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The use of Synthetic Aperture Radar
(SAR) to measure waves

by

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CONTENTS

SUMMARY

This document presents two short notes on the way the aperture synthesising process combines with wave particle motions to smear the picture in the along-track (azimuth) direction. Quantitative assessments are made using the Dornier feasibility study proposals for the European Resources Satellite ERS-1. The first process is the well-known sideways shift of the image of range-travelling objects. It is shown that locally-generated waves will be grossly distorted in the azimuth direction, but that low swell by itself may be clearly seen. The second effect is not well-known. It is the distortion of the pixel surface by waves shorter than the resolving power of the synthetic aperture radar (SAR). An approximate calculation shows that this can produce major azimuthal smearing.

REFERENCE

ALPERS W R, ROSS D B and RUFENACH C L. "On the detectability of ocean surface waves by real and synthetic aperature radar". Journal of Geophysical Research (in press).

## 1. INTRODUCTION

In connection with the proposal to put a synthetic aperture radar for measuring waves on the European Resources Satellite ERS-1, the author at the invitation of ESA, recently attended a presentation by Dornier of their feasibility study followed by a meeting of the ESA Scatterometer Expert Group.

The present document has two purposes. Firstly, to put on record an assessment of the smearing effect of range-travelling wave particle velocities which the author gave at that meeting. The author now has access to more appropriate wave data so that the numbers are not quite the same, and a short section on swell has been added. This part of the note contains nothing new in principle and the presentation was only intended to illustrate simply why the author is sceptical of the ability of a satellite-borne SAR to give useful information about waves. For a full treatment the reader is referred to a paper by Alpers et al (in press).

This theory assumes in effect that the pixel surface retains a constant shape and attitude during the period of aperture synthesis. This is patently untrue, and at the meeting concern was expressed at the effect of the distortions on the aperture synthesis process. On the way home the author thought of a way of assessing this effect, and this is the second part of the document. At present it is only an outline theory, but even so allows an approximate assessment to be made of the magnitude of the effect. It is shown to be important. The author is not sufficiently familiar with the literature to know whether this is a novel approach or not.

Perhaps, however, the very first point to make is that the range resolution is accomplished by a pulse compression technique which operates so fast that surface movements are negligible. Thus, any smearing which takes place is entirely in the azimuth direction, and is due to the significant time of flight required to gather the information for the aperture synthesis process.

## 2. THE EFFECT OF PARTICLE VELOCITIES IN THE LONGER-WAVELENGTH WAVES (THOSE RESOLVED BY THE SAR)

### 2.1 Azimuth offsets due to a constant range-component of velocity

This section is for those not already familiar with the effect. It explains its origin and derives its magnitude by simple geometrical arguments.

The aperture synthesis process can be regarded as follows (see figure 1). As the target comes into the beam of the satellite the range first shortens and then lengthens again. The phase of the echo relative to the transmission is proportional to this range. The synthesis process correlates the received signal with this expected phase pattern and gives maximum output when an exact match is achieved. Thus, what the process examines is the range history of a target.

In the case of a target moving in the range direction the range history is shown in figure 1 (c), where the fixed target with the same range history is shown. Remembering that $\theta$ is small, figure 1 (d) gives the offset $x$ as

$$x = \frac{v \, . \, TOT}{\theta}$$

$$\simeq 91 \, v \qquad 1$$

Note that $\theta / TOT$ is the angular velocity of the satellite as seen by the target and the position of the equivalent fixed target does not change if only part of the beamwidth is used for the synthesis process (as is the case with the Dornier proposal, which is a 9-look system).

## 2.2 Some typical wave conditions

2.2.1 Using wave data measures at Ocean Weather Station INDIA (held by MIAS) a fairly typical severe storm (equalled or exceeded perhaps 5 times a year) would have a significant waveheight (Hs) of 14 m and a zero crossing period (Tz) of 12.5 s.

Taking a wave of this height and period from a regular wave train would give (in deep water)

$\lambda$ = Wavelength = $1.56 \, T^2$ = 244 m

$v_o$ = Orbital velocity = $\pi$ H/T = 3.5 m/s

For range-travelling waves the maximum range-velocity = the orbital velocity.

Remembering that the proposed angle of incidence of the radar beam is $25^o$, for azimuthal travelling wave the maximum range component of velocity is

$$v_o \cos 25^o = .91 \, v_o \qquad 2$$

The range component is thus only slightly dependent on the direction of travel of the waves.

For $v = 3.5$ m/s, equation 1 gives the azimuthal offset as 318 m, or 1.3 wavelengths.

Since Hs = 4 x rms waveheight, an equivalent computation of rms offset would give it as 159 m.

Such offsets would clearly grossly distort azimuthally travelling spectral components, and in particular the shorter wavelength components, over a wide range of directions.

2.2.2 Consider a locally-generated wave system whose dominant wavelength is just within the nominal resolution of the SAR system. The spatial resolution quoted is approximately 30 m with a cut-off wavelength of about 60 m. This gives a zero-crossing period of 6.3 s and a fully-arisen sea gives the following characteristics (using the same logic as in 2.2.1 above).

$$\lambda = 60 \text{ m}$$

$$H_s \simeq 3 \text{ m}$$

$$v_o \simeq 1.5 \text{ m/s}$$

Azimuthal offset for wave of height Hs $\simeq$ 136 m

rms azimuthal offset $\simeq$ 68 m

2.2.3 Considering an azimuthal travelling swell of 12.5 s period, it will give maximum visibility when the offset due to the maximum range-component of orbital velocity is $\lambda/4$ or 61 m. For this

$$v_o \cos 25^\circ = \frac{61}{91} \text{ m/s} = 0.67 \text{ m/s}$$

or $$v_o = .74 \text{ m s}$$

This corresponds to a waveheight $H = \frac{T v_o}{\pi} \simeq 2.9$ m

Such a swell will have its visibility enhanced relative to a range travelling swell by a factor of the order of 4 (see Alpers et al, loc cit).

The offsets due to such a swell will, of course, grossly distort shorter wavelength components of the spectrum.

## 3. EFFECTS OF TILTING AND BENDING OF THE PIXEL SURFACE

### 3.1 The basic theory

One conventional way of looking at the image amplitude of a pixel is to consider it as the vector sum (phase and amplitude) of all the component backscatterers, which in the present application might be thought of as facets or discontinuities in the water surface. The vector sum can then be regarded as equivalent to a single target giving the same amplitude. In effect, the arguments in section 2 use this concept and assume that the equivalent target is carried about by the particle velocities of the waves whose wavelength is long enough to be resolved by the SAR.

Since we are looking at a narrow band of radar wavelengths it is possible to sum the complex Fourier transforms of each target, or if linear superposition can be assumed, we can Fourier transform the water surface elevation. A little thought will soon show that linear superposition in the latter sense cannot be assumed for the present application, however, and for the present purpose the concept of summation of a number of individual targets is convenient (we will call them micro-targets).

The process of aperture synthesis is linear, and thus it is possible to consider it as processing each of the component micro-targets of the pixel individually and summing the results in the system output. This allows us to examine the effect of the fine structure of wave particle velocities on a statistical basis.

For each micro-target its azimuthal offset is given by equation 1, where $v$ is the range component of its velocity.

$$\therefore \quad \overline{x^2} = (91)^2\ \overline{v^2} \qquad (3)$$

Assuming that linear superposition can be assumed for the water waves, the water surface elevation can be described as usual by the sum of a large number of sinusoidal components, the amplitude (crest to mean water level) of the nth being $a_n$. The particle motion due to the nth component is a circular orbit which has a circumference of $2\pi a_n$ and thus a velocity $v_n = 2\pi a_n / T_n = \omega_n a_n$. The component of this in a direction which is in the plane of the orbit is $v_n \cos(\omega_n t + \delta_n)$, with a mean square value of $\frac{1}{2} v_n^2 = \frac{1}{2} \omega_n^2 a_n^2$

For range-travelling waves the range component of velocity is in the plane of the water particle orbits and these results apply. As the direction of wave travel rotates towards the azimuth the range component is reduced until at the azimuth the factor of reduction is $\cos \phi = \cos 25^{\circ} = 0.91$

where $\phi$ is the angle of incidence of the radar beam. For the present approximate evaluation we shall neglect this reduction, allowing us to reach the simple equation (assuming random phases).

$$\text{mean sq. range velocity} \simeq \Sigma \tfrac{1}{2} v_n^2 = \Sigma \tfrac{1}{2} \omega_n^2 a_n^2$$

$$\longrightarrow \int \omega^2 S(\omega)\, d\omega \qquad 4$$

where $S(\omega)$ is the spectral density of wave amplitude.

The integration can be taken over the frequency range appropriate to the problem. If taken over the whole of the spectrum it gives the overall smearing, but for those components resolved by the SAR the movement can be coherent and result in enhanced visibility of the waves, as discussed in section 2 above. However, for those components with wavelengths below the resolution of the SAR there can be no coherent imaging and the result is a simple smearing effect. It is this effect with which we are concerned in this section.

3.2 Evaluation of the magnitude of the effect

From equations 3 and 4 we wish to evaluate

$$\overline{x^2} = 91^2 \int_{\Omega}^{\infty} \omega^2 S(\omega)\, d\omega$$

where $\Omega$ is the frequency corresponding to the wavelength cut-off of the SAR. From section 2.2.2, $\Omega \simeq 1$ in the case of the Dornier proposed system.

The Pierson-Moskowitz spectrum for a fully arisen sea is

$$S(\omega) = \frac{A}{\omega^5} \exp\left(-B\omega^{-4}\right) \qquad 5$$

where (in SI units) $A = 0.78$

$B = 6.9 \times 10^3 \; U^{-4}$

$U =$ wind speed at 19.5 m

$$\therefore \quad \overline{x^2} = 91^2 \int_{\Omega}^{\infty} \frac{A}{\omega^3} \exp\left(-B\omega^{-4}\right) d\omega$$

Put $p = B^{1/2} \omega^{-2}$

Then

$$\overline{x^2} = 91^2 \frac{A}{2B^{1/2}} \int_0^P \exp\left(-p^2\right) dp \qquad 6$$

where $P = B^{1/2} \Omega^{-2}$

$= 83 \quad U^{-2}$

Now $$\int_0^P \exp(-p^2)\,dp = \frac{\sqrt{\pi}}{2} \operatorname{erf}(P)$$

erf(P) is tabulated in standard works.

Putting all the numbers into equation 6 gives

$$\overline{x^2} \text{ (in } m^2\text{)} = 34.45 \quad U^2 \operatorname{erf}(83\ U^{-2}) \qquad 7$$

This gives the following results

| $U$ m/s | erf(83 $U^{-2}$) | $\overline{x^2}$ ($m^2$) | rms $x$ (m) |
|---|---|---|---|
| 5 | 1.000 | 861 | 29.3 |
| 10 | .760 | 2618 | 51.2 |
| 15 | .398 | 3084 | 55.5 |
| 20 | .231 | 3183 | 56.4 |
| 25 | .149 | 3208 | 56.6 |

4. DISCUSSION AND CONCLUSIONS

The effects discussed above both result from the azimuthal offset due to range velocity of the target. There are other effects causing smear, for example the defocussing due to azimuthal velocity, but as far as the present author can see these are all small by comparison.

Assuming this to be the case and the theory in this note to be correct, then we can now draw some reasonably clear conclusions about the ability of SAR to see waves. These conclusions relate specifically to the Dornier proposals, but the relevant characteristics of SEASAT and SAR 580 are such that they also apply qualitatively to these instruments. The conclusions are as follows.

(1) There is an azimuthal smearing due to waves with wavelengths below the resolution limit which grows rapidly with wind speed until it reaches a saturation limit of approximately 50 m rms for winds above about 10 m/s.

(2) For waves generated by local winds and with dominant wavelengths longer than the nominal resolution, the rms azimuthal offsets are comparable to the wavelength, so that any wave pattern which may remain in the azimuthal direction bears little relation to the actual wave pattern

on the sea surface. In fact, with locally-generated waves travelling in the azimuthal direction it is unlikely that any recognisable wave pattern will remain.

(3) For low swell up to a certain critical height (this critical height depending on wavelength), azimuthally travelling waves will actually be enhanced relative to the range-travelling component by a factor of the order of 4 or 5, but only if the local winds are in a limited range and can ruffle the sea surface without generating waves high enough to smear-out the swell.

(4) In terms of the directional spectrum of the waves, these effects change the amplitude of those waves with components of wavenumber in the azimuthal direction. The influence increases as this component wavenumber gets higher. Except for very low swell in the presence of light local winds these effects are, however, drastically non-linear and the author cannot see how the effects can be inverted to recover the true spectrum.

(5) Thus, except in a very limited range of conditions, wave patterns will either be smeared out, or so distorted that the patterns seen cannot be interpreted with any confidence.

FIGURE 1

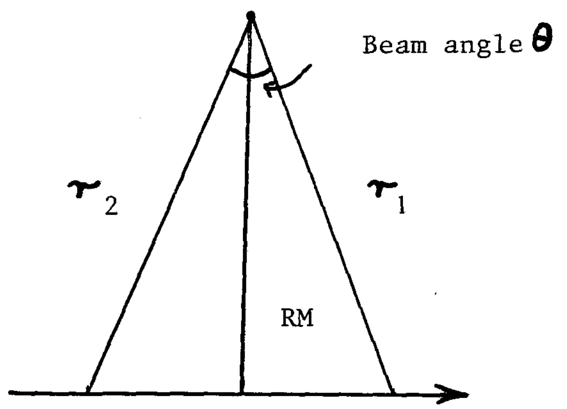


Figure 1 (a). Fixed target seen in the plane containing the target and the flight path

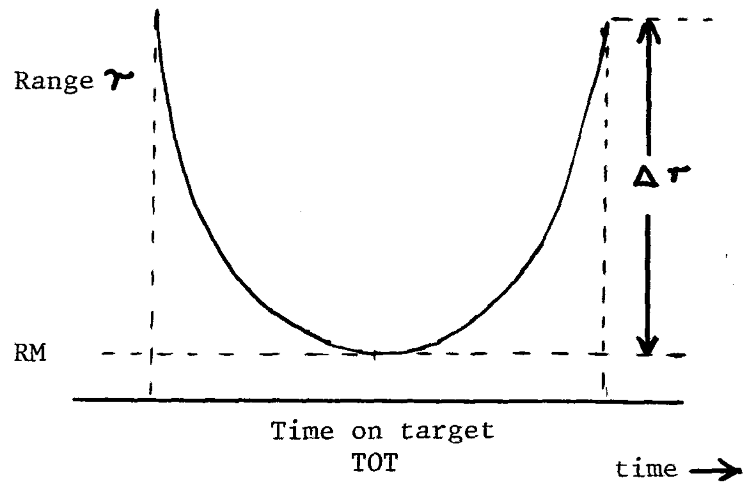


Fig 1 (b). Fixed target range history

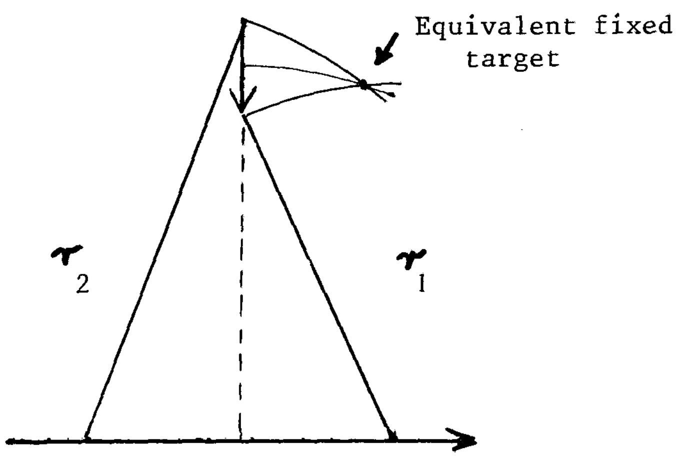


Figure 1 (c). Range history of a target moving in the range direction

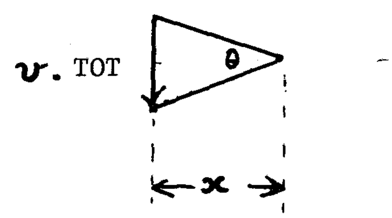


Figure 1 (d). Approximate offset geometry

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For the Dornier proposed system

$\theta = 0.48^{\circ} = 0.0084$ radian

RM $= 688$ km

TOT $= 762$ ms

$\Delta r = 6$ m

Radar wavelength $\simeq$ 6 cm