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NATIONAL INSTITUTE OF OCEANOGRAPHY

WORMLEY, GODALMING, SURREY

The Current System around the British Isles as it relates to Offshore Structures

AN ASSESSMENT

by

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N.I.O. Internal Report No. A. 62

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PART I

The reason for this report

1. Following the Mineral Workings (Offshore Installations) Act 1971 the Department of Trade and Industry has been drawing up detailed regulations for the Safety of Offshore Structures in U.K. waters. For this purpose it set up the Offshore Installations Technical Advisory Committee and NIO has been advising this committee on the oceanographic environment. DTI has also stated that it will request NIO to advise on the validity of the environmental data contained in applications for safety certificates for offshore structures. Although waves are by far the most important environmental parameter, by late 1972 it became clear that currents are also important and that as far as we could (and can) find out, no proper study of them had been made in the present context. NIO therefore undertook a problem definition study as a matter of urgency and started by asking Dr. Carson to consult the experts in order to quantify the importance of currents and to define which characteristics of them needed to be known. His report forms Part II of this publication. Mr. Pitt was then asked to find out what is known about currents and to what extent it is possible to provide the required information. Because of the urgency, it was decided to confine the initial study to the North Sea, which is the scene of the greatest offshore activity at the present time, and for which much more information is available than for the more westerly waters.

PART II

CURRENT FORCES ON OIL RIG STRUCTURES

The National Institute of Oceanography has started a preliminary study of what is known about currents in UK waters, in particular the North Sea, and of what is involved in deriving from this the characteristics required for the design of offshore structures. At the outset, it was not clear what type of current data was required, since the engineering importance of currents was not known to us. The present writer was therefore asked to look into this. Following some discussion with engineers active in this field, it seemed best to calculate some specific examples of loads on typical members in the probable extreme values of waves and current.

The wave force has been calculated, in conventional manner, as the vectorial sum of an inertial and a drag component of force, due to the local particle acceleration and velocity respectively. The current is taken as a simple increase of the velocity term, affecting the drag component only. The wave-induced particle motion is taken from Airy wave theory. The coefficients C_m and C_D for a circular cylinder are taken as 2.0 and 0.5 in accordance with ABS rules: the effect of a greater C_D will be to increase the importance of a current.

The effect of the current can be brought out in three different ways:

(a) We can calculate the force per unit length on a member at a particular depth; this is indicative of the forces on a horizontal member. The particular depth chosen for comparison is the surface layer, where the drag augmentation by current has its greatest effect; many semi-submersible rigs do in fact have tie members in this region.

(b) We can calculate the total side force acting on a vertical pile from the bottom to the surface. This is relevant to mooring forces on a floating rig - since it seems that mooring forces are not to be included in the proposed rig licensing rules, we will neglect this approach.

(c) We can calculate the total moment about the bottom of a vertical pile: this is relevant to the bending strength of piles, and to the overturning moment on a complete platform.

Deep Water

The first case considered is a hypothetical exposed, northern sector location:

Depth	400ft
Design wave height	30m = 98ft
Design wave period	15.8secs
Currents acting over the whole depth	0, 2, 4, 6 kts

Member diameters in the range 0 - 35ft were considered - two examples, 5ft and 30ft diameter have been selected for comparison.

Results

Case (a) Force per unit length in lbs/ft, in the surface layer.

Current (kts)	0	2	4	6
5ft dia	1120	1430	1800	2280
30ft dia	21950	22620	23570	24890

The error introduced by ignoring a 4kt current is 60% in the case of a 5ft diameter member, but only 7% for a 30ft diameter.

Case (b) Moment about base tons ft X 1000

Current (kts)	0	2	4	6
5ft dia	17.0	23.3	32.3	41.7
30ft dia	455	465	484	514

Again the error introduced by a 4kt current is small for the 30ft diameter member (6%) but for the 5ft diameter it is 90%.

Relative Importance of Surface Currents

The moment calculations reported above suggested that the current in the surface layer is more important than mid-water or bottom currents. The reasons for this are as follows:

(a) Since the drag force is proportional to the square of velocity, a given current has a greater effect when superimposed on an existing wave particle velocity. Since wave particle velocity decreases exponentially with depth it follows that a given current has its greatest effect at the surface, and its importance decreases with depth. (It may be of interest to note that, while the wave particle velocity and acceleration both decrease with depth at the same rate, the corresponding drag force decreases at <u>twice</u> the rate of inertial force, due to the $(\text{velocity})^2$ drag law. Thus even for a small diameter member, the inertial force becomes predominant as we descend.)

(b) A force acting at the surface has the greatest moment arm about the bottom; thus the forces in the upper layer are doubly important in calculating the capsizing moment on a fixed rig.

(c) The surface current is in any case generally greater than mid-water currents.

The effect of (c) was investigated by calculating the drag-induced moment M_D due to different current profiles. The profile was assumed to be an exponential of the form

$$U = U_o e^{-\beta z} \quad *$$

U_o = surface current

z = depth coordinate, +ive downwards

and $$\beta = \frac{1}{d} \ln \frac{U_o}{U_D}$$

where d = depth

U_D = bottom current

Values of $\frac{U_D}{U_o}$ = 1, 0.5, 0.1 were investigated.

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* <u>Note</u>: This profile is chosen to demonstrate the importance of the surface currents; very different profiles might be found in practice.

The table below gives the values of $\frac{M_D}{D}$ in tons, where D is the member diameter in feet.

U_o (kts)

$\frac{U_D}{U_o}$	0	2	4	6
1	2596	4149	6112	8483
0.5	2596	3888	5452	7287
0.1	2596	3509	4560	5748

It appears that the effect of tapering a 4kt surface current to 0.4kts at the bottom is to reduce the moment to 75% of its previous value. This confirms an expectation that the greater part of the moment is due to currents in the surface region.

Shallow water

A second hypothetical case was considered, more typical of the southern sector of the North Sea.

Depth	100ft
Design wave height	H = 18m = 59ft
Design wave period	T = 12.5 secs
Currents, acting over the whole depth	0, 2, 4, 6 kts

Case (a) Force per unit length (lbs/ft), in the surface layer.

Current (kts)	0	2	4	6
5ft dia	802	1014	1305	1657
30ft dia	21300	21600	22200	23000

As for the deep water case, the current only affects the smaller diameter member.

Case (c) Moment about base tons. ft. X 100

Current (kts)	0	2	4	6
5ft dia	1.17	1.49	1.99	2.65
30ft dia	34.3	34.8	35.7	37.2

Agsin, the current substantially alters the moment on the smaller member, but not the larger. The conclusions for deep water therefore apply equally in shallow conditions.

Other conclusions

1. The importance of current forces on small diameter members has been stressed; it is worth noting that jack-up platforms generally have lattice frame legs, where the characteristic diameter D is very small; and that semi-submersible rigs have tie-bar members which are also small, in the range of 6 - 9ft diameter.

2. In view of the uncertainty in the value assigned to C_D * it is unrealistic to demand very accurate current measurement.

For example: suppose we know the current to ±20%; the current is at most 1/3rd of the total particle velocity, so we know the particle velocity to ±6.7%. We therefore know the drag force to ±14% (i.e. $(1.067)^2 = 1.14$). This compares favourably with the possible uncertainty in C_D.

3. It has been shown that bottom currents are unimportant in loading a typical structure; never-the-less, bottom current data is needed in connection with foundation scour, and sand-wave formation.

4. Current turbulence

The peak load occurs when the maximum current coincides with the "100-year" design wave. If the current is turbulent, the probability of the peak "gust-velocity" coinciding with the highest wave is very small. We would therefore be justified in using say, a 5-minute average of current. The problem is directly analogous to wind gust loading, where a similar argument is used.

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* See e.g. B.W. WILSON, R.O. REID (1963)
J. Waterways and Harbours Div.
ASCE, Vol. 89 (WW1)

5. Fatigue

Even in a turbulent current, the number of stress cycles will be substantially fixed by the number of wave encounters, irrespective of the current. However, a current does increase the magnitude of each stress cycle. A correction for this could be made in a fatigue life estimate, though it seems doubtful whether it would make much difference to an already approximate calculation.

PART III

THE CURRENT SYSTEM AROUND THE BRITISH ISLES AS IT RELATES TO OFFSHORE STRUCTURES

3.0 Introduction

This report presents some of the more important features of the current system of the North Sea as they relate to offshore structures and as they are presently understood. It also identifies specific areas in which further work is needed.

A large part of the report is devoted to wind-driven currents, as it was felt from an early stage that these would be the most important under extreme weather conditions.

Unfortunately, there exist very few series of observations made during stormy conditions, and although the situation will improve with increased observational effort, methods of extrapolation to extreme conditions are likely to remain a central problem. For this reason some attention is given to several theoretical treatments of wind-driven currents, and to two numerical models.

From the engineering point of view, the near-surface currents are the most important (see Carson, Part II of this report), but these are precisely those which are most difficult to measure reliably (see Pollard 1972). Thus, theoretical treatments will be essential to provide a basis for extrapolation of the observed current structure to the surface.

3.1 The composition of North Sea Currents

The current system of the North Sea represents the combined effect of many components, the principle ones of which are given below:-

(a) Tidal currents
(b) Wind-driven currents
(c) Surge currents
(d) Density currents

The system cannot in general be considered a linear superposition of these components, although in some areas of the North Sea such an approach gives useful results, and all the methods of analysis and prediction of currents so far attempted assume that the various current components can be linearly combined.

3.2 Tidal Currents in the North Sea

3.2.1 The North Sea is often considered to be an area where tidal currents are strong and where the other current components appear as small perturbations on this basic pattern.

This is certainly true of much of the English Channel, a large area of the southern North Sea, around the Orkneys and Shetlands, and generally in coastal regions and estuaries. Here, maximum tidal currents are of the order of 3 knots and exceed 5 knots in places.

However, over most of the central and northern parts of the North Sea, the tidal currents are much weaker ($\sim\frac{1}{2}$ - 1 knot), and in these areas the wind-driven currents are often of the same order as those due to tides.

Tidal currents are well documented in several tidal atlases, and the question arises as to the accuracy of the data obtainable from them.

No extensive or systematic comparison with recent current meter data has been attempted although some spot measurements by MAFF, Lowestoft indicated very good agreement. More generally, a figure of 20% is often quoted, although with reservations, and one of the most important of these is as follows.

Near coasts, and particularly near headlands horizontal gradients of tidal current are large. In coastal areas with extensive indentations, e.g. the Moray Firth, this gradient exists both to landward and seaward of the current; i.e. the current flows as a more or less well-defined stream. The axis of the stream can be shifted by strong winds and so large fluctuations of current occur at a fixed location which may be for part of the time within the tidal stream, but otherwise outside of it.

The same effect can be observed with the quasi-permanent residual circulation of the northern North Sea and is one of the reasons why the current system of this area is difficult to interpret.

3.2.2 Horizontal Variability The strong horizontal gradients of tidal current which occur near coasts have already been mentioned but there is a need for more specific information. There are very few suitable sets of observations and more are definitely required. However, from the available information there do not seem to be marked horizontal sheers except near coasts and in shallow water, where the bottom topography becomes important.

3.2.3 Vertical Variation Since tidal movements are the result of forces acting on the body of the water column, one would not expect a marked variation with depth of the tidal current. This does in fact prove to be the case in the observations of current profiles that have been examined.

These were made from the Total Oil Marine rig Ocean Traveller positioned in the northern North Sea between the Shetland Isles and Norway in 112m of water, and by BP in the Forties Field in a depth of 90m.

It was found that tidal currents vary little with depth except for a thin bottom boundary layer. The bottom boundary layer was about 2-4m deep in the BP observations.

3.3 Density Currents

3.3.1 These are currents which flow in response to local horizontal gradients of water density. They are probably the most persistent non-tidal components of the current regime. Their magnitude is, however, small and a figure of 15cm/sec is often quoted as a typical value.

3.3.2 Long-term Circulation The large-scale, quasi-permanent circulation of the North Sea could be included under the heading of density currents, although in this case the flow is the result of adjustment between density gradients and wind stress on climatic and oceanic, rather than synoptic and local scales.

The main features of this flow as they relate to the northern North Sea are discussed by Dooley (1972).

Inflows of Atlantic water occur to the north of the Shetlands and through the Orkney-Shetland Channel, and the main outflow is in the deep water to the west of Norway.

The Orkney-Shetland current flows south-east to about 57°N and then turns east across the North Sea. This circulation intensifies or weakens in response to weather conditions to the north and west of the North Sea, and not just to the local wind field (see observations).

Dooley gives a maximum value of 2 knots for the inflow to the north of the Shetlands, and values approaching 1 knot have been measured in the Orkney-Shetland flow under strong wind conditions (see observations).

3.4 Wind-driven Currents and Surge Currents

3.4.1 Theoretical models The main results of some of the theoretical treatments of wind-driven currents will now be described. A fuller discussion is contained in the Appendix.

3.4.2 The Ekman Theory A theory of wind-driven currents was first put forward by V.W. Ekman in 1902. In its simplest form (applied to an unbounded, homogeneous ocean of uniform depth) it predicts a current structure in which the flow is 45° to the right of the wind at the surface, and rotates clockwise with depth, its magnitude decreasing rapidly. The magnitude of the current is proportional to the stress of the wind on the sea-surface. Selecting some numerical values which correspond reasonably well with an open North Sea location we find that the predicted surface current is critically dependent on the parameters of the model, and how they vary with windspeed. This applies particularly to the eddy viscosity N.

Taking a 100 knot wind as an example, if we suppose that N does not vary appreciably with windspeed we arrive at a surface current of 5.6 knots, which if it is assumed to vary as the windspeed squared we get 1.2 knots (see Appendix for details).

Relevant observations are few and taken as moderate windspeeds (see below). They do not resolve the problem and observations at higher windspeeds are thus urgently required.

Quite apart from the internal parameters of the model there remains the question of boundary conditions. To solve the problem for a realistic situation, taking account of the shape of coastlines, the bottom topography and the wind field requires that a model of the whole area under consideration be set up.

3.4.3 Computer Models A considerable amount of work has been done to produce computer models of the North Sea and adjacent shelf areas, mainly in connection with the prediction of storm surges. More recently two models specifically designed to compute the current system have appeared (N.S. Heaps, 1971, 1972; F.C. Ronday, 1972). To date, only Heaps' model gives information about the distribution of current with depth.

Heaps (1971) applied his model to a closed rectangular basin of about the same scale as the North Sea. The basin was subjected to a uniform northerly wind of about 42 knots.

The model gave strong south going currents along both N-S coasts. In the interior of the model there were SW currents at the surface with a return flow at depth.

The surface currents in the centre of the model were 30cm/sec in magnitude.

Heaps (1972) went on to apply his model to the Irish Sea, and showed the great importance of the bottom topography in determining the form of the current system.

Ronday's model of the North Sea (1972) is mainly concerned with the long-term residual current system. He used a uniform SW wind of about 15 knots (the long-term average for January), and computed the depth-meaned current field.

Ronday demonstrated the importance of the bottom topography and also of conditions at the boundary with the open sea. These are particularly important in the northern North Sea because of the large inflows and outflows which occur there.

3.4.4 Application of Numerical Models Numerical models of the wind driven current system of the waters surrounding the British Isles are a comparatively new departure. The one model which gives the vertical distribution of current has so far been applied only to the Irish Sea (Heaps 1972). A scheme to make some observations to be used as truth points for the model during the Autumn of 1972 failed through lack of wind, although further observational trials are planned.

To apply Heaps' model to the North Sea would require a great deal of effort to determine the open sea boundary conditions. It would also require from 1 to 3 years work by the scientist doing the computational work.

At this stage numerical models are very useful research tools and provide valuable insight into the problem. More specifically they can be used:-

(a) To help interpret the available observations.

(b) To suggest where future observational effort might best be applied.

However, it will be some time before they can be used to make operationally useful current predictions.

3.4.5 Surges: general The North Sea is subject to surges generated by certain meteorological disturbances passing over the UK Continental Shelf. The 'external surge' is that part generated outside the North Sea and which enters it round the North of Scotland, and the 'internal surge' is that part generated within the North Sea.

Occasionally, as in 1953, they can cause severe flooding on the south-east coasts of Britain and in the Netherlands and Germany, and they have therefore been the subject of intensive study. The measured data available is confined to records of the sea-level from coastal tide-gauges, but the various types of model which have been constructed give depth-meaned currents as a by-product.

I have inspected the current output of two models of North Sea storm surges: those of Ishiguro (1967) and Heaps (1969), both of which give the depth-meaned current field.

3.4.6 Ishiguro's Model This is an electronic analogue model. and the currents were computed for a hypothetical purely external surge of moderate intensity. That is, the surge was supposed to be a free progressive wave running in from outside of the northern boundary of the North Sea, and the current was computed at intervals during its passage down the East Coast of England and its return along the Continental Coast.

The surge gave sea level elevations of about 2 metres along the East Coast of England, and the maximum surge currents were approximately 50cm/sec. These occurred down the western side of the North Sea. They were generally south-going at first but reversed direction during the passage of the surge wave.

The model gives no information about the distribution of current with depth, but as the surge wave is similar to the tidal wave in some respects one can reasonably suppose that the surge current is uniform with depth.

3.4.7 Heaps' Model This is a numerical model and the currents were computed for a combined internal and external surge. This was the major surge of February 1952.

The following estimates of maximum currents were extracted:-

Off North and East Scotland	3 knots
Open northern North Sea	1 knot
Southern North Sea	3 knots
Straits of Dover	4 knots

Note that Heaps' model includes the effect of wind-traction over the whole of the North Sea and adjacent shelf areas.

3.5 Observations and Residual Currents

3.5.1 The most extensive program of current observation carried out in the North Sea is probably that due to the Deutsches Hydrographisches Institüt of Hamburg. The observations are published as tables of $\frac{1}{2}$-hourly means of current with the associated running 24hr 50min mean.

The 24hr 50min mean represents the non-tidal component of the current and is called the diurnal residual.

Values of the diurnal residual were plotted at 12-hourly intervals throughout each series, and also the concurrent windspeed and direction.

Some examples of these graphs are shown in figures 1-6, and a station position map is provided. Table 1 gives further details of the measurements.

3.5.2 Accuracy of Windspeed Estimates To save time, the wind estimates were obtained by inspecting the appropriate charts of the Daily Weather Report published by the Meteorological Office. Using this method, it is highly doubtful if the estimates could be guaranteed to within 5 knots, and in a more thorough-going analysis this difficulty would have to be resolved.

3.5.3 Explanation of figures 1-6 Each figure consists of the observations from one station. The speeds for both wind and current are plotted on the same graph, the scale graduations being in knots for the windspeed and cm/sec for the current.

3.5.4 Discussion of the Observations Inspection of figures 1-6 show two important characteristics.

1. Although the correlation between wind and current at low wind speeds is low, at higher windspeeds the connection between the two is more evident.

2. In those series of observations made in the central parts of the North Sea, the current often flows to the right of the wind at an angle of about 50°, which further north in general it does not, and the relative directions of wind and current have no simple relationship to one another.

3. A further basic point is that the windspeed in knots is of the same order as the current in cm/sec, and one might therefore suggest a linear relationship with a proportionality factor of 0.02. A full wind/current regression analysis of all the available data was not attempted at this time for several reasons:

(a) I consider that the lower values of the residual currents are heavily contaminated by components not directly related to the wind. The results would therefore depend on the higher values of current which are rather few in number.

TABLE 1

Station Number	Water Depth (m)	Measurement Depth (m)	Observation Period
4101	71	10	8 Aug - 27 Aug 1961
4110	24	12	6 Aug - 24 Aug 1961
4203	136	11	18 May - 3 June 1962
4216	87	12	20 May - 4 June 1962
4228	50	12	15 Sept - 3 Oct 1962
4313	89	10	7 Sept - 22 Sept 1963

(b) The errors introduced by our rather poor knowledge of the windspeed would markedly reduce the value of the analysis.

(c) The analysis of groups of data such as those presented in these graphs must take account of the overall circulation of the North Sea. Thus we should not expect the relationship between the wind and current at say, Station 4216, which is in the Orkney-Shetland inflow to be the same as at Station 4110 which is in the centre of the North Sea.

3.5.5 Other Observations I have also inspected some observations from the JONSIS A, B and C moorings for 1971 and 1972, and those made in Whitby Bay during January and February 1970. All these were made by MAFF, Lowestoft, and were considerably deeper than the DHI measurements. However, although the residuals are in general smaller, they show the same sort of relationship with the wind.

The one source of long time series of surface current measurements that is available is due to the Trinity House lightships. These have been taken for many years, in coastal waters and estuaries. Some work has been started at MAFF, Lowestoft on the analysis of these observations and again a clear connection between wind and current is apparent.

3.6 Conclusions

3.6.1 The main conclusions of this report regarding the current system of the North Sea are as follows:

1. The data on tidal currents in the tidal atlases is in general accurate to within 20%, although with the qualifications noted in section 3.3.

2. Wind-driven currents are the most important non-tidal component of the current system, and over much of the central and northern parts of the North Sea can be expected to match or exceed the tidal component under storm conditions.

3. In the central part of the North Sea the wind-driven currents agree broadly with the picture provided by Ekman dynamics in several important respects.

(a) The current flows to the right of the wind.

(b) The current increases with increasing windspeed.

(c) There is appreciable vertical shear.

4. In the northern North Sea where there is a definite large scale residual flow pattern, the effect of increasing winds is to modify and in particular, strengthen the existing currents.

5. Theoretical treatments suffer from uncertainties regarding the basic parameters of the problem, in particular there is not enough relevant data on the growth of turbulence with increasing windspeed. Some observations of currents continuous through periods of high winds should go a long way towards resolving these problems, besides providing a basis for statistical treatments.

6. Numerical models are at present not sufficiently well-developed to be used for current prediction but do provide insight into the problem, particularly as regards the importance of the overall circulation and its relation to the bottom topography.

7. The variation of current with windspeed is not strictly definable on the basis of the investigations underlying this report. However, considering both the observational and theoretical evidence a reasonable description is as follows:

(i) There appears to be a 'threshold' value of windspeed of about 15 knots below which the correlation between wind and current is poor.

(ii) Above this figure the current is related to the windspeed by a relationship which is, over the restricted range of windspeeds for which observations are available, approximately linear, the proportionality factor being around .02, but see below.

(iii) The onset of strong winds after a comparatively quiet period produces transient currents which exceed those above, perhaps by 50%. These transients last for about 6-12 hours.

(iv) There is a local increase of current of perhaps 50-100% where there is a lateral constraint (i.e. a coast or shoaling water) to the right of the wind.

(v) Vertical turbulence and horizontal turbulence increases with windspeed, and with time from the onset of the wind.

Although all these statements are to some extent conjectural, point (ii) is the most crucial, and the one about which there is most doubt.

3.6.2 <u>Further Investigation</u> There are two main approaches to the problem of predicting maximum likely current speeds in the North Sea.

The first one, implicit in much of this report, is to apply what is known about the mechanism of generation, and by using the appropriate inputs (meteorological and otherwise) to calculate the currents for any given situation.

Unfortunately, the theory is inadequate to allow us to apply this method to extrapolate to extreme conditions with any degree of confidence.

The second approach is to apply purely statistical techniques, and use an appropriate distribution function to extrapolate to the 100 year recurrence value.

This approach requires data extending over at least a year, and preferably much longer. Such long time-series of current measurements are not as yet available, and bearing in mind the difficulties and expense of current measurement, the geographical coverage is likely to remain sparse.

The best approach at present seems to be to use a combination of these two in the following way:

(a) There should be a very careful analysis of all the presently available current data to try to improve estimates of the parameters of the theory.

(b) Long time-series of observations in the right location should provide some observations at high wind-speeds and will provide the main basis for improvements in theoretical estimates of maximum currents.

(c) As more observations become available it will become possible to apply statistical techniques. In their turn the statistical estimates can be used to cross-check the other predictions of maximum currents.

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APPENDIX I

Theories of Wind-driven currents

The equations of motion for an element of water are simplified to include only the local acceleration, the coriolis force and the frictional term. If we write the resulting equations in a left-handed rectangular coordinate system with the x and y axis in the sea surface and z vertically down, we get

$$\left.\begin{aligned} \frac{\partial u}{\partial t} &= fv + N\frac{\partial u^2}{\partial z^2} \\ \frac{\partial v}{\partial t} &= fu + N\frac{\partial v^2}{\partial z^2} \end{aligned}\right\} \quad \ldots\ldots\ldots (1)$$

where $f = 2\omega \sin\phi$ is the coriolis parameter and N is the eddy viscosity.

Ekman solved these equations for the steady state and for constant N with the following boundary conditions

(1) The stress is continuous across the sea surface.

(2) The velocity at the sea-bed is zero. The solutions are given below, and apply to a horizontally unbounded, homogeneous ocean of uniform finite depth.

$$\left.\begin{aligned} u &= A \sinh a\zeta \cos a\zeta - B \cosh a\zeta \sin a\zeta \\ v &= A \cosh a\zeta \sin a\zeta + B \sinh a\zeta \cos a\zeta \\ \text{where} \quad A &= \frac{\tau D}{\rho N\pi} \cdot \frac{\text{Cosh } ad \cos ad + \sinh ad \sin ad}{\cosh 2ad + \cos 2ad} \\ B &= \frac{\tau D}{\rho N\pi} \cdot \frac{\text{Cosh } ad \cos ad - \sinh ad \sin ad}{\cosh 2ad + \cos 2ad} \end{aligned}\right\} \ldots\ldots\ldots (2)$$

where $a = \sqrt{\dfrac{f}{2N}}$

$D = \pi/a$, the Ekman layer depth

d = the water depth.

$\zeta = d-z$, i.e. the height above the sea bed.

The ratio $^{d}/D$ is of great importance in the theory. For $^{d}/D > 1$, the solutions are closely similar to those for d finite. In this case the solution is the well-known Ekman spiral. The surface current flows at 45^{o} to the right of the wind and with increasing depth rotates clockwise until at depth D it is flowing in the opposite direction to the surface current.

As the ratio $^{d}/D$ becomes smaller, the current structure changes progressively. The surface current becomes less strong and flows at a progressively smaller angle to the right of the wind. The variation of speed with depth is more nearly linear, and the variation of direction very much less.

Ekman then went on to consider a non-infinite ocean, and in particular the current system off a straight coast-line.

If the wind blows onshore it tends to pile up the water against the coast, while if it is offshore the water level is lowered. In either case the sea surface acquires a slope. This sets up a horizontal pressure gradient in the body of the water which in turn produces its own current system.

Ekman calculated the current due to a sea surface slope, and the solutions follow

$$u = \frac{g \sin \gamma}{f} \left[1 - \frac{\cosh a(d + z) \cos a(d - z) + \cosh a\,(d - z) \cos a(d + z)}{\cosh 2ad + \cos 2ad} \right]$$

$$v = \frac{g \sin \gamma}{f} \cdot \frac{\sinh a(d + z) \sin a(d - z) + \sinh a(d - z) \sin a(d + z)}{\cosh 2ad + \cos 2ad}$$

.... (3)

Near a coast, the current distribution will be given by a combination of solutions of the form of (2) and (3). Sin γ , the sea surface slope, can be eliminated by equating the total (i.e. depth integrated) flow normal to the coast-line to zero.

The requirements of continuity, i.e. the coastal boundary, complicates the current system greatly.

In deep water ($^{d}/D > 2$) a three layer profile is set up with the surface current flowing at an angle which may vary between 0^{o} and 53^{o} to the right of the wind direction depending on the relative directions of wind and coast-line.

In shallow water a two layer system is formed and the surface current can flow to the left of the wind in some circumstances. Also, the presence of a coast-line, in general, increases the speed of the currents considerably.

Because of the complexity of the system, even for an idealized situation, a model of the whole basin is required if it is to be understood.

The numerical models so far constructed follow Ekman's work to some extent but with some important differences.

A central one is the choice of bottom boundary condition. Ekman's model is a "no bottom current" model, i.e. the wind-driven current is set to zero at the sea bed. A boundary condition more often employed in theoretical and engineering calculations on turbulent flows is "no bottom stress".

Nomitsu (1933)worked out the theory for this model and found that while for $^{d}/D$ large it produced a current system similar to Ekman's, for small $^{d}/D$, the solutions differed radically. In particular the current speed, and the angle of flow between the wind and the current increased for decreasing $^{d}/D$.

He therefore proposed a model in which the stress at the bottom was defined in terms of the bottom current velocity. This is an intermediate condition between the other two, and Nomitsu considered that it came closer to the reality of the situation.

He worked out the steady state solutions both a linear and a quadratic friction law, and the transient (i.e. time-dependent) solution for the linear case.

The time-dependent solutions also show important differences from the "no bottom current" model.

There are two numerical models which are specifically designed to gain a better understanding of wind-driven currents in shallow seas. These are those due to N.S. Heaps (1971 and 1972), and F.C. Ronday (1972), and both employ a linear bottom friction law.

The Eddy Viscosity, N

One of the main problems of producing realistic models is the uncertainty concerning the basic physical processes.

The eddy viscosity N parameterises the turbulent exchanges of momentum that occur in the ocean by supposing that the Reynolds stress

$\tau_R = \overline{\rho\, u'\, w'}$ can be replaced by the Newtonian type stress formula:

$$\tau_R = \rho\, N \frac{du}{dz}$$

Leaving aside the question of whether the parameterisation is realistic, there remains the difficulty of deciding upon the correct values of N to be used in given situations. Certainly, the assumption of N constant, or N a function of d (the local sea depth) only, is an oversimplification.

Neumann (1952) shows values of eddy viscosity which vary very strongly with wind speed, and various empirical formulae give $N \propto W^2$, or even W^3.

These formulae are consistent with observations which suggest that the drift current is proportional to the wind speed. Although such results may be of value for the uppermost layers of the open ocean, it is not certain that they can be applied to a shallow, enclosed area like the North Sea.

If the North Sea can be considered a region where tidal currents predominate, then the local value of N is determined by the value of this current and the depth, since these control the vigour and scale of mechanically induced turbulence.

Using dimensional arguments Bowden (1953) suggested,

$$N = K\,u\,d,$$

where $K \simeq 2.5 \times 10^{-3}$

u is the amplitude of the tidal stream, and

d is the depth.

Typical values for an open North Sea location might be

$$u = 0.25 \text{ m/sec}$$

and $d = 80\text{m}$

giving $N = 2.5 \times 10^{-3} \times 0.25 \times 80 = 5.0 \times 10^{-2}\ \text{m}^2\ \text{sec}^{-1}$

In Heaps' model (which uses this type of formulation), N is computed for each mesh but is treated as a constant with respect to time and depth within each mesh.

The strong dependence of N upon windspeed quoted above can be attributed to a combination of mechanical stirring and also to the destabilization of the water as the upper layers are cooled.

These effects will modify the simple picture outlined above, especially in the deeper more northerly parts of the North Sea, and we must expect N to be an increasing function of windspeed.

This can be interpreted in the following way: as the wind speed increases, progressively more of the additional energy fed into the system at the sea-surface is used to increase the turbulent part of the flow, and progressively less to increase the mean flow.

The Drag Coefficient C_D

From dimensional arguments the wind-stress on the sea-surface is usually written

$$\overline{\tau} = C_D\,\rho\,\overline{W}\,|W|$$

where ρ is the air density and C_D is a dimensionless proportionality factor called the drag coefficient.

C_D is itself a rather poorly known function of windspeed, atmospheric stability and the height at which the windspeed is measured.

The value of the drag coefficient when W is measured at the standard height of 10m is called C_{10} and is generally taken as 1.3×10^{-3} for light winds and neutral stability. C_{10} increases for increasing windspeed and decreasing stability, and for the strong wind cases that are of engineering interest it is usually taken as 2.5×10^{-3}.

Numerical Example

(1) Ekman constant N The surface current for $^d/D$ very large is given by

$$V_o = \frac{\tau}{\rho_w N a \sqrt{2}} = W^2 \frac{\rho_a \quad C_D}{\rho_w \quad \sqrt{Nf}}$$

i.e. $$V_o = W^2 \times B$$

where $$B = \frac{\rho_a}{\rho_w} \cdot \frac{C_D}{\sqrt{Nf}}$$

$$\simeq 1.25 \times 10^{-3} \ [\text{sec/m}] \quad \text{for our example.}$$

For a 100 knot wind this gives a current of

$$V_o = (50)^2 \times 1.25 \times 10^{-3} \text{ m/sec}$$

$$= 2.87 \text{ m/sec}$$

i.e. 5.6 knots

which is probably too large.

(2) Thorade - Nomitsu

Assume $N \propto W^2$, and use Thorade's (1914) relation:

$$N = .43\, W^2 \times 10^{-3} \quad [\,\text{m}^2/\text{sec}\,]$$

For our 100 knot wind, $N = 1.075 \text{ m}^2/\text{sec}$

$$D = \pi \sqrt{\frac{2N}{f}} \simeq 400\text{m}, \qquad \text{and}$$

$^d/_D \simeq 0.25$ for a northern North Sea position.

Using Nomitsu's results with a linear bottom friction law we get a surface current of

$$V_o = 1.2 \text{ knots}$$

which is definitely too small.

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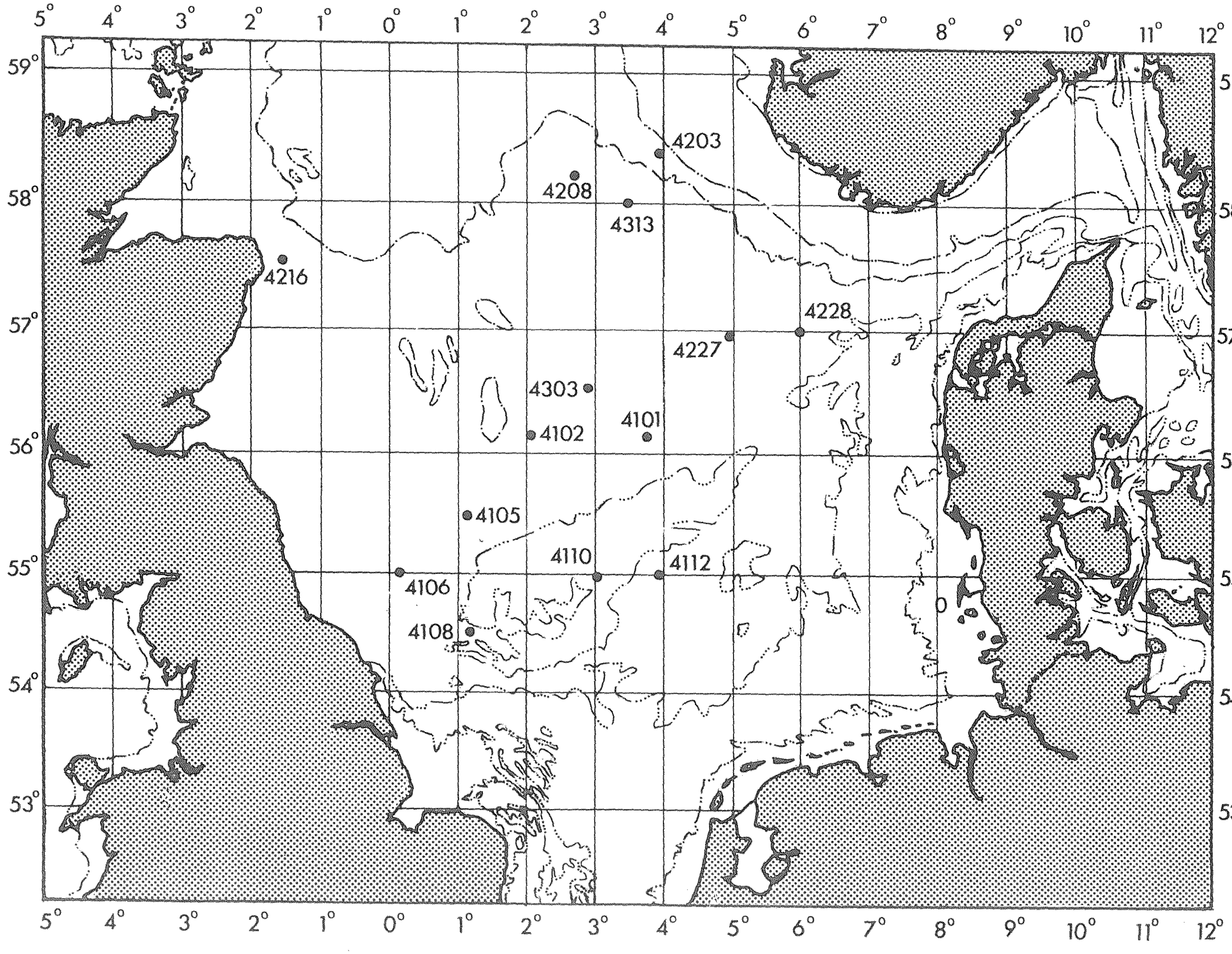
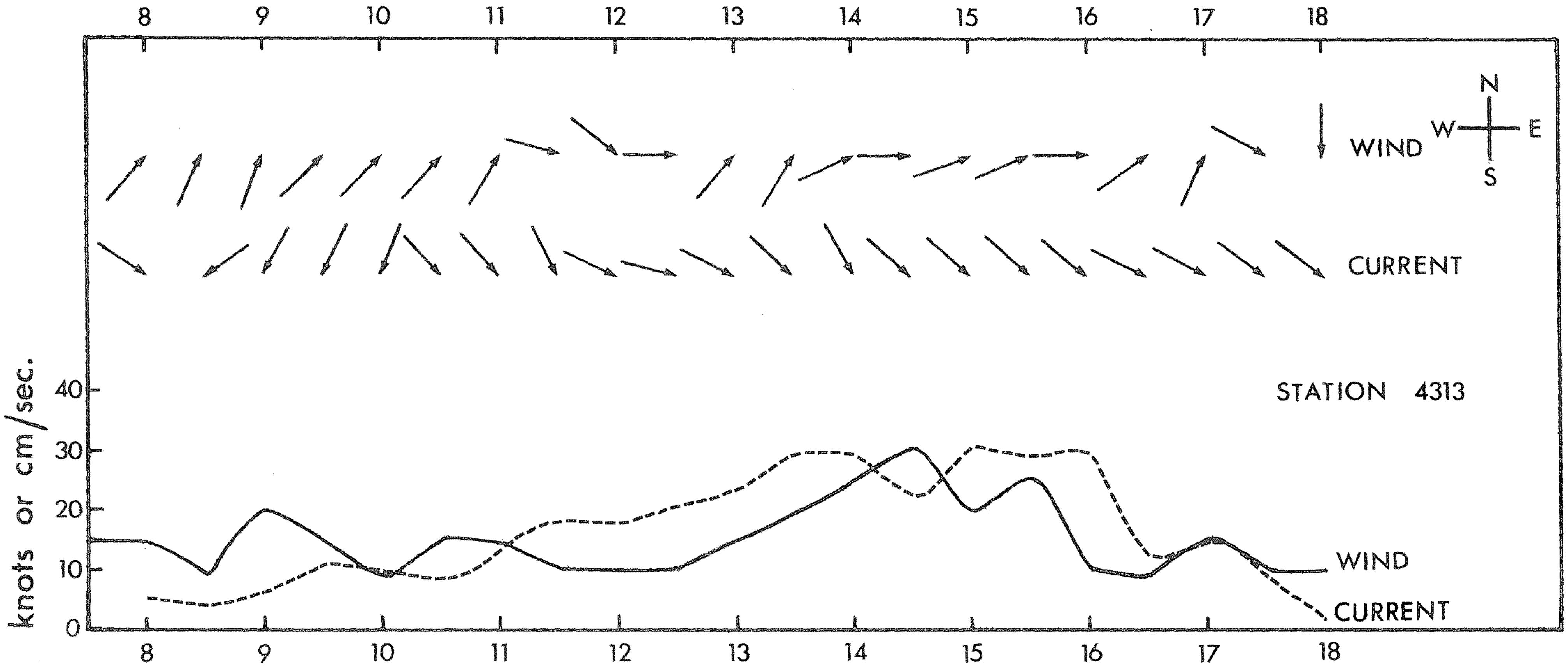
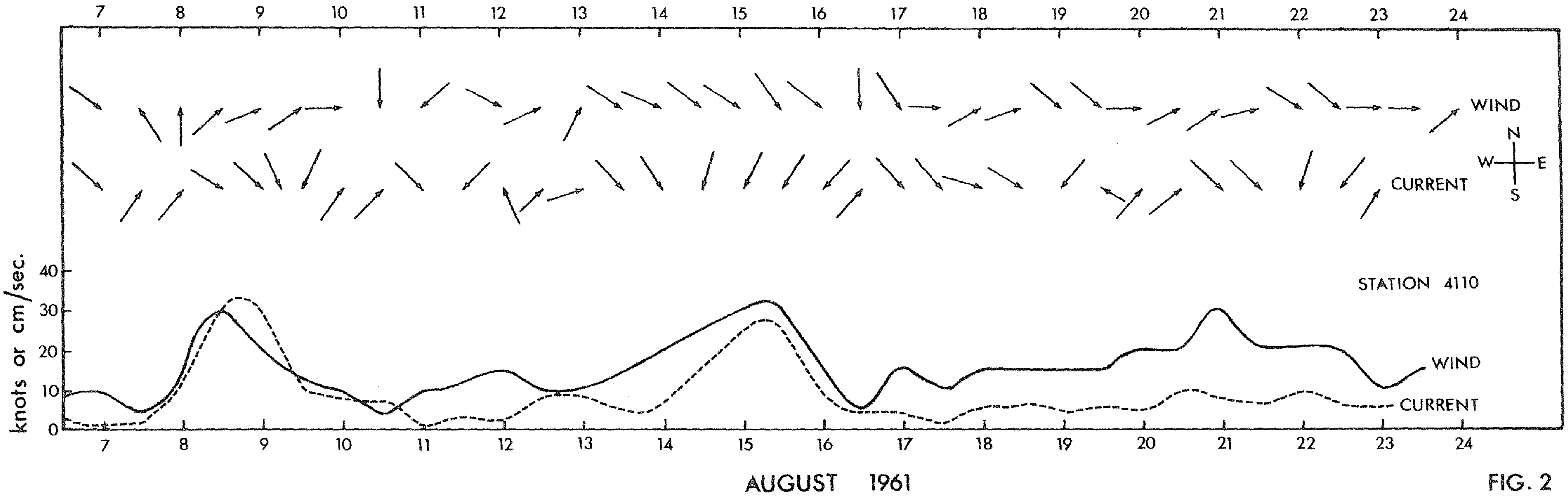


STATION POSITION MAP



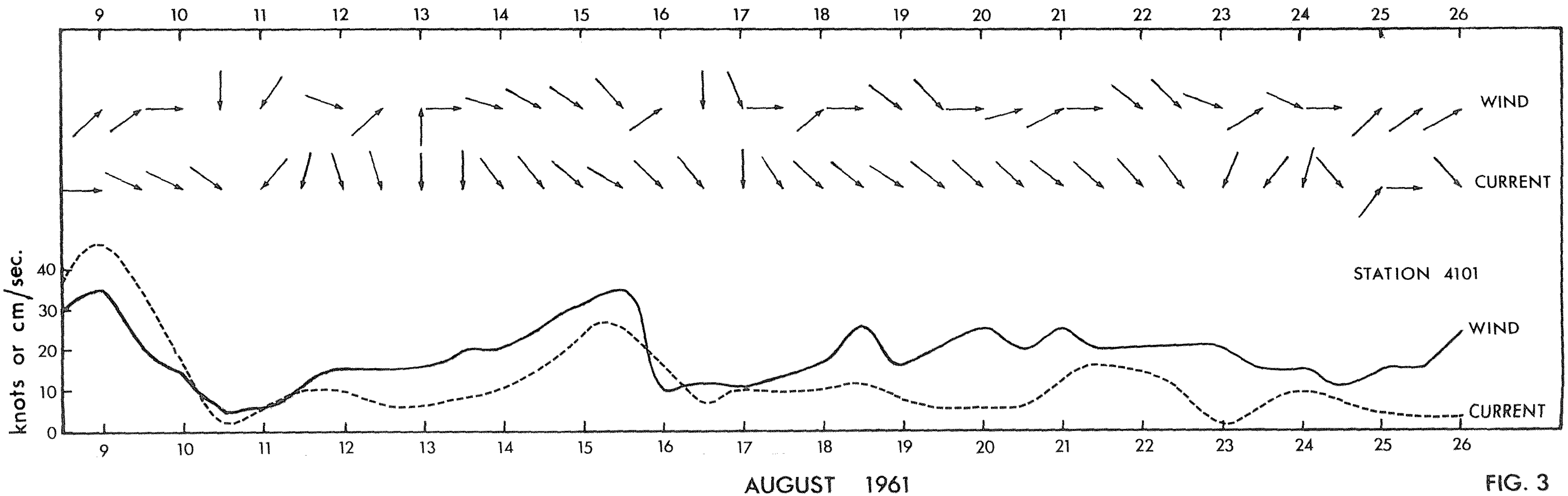
SEPTEMBER 1963

FIG. 1



WIND
CURRENT
N
W
E
S
STATION 4110
knots or cm/sec.
AUGUST 1961

FIG. 2



WIND
CURRENT
STATION 4101
knots or cm/sec.
AUGUST 1961

FIG. 3

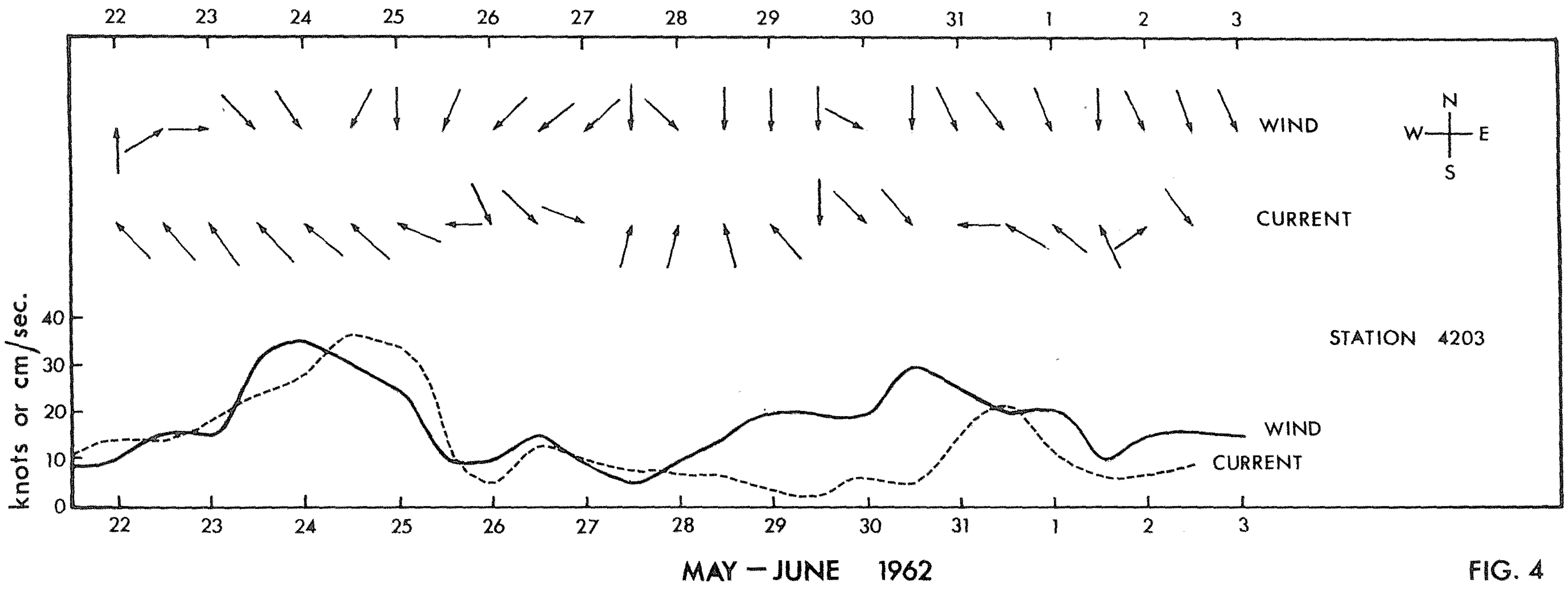

22 23 24 25 26 27 28 29 30 31 1 2 3
WIND
N
W
E
S
CURRENT
STATION 4203
knots or cm/sec.
40
30
20
10
0
WIND
CURRENT
22 23 24 25 26 27 28 29 30 31 1 2 3
MAY — JUNE 1962

FIG. 4

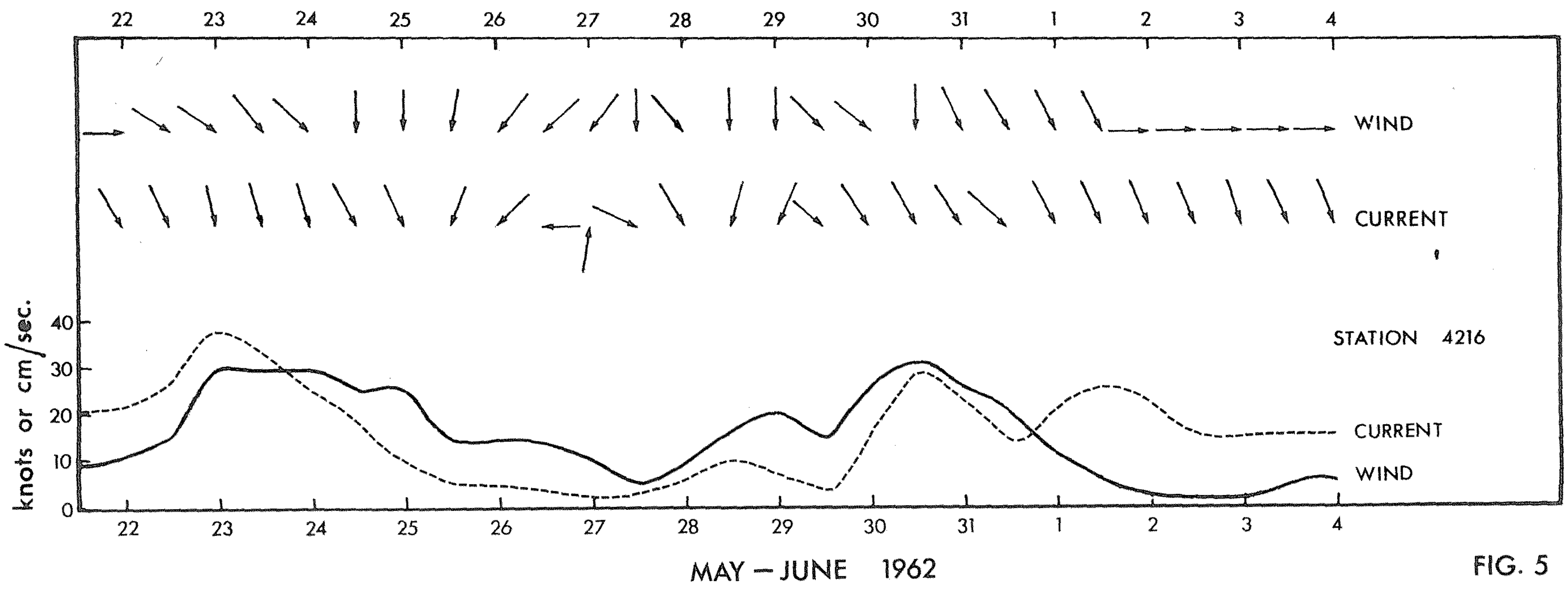
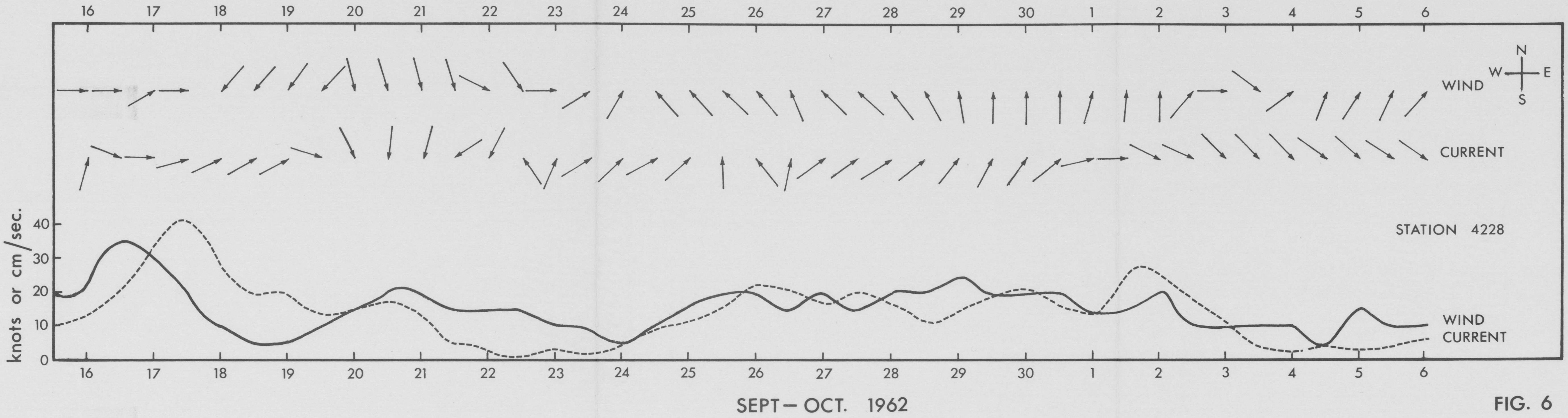

22 23 24 25 26 27 28 29 30 31 1 2 3 4
WIND
CURRENT
STATION 4216
knots or cm/sec.
40
30
20
10
0
CURRENT
WIND
22 23 24 25 26 27 28 29 30 31 1 2 3 4
MAY — JUNE 1962

FIG. 5



16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 1 2 3 4 5 6
N
W
E
S
WIND
CURRENT
STATION 4228
knots or cm/sec.
40
30
20
10
0
WIND
CURRENT
16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 1 2 3 4 5 6
SEPT – OCT. 1962

FIG. 6