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Report on Shape Analysis and Matching and on Semantic Matching

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This state of the art report describes the techniques of shape analysis, and of metadata search that have been already implemented in cultural heritage or we think are useful for the GRAVITATE project. These fields are relatively disjoint, and the research and development challenge of GRAVITATE is precisely to merge them. After the review of the current literature on these fields, we end the report with common remarks on possible or plausible cross-connections that suggest themselves. These considerations will be refined for the Roadmap for Research deliverable.

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1. Executive Summary

In GRAVITATE, two disparate specialities will come together in one working platform for the archaeologist: the fields of shape analysis, and of metadata search. These fields are relatively disjoint at the moment, and the research and development challenge of GRAVITATE is precisely to merge them for our chosen tasks. As shown in chapter 7 the small amount of literature that already attempts join 3D geometry and semantics is not related to the cultural heritage domain. Therefore, after the project is done, there should be a clear 'before-GRAVITATE' and 'after-GRAVITATE' split in how these two aspects of a cultural heritage artefact are treated.

This state of the art report (SOTA) is 'before-GRAVITATE'. Shape analysis and metadata description are described separately, as currently in the literature and we end the report with common recommendations in chapter 8 on possible or plausible cross-connections that suggest themselves. These considerations will be refined for the Roadmap for Research deliverable.

Within the project, a jargon is developing in which 'geometry' stands for the physical properties of an artefact (not only its shape, but also its colour and material) and 'metadata' is used as a general shorthand for the semantic description of the provenance, location, ownership, classification, use etc. of the artefact. As we proceed in the project, we will find a need to refine those broad divisions, and find intermediate classes (such as a semantic description of certain colour patterns), but for now the terminology is convenient – not least because it highlights the interesting area where both aspects meet.

On the 'geometry' side, the GRAVITATE partners are UVA, Technion, CNR/IMATI; on the metadata side, IT Innovation, British Museum and Cyprus Institute; the latter two of course also playing the role of internal users, and representatives of the Cultural Heritage (CH) data and target user's group. CNR/IMATI's experience in shape analysis and similarity will be an important bridge between the two worlds for geometry and metadata. The authorship and styles of this SOTA reflect these specialisms: the first part (chapters 3 and 4) purely by the geometry partners (mostly IMATI and UVA), the second part (chapters 5 and 6) by the metadata partners, especially IT Innovation while the joint overview on 3D geometry and semantics is mainly by IT Innovation and IMATI. The common section on Perspectives was written with the contribution of all.

2. Introduction

A critical activity that CH researchers, anthropologists, and museum curators perform is to carry out comparisons of artifacts within and across collections and institutions. Comparative analysis is a core step in the methodology for classifying and curating artworks; it also enables historians to track changes in techniques, tools, or materials that were associated with the production, decoration, or use of an artifact. What is crucial is to provide archaeological researchers, conservators and curators with the mechanisms to compare objects and fragments; identify patterns and textures; examine material properties, filter noise, and degradation effects on the remains; and take advantage of all data and knowledge acquired to appropriately cluster similar parts.

The GRAVITATE project aims at assisting the expert archaeologist to analyse digitised objects distributed in several collections. Those digital artefacts are typically 3D scans in the form of meshes, of the geometrical object and some of its visual properties (such as colour, decorations, etc.), augmented with structural annotations in one of the standard cultural heritage metadata formats.

Within GRAVITATE, cultural artefacts are typically artefacts, complete or fragmented (rather than entire buildings or excavation sites). Archaeological artefacts are often broken, eroded, worn, or incomplete; their quantity is extremely vast (a dozen fragments of a broken artefact to be reassembled, to hundreds of fragments of a collapsed temple to be reunified across collections, to thousands of artefacts from a similar period to be re-associated), distributed, and fragmented; and there is an intrinsic uncertainty of what data represent and which descriptions are most meaningful. The nature of CH data calls for methods dealing with heterogeneous information in combination (e.g., geometry, texture, reflectance, semantics), which is necessary to effectively group artefacts or their parts into meaningful clusters.

2.1. The GRAVITATE problem statement

The *re-assembly* of fragments has as its purpose to find which of them fit to make a larger subassembly, or even a complete object. Such reassembly is typically attempted on a set of fragments for which there is strong suspicion that they may indeed be capable of being re-assembled across a fracture. Major issues are the combinatorics (if the set of fragments is too large); abrasion of the originally matching facets of a fragment; and the determination of the best way to fit them (both individually and among the set). Re-assembly is based on geometrical clues of the fracture surface, in combination with properties of the 'skin' of the object (similarity in chemical/physical properties, archaeological typology). When many pieces are missing, the re-assembly in the strict sense would become impossible but could be implemented as a virtual reimagining. In this case it may still be based on similar curvature, fitness in space with other fragments so as to complete other fragments in a coherent way, according to some prototype/complete object.

The *re-unification* intends to bring together dispersed fragments (e.g. the head and the torso of a statue); often this dispersion may be across collections. The result can be a potential set of fragments to be re-assembled; but even when re-assembly would be impossible (due to missing parts), it may still be possible to suggest that fragments belong to the same object (for instance, a shoulder and a hand) or propose an approximate position of each fragment along the hypothesised whole. Stylistic shape similarity combined with metadata on material and provenance, for instance, could suffice; or there may be class of objects with a standardized form, which can be parametrized in a manner to allow partial shape matching to an exemplar.

The *re-association* of objects refers to the grouping of fragments across multiple collections and it aims at finding different objects belonging to different collections with one or more features in common (e.g. the school, the age, the pattern). For instance, artefacts produced by the same artist or workshop, or intended for the decoration of a single palace. Here, the combination of geometrically and physically characterizable aspects of style with metadata and semantic descriptions would become a crucial point.

Together, we may refer to these as the Re-X tasks and they provide GRAVITATE with focus to the different roles shape and semantic matching technology can play.

2.2. Glossary of terms

We provide a brief glossary, organized roughly by terms in shape analysis and semantic matching. This terminology allows a reader to understand exactly what each term used in a GRAVITATE document means, and how it should be interpreted in the context of the cross-disciplinary communities GRAVITATE technology touches upon.

2.2.1. Terms related to shape

Geometry: The spatial extension of an artefact. When not used in apposition, 'geometry' is virtually synonymous with 'shape' as geometrical appearance.

Shape: The physical properties of the artefact, including geometry, its visual attributes (colorimetric properties, luminance, etc). Shape is a property of both a set of objects and a particular method of observation, or analysis (Biasotti, Falcidieno, et al. 2014).

Skin: The outer, visible layer of an artefact. Characteristic properties are its colour, colour patterns, a local geometric pattern like forms or texture.

Facet: A part of the object skin that is a natural descriptive unit for functional matching and mating purposes. Examples of geometrical facets are the inside and outside pieces of the skin of a potsherd, and the facet of the fracture surface.

Shape analysis: A set of theories, methods and algorithms that concur to the formalization and computation of properties useful to characterize the shape appearance of objects (Biasotti, Falcidieno, et al. 2014).

Shape description: A concise representation that contains enough information to identify a shape as a member of a class.

Shape retrieval: Refers to the task of finding the models in a database that best match a given query model (Tangelder & Veltkamp 2004; Tangelder & Veltkamp 2008).

(Dis)Similarity: A measure of similarity between objects. Many such measures exist. The challenge is to find the measure or combination of measures that approximate the similarity measure relevant for a specific CH application.

Geometric matching: Matching 3D objects based on geometric features.

Shape matching: Recognition of partial objects whose overall shape is similar by some measure. This measure could be geometric in nature or based on the skin or a combination of them.

Partial shape matching: Recognition of similar sub-parts in objects having different overall shape;

Sub-part correspondence: Recognition of similar sub-parts in objects that are both structurally and globally similar;

Part-in-whole correspondence (matching): Recognition of a whole object as a sub-part of another;

Complementary matching: Matching non-overlapping parts of an object usually close to the place where the parts meet. We will often call this 'shape mating'.

Shape mating: The fitting together of two digitized objects as partial reassembly of the whole object they once formed, based solely on their geometric shape (as opposed to also using skin colour, texture, etc.) It is a major aspect of the re-assembly task.

Shape Completion: Refers to the process of filling any missing part from a digitized dataset, see also (Pavlos Mavridis et al. 2015).

Registration: Given a pair of partially overlapping scans of an object the goal is to recover the relative transformation between the parts.

2.2.2. Terms related to semantics

Semantics: Meaning (grounding of labels to explicit examples in a domain)

Conceptual item: Thing, ideally semantically grounded in the real-world (e.g. Artefact). In CIDOC-CRM an entity is a conceptual item.

Vocabulary: Terms used to label conceptual terms organized in a list, thesaurus or ontology. Vocabulary includes labels for conceptual items and relationships.

Relationship: Property associated with a conceptual item with a literal or concept value (e.g. Artefact hasOwner <person>).

Semantic description: Description of the semantics of a conceptual item typically encoded as a graph (e.g. a RDF graph). Semantic descriptions usually encompass the set of attributes and relationships that when taken together can semantically ground a conceptual item in the context of other conceptual items.

Query: Data query to a database or dataset containing semantic information. Typically a query would follow a standard such as SQL, SPARQL or XQuery.

Inference: Reasoning over data through rules. Types of inference include inferring new concept classes from existing classes and properties (e.g. owl:Restriction), and inferring new concepts and relationships (e.g. SPARQL CONSTRUCT, SPIN rules). Inference can be local or federated across multiple databases or datasets.

Linked data: Semantic description(s) referenceable via a publically HTTP resolvable URI.

Semantic Web: Web of linked data.

Semantic Matching: Matching conceptual items based on similarity of their semantic descriptions.

Geometric semantic description: Semantic description of a geometric item (e.g. body morphology or partonomy labels).

Geosemantics: The conceptual modelling and formal representation of geospatial or geometric information. Examples include contextual descriptions of geometric items (e.g. related parts in a 3D model) and semantic descriptions containing combinations of conceptual items and geometric items (e.g. a semantic description linking the imaging methodology used to make a 3D model with the body morphology parts scanned).

Geosemantic matching: Matching conceptual items based on a contextual descriptions around target conceptual items and/or geometric items.

2.2.3. GRAVITATE terms

GRAVITATE: The name of our project, somehow an acronym of Geometric Reconstruction and Novel Semantic Reunification of Cultural Heritage Objects.

Dashboard: The graphical user interface to the GRAVITATE functionalities, allowing display of artifact images, meshes, and metadata, and their Re-X combinations.

Re-X tasks: The tasks of ReAssembly, ReUnification and ReAssociation, defined in Section 2.1.

ReAssembly: Putting fragments together, a specific GRAVITATE task (see definition in Section 2.1).

ReUnification: Finding fragments across collections, a specific GRAVITATE task (see definition in Section 2.1).

ReAssociation: Re-establishing a long-lost relationship among artifacts, a specific GRAVITATE task (see definition in Section 2.1).

User: An archaeology expert (researcher, conservator or curator) interested in performing a CH task facilitated by GRAVITATE.

3. Descriptors and Similarity

The 3D scanned artefacts in GRAVITATE are available as raw data (scan data) and point clouds. Most of these models are also available as 3D meshes. The computational representation of these shapes has been mainly developed for visualization purposes and virtual reality tasks; pictures of the artefacts are available as separate .jpeg or raw files. No descriptions and coding of the shape properties (2D and 3D patterns, colorimetric histograms, etc.) is currently available. Indeed, shape descriptions aim to find concise yet informative signatures of shape models that are known as *shape descriptors*. These signatures are machine-understandable indexes to the informative content of 3D models. The definition of shape descriptors and the type of attributes (i.e., shape properties) they encode, along with proper comparison strategies, is the basis toward the definition of effective similarity measures and a proper classification of artefacts. In the large literature on shape descriptors, in this chapter we mainly overview the methods that have been adopted in the CH domain plus some descriptions that, even if not yet applied in the CH domain, we think are relevant for the Re-X tasks. In particular, we point out that the best shape descriptor does not exist, each one giving insights on different shape properties; the combination of descriptors, similarity measures, semantics plus the user interaction will be the key elements of the GRAVITATE tasks.

The chapter is organized as follows. The shape descriptors can be local in nature (feature points and their descriptors), region-based or even whole object descriptors (section 3.1). These features can then be used as inputs for similarity measures (section 3.2). This is the geometric basis for Re-X tasks. They can be used to find similar objects or objects with similar artistic styles. Since the notion of similarity is in the eye of the beholder and is task specific, the GRAVITATE user should be able to have some control over which of the similarity measures are used for the task and what importance is assigned to each one. This can be done interactively by the user directly or indirectly by analysing the user response to query results. If the user is able to select the relevant and irrelevant objects from the returned set, or cluster similar objects together, the system might be able to modify the similarity measure using relevance feedback or other learning techniques (section 3.2.3).

3.1. Shape analysis and description

While designing a shape descriptor, the first challenge is to identify the shape properties that better characterize the object under study and are highly discriminative for the task at hand. A good shape descriptor should be robust and endowed with adequate invariance properties. Indeed, robustness requires that small changes in the input data, such as noise or non-relevant details, do not result in substantial changes in the associated shape descriptors. In terms of the Re-X tasks this translates in the robustness of the description with respect to damaged objects, corroded surfaces and colour variations). Invariance properties are related to the application domain. For instance, rotations and translations are the parameters to be determined in shape alignment, so descriptors of the shapes should be invariant to translation and rotation. Scale would become relevant for partial matching if applied to the re-association task: for instance, two heads with different size could have the same geometric characteristics and style.

Having a good shape descriptor at hand, the problem of assessing the similarity between two shapes can be recast into the comparison of the associated descriptors (see section 3.2); in case descriptors come in the form of feature points or regions of interest, these can be used, e.g, for complete or partial matching and are generally based on Euclidean distances.

The use of shape descriptors is largely acknowledged in the Computer Vision and Graphics literature and a variety of methods has been proposed so far (Tangelder & Veltkamp 2004; Del Bimbo & Pala 2006; Biasotti et al. 2008; Tangelder & Veltkamp 2008; van Kaick et al. 2011; Wu et al. 2010). During years, the situation has evolved from 3D descriptors heuristically introduced (Bustos et al. 2005), motivated by techniques and practices inherited from vision (projection-based descriptions), geometry (statistics of surface curvature or geodesic distances) or signal processing (object samples in the frequency domains), to more sophisticated and mathematically sound frameworks to detect a salient shape feature yet showing robustness to noise and different groups of transformations.

In the CH context, the field of shape analysis is wide and mostly focused on geometric information, and covers all applications in which the shape of cultural objects, parts of them, or collections of them, carries valuable qualitative and quantitative information that is used as a basis for further analysis (Pintus et al. 2015). In this Section, we overview the shape analysis and description techniques currently adopted in the CH domain with the inclusion of further methods that would be potentially useful for this domain. However, we highlight that benchmarks for the description and comparison of artefacts with the specific characteristics of the GRAVITATE ones (e.g. damaged and broken objects with corroded surfaces and colour variations) are currently missing. Therefore, the discussion on the suitability of the techniques overviewed in this report can be only based on the authors' claims/results or from a theoretical point of view. Still, because of the time restrictions on GRAVITATE, we will have to concentrate on the use of a targeted combination of these known descriptors, rather than the development of new ones.

3.1.1. Purely geometric descriptions

Most of the existing methods for 3D shape analysis and description focus in the geometric information only (Tangelder & Veltkamp 2008; Bustos et al. 2005; Liu et al. 2013; M. Savelonas et al. 2014; Bustos et al. 2007; G K L Tam et al. 2013; van Kaick et al. 2011). For this reason, most of the methods borrowed to CH from these domains discard colour and texture information and only focus on geometric features. In the very large number of descriptors that has been introduced in the literature there is a wide variety of descriptions (histograms, matrices, graphs, etc.) and type of information stored (punctual, normal vectors, surface or volumetric, possibly with attributes). What all these methods have in common is that they extract geometric information, and use that to derive a concise description. In particular, we characterize them with respect to their locality with respect to the whole model:

1) *Local descriptors (key point-based)*, which are simple and efficient but often not robust to noise and shape perturbations;
2) *Region-based descriptors*, which are well suited to handle clutter and occlusions;
3) *Global descriptors*, which are highly invariant, but sensitive to occlusions and partial models.

3.1.1.1. Local feature descriptions

The problem of defining local feature descriptions is an acute problem in several domains, ranging from vision to the alignment of geospatial data. Starting from the success that local feature descriptions achieved in the analysis of 2D images (Mikolajczyk & Schmid 2005), Guo et al. (Guo et al. 2014; Guo et al. 2015) yield a comparative analysis of several local descriptor methods.

In the CH domain, and specifically for the retrieval of partially broken pottery, (Savelonas et al. 2015; M A Savelonas et al. 2014) extend the Fast Point Feature Histogram (FPFH) by (R B Rusu et al. 2009) to adapt it to the mean point distances of the point cloud query. The idea behind the FPFH description is to pair each oriented point (p, n) with each its oriented neighbours (p_i , n_i) and build a vector with three values: *(i)* the cosine of tangent and the direction vector of the neighbour; *(ii)* the projection of the neighbour normal in the plane spanned normal and *(iii)* the tangent normal at p. In a first step these three values are computed between p and its neighbours and stored in a feature histogram (SPFH); in a second step, the histogram in p is obtained as the weighted sum of the SPFH in p and its neighbours. The FPFH considers only the direct connections between a point p and its neighbours, removing additional links between neighbours. This takes the complexity to $O(nk)$, being k the (constant) size of the neighbour of p. Figure 1 presents with colours the relationship between the a point p_q and its neighbour; red lines represent the connections used to extract the first feature vector (SPFH), black lines represent the connections between the points that influence the second step; some connections (and the corresponding contributions) are counted twice (thicker lines).

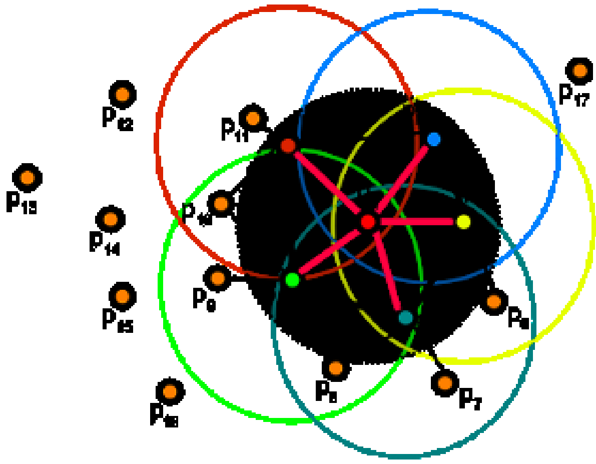
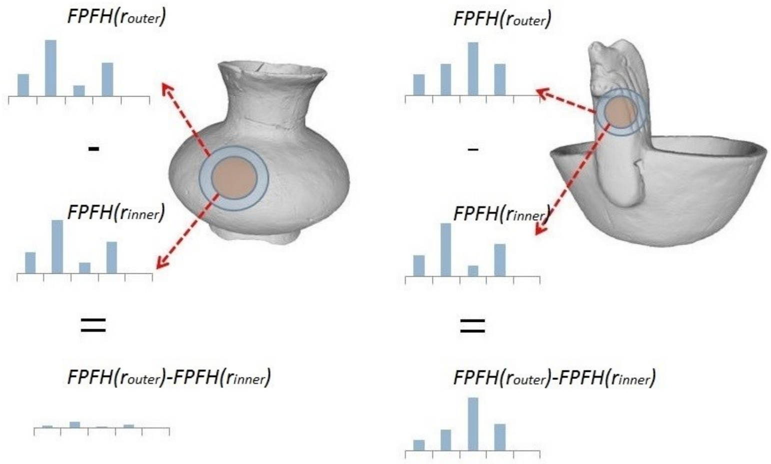


Figure 1. Point pairs established when computing the FPFH for the point p_q (image from http://pointclouds.org)

To make the FPFH description independent of the point cloud densities that may vary from object to object, the authors in (Savelonas et al. 2015) suggest to adaptively estimate the size of the neighbourhood of each point cloud as a linear function of the mean point distance over all k-neighbourhoods. A statistical outlier filter is used to refine the extracted FPFH vectors, filtering out outliers associated with noise. The FPFH vectors are further refined by a filtering component, which discards points surrounded by extremely irregular surfaces, often associated with artefact damages. Of course, this practice makes them unsuitable to the mating task, for which local fractal variation of fracture facets is essential. To further capture local geometric transitions by measuring the differences in feature histograms associated with concentric spheres, the FPFH is concatenated with the difference of the FPFH in a local ribbon around the neighbour. The results description is called dFPFH see Figure 2 for an example.



q

Figure 2 A schematic representation of dFPFH: (a) smooth surfaces result in similar FPFH histograms for the concentric spheres (FPFH(r outer) ≈ FPFH(r inner)) and histogram differences approximating zero, b) irregular surfaces result in much larger differences of the FPFH histograms, from (Savelonas et al. 2015)**.**

The final set of FPFH vectors is used in a bag-of-visual-words (BoVW) context wherein Fisher encoding (Sánchez et al. 2013) is employed. The resulting Fisher vectors are used for partial retrieval of 3D pottery objects.

Another local descriptor considered is the CH domain is the SIFT (Scale Invariant Feature Transform) descriptor, originally defined for images (Lowe 2004) and applied to the PANORAMA views of the 3D objects, i.e. to the projection of the 3D object over a cylinder aligned to the main shape axis that encloses the 3D object (Sfikas et al. 2012). In this case the SIFT descriptor is used in its 2D version, i.e. as a position-dependent histogram of local gradient geometrical directions around the keypoint. Scale invariance is obtained through normalization of the size of the local neighbourhood while rotational invariance is achieved through the identification of the dominant orientation of the neighbourhood. More in general it is possible to extended the SIFT description directly to the 3D domain using the approach proposed in (Wu et al. 2008). SIFT in images is designed to be affinely invariant; for the geometric matching/mating tasks of rigid artefacts, this is undesirable (though it may make sense the visual pattern matching of textures and for the re-association task).

It is worth mentioning that other local descriptors seem to be theoretically suitable for the GRAVITATE, for instance descriptions based on the Difference of Gaussians (DoG) method, like meshHOG (Zaharescu et al. 2012) and the scale space representation proposed in (Castellani et al. 2008). Recently, even if not applied to the CH domain and to objects with the characteristics of the GRAVITATE models, the Local Binary Patterns (mesh-LPB) (Werghi et al. 2015) has been proposed to characterize and recognize 3D patterns (indeed the method is applied to the recognition of facial expressions and 3D textures). **Figure 3** shows some example of 3D patterns on which the mesh-LBP descriptions successfully works. However, the direct application of the method to the GRAVITATE use case seems to be limited by the strong hypotheses that the connectivity of the triangle mesh is very regular and that the shape features are well characterized (not eroded); for this reason it is worth exploring possible extensions of the descriptor to the GRAVITATE context and/or to define new description strategies.

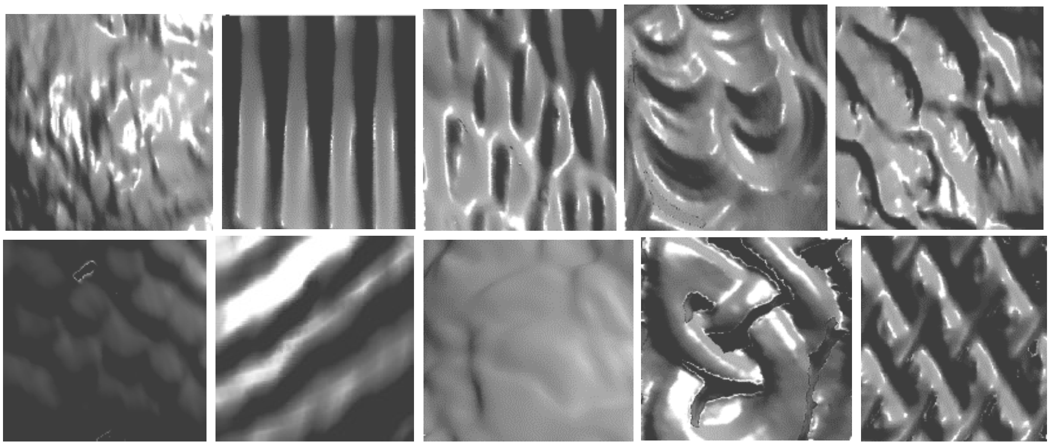


Figure 3 Examples of 3D textures used to test the mesh-LBP description.

3.1.1.2. Region-based descriptions

Region-based descriptors are related to approaches for which the analysis is focused on larger portions of the objects, for example a part of their surface with a well-defined and recognizable geometric shape.

In the CH domain, region-based descriptions can be of help when the aim is to find similar sub-surfaces, since many artefacts are broken. In fact, in this case global techniques such as scaling, alignment, or symmetry cannot be utilized (Tal 2014). An example was proposed in (Gal & Cohen-Or 2006), where the authors investigate the use of curvature for recognizing salient shape features. Once these features have been computed, they are mapped using a geometric hashing mechanism that determines the best transformation between these regions by mean of a voting scheme. The use of the shape curvature promotes the identification of well-detailed and isolated features encouraging the detection of shape details and (almost) flat regions while it discards the whole shape structure, see Figure 4. Therefore, such a technique is suitable for partial matching tasks but suffers of the local definition of "curvature" that could become insufficient when dealing with highly eroded surfaces.

Figure 4 (a): curvature analysis of the Buddha model and the local surface descriptors, blue: low curvature, red is high. (b): the self-similarity of the four lotus flowers as detected in (Gal & Cohen-Or 2006).

To progress in this direction, (Itskovich & Tal 2011) observed that though isolated feature points often do not suffice, their aggregation provides adequate information regarding similarity. In that paper the authors introduce a probabilistic framework in which segmentation and neighbouring feature points allows one. Specifically, at first, the salient points are detected and their similarity is computed using an approach similar to (Gal & Cohen-Or 2006). Considering only a subset of the vertices, rather than the whole set of vertices of the mesh, not only improves the performance, but also enhances the results, since non-distinctive vertices are ignored.

(Attene et al. 2011) proposed the Fast Reject schema to match a template shape with one or more parts of a scene (in this case the scene is defined as a set of 3D objects). The schema adopts the so-called *onion descriptors* because it is an incrementally defined region description based on different levels of detail, see Figure 5. In the implementation in (Attene et al. 2011), three shape descriptors are used to code the single layers: the Spherical Harmonics (SH), a coarse volumetric descriptor, and a surface descriptor based on curvature analysis. Finally, the onion description concatenates all the single layer descriptions. Again, this method seems to be suitable for matching subparts in incomplete objects, even though , as discussed by the authors in their paper, it was designed for features that are geometrically well characterized and rigidly superimposable (this fact depends on the use of the SH description).

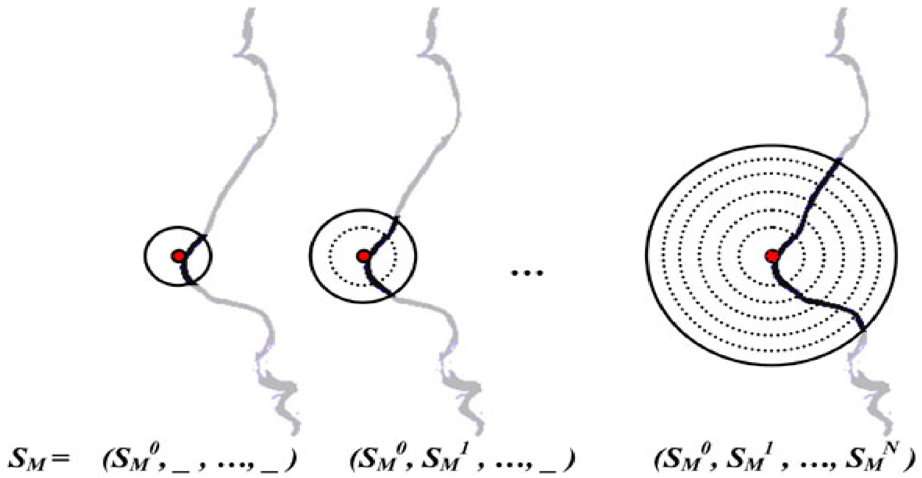


Figure 5 An example of incrementally-defined onion descriptor of a nose (image from (Attene et al. 2011)). From left to right: the first circle encloses a small region around the nose tip; this region defines the first layer. The second circle encloses a slightly larger region defining two layers. The whole nose is enclosed by the largest circle on the right, which defines the complete onion descriptor of the shape.

3.1.1.3. Global descriptions

Within the EROS-3D project (Gorisse et al. 2007), a set of global descriptors, namely the cord histograms (Paquet et al. 2000), the Extended Gaussian Images (EGI), the Complex Extended Gaussian Images (CEGI) and the 3D Hough transform (Hough3D) where explored for 3D shape retrieval and classification. In the case of the EROS-3D project, the 3D models, partially broken and eroded, were not equipped with colour information and represented single 3D models, thus not dealing with re-assembly tasks. Indeed, the project focused on dataset navigation and global matching.

Cord histograms are two normalized histograms that encode the distribution of two features of a set of cords (a cord is a vector from the model centre to a vertex) that are the length of the cord and the angle between the cord and the first principal axis, see Figure 6.

The EGI (Extended Gaussian Images) were first introduced in (Horn 1984). Each object is projected onto a Gaussian sphere, and each point of the sphere is valued with the total area of the object faces of the same orientation, see Figure 6.

The Complex EGI (CEGI) (Kang & Keuchi 1993) is a variant of the EGI able to discriminate between concavities and convexities. The CEGI feature describes the object thanks to two attributes: the face orientation and the distance between the face and the centre of gravity of the object. For each facet of the Gaussian sphere, an accumulation of these two attributes is performed in the complex space. To increase the difference between concavities and convexities, the EGI distance is signed. It is negative when the face is directed towards the object centre and positive else. Finally module and phase are computed and compose the CEGI descriptor.

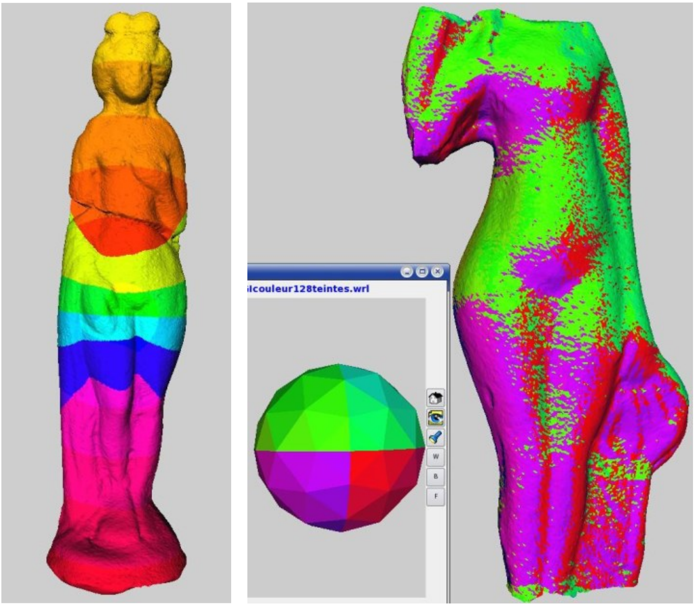


Figure 6 Colour representation of (left) the Cord2D features and (right) the face orientations for the EGI descriptor, images from (Gorisse et al. 2007)

The 3D Hough feature (Hough3D) (Zaharia & Preteux 2002) is an extension of the Hough transform (Hough 1959) adapted to 3D objects. It consists in accumulating the parameters of the planes defining the faces of the object. In spherical coordinates, a plane is uniquely defined by the triplet (s, θ, φ), where s is the distance of the plane to the origin, θ the angle of azimuth and φ the angle of elevation. A 3D histogram is computed for the triplets (corresponding to the coordinates of the face centres), where each face contributes proportionally to its area. (Gorisse et al. 2007) claimed that the Hough3D description as an extension of the EGI in terms of computation of the EGI on the set of faces located at distance s to the object centre and repeat this calculus for a set of distances s.

Among the large set of global descriptions proposed in the literature, (Gregor et al. 2014) proposed a benchmark of artificially fractured 3D artefacts on which three global descriptors are evaluated, namely the DSR description proposed in (Vranic 2004), the Heat Kernel Signature (HKS) (Sun et al. 2009) and the Scale Invariant HKS (Bronstein et al. 2011). The objective of the benchmark is mainly global matching of deformed artefacts.

The *DSR* has been introduced as an hybrid descriptor in (Vranic 2004) crossbreeding three feature vector descriptors based on silhouette, spherical harmonics and depth-buffer images. In particular, in his PhD thesis Vranic showed that this method outperforms the others three shape retrieval methods and that in general the DSR is the best descriptor among those considered in his thesis. However, this descriptor achieved a good performance also in the survey paper (Bustos et al. 2005) even if in the latter paper it was outclassed by the Lightfield descriptor introduced in (Chen et al. 2003).

The heat kernel $h_t(x, y)$ is a fundamental solution of heat equation, with heat source point at x and heat value at y after time t: it represents the amount of heat transferred from x to y in time t due to the diffusion process, see Figure 7. The heat kernel has many nice properties, among which invariance to isometries; being related to the Riemannian metric of the object space, this means that the heat kernel is an intrinsic property of the manifold. Also, the heat kernel is multi-scale: for small values of t, $h_t(x,\cdot)$ only reflects local properties of the manifold around the base point x, while for large values of t it captures the global structure of the space at the scale of t. Finally, the heat kernel is stable under small perturbations of the underlying manifold. All these properties make the heat kernel a good candidate for the definition of informative functions and distances to be used for shape description, such as the *heat kernel signature (HKS)* (Sun et al. 2009; Gebal et al. 2009) and the diffusion function. The HKS at a time t, denoted by HKS_t, is defined as:

$$HKS_t(x) = h_t(x,x),$$

for any x; the diffusion distance d_t between two points x, y at time t is given by

$$d_t^2(x,y) = h_t(x,x) + h_t(y,y) - 2h_t(x,y).$$

The computation of the spectrum of the discrete Laplacian is the main computational bottleneck for the evaluation of the heat kernel, and hence of $HKS_t$and d_t; in fact, it takes from $O(n)$ to $O(n^3)$ operations, according to the sparsity of the Laplacian matrix. Recently, a discrete and spectrum-free computation of the diffusion kernel on a 3D shape (either represented as a triangulation or a point cloud) has been proposed in (Patanè & Spagnuolo 2013), based on the computation of the full spectrum via the Chebyshev approximation of the weighted heat kernel matrix.

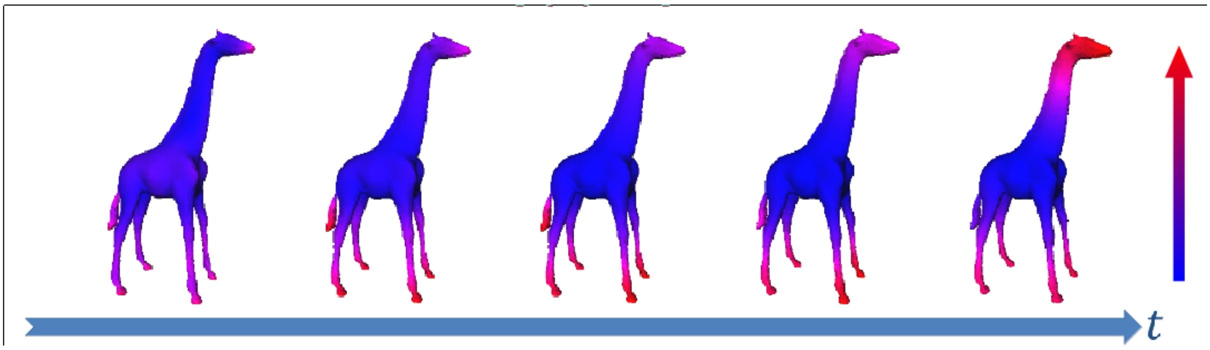


Figure 7 The heat kernel function at increasing values of the time t (the value of the HKS are coloured from blue (minima) to red (maxima)

The *scale-invariant HKS (SI-HKS)* was proposed by Bronstein et al. (Bronstein & Kokkinos 2010) to overcome the scale dependence of the standard *HKS* and inherits its autodiffusion-related structure. Indeed, scale independence is done in terms of scaling and shift in time: scale is obtained from the logarithm of HKS_t and its discrete derivative with respect to time, while the shift is seen as a different phase that is discarded through a complex representation of the discrete Fourier transform. The *SI-HKS* at each point of the shape is approximated through soft quantization by the closest geometric words in a pre-computed vocabulary of 48 elements that is compared using the L^1 distance. The *SI-HKS* fully satisfies intrinsic invariance and scale independence. Moreover, the choice of the discretization scheme for the Laplace-Beltrami operator (e.g. point wise or mesh-based) makes this signature available for different inputs such as point clouds or meshes.

The *Multiscale Heat Kernel* proposed in (Yu et al. 2012) is based on the same theoretical background as *HKS:* in this case the shape features are characterized as local maxima and minima of the Heat kernel function, and the parameter t is the tool used to determine the scale of the feature. A further extension of the Multiscale heat kernel is the Treesha description (Garro & Giachetti 2015) that, even if not yet applied to artefacts, couples the multiscale characterization of maxima and minima of the heat kernel function (ADF in the paper) with a scale-space representation that captures in a tree the evolution of these features over time. When coupled with a texture description (a colour histogram), the Treesha description has been shown to over-perform the existing state of the art methods for global shape matching and therefore seems to be interesting also for the GRAVITATE purposes.

The idea behind the *mutual distance matrix MDM*, originally defined in (Biasotti 2010) and adopted as a geometric descriptor for 3D artefacts retrieval and clustering in (Biasotti, Cerri, et al. 2014; Biasotti et al. 2015a), is to represent the most salient geometric traits of 3D artefacts by building on a set of "basis functions" that concur to model the shape properties of interest. The method has been applied to partial and eroded artefacts and assumes that the 3D model is of a quality good enough to efficiently extract the set of geometric properties. Starting from an initial set $F = \{f_1, \dots, f_n\}$ of n functions defined on a 3D artefact, the MDM stores the inter-distances between all the possible couples of functions, i.e. shape properties, by looking at the behaviour of their gradients. In practice, the $i-th$ row (or column) of the MDM identifies all the distances of f_i with respect to $\{f_1, \dots, f_n\}$. Intuitively, there is the assumption that a relevant discrepancy in the distribution of the gradients of two functions $f_i$and f_j implies that they are significantly different. The minima of the $i-th$ row correspond to functions that are qualitatively similar to f_i, while maxima highlight functions that significantly differ from f_i. Besides the use of the MDM as a geometric descriptor it is also possible to use this matrix to identify a (small) number of functions that are mutually independent and select a subset of scalar functions that qualitatively preserve the descriptive power of the original set. Figure 8 shows a model and the MDM ranging the number of the functions; dark blue colours represent functions that judged similar, while red ones represent values that are close to 1.

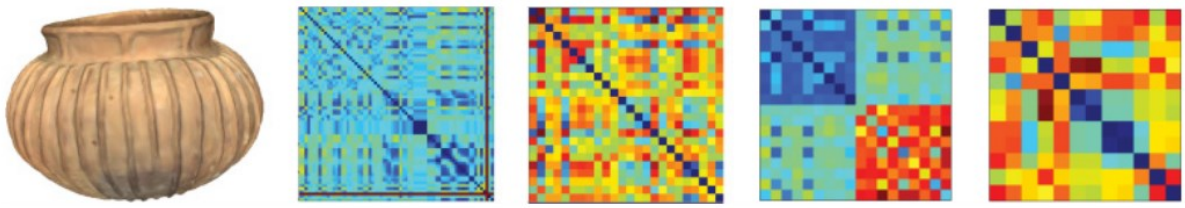


Figure 8 A vase and the MDM signature obtained either with 70, 26, 18 and 12 geometric functions (image from (Biasotti et al. 2015a)**)**

3.1.2. Colour descriptions

The literature on shape analysis proposes several attempts to support matching with colorimetric properties. However, in cultural heritage there are not guarantees of an uniform conservation of the colour properties of the artefacts, therefore the colorimetric analysis should at least combined with other properties.

Colorimetric properties can be represented in different colour spaces, such as the RGB and HSV spaces, however the most popular choice is the CIELab one (Fairchild 2005). Adopting the CIELab colour space is justified by physiological studies, showing that it well represents how the human eye perceives colours. In particular, it has been demonstrated that small colour variations slightly alter the CIELab colour embedding. Moreover, in the CIELab space tones and colours are held separately: the L channel is used to specify the luminosity or the black and white tones, whereas the A channel specifies the colour as either a green or a magenta hue and the B channel specifies the colour as either a blue or a yellow hue.

In most papers, the colours were used as is, in the CIELab space (Cho et al. 2010; Pomeranz et al. 2011; Sholomon et al. 2013; Paikin & Tal 2015; Yang et al. 2011). As no special features were extracted, most of the work is performed in the dissimilarity measure. In (Gallagher 2012; K.Son et al. 2014), the features are the mean distribution of the colour gradients, as well as colour difference (as before).

Other choices are the use of colour histograms, such as the 128 bin colour histogram given by the concatenation of the three colour channels (CIELab) adopted in (Biasotti et al. 2015a). Indeed, these colorimetric descriptors behave well against localized noise, since it does not alter colour distribution too much.

Colour histograms are not enough to recognize patterns, an operation that is necessary in the GRAVITATE project. In this case, by patterns we mean the decoration styles (shapes of lines for example). A few attempts to mix shape and colour and therefore to approach patterns are the heterogeneous descriptors described in the next Section.

3.1.3. Heterogeneous descriptors

In practical applications, a single class of shape descriptors might not be enough to get a sufficiently detailed shape characterization. In this case, multiple categories of descriptions can be used to obtain relevant information from different points of view, and possibly merged a posteriori into a common descriptor or in a single (dis)similarity score. The shape properties of interest involved in this process might represent as many geometrical investigations of the objects under study, including for instance curvature or local height analysis; additionally, extra-geometric information might be taken into account, focusing, e.g., on the presence of decorative patterns or paintings. Additionally, the analysis might involve not only an object as a whole, but also applied to its sub-parts, such as interior, exterior or fracture surfaces. The approaches following this paradigm are referred to as *multi-modal descriptors*.

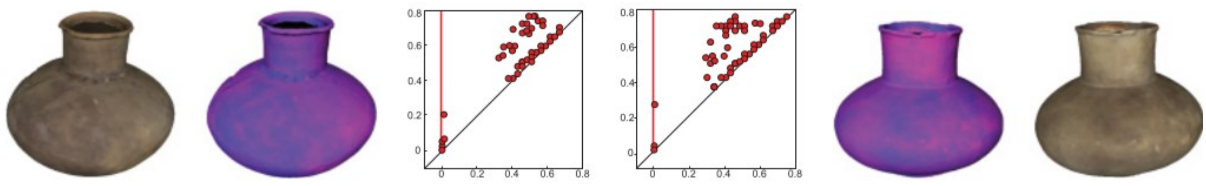
For instance, (Kleber et al. 2010) addressed the reconstruction of Ephesus marble plates and showed how the analysis of a number of purely geometric techniques improved when additional information in the fragments' shape description was included, namely marble texture, thickness of the considered fragments and location at the excavation. Within the overall GRAVITATE platform, some of those descriptors such as volume, area and length, can be gleaned from the metadata, but they would need to be translated to inform the modules performing physical computations.

The authors of (Oxholm & Nishino 2011; Oxholm & Nishino 2013) focus on the problem of reassembling fractured thin artefacts, thus dealing with the re-assembly problem from an image-like point perspective. Their approach involves the fragment boundary contours and photometric information. In particular, the shape of each fragment is described through a scale-space representation of its boundary contour, and encoded in a 2D multi-channel image. In doing this, geometric data is paired with the photometric information acquired from a range sensor, in order to detect and analyse the scale variability of shape and colour for each fragment.

Geometry and colour are jointly analysed in the graph-based, group-wise matching technique proposed in (Zhang & Li 2014) for the reassembly of fragmented 2D images, thus not dealing with 3D models like those of the GRAVITATE project. In that paper, the authors combine geometric and colorimetric information to group fragments in clusters characterized by similar shape properties; only fragments belonging to the same cluster, i.e. sharing similar geometric and colorimetric descriptions, will be considered as possible candidates for being matched together.

Similarly, in (Funkhouser et al. 2011), the heterogeneous approach is used to rank predicted matches between pairs of fresco fragments, by assigning to each candidate matching a measure of confidence. The underlying idea here is that, although combining shape properties involving geometry and colour to assess the reliability of a matching, most methods merely rely on a limited number of considered features, yielding matching confidence measures that are not sufficiently precise. Starting from this observation, a learning technique is introduced to predict the probability of a matching between two fragments, which takes into account 64 shape properties based on geometric and colorimetric information directly computable from the scans of the considered fragments. Even if the direct application of the method is not suitable for the 3D models of the GRAVITATE project, the learning technique is interesting and could be coupled also with 3D shape descriptions.

Finally, we mention the approach proposed in (Biasotti, Cerri, et al. 2014; Biasotti et al. 2015a) to assess the similarity between 3D artefacts. In that paper, the authors propose a multi-modal description for each considered 3D objects, taking a large number of geometric attributes, as well as colorimetric information into account. In particular a hybrid shape description based on the notion of persistence diagrams and spaces (Edelsbrunner et al. 2002; Cerri & Landi 2013) of the topological characteristics of the channels L and AB of over the geometric shape is adopted to complement the purely geometric and colorimetric descriptions. The rationale behind this approach is that, by considering a multitude of shape properties from the very beginning, it should be easier to deal with the variability that can be found across a collection of artefacts scanned in different circumstances, as well as to represent the properties of interest for users that are difficult to characterize in a precise manner. The stability properties of persistence diagrams and spaces imply resistance to noise (in GRAVITATE terms, it means surface degradation or small fractures) when it comes in the form of small perturbations in the L, A, and B values (Cohen-Steiner et al. 2005; d'Amico et al. 2010), Figure 9. In particular, the topological approach ensures robustness even when the small changes in the CIELab colour coordinates are widely spread over the shape model. In practical situations, this may happen when material degradation results in large spots affect the surface artefact, or when different histories lead to colour changes of parts in a re-unification task.



(a) (b) (c) (d) (e) (f)

Figure 9 A model (a); the channel L (b) colour-coded from blue to red; (c) the corresponding persistent diagram; (d-e) the persistent diagram and the channel L of another model (f). From [Biasotti et al 2015a]

3.2. Similarity measures and applications

One of the goals of GRAVITATE is to support end-users in exploring a collection of fragmented objects, in order to boost how archeologists and curators currently deal with the re-unification and re-association tasks. Roughly, the main idea is to map the collection of objects (e.g. fragments) into suitable spaces of similarities that can be explored, e.g., to properly group items according to their geometric and/or colorimetric closeness and possibly to formulate re-association and/or re-unification hypothesis based on such grouping. In this view, a necessary step is then to measure the shape similarity between the objects in the dataset under examination.

Assessing the similarity between shapes can be posed as the problem of defining a suitable function $d: \mathcal{X} \times \mathcal{X} \to \mathbb{R}$, taking a pair of input objects from a universe $\mathcal{X}$ to a real number that represents a similarity score for the two objects (Skopal & Bustos 2011). Such a function d is called a *pairwise similarity function*. A common strategy in shape similarity assessment is to associate the shape of an object with a compact codification of its most salient features, which is usually referred to as a shape descriptor. In this way, shape descriptors can be used in place of the whole model representations to derive some similarity score between the original objects. Nevertheless, a single descriptor might not be enough to get a sufficiently detailed shape characterization. Therefore, batteries of descriptors can be used separately to produce multiple (dis)similarity scores that would be merged a posteriori.

Following the above paradigm, in (Koutsoudis et al. 2010) the authors analyse a collection of (almost complete) pottery vessels with the goal to find, for a target object, the most similar items within the entire collection. To this aim, two of the main morphological features of 3D vessels, namely rotational symmetry and the opposed positioning of appendages, are exploited to derive compact shape descriptors in the form of two families of feature vectors. Descriptors are then compared by means of the Euclidean and the Hamming distances: The comparison results into two different similarity measures, which are eventually put together through a weighted combination to get a final similarity score. Similarly (Biasotti, Cerri, et al. 2014; Biasotti et al. 2015a) define a similarity measure over a collection of 3D artifacts, which can be used to group objects (in a re-association/re-unification fashion) and/or to retrieve items that best match a query object. In this view, two artefacts are compared according to the combination of three distances, namely Manhattan and the L^1 distance for features vectors and histograms, respectively, and the Hausdorff distance for the hybrid descriptors.

3.2.1. Similarity versus dissimilarity

Instead of a similarity function, the inverse concept is often required, namely a *dissimilarity* function δ, where a higher dissimilarity score stands for a lower similarity score, and vice versa. Hence, a dissimilarity δ equivalent to a similarity d must fulfil $d(X,Y) \geq d(X,Z) \Leftrightarrow \delta(X,Y) \leq \delta(X,Z), \forall X,Y,Z \in \mathcal{X}$. The choice between similarity and dissimilarity function mainly depends on the application domain; however, there exist many situations where the formula/algorithm defining the function is available in just one of the two forms, while its manual transformation into the inverse is not straightforward (Skopal & Bustos 2011).

A scenario in which dissimilarity functions are often considered is that related to the re-assembly of 2D puzzles. In (Cho et al. 2010) a dissimilarity measure is introduced in order to quantify the difference between two puzzle patches, and hence to have an evaluation for their compatibility. In this case, the dissimilarity function depends on the colour difference, measured in the L^2-norm, along the abutting boundaries of the considered patches. In (Gallagher 2012) the Mahalanobis Gradient Compatibility is introduced, which is roughly a dissimilarity function measuring the compatibility of two patches by looking at local behaviour of the RGB gradients near the patches' boundaries. Both the above dissimilarity functions are exploited by the algorithm based on "loop constraints" proposed in (K.Son et al. 2014) for re-assembling non-overlapping square-piece jigsaw puzzles. In (Pomeranz et al. 2011; Paikin & Tal 2015) a prediction-based dissimilarity function is used to analyse the compatibility of squared patches as well. Briefly, the last two pixels in each row (column) near the boundary are considered, from which prediction of the first pixel in the adjacent piece is obtained. The dissimilarity measure between pixels in two patches uses the L_q^p-norm in the LAB colour space. In (Pomeranz et al. 2011) $p = 3/10$ and $q = 1/16$, in (Paikin & Tal 2015) the L^1-norm is utilized, which accelerates the computation and was found it to improve the results.

Beyond (a subset of) metric axioms whose details we omit but that guarantee the dissimilarity function is a metric in the mathematical sense and can be useful for dataset indexing, a notion of continuity is often required for a (dis)similarity function. Continuity guarantees robustness with respect to different discretizations of models and small perturbations in the input measurements: this translates in robustness to surface degradation. Last but not least, invariance to some classes (groups) of transformations may be required, thus allowing the similarity assess to be independent, for example, to orientation, scaling or rigid movements of the considered objects. Invariance properties can be directly owned by the considered shape descriptors, as explained in (Biasotti et al. 2015a), or may came after some *a priori* normalization of the objects under study, such as in (Koutsoudis et al. 2010).

3.2.2. (Dis)similarity score versus correspondence

Rather than establishing a shape (dis)similarity score through the use of shape descriptors, another approach is to assess the dissimilarity between shapes by modelling them as suitable spaces, and to formally quantify similarity in terms of the distortion needed to deform one space into the other (Kim et al. 2011; Kovnatsky et al. 2013). The added value in this approach is that similarity can be expressed not only in terms of a single score, but also trough a map between shapes. Despite of the increasing computational complexity, this makes possible to derive either a sparse or a dense shape correspondence, which is particularly useful when analysing variability among shapes or when looking for a matching between shape parts. In practice, a correspondence between shapes is typically given as a point-to-point mapping between samplings of the considered digital representations.

In the CH domain, point-to-point correspondences can be used to solve re-assembly of fractured objects, as proposed in (Mavridis et al. 2015). In that paper, the authors introduce a pipeline for extracting a set of candidate fractured facets from each fragmented object given as input, computing a pair-wise matching score and finally proposing a set of pairwise matches to come up with the final reassembly. In (Itskovich & Tal 2011), point-to-point correspondences are integrated with part-to-part correspondences to address the problem of surface partial matching for archaeological artefacts. The part-to-part correspondence is the final stage and is obtained by considering the similarity between the key-points belonging to corresponding parts.

3.2.3. Grouping and organizing similarities

To facilitate exploration and content search to understand the overall categorization and summarize the content of a collection of models, research is currently moving towards techniques of clustering, relevance feedback and metric learning. The key challenge is that shapes can vary in different ways. Moreover, when a collection possesses rich variation and highly dissimilar objects (Huang et al. 2012) a single measure is not likely to provide a good organization and quantitative measures may be unreliable, or at least not informative enough. In this case, it is possible to use qualitative information derived from multiple quantitative measures or to build a network where shape correspondences are consistently preserved (Huang et al. 2014).

The clustering problem has been very well studied: the k-means (MacQueen 1967), the NCut (Shi & Malik 2000) methods and all their variations are very popular and several surveys on clustering and spectral clustering techniques are available in the literature (Xu & Wunsch D. 2005; Luxburg 2007). Most of these methods require to know a-priori the number of clusters and/or implicitly assume that the space of the elements is a mathematical variety equipped with a Riemannian metric. This may be true for applications to shape reconstruction, but not easy to assume when used to study the structure of a set of descriptors.

For this purpose, methods based on a “proximity” notion of the elements in the cluster seem to be more suitable, like the DBSCAN method (Ester et al. 1996) and its generalization, OPTICS (Ankerst et al. 1999). The main advantage of these methods is its independence on the number and the shape of clusters, the invariance of the order of the elements and the robustness to noise. Unfortunately, a unique choice of the parameters of proximity could not be effective in case the space presents different local densities.

In the CH domain, (Biasotti et al. 2015b) adopted a cluster technique based on the notion of dominant sets, both to group different descriptors (functions) and models. The space clustering is performed iteratively, one cluster at a time and the cohesiveness of the elements in the cluster is updated every time a new element is added to it.

The identification of the features shared by a set of models has been addressed mostly at the level of machine learning techniques, like boosting and support vector machines (Tieu & Viola 2004;

Hou et al. 2005). As an example in the computer graphics community, the co-analysis (and co-segmentation) methods aim at consistently identifying semantic parts of a set of 3D shapes from the same class grouping faces and segments. Co-analysis methods can be classified into supervised, semi-supervised and unsupervised, depending on the learning strategy adopted. The unsupervised setting is more challenging and flexible, since no prior information is given and the entire knowledge must be extracted from the input set (Golovinskiy & Funkhouser 2009; Hu et al. 2012; Wang et al. 2012). These methods generally cluster the shape faces or the elements of the descriptor space; these clusters become the shape segments. Recently, statistical methods such as learning techniques (Daras et al. 2012; Litman et al. 2014) have been investigated to automatically infer knowledge in the loop. These techniques represent a possible solution to automatically determine the weights of the different shape features on the basis of context (e.g. the shape classes of a database) (Laga 2010; V Barra & Biasotti 2013; Biasotti et al. 2015a; Tabia et al. 2013) or design class- or application-specific shape descriptors (Litman & Bronstein 2014; Masci et al. 2015; Boscaini et al. 2015). In CH domain, learning has been proposed for matching of pairs of fresco fragments, mixing 64 geometric and colorimetric properties (Funkhouser et al. 2011). Another solution might be to consider meta-representations able to characterize the configurations that are common across a family of 3D objects and to consider this knowledge when storing the arrangements among shape parts (Fish et al. 2014) thus making possible to explore multiple shape configurations in parallel and to collectively edit sets of shapes.

The tuning of the descriptors and similarities used to the specific context or user needs has been addressed implicitly by relevance feedback techniques (Nguyen et al. 2007; Giorgi et al. 2010). The idea is to adopt weighted algorithms that automatically and dynamically modify the degree of relevance of the descriptors to fit the preferences and information needs of the users via Relevance Feedback (RF's) schemes. Relevance feedback is a widely adopted methodology for query refinement in image retrieval, with many different proposals; see (Zhou & Huang 2003) for a survey paper and (Kersten & Lindstaedt 2012) for a discussion of its application in the cultural heritage domain. In particular, Relevance Feedback is an online learning strategy, which adapts the response of the system by exploiting user interaction. The goal is to automatically adjust an existing query (i.e., the query of the system) using information feedback by the user about the relevance (or irrelevance) of previous results (Kersten & Lindstaedt 2012). In a relevance feedback scheme, the user is able to take part in the process and to evaluate the results. Thus, the system learns from this user's evaluation so that the new responses are closer to the user's perception as far as the alerting is concerned while simultaneously keeping all the already obtained knowledge of the system as unchanged as possible, examples of methods that adopt relevance feedback schemas for 3D object retrieval are (Elad et al. 2002; Leifman et al. 2012; Atmosukarto & Shapiro 2008; Onasoglou & Daras 2008; Akgül et al. 2010).

Within the EROS-3D project, the RETIN search engine was developed (Gosselin & Cord 2006) with the aim of dealing with a collection of artwork 3D models and to visualize, classify and compare them. In particular, the plug-in "RETIN-3D" was performed to yield interactive 3D object classification and retrieval using active learning. The user provides an object as a query and the system has to retrieve similar objects from the database. The similarity is measured thanks to the features and the models are displayed according to their similarity to the query (Figure 10). Because of the great variability in the shapes of the objects within a category (in particular because of broken parts), no set of features is able to represent the category in a unequivocal way. The user leads the search by annotating displayed objects as belonging or not to the searched category. This gives a large flexibility to the system, since the classification is achieved online and according to the user attempts. That is to say, the results obtained by a user searching for vases with two handles will not be the same as those obtained by a user searching for any type of vase. The problem is thus a two-class classification problem, with a semi-supervised learning, actually active learning, since the leaning set is enriched at each iteration by new examples and counter-examples provided to the system thanks to the user annotations. The goal is to separate two classes with a function induced from available examples of both classes and thus to build a classifier that will properly work on unknown objects, for this purpose a Support Vector Machine (SVM) is adopted since it allows a non-linear classification into two classes without requiring explicitly a non-linear algorithm thanks to the kernel theory (a Gaussian kernel was used). In order to perform active learning, the closest objects to the border of the class are displayed in a specific panel at the bottom of the interface ("active learning panel", see Figure 10).

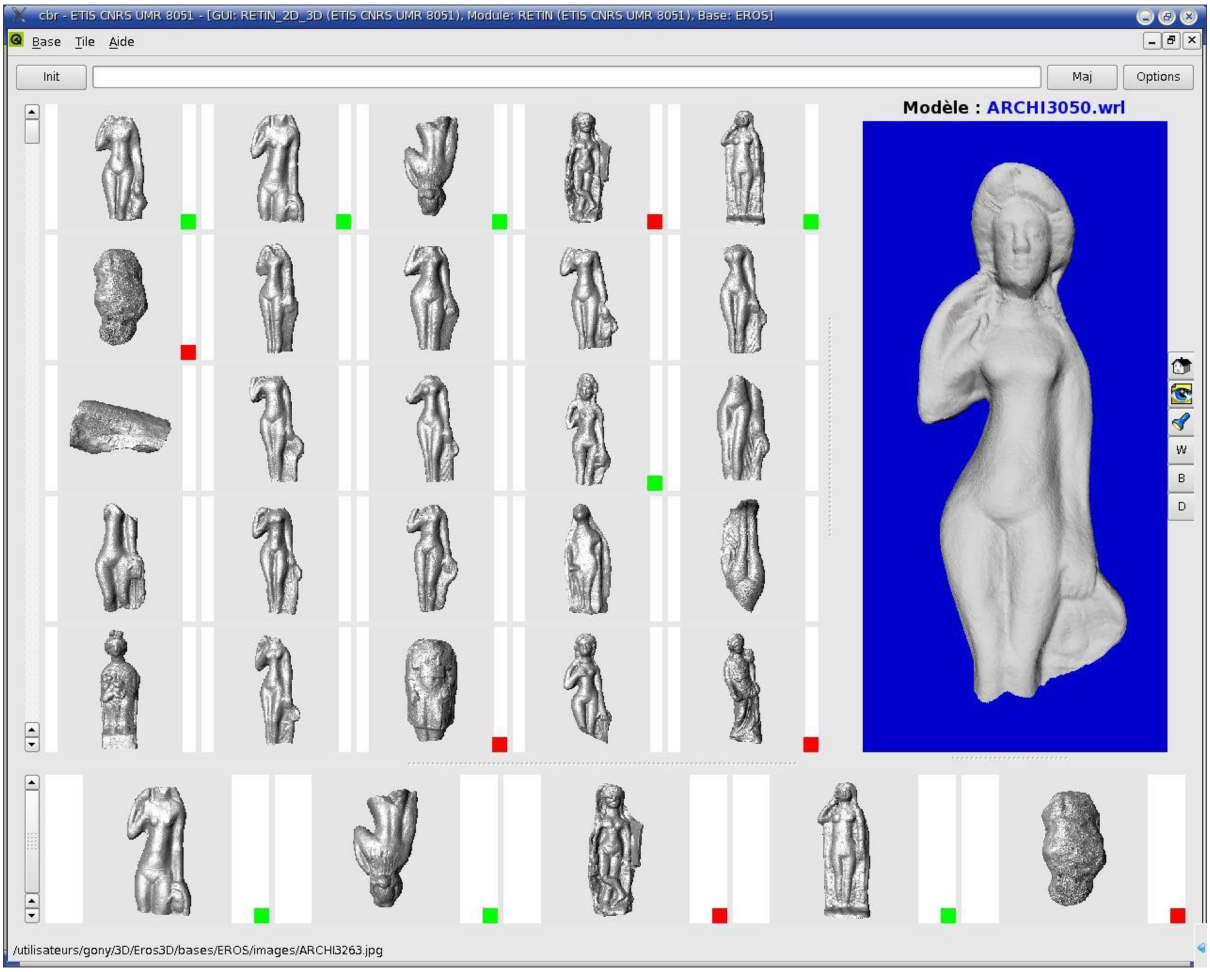


Figure 10 The RETIN-3D interface, image from (Gorisse et al. 2007)**. Left: 3D models ranked by their classification rate, from top to bottom, left to right. Objects are annotated with a green (resp. red) mark if they are relevant (resp. irrelevant) to the request. Right: one of the object of the class (Venus). Bottom: the active learning panel.**

We also briefly describe the method proposed in (Giorgi et al. 2010). Here, the authors assume the user employs a pseudodistance (i.e., a distance without the property assuring that two objects having zero distance are the same object) to compare shapes. Then, the goal is the approximation of this unknown and subjective pseudodistance on a given dataset, according to a partial knowledge on some examples. The solution is to define a technique based on multilevel relevance judgements expressed by the user through an interface. To approximate the unknown user's pseudodistance, a set of known pseudodistances corresponding to a set of shape descriptors is pre-computed, possibly analysing complementary shape properties. The user's judgements are then used progressively to inhibit those pseudodistances that are not compatible with the user judgement; the method penalizes the pseudodistances coming from descriptors that do not reflect the user perception, i.e., the descriptors that perceive to be different for the objects the user perceives to be similar. This is done by a simple scaling procedure which produces a new set of pseudodistances compatible with the user's ideas. These new pseudodistances are merged by considering the max operator. The assumption is that similarity may emerge from the inhibition of differences, i.e., from the lack of diversity with respect to the shape properties taken into account for the comparison (hence the user) at hand. The method also combines multiple descriptors by working in the space of dissimilarities, thus being independent of the nature of the descriptors themselves, which can be feature vectors, graphs and so on.

4. Shape mating and matching techniques for ReX

Algorithmically the ReX GRAVITATE tasks are similar to CH and non-CH tasks that have been studied before. This chapter treats this algorithmic inventory and the corresponding literature. Here we focus on the ReX tasks as considered from the physical properties of the artefacts, often restricting ourselves to the geometry.

The techniques from global and partial shape matching are at the basis of the similarity reasoning and comparison and can be employed in all the ReX tasks, see Section 4.1.

Section 4.2 covers the geometric mating across fracture facets, casting it as algorithmically similar to object registration and shape matching and studying the accuracy of 3D scanning.

The fitting of objects does not only depend on the mating of the fracture; continuity of geometrical or colouring patterns on the skin can also help, notably in reducing the combinatorics of potential matches, see Section 4.3.

When actual ReAssembly is hard or impossible, we may still be able to ReUnify objects belonging to the same artefact. The ReAssociation task will often be based on the metadata characteristics of the artefact. Indeed, the combination of shape matching techniques and metadata interpretation and search should be employed for the interesting ReAssociation task; some recent examples of such usage are discussed in Section 4.4.

There is quite a bit of software to support the manipulation of virtual objects, and many of the algorithms discussed are readily available in coherent software platforms. We give pointers to these, and to CH datasets, in Section 4.5.

4.1. Shape matching techniques

In general, the problem of matching real-world archaeological fragments is a difficult task that heavily involves the evaluation of similarity between artefacts and their parts. Usually, shape comparison is carried out by first abstracting 3D objects as shape descriptors and then performing similarity assessment among descriptors as described in chapter 3. For this reason there are common references between this section and chapter 3. The main difference is the point of view: here we focus on the complete workflow of shape matching, i.e. how the single operations described in chapter 3 are combined together. Being the methods listed in this section of general application, some of them are referred also in the sections related to shape mating and reassembly. In the context of GRAVITATE, these methods are of relevance also for the exploration issues.

A major complication when dealing with artefacts, is that fragments are irregular and can match along any subset of their complete boundary. Additionally, typical real-world fragments introduce a vast variety of possible sources of confusion like (Willis & Cooper 2008):

1) physical degradation of the fragments due to chipping and, if exposed to the elements, erosion;
2) the number of pieces involved in the re-unification may not be known, i.e., we are given a collection of fragments coming from an unknown number of objects;

3) fragments may be missing as they may have yet to be discovered or may have been destroyed by some physical phenomenon.

Moreover, in the CH domain, since dealing with fragments, the challenge is to identify/locate parts of features in portions of models (e.g. a broken eye in a portion of face) thus leading to "double partiality" of the matching.

In the following, we review some of the methodologies that tackles matching between fragments without a specific need of exact correspondence between then fixing the corresponding terminology adopted in the 3D object retrieval community.

Shape retrieval refers to the task of finding the models in a database that best match a given query (Tangelder & Veltkamp 2004; Tangelder & Veltkamp 2008). All methods whose output is a similarity score between couples of shapes, can be adopted for 3D content based retrieval (Fang et al. 2011; Daras et al. 2012; Vincent Barra & Biasotti 2013; Li & Johan 2013; Biasotti et al. 2013). For instance, in (Koutsoudis et al. 2010; Koutsoudis & Chamzas 2011) the shape retrieval paradigm is applied to the analysis of (almost complete) pottery vessels. Two of the main morphological features of 3D vessel replicas, namely rotational symmetry and the opposed positioning of appendages, are exploited to create compact shape descriptors for web-based content-based retrieval search engines. The similarity score between two shapes is then evaluated in terms of distances between the associated descriptors. In (Biasotti, Cerri, et al. 2014; Biasotti et al. 2015a) a shape retrieval pipeline for real-world 3D artefacts is proposed, which concurrently evaluates a number of heterogeneous shape properties. Each 3D model is described by taking into account both geometric and colorimetric information. In practice, a feature vector encoding the geometric shape properties of the object, a colorimetric histogram and a hybrid descriptor, combining geometric and colorimetric information, which come in the form of a set of points in $\mathbb{R}^2$ and $\mathbb{R}^4$. The geometric and photometric descriptions associated with different shapes are then compared through suitable metrics to derive as many distances; these are further combined into a final score for shape similarity assessment.

A variation of the retrieval problem consists in looking for partial shape similarity, for example if some of the considered shapes present missing parts or in case part of a shape has to be considered as background and hence discarded (Julien Tierny et al. 2009; Dey et al. 2010; Areevijit & Kanongchaiyos 2011; Lavoué 2012). This is usually referred to as *partial shape retrieval* (Liu et al. 2013). In the CH scenario this is a standard case, as many artefacts are found broken. In this context, helpful global techniques such as scaling, alignment, or symmetry cannot be utilized. Therefore it is necessary to rely on partial similarity, where given a specific part of an unknown surface, the goal is to detect similar parts on other surfaces, regardless of the global surface this part belongs to (Tal 2014). For example, the method for the retrieval of 3D pottery objects in (Michalis A Savelonas et al. 2014) uses fast persistent feature vectors calculated adaptively to the mean point distances of the point cloud query to feed a bag of visual words scheme. This method only deals with point clouds. On the contrary, the partial 3D pottery object retrieval method proposed in (Savelonas et al. 2015) can be applied on both point clouds and meshes and builds on a hybrid shape comparison scheme, defined so as to account for both local and global shape similarity, as well as to address the partiality of the query object.

Shape matching is usually referred to the task of establishing a correspondence between feature points or regions of different shapes (Zaharescu et al. 2009). Often, this is the result of minimizing the distortion of some shape structure, while mapping one shape to another (Kim et al. 2011; Kovnatsky et al. 2013). Nevertheless, matching two shapes can be expressed in the form of a global similarity score (Boyer et al. 2011), possibly (but not necessarily) obtained as the by-product of a correspondence.

Partial matching is a variant of the shape matching problem (M. Savelonas et al. 2014; Liu et al. 2013), according to which similarity assessment is restricted to shape parts, still in terms of correspondence (Sharma et al. 2011; van Kaick et al. 2013) or numerical score (Shapira et al. 2010; Dey et al. 2010; J Tierny et al. 2009; Bose et al. 2011; Wu et al. 2010).

As stated in (Tal 2014), in (Itskovich & Tal 2011), partial matching is applied in an archeological context, where given a specific part of an unknown surface, the goal is to detect similar parts on other surfaces, regardless of the global surface this part belongs to. The key observation here is that, though isolated feature points often do not suffice, their aggregation provides adequate information regarding similarity. For collections of 3D artifacts represented as surface triangle meshes, salient points are first detected and their similarity is computed. Then, surfaces are segmented into meaningful components and their segments are matched. Next, given the above similarity measures, they are integrated. The goal is to compute consistent correspondences between the feature points. Finally, the similar region(s) in one surface is determined according to the correspondence established in the previous stage. Figure 11shows the results presented in (Itskovich & Tal 2011) for the domain of archaeology, in which the data is very noisy, and hence challenging. Figure 11(a) shows the detection of a Greek letter (A) extracted from Hellenistic stamps even when the letters may differ in shape and the scale ratio is unknown. In Figure 11(b) the query is a cupid from a Hellenistic oil lamp and the method matches this query to the cupids on a different oil lamp. The poses, as well as the shapes of the matched cupids differ, i.e., the query cupid has hair while the matched cupids do not, the matched cupids have wings while the query does not, etc.

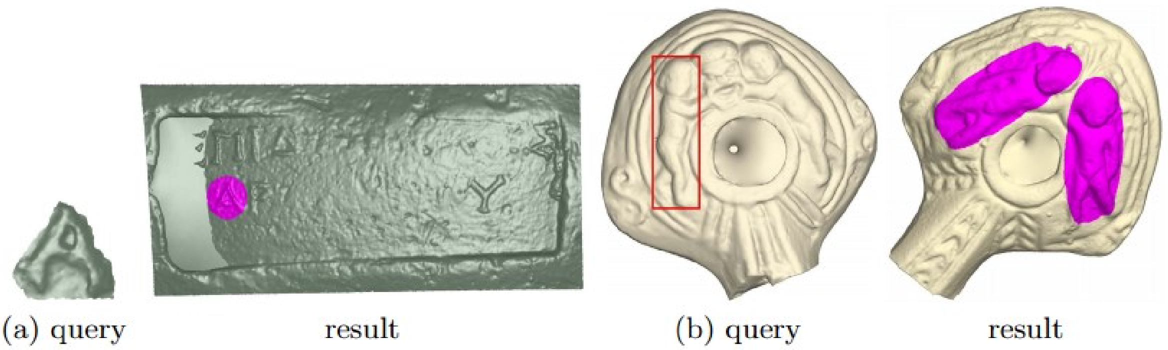


Figure 11 Partial similarity results on non-identical inputs. (a) Detecting a letter extracted from a template; (b) Searching for cupid-like shapes in collections of Hellenistic oil lamps (from (Itskovich & Tal 2011)).

Partial matching is also a key issue for the reassembly and geometric auto-completion of fragmented CH objects. As archaeological fragments are often weathered or chipped, exact geometric quantities like arc-length and differential properties like curvature are very noisy and will not exactly agree (Willis & Cooper 2008). For this reason, it is worth to consider similarity (matrices) between feature curves and surfaces (Willis & Cooper 2008). In (Gregor et al. 2014), a set of existing reconstruction methods is selected according to their potential in processing CH objects; methods are then combined to face with different types of physical defects that are encountered in CH artefacts, by proposing a repair workflow for 3D digital representations of CH objects. The workflow accommodates an automatic reassembly step, which can deal with fragmented input data. It also includes the similarity-based retrieval of appropriate complementary object data, which is used to repair local and global object defects. In (P Mavridis et al. 2015), the reassembly of fractured 3D objects is tackled from a geometric matching perspective, by proposing a pipeline for the automatic solution of the problem, where an efficient and generic three-level coarse-to-fine search strategy is used for the underlying global optimization.

Since the nature of CH data calls for methods dealing with multimodal information in combination (e.g., pattern, texture and reflectance), several works complement the geometric analysis with pattern/colour information about the outer/inner facets, in order to disambiguate the re-assembly between the subparts (Willis & Cooper 2008). For instance, the work (Kleber et al. 2010) combines features extracted from geometry, marble texture, thickness of marble fragments and location at the excavation, to reassemble the Ephesos marble plates. In this way, it is possible to overcome the limitation of considering only exact puzzle techniques that is not suitable for the eroded border and missing fragments (gaps) that characterize this dataset. Figure 12 shows 3 fragments that belong together. In the example, texture synthesis fails when used alone, see for instance the extended (red) lines in Figure 12 that show the predicted and the real grain flow. However, the thickness of the third matching fragment helps to identify the site of fracture and the direction of the grain flow (marble is broken at different rock strata). Indeed, the thickness of the fracture site is declared to be a stable feature due to the manufacturing process. Other approaches that combine geometric and photometric information are (Oxholm & Nishino 2011) and (Arbace et al. 2013).

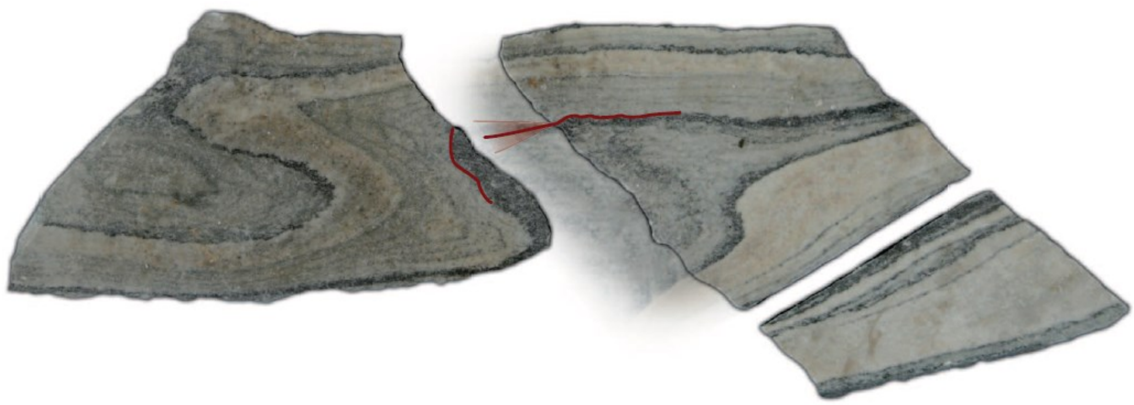


Figure 12 Three fragments of the Ephesos marble plates (image from(Kleber et al. 2010)**)**

Shape Completion can refer to the process of filling any missing part from a digitized dataset, see also (Pavlos Mavridis et al. 2015). In the context of cultural heritage, object completion algorithms can be used to suggest plausible completions for objects with missing parts. When used in conjunction with fractured object reassembly algorithms (Huang et al. 2006)(P Mavridis et al. 2015), this allows for the creation of automated repair systems that can present to the archaeologist a number of plausible reassemblies and completions, in order to provide visual aids for restoration tasks. The digital expansion of the input object can even be exploited for the fabrication of the missing parts to physically complete the artefact. In (Pavlos Mavridis et al. 2015), a method for the completion of partial globally-symmetric 3D objects is presented, based on the detection of partial and approximate symmetries in the incomplete input dataset. Symmetry detection is formulated as a constrained sparsity maximization problem, which is solved efficiently using a robust RANSAC-based optimizer. The detected partial symmetries are then reused iteratively, in order to complete the missing parts of the object.

A further application of shape completion is model restoration and repairing. Optical 3D acquisition systems often produce incomplete datasets, due to occlusions, unfavourable surface reflectance properties or geometric restrictions that prevent capturing the target object from certain angles. For example, in (Harary et al. 2014) a user-guided, semi-automatic approach is presented to complete large holes in a 3D object provided in the form of a triangle mesh. As the reconstruction of the missing features in such holes is usually ambiguous, the user is allowed to indicate constraints by providing four points per important feature curve on the mesh. The algorithm interpret this input as an indication of an important broken feature curve. The completion is formulated as a global energy minimization problem, with user-defined spatial coherence constraints, and allows for completion that adheres to the existing features. Shape completion for architectural artefacts was covered in the PRESIOUS project (Papaioannou et al. 2015); it is not part of GRAVITATE.

4.2. Geometric mating algorithms

Shape mating involves the (preferably exact) fitting of two shapes; within GRAVITATE, this is typically along a fracture facet of the object. It has some similarity with *partial shape matching* (see Table 1) and some of the literature invokes partial shape matching algorithms to perform shape mating of essentially complementary surfaces. Shape mating is also related to *registration of shapes* (again see Table 1) as for instance performed in the alignment of different scans of the same object to produce a single 3D representation, see Section 4.2.24.2.2. The field of *mathematical morphology* described in Section 4.2.3 is possibly best suited to describe shapes in contact.

Table 1 Borrowing from existing problem solutions for reassembly.

Aspect	**Reassembly CH artefacts**	**3D scan registration**	**Partial shape matching**
noise	one-sided, considerable	two-sided small	mesh taken as truth
format	meshes	point clouds/meshes	Meshes
matching likelihood	to be determined	assured	to be determined
data overlap	only on fracture	10-80%	on the part chosen
fit measure	Euclidean with gaps allowed	Euclidean	Euclidean and spectral
initialization	needed	needed	none
number of objects	can be large	small	one-to-several
poses 2D/3D	some useful 2D cases	3D essential	typically 3D
deformation	rigid	rigid	may be flexible

Aspect	Reassembly CH artefacts	3D scan registration	Partial shape matching
surface	fractal	fairly smooth	smooth

Before entering into the constituent contributing fields for fracture mating, we review the current state of the art in CH mating and its successes and shortcomings.

4.2.1. Geometric mating

True 3D geometric mating is hard to do. A system for freshly broken artefacts was developed in (Huang et al. 2006), see Figure 13. The geometric features used were smoothed local curvatures, computed efficiently; geometric hashing techniques were employed to make the search among candidate complementary matches efficient, and the ICP algorithm (Iterative Closest Point, see e.g. (Pottmann et al. 2006) for definition and analysis) for the actual mating. An almost complete reconstruction of a non-abraded broken statue was performed. Though much quoted, the techniques do not appear to have been copied by others. A recent reassembly system for archaeological artefacts (Mellado et al. 2010) relies much more on interactive visualization techniques to assist a restorer digitally.

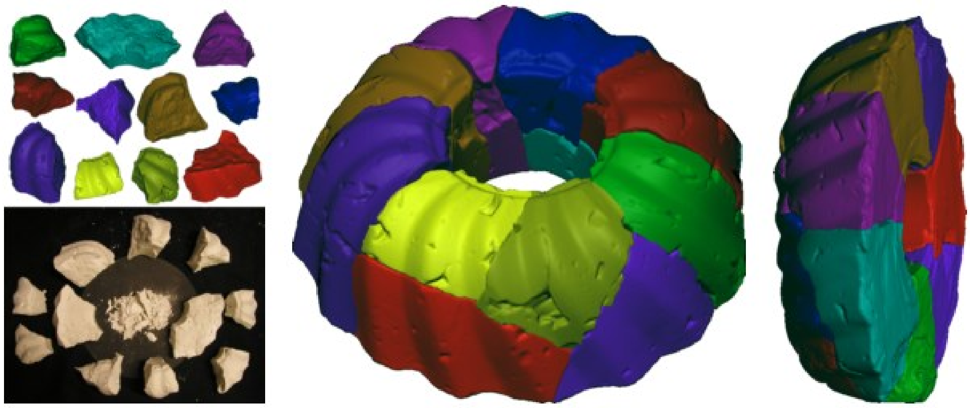


Figure 13 Automatically reassembling a freshly broken object, from (Huang et al. 2006).

The main recent source for up-to-date reassembly references is the PRESIOUS D4.4 Reassembly and Object Repair Methodology Report (Papaioannou et al. 2015); the focus is shape repair of architectural artefacts, with reassembly as a subtask. It refers to a number of very recent posters and short papers to emerge from the PRESIOUS project, of which we now discuss the ones applicable to our artefacts (some of their methods on symmetry completion and shape continuation presuppose architectural elements), plus a selection of results achieved by others before or parallel to PRESIOUS

The results of (P Mavridis et al. 2015) appear to solve the problem of reassembly. Those results are uncommonly good compared to previous literature (see Figure 14) such as (Huang et al. 2006; Papaodysseus et al. 2008; B. J. Brown et al. 2008), and are achieved by a somehow sensibly chosen discretization of the descriptors to allow fast convergence of ICP-like mating algorithms. Unfortunately, so far the algorithms have not been described in great detail, and even though this was a publicly funded project after which one would have hoped for open source code, only binaries are to be delivered (in January 2016). This means that any user who wants to extend these methods would have to re-implement them. Besides implementation details, a statistical analysis or benchmarking is missing, so we cannot assess whether reimplementation will be worth the effort.

Figure 14 Automatic reassembly in the PRESIOUS project (Papaioannou et al. 2015)**.**

It is important to perform the mating only on the fracture facets of an artefact. The segmentation of a mesh into facets is described in the PRESIOUS poster (Andreadis et al. 2014), see Figure 15. It is done by a measure of the median of the variance of the sphere volume integral; moreover, this is computed by ideas from a fast obscurance algorithm (from computer graphics); however, precise details are lacking, and source code is not available. Other references for faceting like (Wang & Yu 2011) typically presume much cleaner, industrial artefacts as their field of application, so are less suited in our CH setting. Earlier work is (Papaioannou et al. 2002), a sensible paper based on minimization of an (incidentally erroneously expressed) slope-based matching error computed with z-buffer techniques and minimized by enhanced simulated annealing (ESA).

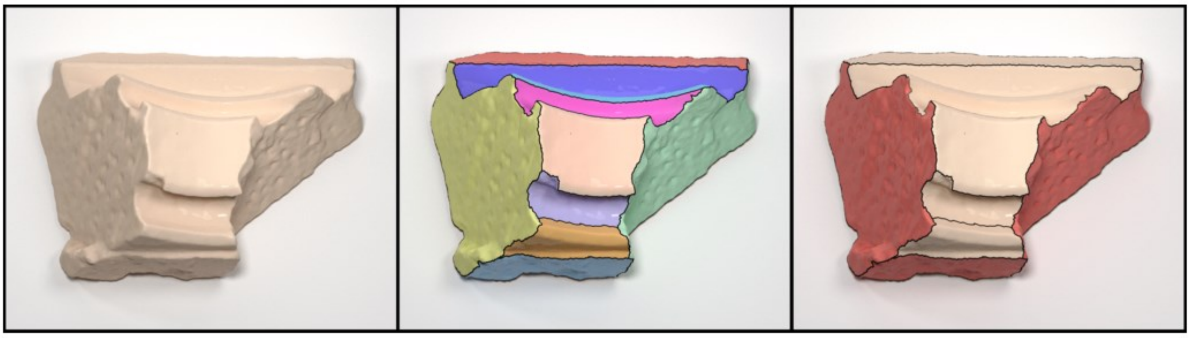


Figure 15 Facet extraction by the method of (Andreadis et al. 2014)**, figure from** (Papaioannou et al. 2015)**.**

(M A Savelonas et al. 2014) deals with partial matching/retrieval for pottery point clouds based on persistent feature histograms (PFH Section 3.1.1.1, by applying machine learning methods (like bag of visual words for Gaussian mixture models) to address the retrieval; the method is evaluated on the Hampson collection and works reasonably well.

(Gregor et al. 2015) suggest the construction of an artificially broken dataset for benchmarking. This may be sensible, though their suggested artificial fracturing process for their pottery vases by noisy spheres still has some unnatural physically improbable features.

Some papers restrict themselves to more special cases of fitting, with a more restricted number of degrees of freedom in the relative positioning: frescos (with fewer 3 DoF) (B. J. Brown et al. 2008;

C Papaodysseus, D Arabadjis et al. 2012) with or without patterning, or pottery with rotational symmetry (2 DoF) (Arbace et al. 2013). These often use patterning information on the 'skin facets' of the artefacts. With frescos, the patterned parts are often already done by humans, and only the non-figurative jigsaw puzzle may remain; for this reason we treat the literature on 2D puzzles in Section 4.3.

4.2.2. Mating as registration of 3D point clouds

Geometric mating shares some algorithmic characteristics with the registration of 3D point clouds. We first summarize that field and its techniques, and then investigate their relevance for GRAVITATE ReAssembly.

When *registering 3D scans*, one typically has the raw data in a point cloud format. The cloud will not form a watertight mesh, due to partial scanning. It may contain disjoint parts of an object (for instance, a part of a jug handle that was not fully seem) that will yet play an important part in the registration. The noise in the data is small relative to the registration displacement, and typically two-sided: noise points may end up slightly in front or in the back of the actual surface. The number of scans to be registered is usually small (4-10), and it is known that they should form a single object. The overlap between matched scans may be anything from 10 to 80 per cent. There is a clear local error measure: find the transformations that minimize the Euclidean distances of overlapping parts; preferably between the actual surfaces, but between closest sample points may be good enough.

An overview of the registration algorithms is (Gary K L Tam et al. 2013), which also includes non-rigid registration not currently interesting to GRAVITATE. Typical algorithms to solve registration are variations of ICP (Iterative Closest Point) (Pottmann et al. 2006), possibly requiring some coarse initial alignment to avoid getting caught in a local minimum of suboptimal registration (P. Mavridis et al. 2015). That initial alignment may be based on some hierarchical structure in shape description by means of local geometric features (such as extended Gaussian images, feature point histograms (Radu Bogdan Rusu et al. 2009)); or a coarse-to-fine fitting process (such as simulated annealing (P. Mavridis et al. 2015)). Matching of the scans appears to be mostly done purely on the basis of geometry, rather than of spectral information such as object colour. The scanning setup is typically chosen such that this is permitted.

When mating two non-abraded fractured shapes, the points on the fracture facets are very much like the point clouds in 3D scan registration: there is partial overlap, because only the local fracture facets need to match to have good mating (P Mavridis et al. 2015). But whereas in scan registration there is simply no data in one of the clouds for the non-matching parts, in mating there is a lot of non-matching point data in both objects. Sometimes one can pre-identify the fracture facets geometrically (Huang et al. 2006) (Andreadis et al. 2014), see Figure 15; but when a fracture facet itself is fractured, knowing 'when to stop mating' is ultimately determined by the mating algorithm itself. This gets harder the more abrasion has occurred. Because of abrasion, one generally expects the middle of a fracture facet to match better than the border.

The local criterion to minimize a good match of fractures is no longer merely the Euclidean distance. Two fracture facets may mate very well even when there are dents in it, caused by abrasion or otherwise. This can be seen as one-sided 'noise', but it is more structural than that, and mathematically not dissimilar from identification of the points contributing to the facets themselves. One should take a method or an error criterion that is (almost) completely indifferent to this 'lack of data'; (P. Mavridis et al. 2015) suggest using non-Euclidean distance measures (Figure 16). On the other hand, extra material sticking out of the fracture should count heavily against the mating. In museum artefacts, remains of plaster from an earlier attempt at restoration could upset the process considerably, we have to assume for now that this does not occur.

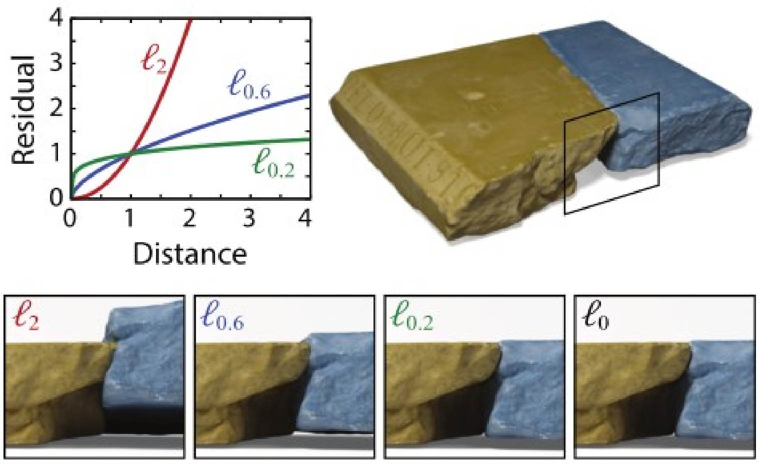


Figure 16 From (P Mavridis et al. 2015)**: the effect of non-euclidean metrics on the part alignment.**

The CH artefacts will mostly be represented as a watertight mesh, though this is not strictly necessary for an effective mating. If one is presented with a point cloud, some local neighbourhood structure permitting the selection of nearby points is rather necessary for computational efficiency. There are standard methods to convert point clouds into meshes, available in all point cloud libraries (such as pointclouds.org) to make the transition to mesh-based libraries (such as Meshlab meshlab.sourceforge.net/ and the references in Section 4.5.1).

Once several parts have been mated and form a 'hyperpart', the fracture surface may become very non-convex and ICP-like minimization of distances may acquire many local minima. On the other hand, intermediate poses may no longer need to be collision-free, since the mating need not take into account an order of assembly that is actually executable. There appears to be little literature on this issue (Huang et al. 2006) (Palmas et al. 2013) see Figure 1Figure 17, but perhaps the methods of (P. Mavridis et al. 2015) or (Combès & Prima 2012) though more generally developed for non-rigid alignment) would help convergence.

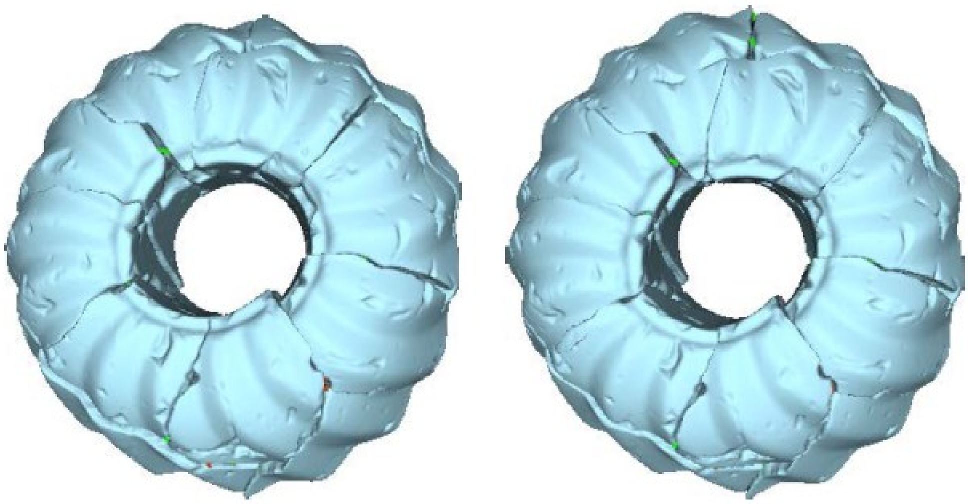


Figure 17 Enforcing global alignment improves individual mating (from (Palmas et al. 2013)**) somewhat.**

When mating an object broken in several pieces, the fact that eventually all fragments have to fit together is a heavy constraint. It may lead one to revisit and readjust earlier likely matches. While this also occurs in the need for closure of a set of 3D scans, the internal mating of fractures makes this a harder process, with possibly greater requirements on accuracy and consistency. Methods for closure of the total fit are treated in (Huang et al. 2006) and (Palmas et al. 2013), but do appear to be capable of improvement (see Figure 17).

4.2.3. The mathematics of contact

The complementary nature of a fit between parts, and the one-sided 'noise' (abrasion) that may occur while not affecting our confidence in a match are ill represented by casting the mating as registration or partial shape matching. The non-linear mathematics of mating just does not correspond to linear methods. Fortunately, this type of non-linearity has been studied before.

The mathematics of ideal shapes in exact contact has been worked out in a field called 'Mathematical Morphology', originating in the 1970s at the École des Mines near Paris as a formal way to describe sieves. As a standard form of 'non-linear filtering' it was algorithmically developed within image processing in the 1980s to build shape selective filters. Its elementary operation between shapes is called 'dilation' (or 'Minkowski sum' or 'offset meshes' in computer graphics and implemented in geometry processing libraries such as CGAL). The opposite of dilation is called 'erosion', this is a shrinking operation on a shape. Both operations also occur in the description of robot path planning, as determining the area a robot of a given shape can still cover despite the presence of certain known obstacles in the scene. From these elementary operations, one can construct a 'mathematics of fitting', with a corresponding suite of algorithms (Boomgaard et al. 1996). This finds application in numerical control of milling machines (where one needs to compute what orbit a finite bit of a certain shape has to move on to carve out a desired shape).

A systems theory of contact has been developed (Dorst & Van den Boomgaard 1994) (Dorst & van den Boomgaard 2000), and fast algorithms to compute dilations are available in various object representations (Gil & Kimmel 2002). There is a morphological scale space, in which objects can be described at various resolutions in a manner that partially preserves their potential contact in a well-understood way. To date, no one seems to have used this structured approach to the description and computation of mating of objects, even though the abrasion is very much a morphological perturbation of the original shape. We can set up hierarchical object descriptions with inheritance of the 'mating' correspondence between shapes, so that the possibility of mating at the most detailed level can be efficiently checked at the coarsest level of representation. The UVA partner intends to explore this possibility to improve existing mating algorithms, but the presence of noise in the data makes this a subject of PhD level original research. It is our hope that the presence of abrasion actually helps to highlight the advantages of morphological methods relative to the common linear ICP methods based on Euclidean distance matches, and the non-Euclidean ICP employed in (P. Mavridis et al. 2015).

4.2.4. The accuracy of CH scanning

What we can do with the scans of CH objects clearly depends on their accuracy. The quantitative analysis required by the ReAssembly task goes beyond the needs for simple visualization and documentation, which appears to be the role of most systems so far. While there is the manufacturer's data of scanners on the point clouds produced, there is only sparse literature on the accuracy of the resulting registered meshes (ter Haar et al. 1999) Figure 13 , and on repeatability (for dentistry scans (Nedelcu & Persson 2014)). To what resolution the fractal fracture facets are dependable and interpretable is unknown; most previous studies are on relatively smooth objects, for which geometric texture may be merely decorative or not quantitatively relevant since they may be merely used as input to realistic 3D gaming engines. An exception is PRESIOUS D2.1 (Pratikakis 2013) which in its erosion studies signalled the need for mating methods that work even in the context of relatively low-resolution scans, and viewed the high-resolution results of 3D-COFORM (Arnold 2013) as unrealistic in practice.

Figure 18 Scanning accuracy comparisons from (ter Haar et al. 1999).

Scanning CH artefacts is complicated by many factors: there is clearly not the possibility to prepare the object for scanning by spraying it with a matte white paint (as is common in scanning for gaming); mounting may preclude full scanning; the material itself may be hard to scan (marble is notorious in this respect (Guidi et al. 2009)), etc. There are some initial efforts to define scanning protocols in an attempt to get comparable accuracy from different sources (Moitinho & Barcelo 2012), and to produce calibration objects (Hess & Robson 2012), mimicking what is common in colour photography of artefacts. GRAVITATE will not deal with these issues directly, but follow the developments closely through her CH partners.

For GRAVITATE, we will initially focus on rather carefully scanned artefacts from museum collections in controlled circumstances (as for our Salamis use case), yet we plan to develop algorithms that can handle varying accuracy and study their performance.

The new H2020 project SCAN4RECO (which started October 2015)[1] is very relevant to these discretization issues, as is apparent from its unabridged title *Multimodal **Scan**ning of Cultural Heritage Assets for their multimodal digitization and preventive conservation via spatiotemporal **4**D **Reco**nstruction and 3D Printing*. We plan to establish a close collaboration with them.

4.3. Combinatorics in 2D puzzles of square pieces

In the shape mating for GRAVTATE's ReAssembly, we may not know beforehand whether the components will fit. This implies that we cannot devote all our computational resources to the detailed mating; it makes more sense to have a pass in which the possible success of mating is checked before actually doing it in detail. This combinatorial problem is partly addressed in a problem which is being studied in computer vision: that of solving a 2D puzzle consisting of one or more pictures cut into same-size square tiles. This way the geometric component of the ReAssembly task is removed, for better and for worse. On the one side, the geometry cannot be used for estimating the probability that components match, making the problem harder. On the other side, once it is determined which edge of one piece matches an edge of another piece, the problem of mating has a trivial solution. The Technion/Haifa partner has been successfully involved in this puzzle, both in the aspects of characterization of the colour variations along the boundaries of the pieces, and in the combinatorics of the order of attempted mating. The algorithms described below were able to deal successfully with large puzzles, generated from challenging images in which the many of the tiles are hard to match using colour descriptors alone.

Reconstructing a 2D image from its pieces is important in many applications, such as image editing and biology; in the cultural heritage domain it has been adopted for fragment re-assembly (B. J. Brown et al. 2008; Koller et al. 2006; Wei et al. 2010), to name a few. See also the state of the art (Willis & Cooper 2008). Though the problem is NP-complete (Altman 1990; Demaine & Demaine 2007), various solutions have been proposed. These are based on shape matching (Wolfson et al. 1988; Kong & Kimia 2001; Goldberg et al. 2002; Webster et al. 1991) or on a combination of shape matching and colour matching (Chung et al. 1998; Nielsen et al. 2008; Kosiba et al. 1994; Yao & Shao 2003; Makridis & Papamarkos 2006; Sagiroglu & Erçil 2006).

Below we focus only on colour matching. The problem was introduced by (Cho et al. 2010) and is still a lively problem of research. The problem can be decomposed into two sub-problems: matching of two parts and placement. In the sequel, we discuss these components separately.

4.3.1. Placement

Most of the existing methods to reassembly the square-piece puzzle focus on reassembly, i.e., composing parts of a single image. Pomeranz et al. (Pomeranz et al. 2011) introduce the first fully-automatic square jigsaw puzzle solver that is based on a greedy placer. Gallagher (Gallagher 2012) generalizes the method to handle parts of unknown orientation. Son et al. (K.Son et al. 2014)

[1] http://scan4reco.eu/scan4reco/

demonstrate a considerable improvement for the case of unknown orientation, by adding ”loop constraints” to (Gallagher 2012).

Rather than pursuing a greedy solver, Sholomon et al. (Sholomon et al. 2013) present a genetic algorithm that is able to solve large puzzles.

In (Paikin & Tal 2015) in the first attempt to handle the case of Re-Unification. In particular, it concurrently solves multiple jigsaw puzzles whose pieces are mixed together. Moreover, it handles puzzles with missing pieces, unknown puzzle size, and unknown orientation of the parts. Within the realm of puzzles, this the closest challenge to the real-world re-unification in archaeology in which the conserver is given a set of fragments which belong to possibly several incomplete objects and has to reassemble them.

We note that in (Cho et al. 2010) the placement of the pieces is done by maximizing a global optimization problem using loopy belief propagation. Since loopy belief propagation can fall into a local minimum, they run loopy belief propagation three times with random seeds and pick the best reconstruction in terms of the reconstruction accuracy.

The algorithm discussed in (Yang et al. 2011) improves the results by using a particle filter.

4.3.2. Matching two parts of the square-piece puzzle

This problem can be divided to three components: the feature vector, the dissimilarity measure, and the way to choose the next piece to assemble, given the former. Hereafter we discuss each of these components.

Choosing the next piece: As most algorithms are greedy, at every stage of the algorithm a “good” piece has to be selected. This is often done by iteratively selecting a piece to be assembled, utilizing a compatibility function.

(Pomeranz et al. 2011) defines the concept of ”best buddy”, which gives a good indication regarding the reliability of the match. Two parts are best buddies if both “agree” that the other part is their most likely neighbour in a certain spatial relation. The best buddies metric for a given approximated solution represents the ratio between the number of neighbours who are said to be best buddies and the total number of neighbours.

It is proposed in (Pomeranz et al. 2011) that in each step the placer chooses the empty slot and unallocated part which “agree” to be best buddies among all empty slots and remaining unallocated parts. If there is more than one possibility, or none at all, the algorithm then chooses the part with the lowest dissimilarity to its candidate neighbours.

In (Paikin & Tal 2015) the selected piece is the one having the highest mutual compatibility function. This piece not only optimizes the best buddy notion, but is also guaranteed to be reliable, as it dissimilarity to the best neighbour is much better than the dissimilarity to the second-best.

4.3.3. The relation to reassembly in cultural heritage

Figure 19 illustrates the importance of colour matching. The Figure shows two mated pieces from the Salamis collection that are now in the British Museum. It can be seen that in order to match these pieces correctly, the colour and the texture are as important as the geometry

Figure 19: Two Salamis fragments mated using colour and geometric features

Even though the problems addressed in the papers above on the square-piece puzzles are not identical to the challenge of reassembly in cultural heritage objects, some of the ideas can inspire solutions to these problems.

This can be viewed at several levels:

1) First, the colour matching techniques can be used as part of the matching component in the algorithm, as done for the matching of the pieces in the figure. The colour matching will be a component of the general matching, which includes other considerations, such as geometry or meta-data.
2) Second, when there are more than two pieces, the order in which matching hypotheses are tested, can be vital to the final composition. This is orthogonal to the manner in which the matching is performed. How to choose the pieces, again, can be inspired by the methods described above.
3) Finally, when there are many pieces, the general strategy of placement should be carefully designed. It should take into account the number of artefacts expected (re-unification is the common setting in excavations) and the fact that there are many holes in the complete artefact. These considerations were also taken into account in developing some of the algorithms above and their relevancy should be studied.

4.3.4. Reassembly attempts of 2D objects in cultural heritage

There have been several attempts in performing reassembly on planar broken archaeological artefacts. The difficulty of the task depends on completeness of the fragmented object, its erosion state, the number of pieces and the colours of the pieces and how easy it is to produce possible matches between them.

In the 2D fitting cases, obvious matches have been found manually by matching the scenes depicted on the fresco or the map; the geometrical mating was to be performed on the remaining bland pieces. The cases studied are highly incomplete, and heavily eroded.

(Koller et al. 2006) treats the ancient planar map of Rome Forma Urbis Romae. In this case only 10% of the map exists, broken into 1186 pieces. The pieces are not coloured and are therefore matched by analysing the incisions and by complementary matching of the fractures. A semi-interactive system was built that proposed matches and displayed the similarity, to aid browsing of the collection for matches. At the time the report was written 20 new high probability joins were found.

(B. Brown et al. 2008) (Brown et al. 2010) devised such a semi-interactive system for frescoes. They give good tips for scanning practices. They characterize the overall fracture curve as a ribbon, with the local variations encoded by smoothed normal vectors which can then be matched with the counterpart fragments using standard image processing techniques. They have tested their system on Theran, Tongeren and Akrotiri frescoes, and on the Roma dataset (this type of benchmarking is rare). The matching algorithm suggested 6103 possible matches. 4000 of them were manually tested using the interactive system, verifying 18 correct matches of which only four were previously known. This is a great improvement compared to performing the matching manually.

In (Skembris et al. 2012), a method was suggested for matching 2D coloured pieces. Pieces are matched using boundary colour information, shape information. The algorithm was tested on fragmented wall paintings excavatec at Akrotiri, Santorini, Greece.

4.4. Current methods for ReAssociation

A recent important trend in CH is the use of 3D morphological analysis for the interpretation, authorship attribution and/or motivation of works of art, as surveyed also in (Pintus et al. 2015). In the following we present some of the current contributions in this direction. They illustrate how a mixture of standard techniques from shape analysis can be used ingeniously in an archaeological context. Our GRAVITATE dashboard design should be inspired by this kind of usage.

The paper (Dellepiane et al. 2007) shows how technologies of 3D graphics can be exploited to support the hypothesis of attribution proposal of a small bronze horse to the Renaissance period. The paper starts from Fondersmith's attribution hypothesis which states that a Leonardo metalpoint is an optically-traced drawing of a small bronze horse which, therefore, can be considered a work of art by Leonardo. Using acquisition (3D Scanning) and visualization techniques, Dellepiane et al. provide a new objective evidence of this attribution hypothesis. In particular, the global shape of a digital model of the small bronze horse is superimposed, via image-to-3D-model mapping techniques, to the silverpoint drawing. The comparison is semiautomatic as it reduces to the problem of registering an uncalibrated image to a 3D model which, in general, requires the user intervention. Indeed, in order to estimate some internal values (such as focal length, lens distortion) of a camera, existing algorithms require some correspondences between the image and the 3D model, i.e. the selection of some corresponding point pairs. Even though no numerical proofs of the quality alignment is provided, the results obtained in a metrically objective framework and the surprising alignment yield new evidence for the continuation of the debate on this attribution hypothesis.

The work (Muzzupappa C. Bruno M. Gallo F Mattano' Ruggiero R.M. 2012) addresses a problem of interpretation of a work of art, the Riace Bronzes, one of the most important masterpieces of Magna Graecia archaeology. Since many years, the interest for the Bronzes is not only related to their extraordinary workmanship and highly refined taste, but also to the great mystery that surrounds their history and origin. It is not yet certain who was the author (or the authors) of these works of art, nor whom the two statues portray. An important detail for identifying their identity is represented by a controversial part of the Bronzes: the right hand of bronze "A". From a detailed analysis of its position, enjoying the support of various iconographic/historical studies, the most accredited theory is that the right hand is grasping a lance (or a sceptre or a staff) leading to the hypothesis that bronze "A" is a warrior or a famous person. In this paper, 3D scanning techniques are exploited to study this hand detail. In particular, a digital model of the right hand is provided via an acquisition procedure, carried out by 3D scanner, and a subsequent 3-dimensional reconstruction. A detailed geometrical analysis of the 3D model is performed manually to provide archaeologists some quantitative evidence about the hand shape and the grasped object. Namely, measurements of cross-sections, deformations and internal volumes are registered. Such measurements lead to the conclusion of the presence in the hand of a cylindrical element, with an about 30 mm maximum diameter, always contained inside the volume defined by 14 circumferences. Starting from these data, the most plausible object reconstruction is that of a spear, 2200 mm long with a 30 mm diameter, which is shown in the paper. The described approach thus helps to enforce the hypothesis about the object in the hand, which is one of the fundamental steps to establish the statue's identity.

(Nadel et al. 2015) uses a combination of multiview reconstruction and geometrical analysis for addressing archaeological study of prehistoric bedrocks. Bedrock features such as hewn mortars and cupmarks are contained in the archaeological records of the southern Levant since at least one century. Their diversity has been related to chronological as well as functional variations (changes in subsistence patterns, tradition from nomadic hunting to sedentism). Traditionally, archaeological documentation for bedrock features has been limited to hand drawings supported by photography, yielding a graphical description of the feature, usually not very accurate. In (Nadel et al. 2015)] a protocol for high resolution recording, documentation and characterization of bedrock features is presented, which exploits photogrammetry, 3D model generating and geometrical analysis. A dense 3D model (with a millimetre resolution) is constructed in each case and characterized with geometrical measurements, such as volume, sections and vertical and horizontal reflective symmetries. Measurements of dimensions and volumes coupled with a geometrical analysis of shape and symmetries yield more information about each feature, thus bringing new valuable intra- inter-site comparisons and a better understanding of their past functionalities.

(Docchio et al. 2005) addresses the problem of the collocation in the right temporal and spatial framework of the "Vittoria Alata", a 2m-high, bronze statue located at the Civici Musei di Arte e Storia in Brescia. Since its discovery in 1826, there has been no doubt that the "Vittoria Alata" was originally an Aphrodite fused in Rome during I Century a.C., and successively (during the

Vespasian age) transformed into a Victory by adding the wings. Recently, a new conjecture on the correct temporal and spatial collocation of the statue has been proposed by Moreno (see (Moreno 2001)). By comparing the statue with those of Aphrodite of Capua, the Venus of Milo and the Aphrodite Urania of Fidia, on the one hand, and those of an Aphrodite described by Apollonio Rodio between 250-240 b.C. on the other, Moreno suggested that the "Vittoria Alata" could be an Aphrodite fused in Greece during III century b.C. and successively carried to Rome as war spoil where it was transformed into a Victory by adding the wings. In order to verify this new hypothesis, archaeologists suggested to study the overall proportions of the statue in order to infer the archetype from which they have been generated. (Docchio et al. 2005) addresses the problem using 3D modelling technique. To this aim, the acquisition of the statue is performed by using a three-dimensional vision system, called OPL-3D, which is based on active triangulation and on the projection of non-coherent light. In order to fulfil the archaeologists' requirement of a very high-definition of some details (such as the head) despite the very big extension of the surfaces to be measured (if we consider, for instance, the overall height of the statue or the dimension of the wings), the authors choose to diversify the geometry of the system to obtain different resolutions for the various part of the statue. A very detailed triangle mesh (16 million triangles) is created and edited for optimal performance. With this representation, a list of measurements between triangle pairs is straightforward to obtain and fundamental to facilitate archaeologists' study of the statue's proportions.

4.5. Software and datasets for ReX tasks

We include here a set of pointers to resources used in the world of shape analysis. The choice of items has been based on the authors' experience and it is meant as a starting point. The list below includes software, repositories of 3D models. Finally we mention that several benchmarking initiatives exist in the multimedia domain, such as TRECVID (Smeaton et al. 2009; Smeaton et al. 2006; Over et al. 2012) and IMAGECLEF (Mueller et al. 2010), in the 3D shape retrieval SHREC (Veltkamp et al. 2006), while much less is done in the cultural heritage domain.

4.5.1. Software

Quite a number of readily available tools exist for the interaction with and computation on 3D meshes (and point clouds). Many of the descriptors of Chapter 4 are represented by efficient methods computing them, and even shape matching functionality as discussed in Chapter 5 is often provided. We briefly mention the most popular or most suitable platforms.

Point Cloud Library (PCL): it contains several local descriptors and global descriptors such as Spin Images (Johnson & Hebert 1998), Signature Histogram of Orientations (SHOT) (Salti et al. 2014), Fast Point Feature Histogram (FPFH) (R B Rusu et al. 2009) and Rotational Projection Statistics (RoPS) (Guo et al. 2013) (http://pointclouds.org).

MeshLab: it is an open source system for the processing and editing of unstructured 3D triangular meshes (http://meshlab.sourceforge.net).

Compare: it is a 3D point cloud (and triangular mesh) processing software, originally designed to perform comparison between two 3D points clouds (such as the ones obtained with a laser scanner) or between a point cloud and a triangular mesh. Afterwards, it has been extended to include many processing algorithms (registration, resampling, colour/normal/scalar fields handling, statistics computation, sensor management, interactive or automatic segmentation, display enhancement, etc.). CloudCompare is a free software, available under the GNU General Public License (GPL) (http://danielgm.net/cc/).

CGAL: the computational geometry algorithm library (http://www.cgal.org).

meshLPB: a local shape descriptor currently proposed for 3D texture analysis and recognition, the code is freely available at http://www.mathworks.com/matlabcentral/fileexchange/48875-mesh-lbp.

ReMESH: an editor to repair and edit triangle meshes (http://remesh.sourceforge.net/).

meshFIX: it converts a raw digitized triangle mesh to a clean mesh where all the occurrences of a specific set of "defects" are corrected (http://sourceforge.net/projects/meshfix/).

OpenMesh: is a data structure for representing and manipulating polygonal meshes, not only triangle meshes that can be included in more sophisticated tools for 3D object representation and manipulation (http://www.openmesh.org/).

OpenFlipper: is an example of geometry modeling and processing framework developed on the top of OpenMesh (http://www.openflipper.org/).

Toolbox graph: http://www.mathworks.com/matlabcentral/fileexchange/5355-toolbox-graph. This toolbox contains useful functions to deal with graphs and triangulations, in particular it allows the computation of mean and Gaussian curvatures and offers several tools originated from spectral theory (Laplace-Beltrami operators, diffusion kernels, etc.).

MeshLP: Approximating Laplace-Beltrami Operator from Meshes, http://www.geomtop.org/software/meshlp.html. MeshLP offers an implementation of the mesh Laplace operator which approximates the Laplace-Beltrami operator of the surface approximated by the input triangle mesh. On the top of MeshLP, the **Heat Kernel Signature (HKS)** is publicly available at http://www.geomtop.org/software/hks.htm

Sparse ICP: http://lgg.epfl.ch/sparseicp is a tool for rigid registration of two geometric data sets represented as point clouds.

Multi-Scale Segmentation of Triangulated Surfaces: This tool implements the multi-scale curvature characterization of a closed surface and automatically extracts the features that can be described as generalized cylinders or cones, http://saturno.ge.imati.cnr.it/ima/smg/plumber-web/plumber-web.html.

SI-HKS & ISC: http://vision.mas.ecp.fr/Personnel/iasonas/descriptors.html. These tools are freely available examples of non-rigid shape descriptions that have been applied to similarity analysis of images and 3D shapes.

TransforMesh, **MeshHOG** and **MVViewer**: http://mvviewer.gforge.inria.fr/. This repository contains an implementation of a 3D feature detector (MeshDOG) and a 3D feature descriptor (MeshHOG) for uniformly triangulated meshes, invariant to changes in rotation, translation, and scale. Both the detector and the descriptor are able to capture the local geometric and/or photometric properties.

4.5.2. Relevant Datasets

There is a growing number of openly available datasets with scanned 3D CH artefacts. We are especially interested in broken objects (rather than for instance the David scan in Digital Michelango) since we can use those to test our algorithms. We mention a few, with links.

- Vienna data set of 3D puzzles, in cdm and pcd formats
- http://www.dmg.tuwien.ac.at/fg4/3dpuzzles.html
- A 3D Pottery Content Based Retrieval Benchmark Dataset (overview and links to 17 datasets on pottery) http://www.ipet.gr/~akoutsou/benchmark/
- Stanford Digital Forma Urbis Romae Project (but permission required for actual hi-res data) http://formaurbis.stanford.edu/
- EUROPEANA: the continuing repository of cultural heritage of diverse kinds, with some metadata attached to enable search. Most objects in this collection are not broken. http://www.europeana.eu/portal/
- PRESIOUS project data sets, pieces of Nidaros Cathedral. Contains the fragments of a broken arch. http://www.presious.eu/resources/3d-data-sets
- The Shape repository of the Visualization Virtual Services (VVs) "VISIONAIR": http://visionair.ge.imati.cnr.it/ontologies/shapes/ , that also offers services for preparing digital shapes for visualization purposes, and shape search facilities.
- Virtual Hampson Museum (VHM, http://hampson.cast.uark.edu). The dataset is composed of 442 models (coloured 3D triangle meshes), 395 of them available for download, representing many American Indian artefacts largely produced from 350 to 600 years ago. Most of models are grouped into three geometric classes, namely bottles (189 elements), bowls (112 elements), and jars (73 elements). Smaller groups have also been detected by VHM experts, for a total of 13 geometric classes.
- Cyprus Institute database of artefacts, containing also the Salamis collection of our use case: http://oc.cytera.cyi.ac.cy/index.php/apps/files?dir=//Cyprus%20Museum

4.6. Discussion

To summarize our findings in this 'Shape' part of the survey, there is a rich variety of techniques for describing shape. Although these were often designed for other purposes than the GRAVITATE Re-X tasks, they can be readily employed within the project.

Some descriptors represent rather closely the 'natural' human perception of shape, and they may well correspond to the objective descriptions an archaeologist would give. Often those are compactly abridged in computational structures that would likely be hidden from the GRAVITATE user, and merely employed internally in efficient algorithms processing the shape (mating and matching). No descriptors as yet reflect the utilitarian classification an archaeologist might give of the shape of an object, so such translations to its semantics will have be studied within GRAVITATE. Similarly, the probably useful classification of parts of a digital mesh as for example an ‘ear’ (be it of a jug or of a human) could be developed based on the more elementary descriptors, augmented by some user feedback within the GRAVITATE dashboard user interface. This would provide a natural bridge between the ‘shape’ and the ‘semantics’ of an object, and would be a focus of research interest for the GRAVITATE partners, see also discussions in Chapter 7.

For the mundane task of ReAssembly of a collection of broken abraded objects, there appear to be techniques available in literature when the abrasion is small, and the parts not too many. However, many details are missing in the publications, the software methods are not publically available, and they appear not to have been tested thoroughly. We will have to redevelop part of this field, taking into account abrasion, and permitting template based reassembly for not truly complementary shapes.

5. State of the Art in Semantic Matching

In this section a selection of techniques relevant to the semantic matching problem are reviewed. Graph matching is a well-studied area that can be applied to geometric matching as well as semantic matching, as long as the domain data be presented as a set of directed graphs. As such the review of graph matching is wider than just the cultural heritage sector. For other areas, including natural language processing, 3D object semantic descriptors, semantic mapping and semantic alignment, only the cultural heritage target domain is considered.

5.1. Graph matching

The fields of pattern recognition and natural language processing have both seen an uptake in recent years of graph matching approaches (Foggia 2014; Mills 2014). This has a great relevance to the semantic matching problem in GRAVITATE since many of the metadata structures used by this project can be represented in a graph-like structure. In addition free-text fields within metadata could be processed using a combination of natural language processing and domain vocabulary lookup to further enrich the existing metadata graphs.

Looking across the range of approaches in this section there is in general a trade-off between the low information loss but computationally expensive graph edit distance approaches, and the high information loss but computationally inexpensive vector-based approaches. This trade-off needs to be considered in the context of GRAVITATE dataset sizes and the complexity of metadata graphs.

5.1.1. Exact graph matching

Exact graph matching is the search for a mapping between the nodes of two graphs which is edge preserving, in the sense that if two nodes in the first graph are linked by an edge, the corresponding nodes in the second graph must have an edge, too. Many techniques for exact matching have existed for years. However most work on problems such as pattern recognition require support for inexact matching, and the ability to both handle some level of noise and scale up to support matching of complex graph structures.

Examples of exact matching algorithms include tree search variants (the main approach), random walk and decision trees. An example of a tree search algorithm can be found in (Ullmann 2011), where the classic subgraph isomorphism approach is improved using ideas motivated by the wider binary constraint satisfaction problem. Random walk approaches such as (Gori 2005), inspired by Google's PageRank algorithm, can enhance a graphs topological features at node level. Decision trees (Weber 2011) can be used to significantly reducing the spatial complexity for graphs whose nodes have a small number of different labels.

5.1.2. Inexact graph matching

For inexact matching either (a) the concept of a mapping function is extended to allow mapping to a null node or (b) weighted graphs matching is supported with graphs represented through adjacency matrices and elements in these matrices expressed using a continuous weight for each relation.

Many inexact matching algorithms are formulated as an approximate way to compute the graph edit distance (GED). Graph edit distance computes the distance between two graphs on the basis of the minimal set of edit operations (e.g. node additions and deletions, etc.) needed to transform one graph into the other one. In practice most inexact methods compute the graph edit distance by finding a matching for the nodes that are preserved by the edit operations (i.e. those that are not added or removed, but possibly have their label changed). Usually the outcome of the matching algorithm is not only an indication of the distance between the graphs, but also the matching that is supposed to minimize the value of this distance.

Examples of inexact matching include tree search variants, continuous optimization approaches and spectral methods. Tree search variants have been around for many years. A typical example is (Sanfeliu 2004) which matches attributed relational graphs based on graphs enriched with constraints on the joint probabilities of nodes and edges. The graph matching problem can be reformulated as a continuous problem by relaxing some constraints, allowing optimization algorithms to be used in an otherwise discrete domain. Recent examples include (Chang 2011), where a graduated assignment graph matching algorithm is extended to work on hypergraphs instead of graphs, and (Sanromà 2012) where a probabilistic graph matching method is used based on the Expectation Maximization (EM) algorithm. Spectral matching methods are based on the observation that matrix representations of two isomorphic graphs (e.g. adjacency matrices) have the same eigenvalues. Recent examples include (Escolano 2011) using graphs represented as a bag of partial node coverages using spectral features and (Duchenne 2011) using spectral features in hypergraphs.

5.1.3. Graph embedding and kernel approaches

Graph embedding means either a technique that (a) maps the nodes of a graph onto points in a vector space, in such a way that nodes having similar structural properties will be mapped onto points which are close in this space, or that (b) maps whole graphs onto points in a vector space, in such a way that similar graphs are mapped onto close points. The second approach is the most popular. Embedding approaches can be isometric, spectral, sub-pattern or prototype-based.

Isometric embedding starts from a distance or similarity measure between graphs, and attempts to find a mapping to vectors that preserve this measure. An example (Bonabeau 2002) is the self-organizing map (SOM) unsupervised neural network approach, which uses competitive learning to map graphs onto a bi-dimensional plane.

Spectral embedding uses properties related to the eigenvalues and eigenvectors of matrices representing the graphs, such as the adjacency matrix. An example (Xiao 2011) is an embedding for hierarchical graphs, obtained by a hierarchical segmentation of images with spectral features vectors computed for different levels of the hierarchy.

Sub-pattern embedding works through detection of specific types of sub-patterns within the graphs to be embedded. An example is (Gibert 2011) which uses a graph embedding based on graphs of words, an extension of bag of words approaches. Graphs are actually obtained from images, with nodes corresponding to salient points, and node attributes corresponding to visual descriptors of the points. A histogram of frequency counts from intermediate graphs in the attribute space is used as the embedding.

Prototype embedding uses a set of prototype graphs, with each target graph mapped onto a vector space from which a distance function is used to calculate similarity to the prototypes. An example is (Riesen 2009) using Lipschitz embedding as a method to construct a graph embedding. Vectors are computed from a set of prototype graphs and a graph edit distance measure used alongside a k-medoids clustering algorithm.

A graph kernel is a function that maps a couple of graphs onto a real number, and has similar properties to the dot product defined on vectors. Once represented as a kernel techniques such as support vector machine (SVM) classification and principal component analysis (PCA) can be applied. An example (Gaüz 2012) uses a Laplacian kernel, working with both a graph edit distance function and treelet approach, with the kernel computed from an occurrence count.

5.1.4. Graph clustering

Graph clustering can mean either clustering of graphs or graph-based clustering. Clustering of graphs is where a set of graphs are represented as objects to be clustered. Graph-based clustering is more common and is where a single graph is used to represent the structure of the space to which the objects belong, with a node representing each object and edges encoding the relationships between pairs of objects.

Approaches for clustering of graphs can make use of any of the myriad of clustering algorithms available today. An example is (Jain 2011) who uses an approach based on vector quantization, based upon a graph edit distance measure, and a k-Means algorithm.

Approaches for graph-based clustering include use of discrete numeric ranges to label nodes, partitioning objects from a manually chosen root node, approximate nearest neighbour graphs, connectivity graphs and hypergraphs based on spectral features. An example is (Mimaroglu 2011) where a number of clustering algorithms are first run to generate a basis for graph creation. The subsequent graph is built with nodes corresponding to data points, and edges encoding where the number of clustering algorithms that have assigned two data points to the same cluster.

5.1.5. Graph learning

Graph learning can mean either the learning of graphs or graph-based learning. The learning of graphs is where individual patterns are represented as graphs to allow graph matching. Graph-based learning where a graph structure represents the whole input space, with nodes corresponding to individual patterns, and edges representing some sort of proximity or similarity relation.

An example for learning of graphs is (Serratosa 2011) where a method is devised for learning the edit costs based on graph edit distance. An adaptive learning algorithm is used with feedback based on comparison of iterative results to a manually labelled dataset of examples. A lot of examples of graph-based learning use supervised learning or semi-supervised learning approaches. A k-regular nearest neighbour graph (k-RNN) technique is used by (Wang 2012), where k is the average number of neighbours, and the graph is constructed to minimize distances of the edges.

5.1.6. Graph mining

The process of extracting new and useful knowledge from graph data is known as graph mining. In the field of graph mining frequent subgraph pattern mining plays an important role. Typically a graph representation is created, such as an adjacency matrix or list, and sub-graphs generated according to some selection criteria. Lastly, frequency counting is performed on sub-graphs to remove duplicates. An example can be found in (Dinari 2014). Other popular graph mining approaches include PageRank, Random Walk with Restart and diameter/radius estimation.

Recently interest has turned to big graph mining of very large datasets. Typical approaches use a MapReduce problem formulation (e.g. using Apache Hadoop and HBase). An example is the Pegasus (Kang 2012) big graph mining approach. However there are interesting alternatives (Batarfi 2015) such as GraphLab, Pregel, and Trinity. Pegasus is built on top of a MapReduce data processing platform and supports PageRank and other graph mining algorithms. GraphLab uses distributed machine learning with shared memory. Google's Pregel is a large scale graph processing system where stateful graph vertices are stored in memory and can exchange messages to connected verticies. Trinity is a memory-based distributed database. Whilst it is unlikely GRAVITATE data will become large enough to need a MapReduce strategy it is interesting to consider the value of parallelization for potentially slow graph index calculations.

5.2. Natural language processing for cultural heritage enrichment

A significant amount of work has been carried out (Aletras 2012) on the problem of determining the semantic similarity between text falling broadly into two categories, knowledge-based methods and corpus-based methods. Knowledge-based methods include algorithms based on structured knowledge resources (e.g. thesauri, dictionaries, and semantic networks) to measure similarity. Corpus based methods make use of information about occurrences of words in a collection of documents. The words that occur in a document are used as features to represent it and approaches used to compare pairs of documents.

The current state of the art is for high level entity extraction (e.g. person, location) followed by relational extraction between entities at this level. Detailed fact extraction approaches providing context to these entities is much less common in the literature. In the GRAVITATE project users include highly skilled curators so some level of fact extraction should be considered to provide annotations on extracted free text fields at a level of detail useful for realistic end user search queries.

5.2.1. Knowledge-based methods

The Archaeotools project employed NER and knowledge-based rules for fact extraction based on relational extraction. This enabled access to site metadata and archaeological reports via a faceted classification (Jeffrey 2009). Entities extracted included subject, location, time and bibliographic references. The similar task of entity detection (Ore 2009) has been tried using CIDOC-CRM databases, manually mapping entries to an event-oriented model.

Wikipedia has been used (Grieser 2011) to compute the similarity between museum exhibits. This work was based on the Melbourne Museum collection and wikipedia entries for each exhibit. The document category label, wikipedia article term similarity and the physical distance between exhibits at the museum was used as a basis for similarity between items. The hypothesis behind physical distance is that curator's position related exhibits physically near each other. Ontological similarity approaches, such as the Related Article Conceptual Overlap (RACO) measures was used to calculate similarity.

In the context of assisting information discovery Wikipedia entries have been (Hall 2012) used to create a glossary of terms, each with a link back to a Wikipedia web page, and mapped to dc:title record entries within the Europeana database. This allowed Europeana indexes to be annotated with Wikipedia links to concepts such as 'roman coins'.

The STAR project[2] has worked (Vlachidis 2015) on information extraction approaches using a combination of CIDOC-CRM metadata and glossary terms extracted from the English Heritage excavation recording manual (CRM-EH). This work implements a pipeline of hand-crafted rules that employ syntactical structures for the detection of textual phrases potentially useful to archaeological research. The extraction method follows a shallow parsing strategy based on part of speech (POS) tagging, named entity recognition/specialization, negation detection and relational extraction. The ultimate goal was to detect explicit or implicit phrases that can be modelled as CRM-EH events or properties, moving beyond generic event detection and recognition. The General Architecture for Text Processing (GATE)[3] was used for all NLP work and entities were represented as RDF. The glossaries were imported as SKOS RDF graphs, allowing entities to be assigned expanded semantic relationships via SKOS defined synonyms (i.e. same concept different terms for it), hyponyms (i.e. specific sub-classes of a concept) and hypernyms (i.e. general super-classes of a concept) relationships. Word sense disambiguation (WSD) could then be performed on all extracted entities.

The STAR project created a manually annotated (by 12 archaeologists) corpus of text where archaeological entities were labelled. This gold standard dataset contained overall 1,215 annotations: 120 Physical Object, 309 Time Appellations, 511 Place, 61 Material, and 214 Relation Phrases (128 Context Event, 42 Production Event, 21 Deposition Event, and 23 Consists of Material). This dataset is available[4] for other researchers to download and benchmark against.

The STAR project benchmark results (best case) are shown below in Figure 20.

CRM Entity	Recall	Precision
Physical Object	0.84	0.64
Time Appellation	0.99	0.97
Place	0.92	0.76
Material	0.68	0.49

Figure 20. STAR project benchmark CRM entity extraction results

[2] http://hypermedia.research.southwales.ac.uk/kos/star/
[3] https://gate.ac.uk/
[4] http://www.andronikos.co.uk/evaluation.php

5.2.2. Corpus-based methods

In the cultural heritage domain, named entity recognition (NER) is evident in a range of projects. (Grover 2008] applied NER techniques over historical texts from the House of Lords, dating to the 18th century and digitized using optical character recognition (OCR). The project employed a rule-based approach supported by lexicons (gazetteers) for the identification of person and place names. (Byrne 2007) focused on NER from historical archive texts, originating from the Royal Commission on the Ancient and Historical Monuments of Scotland (RCAHMS) via a machine learning approach based on a maximum entropy classifier.

Data mining has also been applied to cultural heritage sector, such as work (Kauppinen 2009] on the Finnish CultureSampo RDF dataset. Geoparsing and an artefact partonomy was used to connect data in terms of where different parts of an artefacts were manufactured and used. A WEKA-based association rule mining approach was then used to associate occurrences of locations.

The use of WordNet for word sense disambiguation is popular in the general text processing literature. A detailed study (Agirre 2009) compared graph based approaches using WordNet to distributional similarities approaches exploiting co-occurrence statistics between words. Training data used was a web corpus in excess of 1 Terabytes in size. These techniques are very effective at word disambiguation, and are exploited commercially by applications such as search engines, but do not exploit any cultural heritage domain metadata and require very large training datasets.

5.3. Semantic mapping and alignment for cultural heritage datasets

Ontology matching (Otero-Cerdeira 2015) is a complex process that helps in reducing the semantic gap between different overlapping representations of the same domain. When heterogeneous dataset schema are represented using ontologies, the solution to semantic mapping between them typically involves the use of ontology matching techniques. Ontology matching techniques (Euzenat 2013) can be categorized into either context-based & content-based approaches, or categorized as element-level or structural-level approaches. The sub-categorization proposed by (Euzenat 2013) can be seen in Figure 21. The features used for each matching approach fall into semantic (i.e. semantic descriptions and reasoning), syntactic (i.e. limited set of instructions), structural (i.e. entity class and relationship structures), terminological (i.e. vocabularies) or extensional (i.e. class instance set correlations) types.

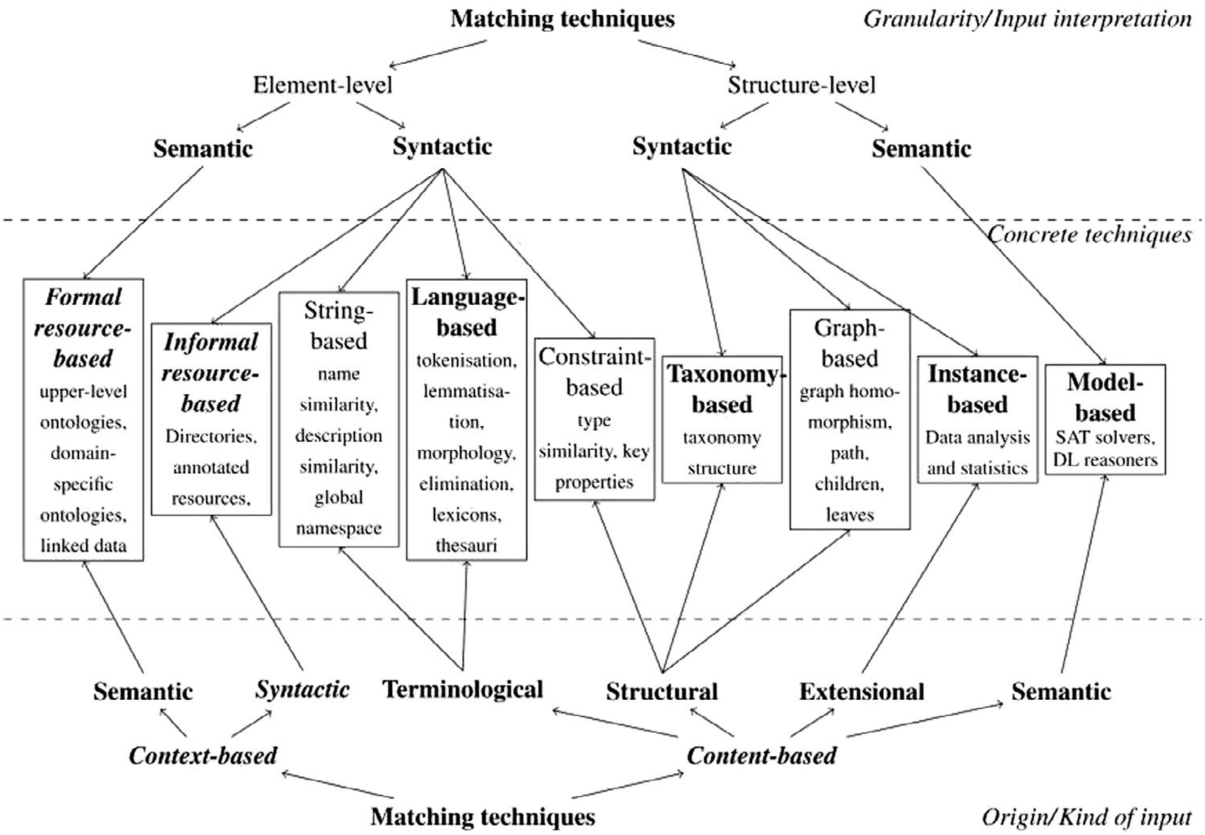


Figure 21. Ontology matching technique classification - source (Euzenat 2013).

Formal resource-based ontology matching re-uses existing formal alignments between global or domain ontologies to perform matching. An example is (Scharffe 2013) where concept matching rules are encoded as design patterns in an XML schema language. A previously created pattern library is loaded and can be executed to semantically align concepts between two new ontologies. Informal resource-based techniques are a sub-type of formal approaches, using ad-hoc or dynamically generated 'informal' resources in addition to published 'formal' resources.

String-based and language-based matching approaches look for textual similarity between entity names and descriptions. This has a strong overlap with the natural language processing based techniques described in detailed in section 5.2. Typically a domain thesaurus, glossary or dictionary would be used as an external resource.

Constraint, graph and taxonomy based approaches use similarity measures that work across relationships between entities. Constraint-based approaches use the allowed values in property domain and ranges. Graph-based approaches are discussed in detail in section 0, with taxonomy-based approaches a sub-set of graph-based approaches.

Instance-based approaches look at similarity of individuals of known classes. Typically they follow with either set-theoretic principles or look to find statistical correlations. An example is (Loia 2013) where string and link based similarity metrics are used with a latent semantic indexing algorithm to create SKOS-based RDF concepts for datasets. The SKOS narrower and broader relationships are used to weight potential inter-dataset concept matches. Results show mean precision of 0.6 with a mean recall 0.7 for the proposed matching strategy. A similar approach (Wang 2012) on a

Dutch cultural heritage dataset produced precision of 0.4 recall 0.5 with 10,000 mappings, getting worse as mapping volume increases.

Lastly model-based approaches exploit description logics and reasoning to infer mappings between concepts. An example is (Sánchez-Ruiz 2011) where description logics are manually created to first compare similarity based on the most specific concepts from available individuals, and then on the least common subsumer of available individuals. Results show a classification accuracy of up to 80%.

Semantic matching has been used to handle semantic interoperability between heterogeneous cultural heritage datasets to identify common concepts that could be semantically mapped. One approach (Mäkelä 2012), an evolution of the CultureSampo prototype, used a SKOS-based ontology of Finnish vocabulary to expand concepts prior to automatic ontology alignment. Additional semantic matching was then performed using subsumption and equivalence tests, along with locations matching based on geotags being contained by bounding boxes. Another approach (Nasir 2013) found that for a cultural heritage dataset, based on CIDOC-CRM and a domain ontology, the mapping tool COMA[5] had the best accuracy 0.38 and S-Match[6] the best recall 1.0.

[5] http://dbs.uni-leipzig.de/Research/coma.html
[6] http://semanticmatching.org/

6. Domain Vocabulary and Semantics

The Cultural Heritage domain covers a wide range of datasets (Libraries, Archives and Museums) using many different data and metadata standards, some of which are highly specialised. In the museum sector the dominant standards are SPECTRUM and CDWA/Lite (Categories for the Description of Works of Art) which both cover the curation and description of works of art and material culture. Other standards like EAD (Encoded Archival Description) and ISAD(G) (Internal Standard Archival Description – General) are designed specifically for archives and MARC (Machine Readable Cataloguing) and FRBR (Functional Requirements for Bibliographic Records) are designed and used by Librarians and Library systems. Others standards like METS (Metadata Encoding and Transmission Standard) attempt to provide support for more complex and diverse records, particular for digital content, that might appear in digital libraries. The CIDOC CRM (Conceptual Reference Model) is a semantic framework capable of providing semantic framework for integrating across archive, library and museum records as well as specialised research datasets in different disciplinary areas.

The issues related to integrating cultural heritage data over the last 20 years has been reviewed by (Oldman et al, 2014)[7]. The key relevant conclusions in this document were that reuse of data for research, but also other educational and engagement purposes, requires that data be recorded with a much greater correspondence with the implicit context missing from unified catalogues and raw data exports. In order for cultural heritage data to become a research object that can also be integrated with the wide range of heterogeneous datasets that cultural heritage organisation produce, its representation needs to incorporate the semantics that were never part of internal closed world information systems and this fact is preventing research projects from producing quality and long lasting outcomes.

Some of the relevant standards for cultural heritage Linked Data are summarised in (Hyvönen 2012)[8].

6.1. Dublin Core (DC)

Dublin Core (DC) is a metadata model and framework widely used in libraries and other organizations. DC can be used to describe a wide range of objects, such as books, photos, videos, Web pages, and artworks.

There are 15 standardized properties in the DC Metadata Element Set[9].

```
DC Metadata Element Set
namespace: http://dublincore.org/documents/dces/
```

[7] Oldman, Dominic, Martin Doerr, Garald de Jong, Barry Norton, and Thomas Wikman. 'Realizing Lessons of the Last 20 Years: A Manifesto for Data Provisioning & Aggregation Services for the Digital Humanities (A Position Paper)'. D-Lib Magazine, 20, no. 7/8 (August 2014). doi:10.1045/july2014-oldman.

[8] Hyvönen, Eero. Publishing and Using Cultural Heritage Linked Data on the Semantic Web. Morgan & Claypool Publishers, 2012.

[9] http://dublincore.org/documents/dces/


```
properties: title, creator, subject, description, publisher, contributor, date, type,
format, identifier, source, language, relation, coverage, rights
```

In common usage is also the extended version of the core elements called DCMI (Metadata) Terms[10]. It has become one of most popular RDF vocabularies in use. DCMI Terms contain 22 classes and 55 properties.

```
DCMI (Metadata) Terms
namespace: http://dublincore.org/documents/dcmi-terms/

classes: Agent, AgentClass, BibliographicResource, FileFormat, Frequency, Jurisdiction,
LicenseDocument, LinguisticSystem, Location, LocationPeriodOrJurisdiction, MediaType,
MediaTypeOrExtent, MethodOfAccrual, MethodOfInstruction, PeriodOfTime, PhysicalMedium,
PhysicalResource, Policy, ProvenanceStatement, RightsStatement, SizeOrDuration, Standard

properties: abstract, accessRights, accrualMethod, accrualPeriodicity, accrualPolicy,
alternative, audience, available, bibliographicCitation, conformsTo, contributor, coverage,
created, creator, date, dateAccepted, dateCopyrighted, dateSubmitted, description,
educationLevel, extent, format, hasFormat, hasPart, hasVersion, identifier,
instructionalMethod, isFormatOf, isPartOf, isReferencedBy, isReplacedBy, isRequiredBy,
issued, isVersionOf, language, license, mediator, medium, modified, provenance, publisher,
references, relation, replaces, requires, rights, rightsHolder, source, spatial, subject,
tableOfContents, temporal, title, type, valid
```

6.2. SPECTRUM

Standard Procedures for Collections Recording Used in Museums (SPECTRUM)[11] is an open collections management standard managed by the UK charity The Collections Trust. It is recognised nationally and internationally, as the primary specification for collections management activity in museums. It is used by more than 23,000 museums in 40 countries to improve the management, sustainability and use of Collections. SPECTRUM is developed and maintained by the SPECTRUM Community and brings together the knowledge, expertise and experience of hundreds of museums, from nationals to volunteer run societies.

SPECTRUM defines best practice for museum policies and practices and a set of “units of information” that underpin them. The process areas are:

Collections Development – The lifecycle of an object before, during its participation with a collection.

Collections Information – The documentation about an object including descriptions, movement, value, insurance, audit and rights.

[10] http://dublincore.org/documents/dcmi-terms/
[11] http://www.collectionstrust.org.uk/collections-link/collections-management/spectrum

Collections Access – The use of the collection both inside and outside the museum including loans.

Collection care and Conservation – The state of an object and its care and security both within the museum environment and externally if it is transported.

The corresponding units of information are split into different groups which are:

Object Groups	**Procedure groups**	**Record management groups**
Audit information Object collection information Object condition and technical assessment information Object conservation and treatment information Object description information Object history and association information Object identification information Object location information Object owner's contribution information Object production information Object requirement information Object rights information Object rights in information Object rights out information Object use information Object valuation information Object viewer's contribution information	Common Procedural Units Acquisition information Audit information Condition check/technical assessment information Conservation and treatment information Disposal information Indemnity information Insurance information Loan in information Loan out information Loss/damage information Movement information Object entry information Object exit information Process information Valuation information Use of collections information Address information Date information Location information Organisation information People information Person information Place information	Amendment history Use and provision of information Record information Reference information

An example definition for a unit of information is:

> **Object production person**
>
> **Definition:** A Person involved in the design, creation or manufacture of an object. This may include the commissioner of an object.
>
> **How to record:** It will be necessary to record several units of information, including, for example, a surname and a forename as well as the nature of their involvement with the production process. The descriptions for these information units are gathered together under the Person heading. The organisation may have standard forms of names for use.
>
> **Examples**: See under Person.
>
> **Use**: As many times as required for an object.
>
> **Information group:** Object production information

6.3. CIDOC Conceptual Reference Model (CRM)

CIDOC[12] is the ICOM's International Committee for Documentation and provides the museum community with advice on good practice and developments in museum documentation. The CIDOC Documentation Standards Working Group[13] (DSWG) have since 1999 created and maintained the CIDOC Conceptual Reference Model[14] (CRM) (ISO 21127:2014). The CIDOC-CRM provides definitions and a formal structure for describing the implicit and explicit concepts and relationships used in cultural heritage documentation.

The CIDOC-CRM is schema independent but our focus is on the W3C RDFS encoded schema of CIDOC-CRM (e.g. v6.2[15]).

```
CIDOC-CRM Class Hierarchy (simplified)
E1 CRM Entity
  E2 Temporal Entity
    E3 Condition State
    E4 Period
      E5 Event
        E7 Activity
        E63 Beginning of Existence
        E64 End of Existence
  E77 Persistent Item
    E70 Stuff
      E72 Legal Object
        E18 Physical Stuff
        E73 Information Object
      E71 Man-Made Stuff
    E39 Actor
    E41 Appellation
    E51 Contact Point
  E52 Time-Span
  E53 Place
  E54 Dimension
E59 Primitive Value
  E60 Number
  E61 Time Primitive
  E62 String
```

An example (Dörr 2012) of a CIDOC-CRM person entity is below.

```
CIDOC-CRM example (E21 Person)
identifiedBy = Rodin, Auguste
hasType = artists
hasType = sculptors
```

12 http://network.icom.museum/cidoc/home/
13 http://network.icom.museum/cidoc/working-groups/crm-special-interest-group/
14 http://www.cidoc-crm.org/scope.html
15 http://www.cidoc-crm.org/rdfs/cidoc_crm_v6.2-draft-2015August.rdfs

```
was born (E67 Birth)
    identifiedBy = Rodin's birth
    hasDate = 1840
died in (E69 Death)
    identifiedBy = Rodin's death
    hasDate = 1917
...
```

An example (Dionissiadou 2007) of a CIDOC-CRM physical object entity is below.

```
CIDOC-CRM example (E19 Physical Object)
identifiedBy = Epitaphios
hasType = ecclesiastical embroidery
hasType = liturgical cloth
hasCurrentOwner (E40 Legal_Body)
    identifiedBy = Museum Benaki
    hasType = private museum
    hasContactPoint (E51 Contact_Point)
        identifiedBy <Ifigenia Dionissiadu> ifi@benaki.gr
    hasContactPoint (E45 Address)
        identifiedBy = Koumbari Street 1, Athens
...
```

The CIDOC CRM provides a number of specialisations which are derived from core CIDOC CRM Classes Figure 22. This family of ontologies includes:

- CRMdig : a model for provenance metadata
- CRMgeo : a Spatiotemporal model
- CRMsci : a Scientific Observation model
- CRMarchaeo : an Excavation model
- CRMinf : an Argumentation model
- MarineTLO : a top-level ontology for the Marine Domain[16]
- FRBRoo : a model for bibliographic material and the performing arts

[16] See http://www.ics.forth.gr/isl/index_main.php?l=e&c=252

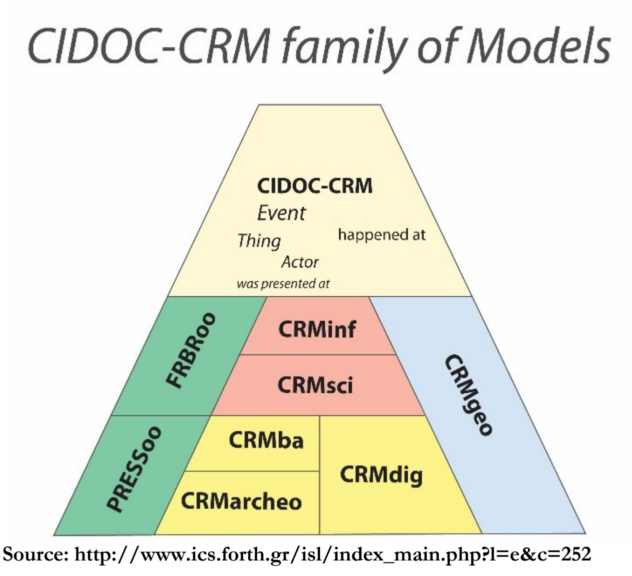


Figure 22. CIDOC-CRM family of models

The CRMinf model is of particular interest as is can represent beliefs and viewpoints associated with conceptual items such as artefacts. This is different from more definitive factual assertions in W3C PROV about authorship and attribution. This allows a curator, for example, to express an opinion about an artefact citing any available evidence to reinforce this opinion. Another curator might annotate a conflicting alternative viewpoint. An algorithm searching for semantic annotations around this artefact can choose to use one viewpoint, no viewpoints or handle the conflicting information in some way. CRMinf might also be useful when using natural language processing to extract factual information from free text fields, since these potentially error prone facts can be labelled as a viewpoint which a client might or might not consider in a semantic matching operation.

An example of properties supported by CRMinf is below.

```
CIDOC-CRMinf argumentation model properties
<I5 Inference Making> used as premise (was premise for) <I2 Belief>
<I1 Argumentation> concluded that (was concluded by) <I2 Belief>
<I5 Inference Making> applies (was applied by) <I3 Inference Logic>
<I2 Belief> that (is subject of) <I4 Proposition Set>
<I2 Belief> holds to be <I6 Belief Value>
<I7 Belief Adoption> adopted (adopted by) <I2 Belief>
```

6.4. Europeana Data Model (EDM)

The Europeana Data Model (EDM)[17] has replaced the previous model Europeana Semantic Elements (ESE). It is a more flexible and precise model than ESE, and offers the opportunity to attach every statement to the specific resource it applies to, and to include some basic provenance

[17] http://labs.europeana.eu/api/linked-open-data-data-structure

information. EDM re-uses elements coming from already-established vocabularies, including Dublin Core, OAI-ORE, SKOS and CIDOC-CRM.

The main EDM requirements include:

- Distinguishing between a 'provided item' (painting, book) and its digital representations
- Distinguishing between an item and the metadata record describing it
- Allowing the ingestion of multiple records for the same item, which may contain contradictory statements about it

The vast majority of external links in the EDM come from semantic enrichment of places (linking to GeoNames entities), concepts (linking to the GEMET thesaurus), persons (linking to DBpedia records) and time periods (using an adhoc time period vocabulary).

```
EDM example (html)
Identifier = 007000000000001U00031000; Maps.Crace I
Title = the Cittie of London 31
Description =
This map has been attributed to Augustus Ryther, an engraver who prospered between 1572 and
1592, contributing to Saxton's Atlas of 1579. This plan was produced to satisfy a European
market, and contains certain inaccuracies which a native Londoner would not have tolerated.
The streets appear very much wider than they were in actuality. Houses are depicted as
having large gardens, when these had, in fact, begun to disappear from London two centuries
before. The map details the gap at the north end of London Bridge, caused by a fire in
1632.
Creator = Cartographer : Ryther, Augustus
Geographic coverage = 531500 , 181500; London, City of London
Date = 1633; Part of: Mid 17th century; From: 01-01-1633 – To: 31-12-1633
Date of creation = 1633
Type = map
Format = jpeg; 368; 464; Millimetres; Engraving
Subject = River Thames, London Bridge; London (England) -- Maps; 912
Is part of = Crace Collection Of Maps Of London. Collect Britain
Data provider = The British Library
Provider = The European Library
Providing country = United Kingdom
```

```
EDM example (RDF)
<rdf:RDF ...>
  <ore:Proxy ...>
    <dc:date>1633</dc:date>
    <dct:medium>Engraving</dct:medium>
    <edm:type>IMAGE</edm:type>
    <dct:extent>Millimetres</dct:extent>
    <dc:description>This map has been attributed to Augustus Ryther, an engraver who
prospered between 1572 and 1592, contributing to Saxton's Atlas of 1579. This plan was
```

```
produced to satisfy a European market, and contains certain inaccuracies which a native
Londoner would not have tolerated. The streets appear very much wider than they were in
actuality. Houses are depicted as having large gardens, when these had, in fact, begun to
disappear from London two centuries before. The map details the gap at the north end of
London Bridge, caused by a fire in 1632.</dc:description>
    <dc:title>the Cittie of London 31</dc:title>
    <dc:creator>Cartographer : Ryther, Augustus</dc:creator>
    <dct:created>1633</dct:created>
    <dc:subject>912</dc:subject>
    <ore:proxyIn>
      <ore:Aggregation ...>
        <edm:provider>The European Library</edm:provider>
        <dc:rights>Copyright © British Library Board</dc:rights>
        <edm:dataProvider>The British Library</edm:dataProvider>
        <edm:object
rdf:resource="http://ogimages.bl.uk/images/007/007000000000001U00031000%5BSVC1%5D.jpg"/>
        <edm:rights rdf:resource="http://www.europeana.eu/rights/rr-f/"/>
      </ore:Aggregation>
    </ore:proxyIn>
    <dct:isPartOf>Crace Collection Of Maps Of London. Collect Britain</dct:isPartOf>
    <dct:extent>368</dct:extent>
    <dc:subject>River Thames, London Bridge</dc:subject>
    <dc:subject>London (England) -- Maps</dc:subject>
    <dct:extent>464</dct:extent>
    <dct:spatial>London, City of London</dct:spatial>
    <dc:identifier>007000000000001U00031000</dc:identifier>
    <dct:spatial>531500 , 181500</dct:spatial>
    <dc:type>Map</dc:type>
    <dc:format>jpeg</dc:format>
    <dc:type>StillImage</dc:type>
    <dc:identifier>Maps.Crace I</dc:identifier>
  </ore:Proxy>
</rdf:RDF>
```


EDM is defined at a level of generalisation above that of the CIDOC CRM and Dublin Core and has not been created to provide the same level of semantic information. Influence by Digital library considerations it retains Dublin Core at its core and it is less able to document objects of art and material culture which have more complex requirements as indicated by the SPECTRUM standard. This creates an issue with adequately describing and integrating objects from different cultural heritage organisations and has resulted in limitations apparent on the Europeana aggregation portal in terms of utility for researchers.

6.5. IFLA Study Groups on Functional Requirements for Bibliographic Records (FRBR), Authority Data (FRAD) and Subject Authority Data (FRSAD)

The IFLA Study Group on the Functional Requirements for Bibliographic Records (FRBR) developed a conceptual model showing the entities and relationships of the bibliographic universe in 1997. This has periodically been incrementally updated with the latest versions published between 2008 and 2010. The purpose of the FRBR model is to identify the functional requirements of information in bibliographic records with a focus on use by national bibliographic agencies.

The FRBR reports represent an entity-relationship model, identifies entities and their attributes, and defines relationships among entities. Although in the FRBR model the entities of all three groups are defined, the main focus is on the first group. The developers of FRBR envisioned that its extensions would cover the additional data that are normally recorded in authority records.

```
FRBR Entities
<work> is_realized_through <expression>
<expression> is_embodied_in <manifestation>
<manifestation> is_exemplified_by <item>
<frbr_entity> is_created/realized/produced/owned_by <person> | <corperate_body>
<work> has_as_subject <frbr_entity>

FRAD Entities
<frbr_entity> is_appellation_of <name>
<frbr_entity> is_assigned_to <identifier>
<name> | <identifier> is_basis_for <controlled_access_point>
<rules> govern <controlled_access_point>
<agency> created <controlled_access_point>

FRSAD Entities
<frbr_entity> has_appellation <nomen> (i.e. set of symbols or name)
```

The FRBR Entity Relationship model is a conceptual model which is not fully defined in terms of a data model. It defines the relationships between different bibliographic entities (Works, Expressions, Manifestations and Items) and related concepts like publication, translations, formats etc. The CIDOC/IFLA FRBRoo model defines an object orientated version of FRBR that can be more easily implemented as a data model and is starting to be used in the community. FRBR is also used as a guideline for using existing bibliographic formats like MARC to implement the conceptual FRBR model and is a component of other frameworks such as RDA.

6.6. Advanced Research Infrastructure for Archaeological Dataset Networking in Europe (ARIADNE)

The goal of the ARIADNE infrastructure[18] is to integrate data and metadata from different providers into one common schema, and also to provide semantic integration along different axes (e.g. subject, space, time). This integration intends to provide useful and user-friendly information services for archaeology. The services are intended to be available not only to researchers and related stakeholders, but also to a wider range of potential users requiring access to collections and datasets.

[18] http://www.ariadne-infrastructure.eu

Metadata integration will be facilitated by the ARIADNE catalogue. Vocabulary integration will focus on event types, topics and artefacts using a SKOS mapping behind the scenes to record concept inter-relationships. The project aims to integrate many heterogeneous data sources so various vocabularies will be imported using manually defined semantic mappings and a tool such as the X3ML Framework, which is based on the Mapping Memory Manager (3M) from Institute of Computer Science, Foundation for Research and Technology - Hellas (FORTH), to generate CIDOC-CRM RDF graphs.

All catalogue information is stored using a global ontology called the ARIADNE catalogue data model (ACDM). This is based on the W3C data catalogue vocabulary (DCAT)[19] recommendation which is built itself upon W3C SKOS. The ACDM will provide proprietary extensions to support the CIDOC-CRM data model.

An example of an ACDM catalogue entry is below:


```
<acdm:dataResource ...>
  <dcterms:title>Bureauonderzoek ten behoeve van het leidingtracé ... </dcterms:title>
  <dcterms:description>In april 2009 ... </dcterms:description>
  <dcterms:issued>2009X03X25</dcterms:issued>
  <dcterms:modified>2009X11X19</dcterms:modified>
  <acdm:originalId preferred="false">AIP_ID twips.dans.knaw.nlX4920366299359862317X
1258618514487</acdm:originalId>
  <acdm:originalId preferred="false">eDNAXproject a11267</acdm:originalId>
  <acdm:originalId preferred="false"> 2009/43 (rapportnr)</acdm:originalId>
  <acdm:originalId preferred="false"> 197685 (projectnr)</acdm:originalId>
  <acdm:originalId preferred="true">urn:nbn:nl:ui:13Xh73Xjys</acdm:originalId>
  <acdm:originalId preferred="false">Archis_onderzoek_m_nr 34302</acdm:originalId>
  <acdm:originalId preferred="false">DMO_ID easyXdataset:11244</acdm:originalId>
  <dcterms:language>nl</dcterms:language>
  <dcterms:landingPage>http://www.persistentXidentifier.nl/urn:nbn:nl:ui:13Xh73X
jys</dcterms:landingPage>
  <dcterms:accessRights>Access restricted to registered members.</dcterms:accessRights>
  <dcterms:isPartOf>DANS</dcterms:isPartOf>
  <dcterms:creator>Oranjewoud BV</dcterms:creator>
  <dcterms:creator>Spoelstra, A.</dcterms:creator>
  <dcterms:creator>Kaptein, I.</dcterms:creator>
  <acdm:ariadneSubject>Fieldwork databases</acdm:ariadneSubject>
  <acdm:SpatialRegion>
     <acdm:lat>53.15612478</acdm:lat>
     <acdm:lon>6.23994777</acdm:lon>
<acdm:coordinateSystem>http://www.opengis.net/def/crs/EPSG/0/4326</acdm:coordinateSystem>
  </acdm:SpatialRegion>
  <dcat:keyword>06H</dcat:keyword>
  <dcat:keyword>Marum</dcat:keyword>
  <dcat:keyword>Groningen</dcat:keyword>
```


[19] http://www.w3.org/TR/vocab-dcat

```
</acdm:dataResource>
```

The ARIADNE infrastructure is an active project at the time of writing and publically available detailed examples are limited. It is focusing on catalogue level integration, supports manual semantic mapping and the creation of a final records formatted according to the CIDOC-CRM standard. In GRAVITATE we will be looking at a level of granularity lower than this so the ARIADNE project, whilst contextually relevant, is not directly something that we should be considering integration with. It is also not clear at this stage if material plans are in place to ensure the ARIADNE schema will become an internationally agreed standard, or even a *de facto* standard, and as such an interesting example of a proof of concept approach to catalogue level semantic integration.

6.7. W3C RDF Schema (RDFS)

The RDF Schema language is a mechanism for manging and controlling the use of RDF (Resource Description Format). It does this by defining and applying classes and subclasses and constraining the use of these classes using domain and range constraints that determine how different classes can be used together in an RDF triple statement. This provides the building blocks for formal ontologies by defining a basic level of semantics and defining basic inference rules for RDF data. Simple inferencing works through the definition of sub-classes and sub-properties and through typing data which determines how it can be used.

```
namespace: http://www.w3.org/2000/01/rdf-schema#
classes: Resource, Class, Literal, Datatype, langString (rdf), HTML (rdf), XMLLiteral
(rdf), Property (rdf)
properties: range, domain, type, subClassOf, subPropertyOf, label, comment
```

The CIDOC CRM ontology is defined in RDFS by the standards committee that manages it and as cultural institutions become more familiar with Linked Data, RDFS is probably the most familiar schema to those cultural heritage organisations that use Linked Data, however, familiarity with any Linked data technologies should not be over-estimated with many institutions relying on external expertise and platforms.

6.8. W3C Web Ontology Language (OWL)

Although RDFS provides some basic semantics, OWL provides far greater semantic expressiveness by supporting formal logic systems. OWL refers to a number of different OWL flavours:

OWL Lite – provides the same features as RDFS above but also provides some limited cardinality constraints and additional property characteristics. It includes the following:[20]

RDF Schema Features:	**(In)Equality:**	**Property Characteristics:**
• Class (Thing, Nothing)	• equivalentClass	• ObjectProperty

[20] Table taken from - http://www.w3.org/TR/2004/REC-owl-features-20040210/#s2.1 OWL Web Ontology Language Overview

• rdfs:subClassOf • rdf:Property • rdfs:subPropertyOf • rdfs:domain • rdfs:range • Individual	• equivalentProperty • sameAs • differentFrom • AllDifferent • distinctMembers	• DatatypeProperty • inverseOf • TransitiveProperty • SymmetricProperty • FunctionalProperty • InverseFunctionalProperty
Property Restrictions: • Restriction • onProperty • allValuesFrom • someValuesFrom	**Restricted Cardinality:** • minCardinality (only 0 or 1) • maxCardinality (only 0 or 1) • cardinality (only 0 or 1)	**Header Information:** • Ontology • imports
Class Intersection: • intersectionOf **Datatypes:** • xsd datatypes	**Versioning:** • versionInfo • priorVersion • backwardCompatibleWith • incompatibleWith • DeprecatedClass • DeprecatedProperty	**Annotation Properties:** • rdfs:label • rdfs:comment • rdfs:seeAlso • rdfs:isDefinedBy • AnnotationProperty • OntologyProperty

OWL DL and FULL have the former and the additional functions below, although implement some of the properties in different ways:

Class Axioms: • *oneOf, dataRange* • *disjointWith* • *equivalentClass* (applied to class expressions) • *rdfs:subClassOf* (applied to class expressions)	**Boolean Combinations of Class Expressions:** • *unionOf* • *complementOf* • *intersectionOf*
Arbitrary Cardinality: • *minCardinality* • *maxCardinality* • *cardinality*	**Filler Information:** • *hasValue*

Cultural Heritage organisations are likely to have a very limited knowledge of OWL and again would be most familiar with it (in specialised groups) as the form that ontologies like the Europeana Data Model and CIDOC CRM are implemented in (CIDOC CRM having an approved OWL version) without fully understanding its purpose.

6.9. W3C Simple Knowledge Organization System (SKOS)

SKOS specifies a schema that supports conceptual terminologies (as oppose to instances of real world things) such as those used in thesauri and other terminology authority schemes, but defined within the framework of the Internet and the Semantic Web. The W3C state that:

"Concepts are the units of thought — ideas, meanings, or (categories of) objects and events— which underlie many knowledge organization systems. As such, concepts exist in the mind as abstract entities which are independent of the terms used to label them."[21]

SKOS enables cultural heritage organizations to transfer their terminology schemes easily to the Web with much of their original structures intact. Many of the properties will be familiar to cultural heritage organizations employing thesauri standard ISO 25964. A summary of the scheme is:

SKOS Vocabulary Organized by Theme[22]					
Concepts	**Labels & Notation**	**Documentation**	**Semantic Relations**	**Mapping Properties**	**Collections**
Concept	prefLabel	note	broader	broadMatch	Collection
ConceptScheme	altLabel	changeNote	narrower	narrowMatch	orderedCollection
inScheme	hiddenLabel	definition	related	relatedMatch	member
hasTopConcept	notation	editorialNote	broaderTransitive	closeMatch	memberList
topConceptOf		example	narrowerTransitive	exactMatch	
		historyNote	semanticRelation	mappingRelation	
		scopeNote			

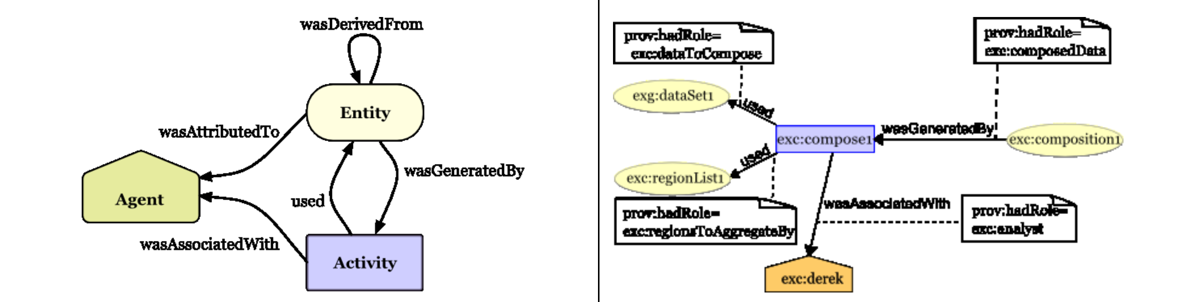
SKOS is a more familiar cultural heritage format because of the importance attached to thesauri in collection management systems and the association with Linked Data versions, including the implementation of Getty Institute vocabularies (for example the Art & Architecture Thesaurus) which is used in many cultural heritage information systems. It is important to note that while the CIDOC CRM ontology incorporates SKOS, it is not useful in itself for data harmonisation because the widely variable nature of cultural heritage vocabularies. However, semantic frameworks based on real world universal (generalised) concepts, allows integration that supports a comparison of terminologies embedded into different datasets.

6.10. W3C Provenance (PROV)

The PROV ontology at its basic level provides a schema that allows provenance information such as attribution, association, derivation and generation information to be added to Linked Data. In other words in allows data owners to describe how information came into being in terms of the activities and people that created it. These basic functions are further enhanced by including the ability to define the roles, procedures and time periods for these provenance activities. The diagrams in Figure 23 are taken from the W3C PROV Model document.

[21] W3C SKOS Primer

[22] Table taken from Wikipedia - https://en.wikipedia.org/wiki/Simple_Knowledge_Organization_System



Belhajjame et al
http://www.w3.org/TR/prov-primer/
http://www.w3.org/Consortium/Legal/2015/doc-license

Belhajjame et al
http://www.w3.org/TR/prov-primer/
http://www.w3.org/Consortium/Legal/2015/doc-license

Figure 23. PROV model

PROV will be more familiar to general Linked Data practitioners than Cultural Heritage organisations. The CIDOC CRM provides provenance models more appropriate to cultural heritage data within core CRM, CRMDig and CRMInf, above.

6.11. Summary of domain semantics

The major characteristic of Cultural Heritage data is that it is highly variable being made up of many specialist classification systems and standards which have developed within different parts of the cultural heritage world. High levels of variation are also apparent between similar institutions, particularly museums. Museum's will have different and customised schema based on different historical backgrounds, disciplinary bias, geographical location as well as many other factors. Despite central vocabularies being available for some time they employ very different terminology schema for the very same reasons. This variability and uniqueness is what provides the richness of cultural heritage data which is an important element to be preserved rather than homogenised because each collection dataset is a research object in its own right when packaged with its original context. This has been a feature of the domain for the last 250 years and arguably much longer.

The collection systems used by the museum partners are:

- MUSIMS – British Museum
- ADLIB – Fitzwilliam
- MuseumPlus – Ashmolean

All three system are SPECTRUM compliant but because SPECTRUM is not a fully defined data model each implementation will be different and have various customisations. The same type of entities will be documented and the units of information taken from SPECTRUM will overlap but will not necessarily be represented in the underlying data models in the same way. Different parts of the SPECTRUM model are used in different museums depending upon documentation priorities. In addition the vocabularies used by each institution will also be different albeit with overlaps and similarities. This is the reason why the GRAVITATE project requires a semantic framework to integrate the different implementations and data models used in each of the different system.

While the datasets will be different they will record the same types of entity and the common groups derived from SPECTRUM. For example, the concepts of Object, People, Places, Time, and so on, are common to all the datasets regardless of the individual fields and details. Therefore an ontology that exposes these entities will provide the commonality needed to make comparisons at the detailed field value level. The datasets themselves will have descriptions that work at different levels of specialisation. The CIDOC CRM ontology acknowledges these differences in specialisation and provides a hierarchy of class specialisation/generalisation that reflects this. This means that the different ways in which the main entities are described will still be integrated and provide a basis for analysis.

6.12. Access protocols to cultural heritage resources

Of interest to, but not in direct scope of, this report are the schema and query languages used within the cultural heritage sector to access data once it is stored under a specific metadata schema. This provides context to the landscape in which any semantic matching technology must exist.

The CIDOC Lightweight Information Describing Objects (LIDO)[23] is an XML schema intended for delivering metadata, for use in a variety of online services, from an organization's online collections database to portals of aggregated resources, as well as exposing, sharing and connecting data on the web. It is not intended to be used as a basis for a collection management system or to support loan and acquisition activities. However it is problematic for details dataset comparison because it while it is CIDOC CRM compatible it generalises to a particular level which removes some of the semantic information required for detailed analysis. It aggregates a particular generalisation which would be inappropriate for the objectives of GRAVITATE. The Europeana project supports the use of LIDO for aggregation to the EDM which itself supports a more fixed and higher level generation for its portal objectives.

LIDO supports multilingual portal environments. It does this by having a language attribute that can be associated with each element, or more generally, with the group of descriptive elements for fully multilingual resources.

The Categories for the Description of Works of Art (CDWA) Lite schema has been enlarged and integrated into the LIDO schema. CDWA is a metadata model targeted for cataloguing works of art.

In addition to metadata delivery schema there are a number of search protocols that have been used such as Z39.50, SRU/SRW, OpenSearch, OAI-PHM and SPARQL. Both Z39.50 and SRU/SRW are now effectively superseded by the W3C SPARQL recommendation.

OpenSearch is a collection of simple formats for the sharing of search results. The OpenSearch description document format can be used to describe a search engine so that it can be used by search client applications. As such repository owners who create OpenSearch descriptions make it easier for OpenSearch aware clients to query their data.

[23] http://network.icom.museum/cidoc/working-groups/lido/lido-technical/specification/

The Open Archives Initiative Protocol for Metadata Harvesting[24] (OAI-PMH) is a low-barrier mechanism for repository interoperability. Data Providers are repositories that expose structured metadata via OAI-PMH. Service Providers then make OAI-PMH service requests to harvest that metadata. OAI-PMH is a set of six verbs or services that are invoked within HTTP. The OAI-PMH schema supports DC metadata.

```
OAI-PMH Concepts
classes: Harvester, Repository, Item, Unique Identifier, Record, Deleted records, Set
```

SPARQL is the query language for RDF (Resource Description Framework) data and is not implemented in a wide range of databases that support RDF. In some respects it resembles the SQL query language for relational databases and uses many of the same functional keywords, but queries are based on matching RDF triple patterns within a dataset. The current version is 1.1 which provides some powerful functionality of defining paths.

An example SPARQL query for the British Museum's open dataset is:

```
PREFIX crm: <http://erlangen-crm.org/current/>

SELECT DISTINCT ?obj
{ ?obj crm:P102_has_title ?title .
  ?title rdfs:label ?label .
  FILTER(STR(?label) = "Hoa Hakananai'a")
}
```

Which retrieves the URI of an object (Easter Island statue) with a specific title.

SPARQL queries can be federated by providing the name of the different services of different SPARQL Endpoints:

For example,

```
PREFIX dbo: <http://dbpedia.org/ontology/>
PREFIX bmo: <http://collection.britishmuseum.org/id/ontology/>
PREFIX crm: <http://erlangen-crm.org/current/>
PREFIX skos: <http://www.w3.org/2004/02/skos/core#>

SELECT  ?dbabstract ?bmobjecttypetext

WHERE
{
   SERVICE <http://dbpedia.org/sparql>
   {
   <http://dbpedia.org/page/Rosetta_Stone>  dbo:abstract  ?dbabstract .
   }
```

24 https://www.openarchives.org/pmh/


```
   SERVICE <http://data.britishmuseum.org/sparql>
   {
   <http://collection.britishmuseum.org/id/object/YCA62958>  bmo:PX_object_type  ?objecttype .
   ?objecttype skos:prefLabel ?bmobjecttypetext .
   }
}
```

This would retrieve the abstract from DBpedia and the object type from the British Museum Endpoint in one query, for the Rosetta Stone.

7. Coupling geometric to semantics descriptions of 3D cultural heritage datasets

The current methodology for the documentation of a heritage asset includes, in addition to peculiar metadata, a reference to 2D/3D digital content, which serves mainly for visualisation purposes. The information implicit in the digital model is not exploited, while it could enrich the documentation –and consequently the sharing, search and retrieval- in a powerful way. Indeed, part-based semantic annotation of 2D/3D data would help categorize the object, or parts within an object, and potentially augment shape search and matching algorithms to improve results.

However, geometric representations do not give explicit information about the content's semantics, which you can grasp only by viewing the object. In order to make machines understood the content of digital 3D media, tools are needed to classify automatically objects in semantic classes, extract salient features, and segment the representation into meaningful parts. Shape analysis tools are devoted to solve such issues and some of them have been presented in the previous sections (i.e. different types of shape descriptors).

Shape segmentation and feature extraction are the computational tools needed to select the portion of interest in a model. Selecting regions of interest in the manual annotation of 2D media is rather simple in terms of user interface: dragging a selection box or lasso tool over an image achieves the necessary functionality. The same simplicity does not hold for 3D media, where parts might be out of reach for mouse interaction, and bounding a part can be rather complex. Shape segmentation is therefore complemented by shape annotation, which is typically the means to document content with contextual knowledge, either manually or automatically (Spagnuolo & Falcidieno 2009).

It is still a challenge how to deal with the transient nature of geometric primitives with respect to persistency of meaning, as described in (Havemann & Fellner 2007). In particular, there is not a standard approach to linking annotations to 3D media yet. Current standards for expressing geometric data, such as X3D or XML3D are evolving towards the so called generalised 3D documents, but part-based annotation is still an issue.

The ShapeAnnotator (Attene et al. 2009) was the first tool tackling part-based ontology-driven annotation. It allows to perform non-trivial segmentations of 3D surface meshes and annotate the detected parts through concepts expressed by an ontology. Different segmentation methods are included and the user can select the segments he/she prefers independently of the tool that computed it. Then, each part is connected to an instance that can be stored in a knowledge base to ease the retrieval process based on semantics. Through an intuitive interface, users create such instances by simply selecting proper classes in the ontology; attributes and relations with other instances can be computed automatically based on a customizable analysis of the underlying topology and geometry of the parts. Each instance is related to one part of the model, and it is defined by its URI, its type (the class the feature belongs to) and other attribute values and relations. In particular, the ShapeAnnotator saves the geometric representation, augmented with information about the segmentation, into a single file, and saves the instances as a separate OWL file that imports the domain ontology.

More recently, in (Banerjee et al. 2015) an extension of the W3C Open Annotation data model has been introduced in a medical application scenario: the goal was to handle the annotation of varying dimensional 3D parts (i.e. areas, edges and points) and to support the annotation of an instance with different types of resources (e.g. free text, images). Moreover, a simple and effective file format, .sem3D, has been defined to ease the storage of part-based 3D annotated models.

Nevertheless, the problem of defining a stable 3D mark-up remains. Concerning the geometric primitives issue, annotations or tags attached to parts of 3D models should survive changes in the geometric representation. This task is not trivial, even if we consider just one representation type, such as triangle meshes. Think of part-based annotation of a 3D model representing a statue or a complex artifact. For visualization, we need to simplify the models; that is, we need to remove a number of vertices and triangles. It is not solved yet how to keep annotations consistent across resolution changes. The problem gets even more complicated if we change the representation type- for instance, switching from triangle to quadrilateral meshes, or even to continuous representations such as NURBS. The shape of the statue, together with its relevant features, remains the same, and the annotations should follow the scale changes accordingly and smoothly.

Ontologies are an expressive and flexible tool to formalise contextual knowledge, to make it machine-understandable, and share it after the agreement of the reference community. Considering the specific context of the conceptualisation of the parts of an object, in the cultural heritage domain only a few published works have attempted to create ontologies for artefact parts, and all of these have done so in the context of very specialized collections of artefacts. Ontologies have been published for Greek vases (Yo 2013a) (Yo 2013b) and architectural columns (Lo Buglio 2013). These two work represent high level and low level partonomies, with semantic grounding via either expert manual labelling or automatically derived numerical signatures based on 3D scanning data.

The work published (Yo 2013a) (Yo 2013b) is particularly relevant to GRAVITATE, defining an OWL ontology, derived from CIDOC-CRM superclasses. This work defined a Greek vase ontology (GVO) representing conceptual items for a vase partonomy. This partonomy was used to manually annotate 3D objects at a high level, labelling parts of the 3D objects such as ‘neck’, ‘head’, ‘rim’ and ‘stem’. In addition to a partonomy an artefact relationship ontology (ARO) is defined, allowing comparative, dimensional and colour relationships to be labelled between parts. The authors used these ontologies to perform semantic searches via SPARQL queries, evaluating results using a qualitative analysis involving user satisfaction of an implemented search website for a collection of 10 vases (50 parts). This ontology is limited to Greek vases, and has not been adopted by any international standards body (e.g. W3C or CIDOC). Examples of GVO parts and ARO relationships are below:

```
Partonomy
artefactA_part1 rdf:type gvo:Stem
artefactA_part2 rdf:type gvo:Base
artefactA_part3 rdf:type gvo:Handle
artefactB_part1 rdf:type gvo:Stem
artefactB_part2 rdf:type gvo:Base
```

```
Comparative relationships
artefactA_part1 aro:isSameAs artefactB_part1
artefactA_part1 aro:isCopyOf artefactC_part3
artefactA_part1 aro:isVariationOf artefactD_part1

Dimensional relationships
artefactA_part1 aro:isTallerThan artefactD_part1
artefactA_part2 aro:isDeeperThan artefactD_part2

Colour relationships
artefactA_part3 aro:isBrighterThan artefactB_part3
artefactA_part3 aro:isDarkerThan artefactD_part3
```

Another interesting cultural heritage ontology (Lo Buglio 2013) used low-level descriptions of architectural elements relating to columns of Saint-Michel de Crux. In this work 3D scanning results were used to create morphological signatures, extracted using line detection, step extraction and statistical averaging, for different column profiles containing artistic reliefs of interest for cultural heritage analysis. Whilst not explicit in an OWL ontology, these low-level structural classes are automatically derived and empirically grounded to real-world structural object measurements via the signature profiles.

Outside of the cultural heritage area there are a number of works relating to semantic descriptions of different aspects of partonomies. Motivated by classic work dating back to the 1980's (Jain 2012) create a high level partonomy for part-of relationships (e.g. 'component of object', 'member of collection', 'place in area' etc.) for use in the drug discovery domain. An OWL ontology for 6 different part-of relationship variants is defined, then similarity of part-of relationships tested on DBpedia after feature extraction using WordNet and parts of speech (POS) tagging for entity disambiguation.

Returning to geometric approaches, partonomies (inter-part structure relationships and shape characteristics) are typically addressed by shape analysis techniques in more general contexts. Indeed, the ultimate goal in shape analysis is to capture as higher as possible semantic concepts embedded in the data. Traditional techniques often extract geometric features and use them to learn the semantic concepts in a supervised manner using annotated data. Recent advances outlined that the arrangement and relations between shape parts provide important cues for learning the semantics of shapes. In particular, there are methods for extracting and modelling part structures and methods for comparing shapes and shape parts using their structure (Biasotti, Laga, et al. 2014). Thanks to structural analysis techniques, it is possible to: (1) find partwise semantic correspondences between 3D shapes in a non-supervised manner and without relying on user-specified textual tags, and (2) design classifiers that learn in a supervised manner the functionality of the shape components. Just to mention a few works, in (Laga 2013) part connectivity graphs between parts discovered via automatic segmentation can be created, and the spatial positioning of these parts used for subsequent classification (e.g. horizontal support relationships for parts such as wings on an aeroplane 3D model). Classifiers can also be created (Gong 2012) to learn simple semantic signatures for object shape properties such as 'circular', 'thin', 'long' and 'natural'. Such techniques are more powerful when they are context-aware and then the potential to experiment them in a specific domain, such as the GRAVITATE one, and couple them with explicit knowledge technologies becomes obvious and stimulates challenging research questions.

8. Perspectives

We conclude this report with some initial remarks on the issues that seem most promising and most worthy of investigation to turn the state of the art as described into useful elements of GRAVITATE. When we present those for the two main parts of this survey, these are typically in the line of expectation for the natural development of the individual fields of research. But we also begin to see the exciting opportunities GRAVITATE offers for cross-fertilisation of those contributing fields. These initial remarks will inform the Research Roadmap (GRAVITATE deliverable D3.2 in PM13), which will also have the inventory of the user requirements as its input (GRAVITATE deliverable D2.1) to guide the research in the most fruitful direction for GRAVITATE.

1) From the literature, we learn that the joint analysis of geometry and skin texture improves the shape matching performance and that the CIELab space is the most appropriate space for dealing with colour information (Section 3.1.3).
2) Current techniques for the characterization and recognition of 3D patterns base on the strong hypothesis the 3D mesh is regular (the cardinality of the vertices is everywhere 6); to recognize characteristic 3D patterns in the GRAVITATE fragments it will be necessary to extend the current methods or to investigate new descriptions (Section 3.1.1.1).
3) The solutions in the literature to the matching problem do not fully satisfy the GRAVITATE needs; even when dealing with partial correspondence between objects it is assumed that the "features" of interest are complete (and this is not true when the problem is to locate features that are partially preserved in broken artefacts), (Section 4.1).
4) Most of techniques limit themselves to specific classes of objects (e.g. pottery) and assume specific symmetries and rotation invariance that the GRAVITATE use case does not have; this implies an extension of the literature and the definition of new "shape templates".
5) Scalability and robustness to noise are important issues in many of the descriptions in the literature; in particular, methods based on the mesh connectivity (and that use spectral descriptions) depend on the quality of the connection between the mesh vertices; in addition, they often use tools of linear algebra whose computation complexity is quadratic.
6) Approximate placement of fragments of an artefact of a known class may be usefully guided by a parametrized mannequin/exemplar shape; this may reflect fairly well how a conservator works. There are indications that exemplar-based reconstruction occurs in various subfield of archaeological research.
7) We need detailed information and source code (rather than binaries) of the apparently successful object mating performed by PRESIOUS and assess its robustness under the abrasion expected in GRAVITATE (Section 4.2.1).
8) Incorporating the particular asymmetry of successful mating despite noise by means of software based on mathematical morphology should be investigated as a novel and potentially robust approach. (Section 4.2.2 and 4.2.3).
9) Reassembly should not only based on the fracture facets, but also on some continuity of the skin, both in shape and colorimetric properties. Volumetric properties may be less relevant for terracotta, but this should be investigated.

10) We want to cooperate with the concurrent SCAN4RECO project to assess the accuracy of digitally scanning artefacts, and the subsequent meshes (Section 4.2.4).
11) We envision the creation of benchmarks, either in terms of 3D models and tools, so that the methods in the literature and the ones developed by GRAVITATE can be compared on a common basis.
12) We should understand how potential findings of the 2D square tiles puzzle reassembly inform the GRAVITATE ReAssembly task (Section 4.3).
13) An incremental approach may be adopted to semantic matching starting with a weighted graph matching algorithm (see section 5.1.2). This would be able to pick up and use later NLP derived annotations. A pre-computed set of indexes could then be built each specialized to support certain query types. Later on support for matching semantic annotations from partonomies (e.g. body parts) and description of binary annotations (e.g. colour labels associated with frequency ranges in colour histograms) could be added to move towards mixed semantic/geometry queries.
14) Of relevance to GRAVITATE, (Otero-Cerdeira 2015) considers the future challenges for semantic mapping / ontology alignment to include (a) improving semantic mapping through semi-automatic machine learning and (b) integrating domain knowledge into alignment techniques. The semantic matching algorithms used within GRAVITATE could advance the state of the art in both these areas.
15) We should use NLP to enrich the metadata available from intelligent processing of free text fields in expert descriptions of archaeological artefacts. The entity glossary from the CRM-EH English Heritage vocabulary, as used by the STAR project, should be considered as a gold-standard labelled dataset suitable for benchmarking purposes.
16) The CIDOC-CRMinf schema should be considered for representing different annotation viewpoints, both from NLP (machine viewpoint with implied error) and curators (human viewpoints with implied organizational and cultural bias).
17) Focussed NLP work on deeper fact extraction (i.e. specialized case by case not in a generic low-precision way), rather than 'surface' topic level entity extraction, can be used in GRAVITATE. This would require encoding aspects of the CIDOC-CRM concepts onto the grammars developed, but should produce annotations that are more useful to real world queries (e.g. by curators looking for pieces of an artefact).
18) There may be opportunities to use shape matching techniques on mannequins to inform the automatic classification of new object, and partially fill in the metadata.
19) Non-annotated constituents of an object can still be found in shape-driven queries, which could be formulated by means of exemplars.
20) Metadata description of obviously geometrical or colorimetric textures (waves of hair, rhythmic hashing) can be made quantitative by using shape descriptors.
21) Using shape queries we may provide a more intuitive, or at least additional, interface to the associative searches for CH artefacts.

8.1. Postscript

As we are finishing this report, the parallel GRAVITATE effort on user requirements (WP2) is providing interesting information on how different kinds of archaeologists – researchers, conservators and curators – are currently interacting with the artefacts, and how GRAVITATE might provide a digital dashboard to simulate, stimulate or enhance these interactions. The remarks above do not yet reflect these new insights.

This state-of-the-art report forms the start of a close interaction between the fields of shape processing and semantic search. Interesting scientific and computational questions arise already, and many should be directly relevant to the GRAVITATE platform and its users. The Research Roadmap will contain our plan of how to address these.

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