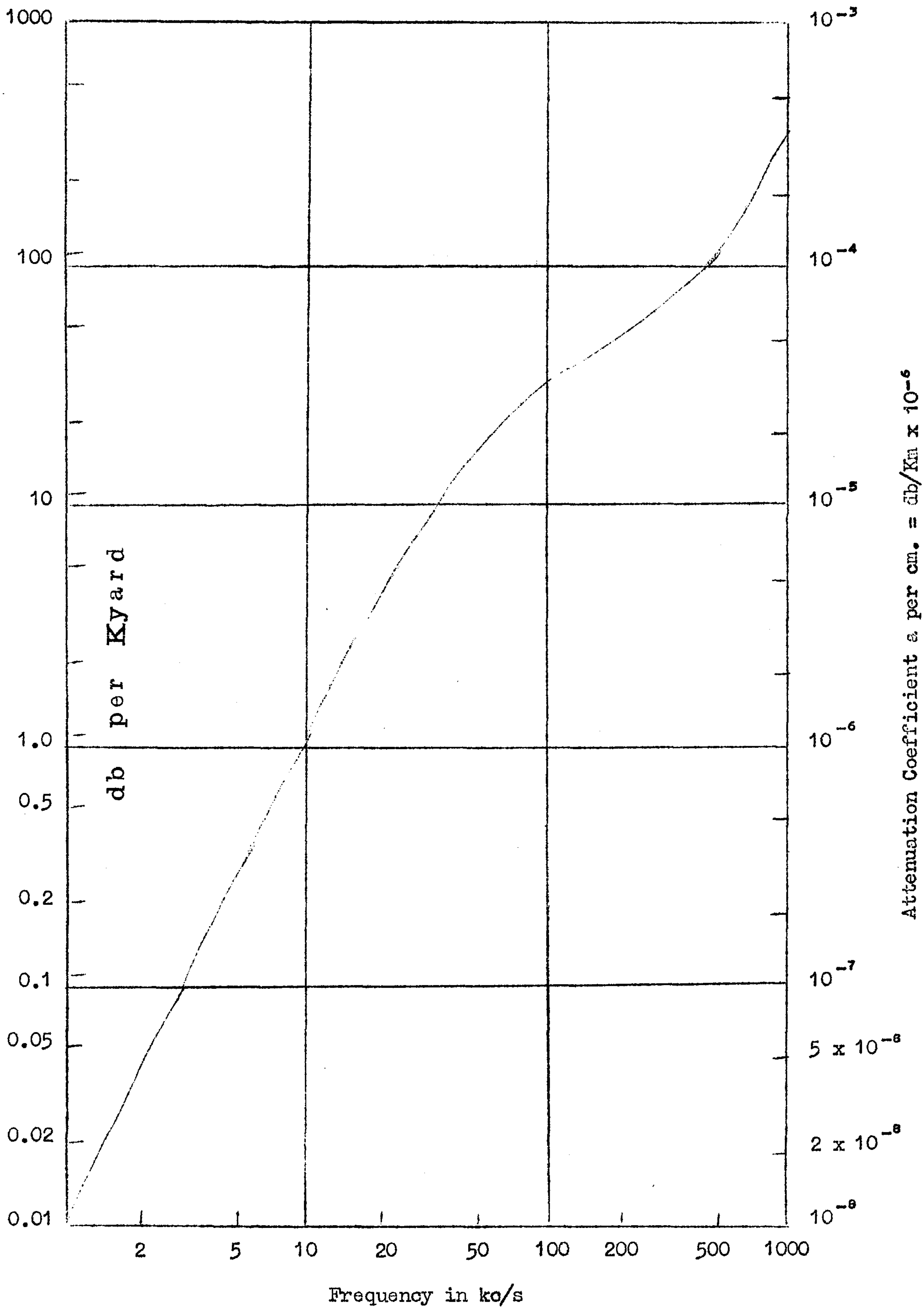


Figure 1 Graph of attenuation coefficient as a function of frequency for clear, clean sea-water. These values must be treated as approximate, since the value at any particular time and place will vary with local conditions. In particular, the attenuation increases greatly in the upper layers during a storm, owing to aeration.

The figures for this graph are taken from "Fundamentals of Sonar" by J.W. Horton (U.S. Naval Institute, 1957).

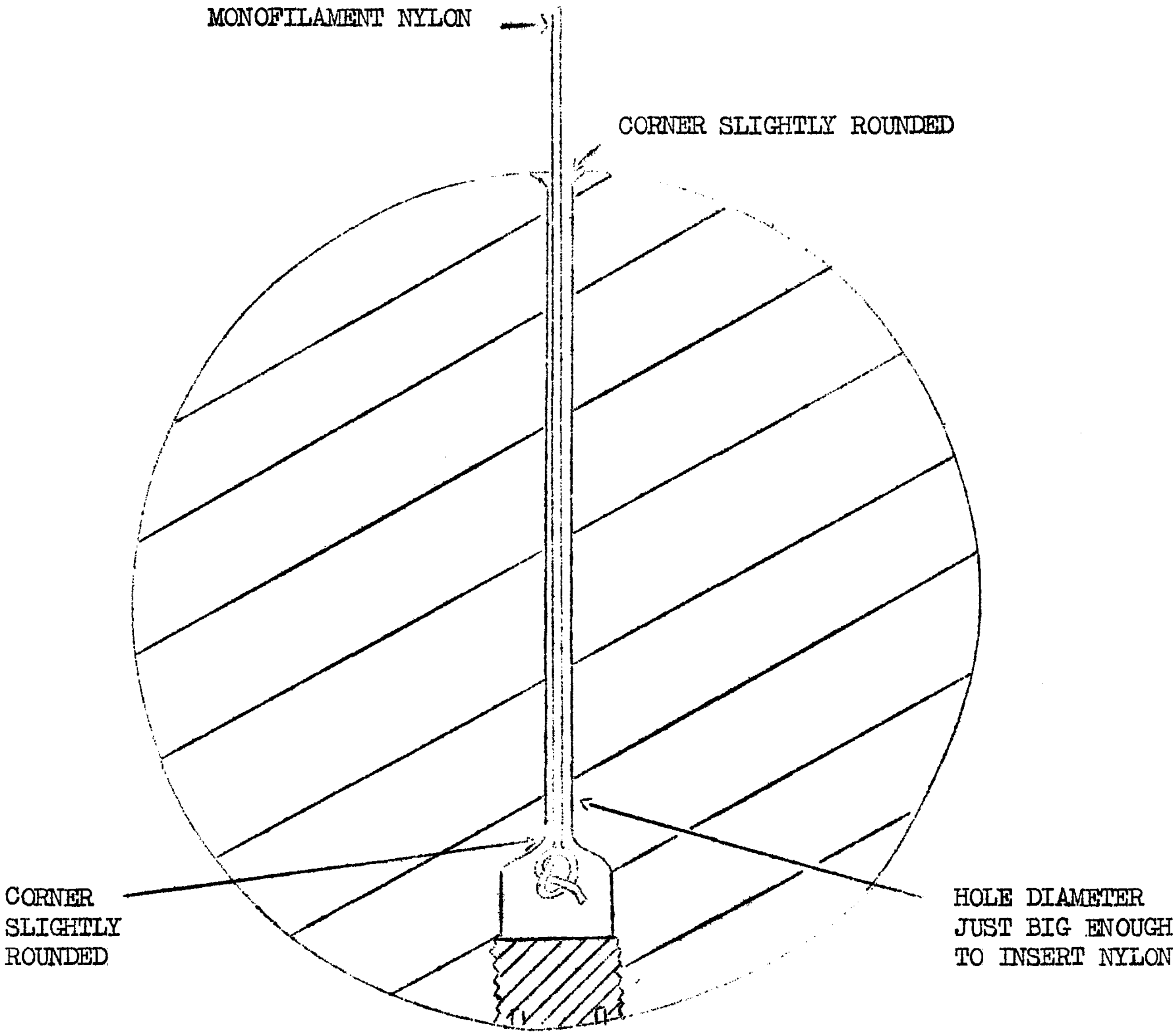
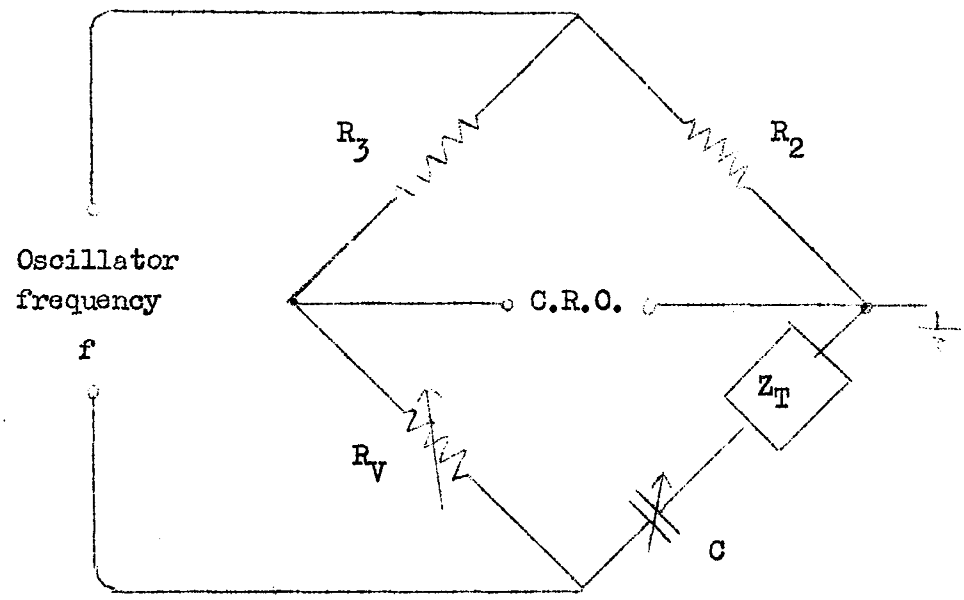
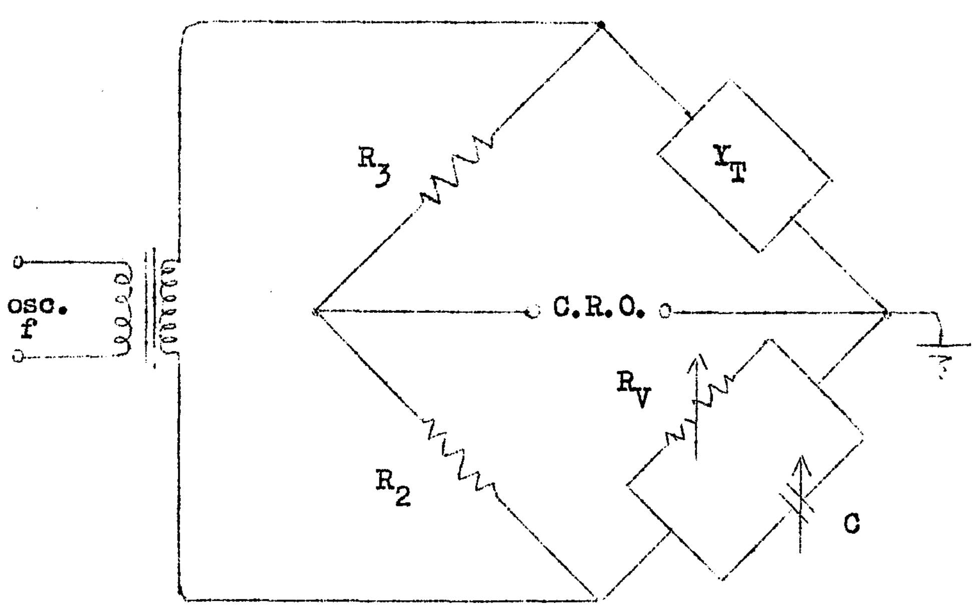


Figure 2 Method of suspending a solid metal sphere for use as a standard target.
(Piano wire may also be used and the end brazed to a ferrule for insertion in the counter bore.)

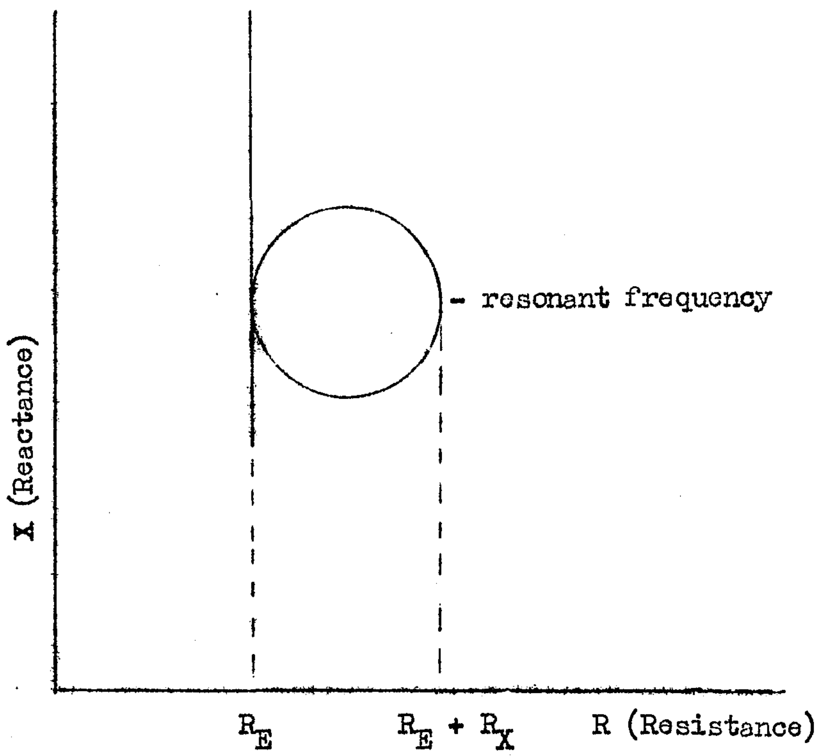


a) Magnetostriction transducer (If necessary due to earthing difficulties, a transformer may be connected between the oscillator and the bridge.)

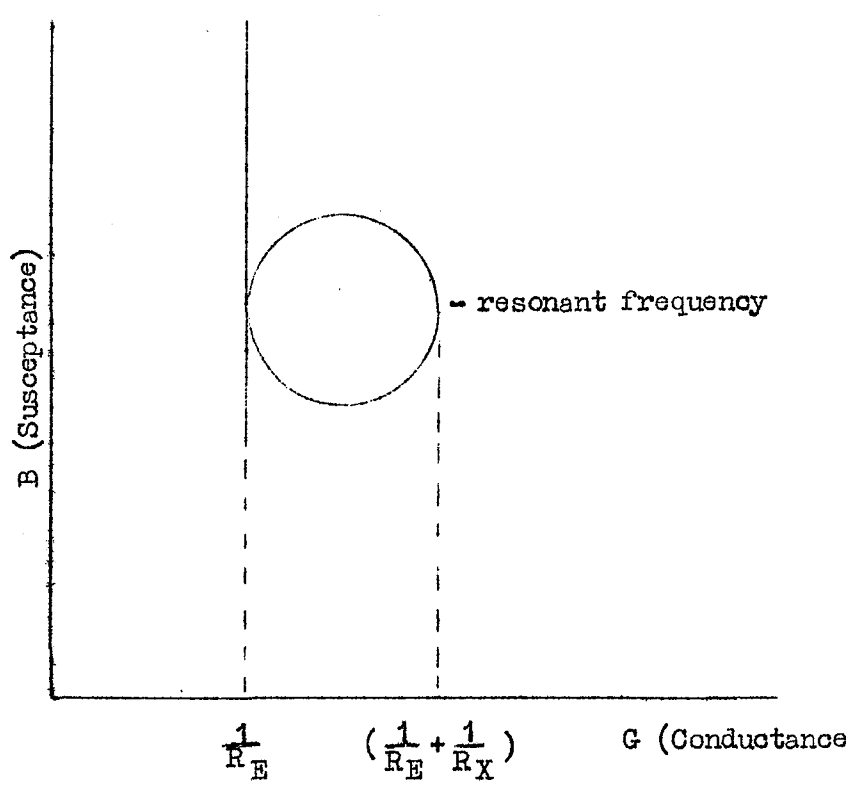


b) Piezo electric transducer

Figure 3 Circuits for measuring the impedance of a transducer at various frequencies.



(a) Magnetostriction transducer $\left(\frac{1}{R_X} = \frac{1}{R_A} + \frac{1}{R_I} \right)$



(b) Piezo-electric transducer $(R_X = R_A + R_I)$

Figure 4 Idealised diagrams of transducer impedance and admittance.

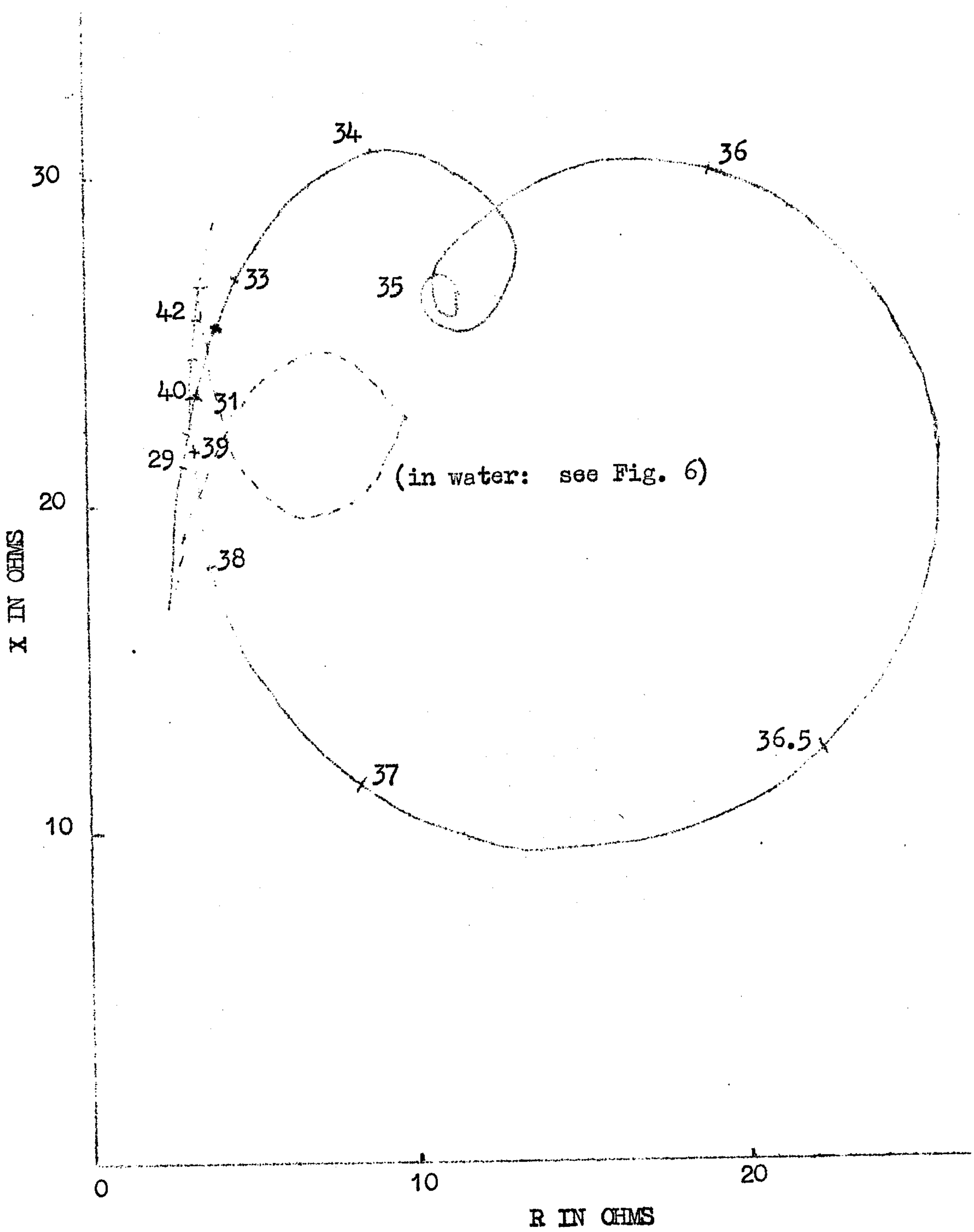
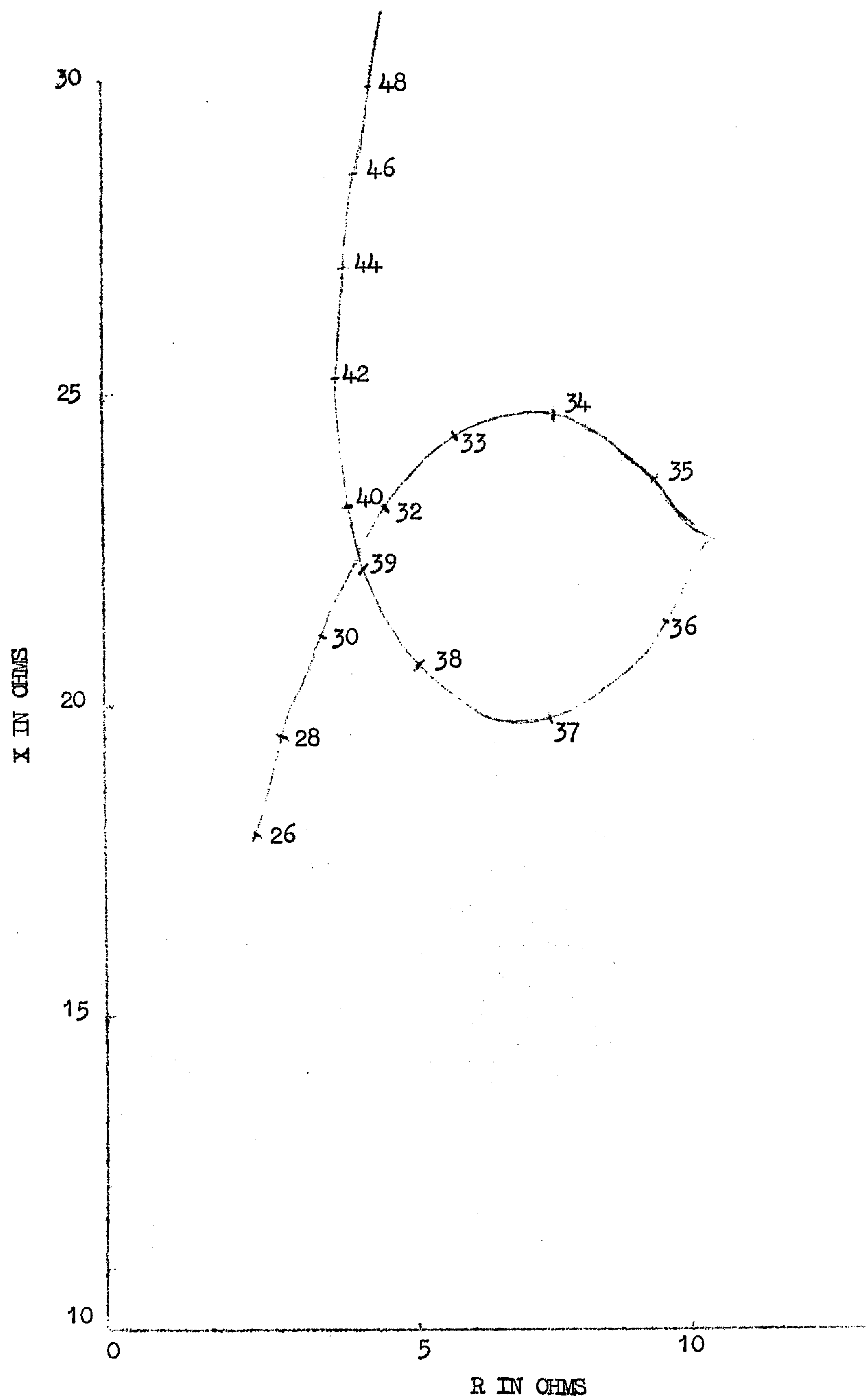


Figure 5 Impedance diagram of a real transducer in air (figures are frequency in kc/s).



<u>Figure 6</u> Impedance diagram of a real transducer in water (Note: to twice the scale of previous diagram).