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Parameter dependent Lyapunov function based robust iterative learning control for discrete systems with actuator faults

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SUMMARY

This paper considers iterative learning control for a class of uncertain multiple-input multiple-output discrete linear systems with polytopic uncertainties and actuator faults. The stability theory for linear repetitive processes is used to develop control law design algorithms that can be computed using linear matrix inequalities. A class of parameter dependent Lyapunov functions are used with the aim of enlarging the allowed polytopic uncertainty range for successful design. The effectiveness and feasibility of the new design algorithms is illustrated by a gantry robot case study. Copyright © 2015 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Iterative Learning Control (ILC) was developed for industrial applications where the plant completes the same finite duration task over and over again. A generic example is a gantry robot executing a pick and place task, such as transporting and placing objects on a moving conveyor where the sequence is: a) collect an object from a fixed location, b) transport it over a finite time duration, c) place it at a fixed location or on a moving conveyor, d) return to the starting location and e) repeat steps a)-d) as many times as required or the maximum possible before a halt for maintenance or other reasons is required. Each execution is termed a trial and the finite duration the trial length.

On completion of each trial, all information generated during its completion is available for use by the control law used to construct the input for the next trial. The basic idea of ILC is to improve performance from trial-to-trial and non-causal temporal information can be used provided it is generated on a previous trial. For example, at instance, say t, on trial $k+1$, $k \geq 0$, information at $t+\lambda$, $\lambda > 0$, on trial k can be used in the computation of the control input. The use of such information is the main distinguishing feature of ILC.

The literature on ILC research is widely recognized to start with [1], which considered a simple first order linear servomechanism system for speed control of a voltage-controlled dc-servomotor. Since this first work, ILC research and applications has broadened and the survey papers [2] and [3]

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are one source for an overview of developments up to the time of their publication. This area of control systems research continues to grow both in theory and applications, where recent new applications areas include robotic-assisted stroke rehabilitation for the upper limb [4], large dynamic range nanopositioning control [5], traffic management [6] and highly coupled electromechanical systems [7].

One setting for ILC design is to make use of its 2D systems structure, where one direction of information is from trial-to-trial indexed by the subscript k and the other along a trial indexed by p. In early work [8] used the Roesser state-space model for 2D discrete linear systems. Subsequent research has included uncertainty in the dynamic model and uncertain initial conditions at the start of each trial [9]. Also there has been research reported on an extension to nonlinear batch processes operating in a series of piecewise affine operating regions and, with industrial applications in mind, allowing the presence of constraints on the input and output [10].

The Roesser model describes 2D discrete linear systems recursive over the upper right quadrant of the 2D plane but ILC operates over a subset of this plane. Repetitive processes are a class of 2D systems where the temporal variable is defined over a finite duration. These processes model physical systems where a series of sweeps, or passes, are executed through a set of dynamics defined over a finite duration known as the pass length. Once each pass is complete the process resets to the starting position.

On each pass an output, termed the pass profile, is generated and also acts as a forcing function on and hence contributes to the dynamics of the next pass. As a result, oscillations can occur in the sequence of pass profiles that increase in amplitude from pass-to-pass. It is not possible to control these processes by standard control action and in response a stability and control theory for them has been developed. A detailed treatment of the dynamics of these processes, including their origins in the modeling of mining operations, can be found in [11].

Given the finite duration of the temporal variable, repetitive processes form a more natural setting for ILC design. Research in this area has resulted in ILC laws that have been experimentally verified on a gantry robot, e.g., [12]. In this last paper, a constant parameter Lyapunov function based sufficient condition for stability along the pass was used to derive ILC control law matrices, with more recent follow on results in, for example, [13, 14, 15]. Use of the repetitive process for analysis and design applies to both analog and digital model based design in contrast to the lifted setting, see, for example, the relevant papers cited in [2, 3], which applies only to digital design. Also it is possible to design a control law for both trial-to-trial error convergence and transient response along the trials. In the lifted setting, if the system to be controlled by ILC is unstable then a stabilizing control law must first be designed and ILC applied to the resulting dynamics.

In many applications, the sensors and actuators will be required to operate in industrial environments that increase the possibility that a fault will occur. If a fault is not detected and corrected promptly, performance could degrade to an unacceptable level. Fault tolerant control (FTC) schemes are designed to guarantee overall systems stability and acceptable performance in the presence of faults. This general problem is an active area of control systems research with recent work including [16, 17, 18, 19, 20, 21].

Fault-tolerant control is a critical issue for ILC and there has been some research reported, such as [22, 23, 24]. This paper gives new results on fault tolerance for ILC design using repetitive process stability theory where a parameter dependent Lyapunov function was used in an attempt to admit a larger uncertainty range than its constant counterpart. The rationale is that the parameter dependent function allows the use of different Lyapunov functions at various points in the polytope defining the uncertainty [25].

The next section gives the required background on repetitive processes and formulates the general problem considered, including the representation of the faults. Section 3 considers Lyapunov-based robust ILC design. In Section 4 simulation based examples are given to show the superiority of parameter dependent Lyapunov-based design. Finally, conclusions are give in Section 5 and possible future research briefly discussed.

Throughout this paper, the null and identity matrices of compatible dimensions are denoted by 0 and I respectively. Also $X > 0$ and $X < 0$ denotes a symmetric positive definite, respectively,

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negative definite, matrix. The symbol $\star$ represents the transposed elements in a symmetric matrix and $\rho(\cdot)$ denotes the spectral radius of its matrix argument.

2. BACKGROUND AND PROBLEM FORMULATION

2.1. Basics of Repetitive Process ILC Design

Repetitive processes are characterized by a series of sweeps, termed passes, through a set of dynamics defined over a finite duration known as the pass length. The analysis and ILC design in this paper is based on discrete linear repetitive processes whose state-space model over $0 \leq p \leq \alpha - 1,\ k \geq 0$, where α is the number of samples over the pass length, is

$$\begin{aligned} x_{k+1}(p+1) &= \mathbb{A}x_{k+1}(p) + \mathbb{B}u_{k+1}(p) + \mathbb{B}_0 y_k(p), \\ y_{k+1}(p) &= \mathbb{C}x_{k+1}(p) + \mathbb{D}u_{k+1}(p) + \mathbb{D}_0 y_k(p), \end{aligned} \tag{1}$$

where on pass k, $x_k(p) \in \mathbb{R}^n$ is the state vector, $y_k(p) \in \mathbb{R}^m$ is the pass profile vector and $u_k(p) \in \mathbb{R}^r$ is the control input vector. The boundary conditions are the state initial vector on each pass and the initial pass profile. In this work the boundary conditions can be taken as zero without loss of generality.

Let $\{y_k\}$ denote the sequence of pass profiles generated by (1). Then the unique control problem for these processes is that this sequence can contain oscillations that increase in amplitude from pass-to-pass. Hence the stability of these processes is defined in terms of the contribution of the previous pass profile to the next pass profile. For processes described by (1) the pass-to-pass coupling is described by the convolution operator, denoted by L_α, for a discrete standard linear system with state-space model matrices $\{\mathbb{A}, \mathbb{B}_0, \mathbb{C}, \mathbb{D}_0\}$. Hence the contribution from pass k to pass $k+1$ can be written as $y_{k+1} = L_\alpha y_k,\ k \geq 0$. Also let $y_k \in E_\alpha$, where E_α is a suitably chosen Banach space with norm denoted by $||\cdot||$ and let the same symbol denote the induced norm on the bounded linear operator L_α. The stability theory for linear repetitive processes is formulated in terms of this Banach space representation [11].

Two forms of stability can be defined in this setting, which are termed asymptotic stability and stability along the pass, respectively. Asymptotic stability requires that a bounded initial pass profile produces a bounded sequence of pass profiles over the finite and fixed pass length whereas stability along the pass is stronger by requiring this property for all possible values of the pass length, which can be analyzed mathematically by considering $\alpha \to \infty$. Stability along the pass requires the existence of finite real numbers $M_\infty > 0$ and $\lambda_\infty \in (0, 1)$ such that $||L_\alpha^k|| \leq M_\infty \lambda_\infty^k$, where $||\cdot||$ denotes both the norm on E_α and the induced operator norm. (For asymptotic stability these numbers are allowed to be functions of α).

In the case of processes described by (1), asymptotic stability requires that $\rho(\mathbb{D}_0) < 1$. The necessary and sufficient conditions for stability along the pass are (i) $\rho(\mathbb{D}_0) < 1$, (ii) $\rho(\mathbb{A}) < 1$, and (iii) all eigenvalues of $G(z) = \mathbb{C}(zI - \mathbb{A})^{-1}\mathbb{B}_0 + \mathbb{D}_0$ have modulus strictly less than unity for all $|z| = 1$. Next, how ILC can be formulated in a repetitive process setting is summarized, where in reference to repetitive processes pass is replaced by trial to conform with most of the ILC literature.

In the case of discrete dynamics, let $y_{ref}(p)$ be a vector valued reference representing desired output behavior. Use also the notation $y_k(p),\ 0 \leq p \leq \alpha - 1,\ k \geq 1$, where y is a vector or scalar valued variable, $\alpha < \infty$ denotes the number of samples over the trial duration and the nonnegative integer k the trial number. Then the error on trial k is

$$e_k(p) = y_{ref}(p) - y_k(p),\ 0 \leq p \leq \alpha - 1. \tag{2}$$

Moreover, the construction of a sequence of input functions that improves performance from one trial to the next is equivalent to the following convergence conditions on the input and error

$$\lim_{k\to\infty} ||e_k|| = 0,\ \lim_{k\to\infty} ||u_k - u_\infty|| = 0, \tag{3}$$

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where $||\cdot||$ is a signal norm in a suitably chosen function space with a norm-based topology and u_∞ is termed the learned control. These conditions ensure trial-to-trial performance and if, as in this paper, the dynamics along the trial are discrete, a commonly used setting for design is based on a form of lifting that enables the dynamic plant model in $\mathbb{R}$ to be treated, for single-input single-output (SISO) systems with a natural extension to the multiple-input multiple-output (MIMO) systems, as a static system in $\mathbb{R}^\alpha$. Once the ILC law is applied, the propagation of the error dynamics from trial-to-trial is described by a linear difference equation in k and this is the starting point for analysis and design.

A general form of ILC law is

$$u_{k+1}(p) = u_k(p) + \Delta u_{k+1}(p), \tag{4}$$

i.e., on pass $k+1$ the control is that used on the previous pass plus a correction term $\Delta u_{k+1}(p)$ constructed using the previous pass error. For analysis purposes only, introduce

$$\eta_{k+1}(p+1) \quad = \quad x_{k+1}(p) - x_k(p). \tag{5}$$

Consider also the case when

$$\Delta u_{k+1}(p) = K_1\eta_{k+1}(p+1) + K_2 e_k(p+1), \tag{6}$$

which is a combination of state feedback action on the current trial plus a component from the previous trial error $e_k(p+1)$ where the current trial error $e_k(p)$ is defined in (2). Introducing

$$\begin{array}{rclrcl} \hat{\mathbf{A}} & = & \mathbb{A} + \mathbb{B}K_1, & \hat{\mathbf{B}}_0 & = & \mathbb{B}K_2, \\ \hat{\mathbf{C}} & = & -\mathbb{C}(\mathbb{A} + \mathbb{B}K_1), & \hat{\mathbf{D}}_0 & = & I - \mathbb{C}\mathbb{B}K_2, \end{array} \tag{7}$$

enables the controlled ILC dynamics to be written as

$$\begin{array}{rcl} \eta_{k+1}(p+1) & = & \hat{\mathbf{A}}\eta_{k+1}(p) + \hat{\mathbf{B}}_0 e_k(p), \\ e_{k+1}(p) & = & \hat{\mathbf{C}}\eta_{k+1}(p) + \hat{\mathbf{D}}_0 e_k(p). \end{array} \tag{8}$$

The state-space model (8) is a particular case of the discrete linear repetitive process (1) with trial state vector η and trial profile vector e and zero current trial input. Convergence of the sequence $\{e_k\}_{k\geq 0}$, i.e., trial-to-trial error convergence of ILC dynamics can now be studied by application of repetitive process stability theory and results in ILC law design algorithms, see e.g. [12].

In particular, formulas for constructing K_1 and K_2 in (6) are obtained in an LMI setting starting from the conditions given above for stability along the trial. The control law can also be written as the sum of two terms, the first involving current trial state feedback and the second the MIMO output version of phase-lead ILC in terms of the previous trial error. This control law regulates the dynamics along the trials (first term) and trial-to-trial error convergence (second). Recalling the definition of stability along the trial in terms of the abstract model, i.e., the requirement that $||L_\alpha^k|| \leq M_\infty \lambda_\infty^k$, for some $M_\infty > 0$ and $\lambda_\infty \in (0,1)$, it follows that this property enforces monotonic convergence of the trial-to-trial error to zero.

2.2. *Problem Formulation*

The systems considered in this paper have the following state-space model in the ILC setting

$$\begin{array}{rcl} x_k(p+1) & = & A(\xi(k,p))x_k(p) + B(\xi(k,p))u_k(p), \\ y_k(p) & = & Cx_k(p), 0 \leq p \leq \alpha - 1, \end{array} \tag{9}$$

where on trial k, $x_k(p) \in \mathbb{R}^n$ is the state vector, $y_k(p) \in \mathbb{R}^m$ is the output vector and $u_k(p) \in \mathbb{R}^r$ is the control input vector. The model matrices A and B of (9) are not precisely known, but belong to

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a convex bounded uncertain domain denoted by Ω:

$$\begin{aligned}\Omega &= \left\{[A(\xi(k,p)), B(\xi(k,p))]\middle|[A(\xi(k,p)), B(\xi(k,p))]\right.\\ &= \left.\sum_{i=1}^{N}\xi_i(k,p)(A_i,B_i), \xi_i(k,p)\geq 0, \sum_{i=1}^{N}\xi_i(k,p)=1\right\},\end{aligned} \quad (10)$$

where A_i, B_i are the corresponding matrix vertices and N denotes their number. This so-called polytopic representation is standard in robust control where for analysis and design the uncertainty is assumed to belong to a model class. An alternative is norm-bounded uncertainty and extending the results developed in this paper to this alternative model is straightforward. The strong structural links between discrete linear systems and discrete linear repetitive processes strongly suggests that such a model is an appropriate place to start for robust control in the presence of actuator faults for these processes.

Let $u_{k,i}(p)$, $1 \leq i \leq r$, denote the entries in $u_k(p)$ and let $u^F_{k,i}(p)$ denote an input, or actuator signal, with possible failures. Then the failure model used in this paper is [22, 26]

$$u^F_{k,i}(p) = \gamma_i u_{k,i}(p), \ \ i = 1, 2, \cdots, r, \quad (11)$$

where

$$0 \leq \underline{\gamma}_i \leq \gamma_i \leq \overline{\gamma}_i, \ \ i = 1, 2, \cdots, r.$$

The scalars $\underline{\gamma}_i(\underline{\gamma}_i \leq 1)$ and $\overline{\gamma}_i(\overline{\gamma}_i \geq 1)$ in this failure model are assumed to be known. The scalar γ_i is unknown but assumed to vary within a known range. If $\underline{\gamma}_i = \overline{\gamma}_i$ the fault-free case is recovered, i.e., $u^F_i = u_i$. Also $\gamma_i = 0$ is the case when the actuator experiences complete failure and $\gamma_i > 0$ corresponds to a partial failure.

Introduce the notation

$$\Gamma = diag[\gamma_1, \gamma_2, \cdots, \gamma_r], \quad (12a)$$

$$\overline{\Gamma} = diag[\overline{\gamma}_1, \overline{\gamma}_2, \cdots, \overline{\gamma}_r], \quad (12b)$$

$$\underline{\Gamma} = diag[\underline{\gamma}_1, \underline{\gamma}_2, \cdots, \underline{\gamma}_r] \quad (12c)$$

to give

$$u^F_k = [u^F_{k,1}, u^F_{k,2}, \cdots, u^F_{k,r}]^T = \Gamma u_k. \quad (12d)$$

Hence a system described by (9) with actuator faults of the form considered in this paper can be described by the state-space model

$$\begin{aligned}x_k(p+1) &= A(\xi(k,p))x_k(p) + B(\xi(k,p))\Gamma u_k(p),\\ y_k(p) &= Cx_k(p), 0 \leq p \leq \alpha - 1.\end{aligned} \quad (13)$$

The control design problem is to determine a fault-tolerant ILC law such that trial-to-trial error convergence occurs in k, where the ILC law is again given by (4) and using (2), (5) and (6) the controlled ILC dynamics are described by the state-space model

$$\begin{aligned}\eta_{k+1}(p+1) &= \widehat{A}(\xi(k,p))\eta_{k+1}(p) + \widehat{B}(\xi(k,p))e_k(p)\\ e_{k+1}(p) &= \widehat{C}(\xi(k,p))\eta_{k+1}(p) + \widehat{D}(\xi(k,p))e_k(p),\end{aligned} \quad (14)$$

where

$$
\begin{aligned}
\widehat{A}(\xi(k,p)) &= A(\xi(k,p)) + B(\xi(k,p))\Gamma K_1 = \sum_{i=1}^{N} \xi_i(k,p)\widehat{A}_i, \\
\widehat{B}(\xi(k,p)) &= B(\xi(k,p))\Gamma K_2 = \sum_{i=1}^{N} \xi_i(k,p)\widehat{B}_i, \\
\widehat{C}(\xi(k,p)) &= -C[A(\xi(k,p)) + B(\xi(k,p))\Gamma K_1] = \sum_{i=1}^{N} \xi_i(k,p)\widehat{C}_i, \\
\widehat{D}(\xi(k,p)) &= I - CB(\xi(k,p))\Gamma K_2 = \sum_{i=1}^{N} \xi_i(k,p)\widehat{D}_i
\end{aligned}
$$

and

$$
\begin{aligned}
\widehat{A}_i &= A_i + B_i\Gamma K_1,\ \widehat{B}_i = B_i\Gamma K_2, \\
\widehat{C}_i &= -C(A_i + B_i\Gamma K_1),\ \widehat{D}_i = I - CB_i\Gamma K_2.
\end{aligned}
$$

The state-space model (14) is that of a discrete linear repetitive process with the input term deleted from both the state and trial profile updating vectors. On trial $k+1$, the ILC error $e_{k+1}(p)$ is the trial profile vector and $\eta_{k+1}(p)$ is the current state vector.

3. ILC DESIGN

In this section, both constant and parameter dependent Lyapunov functions are used to design a robust fault-tolerant ILC law, where the second approach can reduce conservativeness.

3.1. Constant Parameter Lyapunov Approach

Introduce the notation

$$\Sigma = diag[\sigma_1, \sigma_2, \cdots, \sigma_r], \tag{15a}$$

$$\Sigma_0 = diag[\sigma_{10}, \sigma_{20}, \cdots, \sigma_{r0}] \tag{15b}$$

with

$$\sigma_i = \frac{\overline{\gamma}_i + \underline{\gamma}_i}{2}, i = 1, 2, \cdots, r, \tag{16a}$$

$$\sigma_{i0} = \frac{\overline{\gamma}_i - \underline{\gamma}_i}{\overline{\gamma}_i + \underline{\gamma}_i}, i = 1, 2, \cdots, r. \tag{16b}$$

Also from (12) and (15), an unknown matrix Γ_0 exists such that

$$\Gamma = (I + \Gamma_0)\Sigma \tag{17}$$

and

$$|\Gamma_0| \leq \Sigma_0 \leq I, \tag{18}$$

where

$$\Gamma_0 = diag[\gamma_{10}, \gamma_{20}, \cdots, \gamma_{r0}] \tag{19}$$

and

$$|\Gamma_0| = diag[|\gamma_{10}|, |\gamma_{20}|, \cdots |\gamma_{r0}|]. \tag{20}$$

The analysis that follows will make extensive use of the Schur's complement formula and the following result.

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Lemma 1
[27] Assume $X, Y, Z = Z^T$ are real matrices with compatible dimensions. Then for any matrix Δ satisfying $\Delta^T\Delta \le I$, the inequality

$$Z + X\Delta Y + Y^T\Delta^T X^T < 0$$

holds if and only if there exists a positive scalar ε such that

$$Z + \varepsilon XX^T + \varepsilon^{-1}Y^TY < 0.$$

Introduce the following Lyapunov function for the ILC dynamics (14),

$$V_k(p) = V_k^1(p) + V_k^2(p), \tag{21}$$

where

$$\begin{array}{rcl} V_k^1(p) & = & \eta_{k+1}^T(p)P_1\eta_{k+1}(p), \\ V_k^2(p) & = & e_k^T(p)P_2e_k(p) \end{array}$$

with $P_i > 0, i = 1, 2$ and increment

$$\begin{array}{rcl} \Delta V_k(p) & = & \Delta V_k^1(p) + \Delta V_k^2(p) = V_k^1(p+1) - V_k^1(p) + V_{k+1}^2(p) - V_k^2(p) \\ & = & \left[\begin{array}{cc} \eta_{k+1}^T(p) & e_k^T(p) \end{array}\right] \times (\Phi^T P\Phi - P) \left[\begin{array}{c} \eta_{k+1}(p) \\ e_k(p) \end{array}\right], \end{array} \tag{22}$$

where

$$\Phi = \left[\begin{array}{cc} \widehat{A} & \widehat{B} \\ \widehat{C} & \widehat{D} \end{array}\right], \; P = diag(P_1, P_2).$$

Then, (14) is stable along the pass [11] if there exists $P > 0$ such that

$$\Phi^T P\Phi - P < 0. \tag{23}$$

The following results gives control law design formulas and is extended in the next section to parameter dependent Lyapunov function based control law design in the presence of faults.

Lemma 2
[28] The discrete linear repetitive process representation of the ILC dynamics (14) is stable along the trial if there exist compatibly dimensioned nonsingular matrices $G = diag(G_1, G_2)$, $P = diag(P_1, P_2) > 0$ such that the following LMI is feasible:

$$\left[\begin{array}{cc} -P & \star \\ M & P - G - G^T \end{array}\right] < 0 \tag{24}$$

with
$M = \Phi G = \left[\begin{array}{cc} A_iG_1 + B_i\Gamma K_1G_1 & B_i\Gamma K_2G_2 \\ -C(A_i + B_i\Gamma K_1)G_1 & G_2 - CB_i\Gamma K_2G_2 \end{array}\right]$, for $i = 1, 2, \cdots, N$.

Theorem 1
The discrete linear repetitive process representation of the ILC dynamics (14) is stable along the trial if there exists a scalar $\lambda > 0$ and matrices $P_1 > 0$, $P_2 > 0$, G_1 and G_2, R_1 and R_2 such that the following LMI is feasible:

$$\left[\begin{array}{cccccc} -P_1 & \star & \star & \star & \star & \star \\ 0 & -P_2 & \star & \star & \star & \star \\ A_iG_1 + B_i\Sigma R_1 & B_i\Sigma R_2 & P_1 - G_1 - G_1^T & \star & \star & \star \\ -CA_iG_1 - CB_i\Sigma R_1 & G_2 - CB_i\Sigma R_2 & 0 & P_2 - G_2 - G_2^T & \star & \star \\ 0 & 0 & \lambda\Sigma_0 B^T & -\lambda\Sigma_0 B_i^T C^T & -\lambda I & \star \\ \Sigma R_1 & \Sigma R_2 & 0 & 0 & 0 & -\lambda I \end{array}\right] < 0 \tag{25}$$

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for $i = 1, 2, \cdots, N$.

If this LMI is feasible, the control law matrices K_1 and K_2 can be computed using

$$K_1 = R_1 G_1^{-1}, K_2 = R_2 G_2^{-1}. \tag{26}$$

Proof

An obvious application of the Lemma 2 yields

$$\begin{bmatrix} -P_1 & \star & \star & \star \\ 0 & -P_2 & \star & \star \\ A_i G_1 + B_i \Gamma K_1 G_1 & B_i \Gamma K_2 G_2 & P_1 - G_1 - G_1^T & \star \\ -CA_i G_1 - CB_i \Gamma K_1 G_1 & G_2 - CB_i \Gamma K_2 G_2 & 0 & P_2 - G_2 - G_2^T \end{bmatrix} < 0. \tag{27}$$

Also from (17)

$$\Lambda + H\Gamma_0 F + F^T \Gamma_0^T H^T < 0, \tag{28}$$

where

$$\Lambda = \begin{bmatrix} -P_1 & \star & \star & \star \\ 0 & -P_2 & \star & \star \\ A_i G_1 + B_i \Sigma K_1 G_1 & B_i \Sigma K_2 G_2 & P_1 - G_1 - G_1^T & \star \\ -CA_i G_1 - CB_i \Sigma K_1 G_1 & G_2 - CB_i \Sigma K_2 G_2 & 0 & P_2 - G_2 - G_2^T \end{bmatrix},$$

$$H = \begin{bmatrix} 0 & 0 & B_i^T & -B_i^T C^T \end{bmatrix}^T,$$

$$F = \begin{bmatrix} \Sigma K_1 X_1 & \Sigma K_2 X_2 & 0 & 0 \end{bmatrix}.$$

From Lemma 1, it follows that (28) holds if there exists a positive scalar $\lambda > 0$ such that

$$\Lambda + \lambda H \Sigma_0^2 H^T + \lambda^{-1} F^T F < 0, \tag{29}$$

or, equivalently,

$$\Lambda + \begin{bmatrix} \lambda^{\frac{1}{2}} H \Sigma_0 & \lambda^{-\frac{1}{2}} F^T \end{bmatrix} \begin{bmatrix} \lambda^{\frac{1}{2}} \Sigma_0 H^T \\ \lambda^{-\frac{1}{2}} F \end{bmatrix} < 0. \tag{30}$$

Application of the Schur's complement Lemma gives that (30) is equivalent to

$$\begin{bmatrix} \Lambda & \star \\ \begin{bmatrix} \lambda^{1/2} \Sigma_0 H^T \\ \lambda^{-1/2} F \end{bmatrix} & -I \end{bmatrix} < 0. \tag{31}$$

Finally, pre- and post-multiplying (31) by $diag(I, \lambda^{1/2} I)$ and introducing

$$R_1 = K_1 G_1, \; R_2 = K_2 G_2 \tag{32}$$

completes the proof. □

The LMI in this last result, if feasible, produces a family of control law matrices K_1 and K_2. Some of these solutions may result in slow convergence, excessive control action etc. Hence a standard approach to avoid such solutions is to parameterize them using following optimization procedure

$$minimize \left(\sum_{i=1}^{2} tr(P_i) + \sum_{j=1}^{2} se(G_j) \right) \tag{33}$$

subject to (25). where $tr(\cdot)$ is the trace of a matrix and $se(\cdot)$ denotes the sum of all elements in a matrix.

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3.2. Parameter Dependent Lyapunov Approach

The result of Theorem 1 uses a constant parameter Lyapunov function and may give overly conservative results. This section develops a new control law design using a parameter dependent Lyapunov function as used in standard linear systems theory [25], where the candidate function has the form of (21) with $V_k^1(p)$ and $V_k^2(p)$ replaced by

$$\begin{aligned} V_k^1(p) &= \eta_{k+1}^T(p)P_1(\xi(k,p))\eta_{k+1}(p), \\ V_k^2(p) &= e_k^T(p)P_2(\xi(k,p))e_k(p), \end{aligned} \tag{34}$$

where

$$P_1(\xi(k,p)) = \sum_{i=1}^{N} \xi_i(k,p)P_{1i} > 0,$$
$$P_2(\xi(k,p)) = \sum_{i=1}^{N} \xi_i(k,p)P_{2i} > 0$$

with increment

$$\begin{aligned} \Delta V_k(p) &= \Delta V_k^1(p) + \Delta V_k^2(p) \\ &= V_k^1(p+1) - V_k^1(p) + V_{k+1}^2(p) - V_k^2(p) \\ &= \eta_{k+1}^T(p+1)P_1(\xi(k,p+1))\eta_{k+1}(p+1) - \eta_{k+1}^T(p)P_1(\xi(k,p))\eta_{k+1}(p) \\ &\quad + e_{k+1}^T(p)P_2(\xi(k+1,p))e_{k+1}(p) - e_k^T(p)P_2(\xi(k,p))e_k(p) \end{aligned} \tag{35}$$

Also (23) can be written as

$$\Phi(\xi(k,p))^T\mathcal{P}^+\Phi(\xi(k,p)) - \mathcal{P} < 0, \tag{36}$$

where

$$\Phi(\xi(k,p)) = \begin{bmatrix} \widehat{A}(\xi(k,p)) & \widehat{B}(\xi(k,p)) \\ \widehat{C}(\xi(k,p)) & \widehat{D}(\xi(k,p)) \end{bmatrix} = \sum_{i=1}^{N} \xi_i(k,p) \begin{bmatrix} \widehat{A}_i & \widehat{B}_i \\ \widehat{C}_i & \widehat{D}_i \end{bmatrix} = \sum_{i=1}^{N} \xi_i(k,p)\Phi_i$$

$$\mathcal{P} = \begin{bmatrix} P_1(\xi(k,p)) & 0 \\ 0 & P_2(\xi(k,p) \end{bmatrix} = \sum_{i=1}^{N} \xi_i(k,p) \begin{bmatrix} P_{1i} & 0 \\ 0 & P_{2i} \end{bmatrix} = \sum_{i=1}^{N} \xi_i(k,p)P_i > 0,$$

$$\mathcal{P}^+ = \begin{bmatrix} P_1(\xi(k,p+1)) & 0 \\ 0 & P_2(\xi(k+1,p) \end{bmatrix} = \sum_{j=1}^{N} \varsigma_j(k,p) \begin{bmatrix} P_{1j} & 0 \\ 0 & P_{2j} \end{bmatrix} = \sum_{j=1}^{N} \varsigma_j(k,p)P_j > 0.$$

The following result aims to minimize the conservativeness in design that can result from the use of a sufficient, but not necessary, stability condition.

Theorem 2
The ILC dynamics (14) are stable along the trial if there exist compatibly dimensioned nonsingular matrix G and the matrices $P_i, i = 1, 2, ..., N$ such that the following LMI are feasible

$$\begin{bmatrix} -P_j & \star \\ \Phi_i G & P_i - G - G^T \end{bmatrix} < 0 \tag{37}$$

for $i, j = 1, 2, ..., N$.

Proof
Assume that (37) is feasible for all $i = 1, 2, ..., N$. Then

$$P_i - G^T - G < 0 \tag{38}$$

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and, since G and $P_i > 0$ are nonsingular,

$$(P_i - G)^T P_i^{-1}(P_i - G) \geq 0, \tag{39}$$

which is equivalent to

$$P_i - G - G^T \geq -G^T P_i^{-1} G. \tag{40}$$

Also if (37) holds

$$\begin{bmatrix} -P_j & \star \\ \Phi_i G & -\,G^T P_i^{-1} G \end{bmatrix} < 0. \tag{41}$$

Next, pre and post-multiply this last matrix by $diag(GP_j^{-1}, I)$ and its transpose, respectively, to obtain

$$\begin{bmatrix} -GP_j^{-1}G^T & \star \\ \Phi_i G P_j^{-1} G^T & -\,G^T P_i^{-1} G \end{bmatrix} < 0. \tag{42}$$

Letting $S_j = GP_j^{-1}G^T$ and $S_i = GP_i^{-1}G^T$ gives

$$\begin{bmatrix} -S_j & \star \\ \Phi_i S_j & -\,S_i \end{bmatrix} < 0. \tag{43}$$

Further, for each i, multiply the corresponding inequalities for $j = 1, 2, ..., N$ by $\varsigma_j(k,p)$ and sum over j to obtain

$$\begin{bmatrix} -\sum\limits_{j=1}^{N} \varsigma_j(k,p) S_j & \star \\ \Phi_i \sum\limits_{j=1}^{N} \varsigma_j(k,p) S_j & -\,S_i \end{bmatrix} < 0. \tag{44}$$

Noting that $\sum\limits_{j=1}^{N} \varsigma_j(k,p) S_j = \mathcal{S}^+$, multiplying the resulting inequalities by $\xi_i(k,p)$ for $i = 1, 2, ..., N$ and summing over i gives

$$\begin{bmatrix} -\mathcal{S}^+ & \star \\ \sum\limits_{i=1}^{N} \xi_i(k,p)\Phi_i \mathcal{S}^+ & -\sum\limits_{i=1}^{N} \xi_i(k,p) S_i \end{bmatrix} < 0, \tag{45}$$

or, equivalently,

$$\begin{bmatrix} -\mathcal{S}^+ & \star \\ \Phi(\xi(k,p))\mathcal{S}^+ & -\,\mathcal{S} \end{bmatrix} < 0. \tag{46}$$

Application of the Schur's complement formula gives

$$\Phi(\xi(k,p))\mathcal{S}^+\Phi(\xi(k,p))^T - \mathcal{S} < 0, \tag{47}$$

which is equivalent to (36). □

This last result is not expressed in terms of an LMI due to its implicit form with respect to the control law matrices K_1 and K_2 and cannot be used directly for robust control law design.

Theorem 3

The repetitive process representing the ILC dynamics (14) is stable along the trial if there exists a scalar $\lambda > 0$ and matrices $P_{1,j} > 0$, $P_{2,i} > 0$, $i, j = 1, 2, ..., N$, G_1 and G_2, R_1 and R_2 such that the following LMI is feasible:

$$\begin{bmatrix} -P_{1,j} & \star & \star & \star & \star & \star \\ 0 & -P_{2,j} & \star & \star & \star & \star \\ A_iG_1 + B_i\Sigma R_1 & B_i\Sigma R_2 & P_{1,i} - G_1 - G_1^T & \star & \star & \star \\ -CA_iG_1 - CB_i\Sigma R_1 & G_2 - CB_i\Sigma R_2 & 0 & P_{2,i} - G_2 - G_2^T & \star & \star \\ 0 & 0 & \lambda\Sigma_0 B_i^T & -\lambda\Sigma_0 B_i^T C^T & -\lambda I & \star \\ \Sigma R_1 & \Sigma R_2 & 0 & 0 & 0 & -\lambda I \end{bmatrix} < 0 \tag{48}$$

for $i, j = 1, 2, ..., N$.

If this LMI is feasible, the control law matrices K_1 and K_2 are given by $K_1 = R_1 X_1^{-1}$ and $K_2 = R_2 X_2^{-1}$.

Proof

The proof of this result is omitted since, given Theorem 2, it follows identical steps to the proof of Theorem 1. □

The control law matrices in Theorem 3 can (as before) be parameterized using following optimization procedure

$$minimize \left(\sum_{i=1}^{3} tr(P_{1,i} + P_{2,i}) + \sum_{j=1}^{2} se(G_j) \right) \tag{49}$$

subject to (48).

For implementation, the current trial input is given by

$$u_k(p) = u_{k-1}(p) + K_1(x_k(p) - x_{k-1}(p)) + K_2(y_{ref}(p+1) - y_{k-1}(p+1)) \tag{50}$$

where the second term on the right-hand side is ILC phase-lead. The first term on the left-hand side is a stabilizing term actuated by the difference between the current and previous trial state vectors.

4. CASE STUDY

A multi-axis gantry robot [29], which executes the pick and place task, has been used to test the performance of ILC laws designed in the repetitive process setting [12, 13, 14]. The robot has three separate axes that are mounted perpendicular to each other and hence each of them can be treated as a single-input single-output system. Each axis has been modeled based on frequency response tests, resulting in the following transfer-function for one of them

$$G_z(s) = \frac{15.8869(s + 850.3)}{s(s^2 + 707.6s + 3.377 \times 10^5)}. \tag{51}$$

Discretization using a zero-order hold with a sampling period of $T_s = 0.01$ s gives the following z transfer-function for design, i.e., the model of the nominal dynamics

$$G_z(z) = \frac{0.0003648z^2 + 3.572 \times 10^{-5}z + 2.171 \times 10^{-6}}{z^3 - 0.9941z^2 - 0.005077z - 0.0008451}. \tag{52}$$

The numerator and denominator coefficients in (52) have been randomly changed to obtain the polyhedric uncertainties

$$G_{z1}(z) = \frac{0.000357z^2 + 2.878 \times 10^{-5}z + 5.716 \times 10^{-6}}{z^3 - 0.9848z^2 - 0.004235z - 0.0007701}, \tag{53}$$

$$G_{z2}(z) = \frac{0.0003226z^2 + 1.43 \times 10^{-5}z + 1.131 \times 10^{-6}}{z^3 - 0.9911z^2 - 0.005144z - 0.0007753}, \tag{54}$$

$$G_{z3}(z) = G_z(z) = \frac{0.0003648z^2 + 3.572 \times 10^{-5}z + 2.171 \times 10^{-6}}{z^3 - 0.9941z^2 - 0.005077z - 0.0008451}. \tag{55}$$

Remark 1

The transfer-functions used to describe the vertices are strictly proper and this choice agrees with the dynamics of mechanical systems such as the considered gantry robot.

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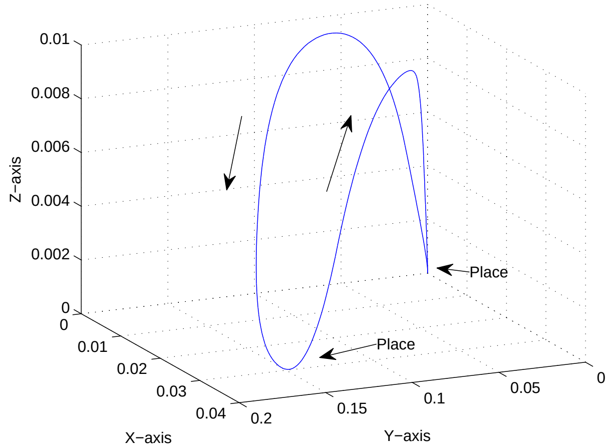

Figure 1. 3D reference trajectory for the gantry robot.

On constructing minimal state-space realizations for the transfer-functions (53), (54) and (55) it can be verified that they do not form a convex hull but it is a routine task to form such a hull with Vertex 1:

$$A_1 = \begin{bmatrix} 0.9848 & 0.0339 & 0.0986 \\ 0.1250 & 0 & 0 \\ 0 & 0.0625 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0.03125 \\ 0 \\ 0 \end{bmatrix}, C_1 = \begin{bmatrix} 0.0114 & 0.0074 & 0.0234 \end{bmatrix},$$

Vertex 2:

$$A_2 = \begin{bmatrix} 0.9911 & 0.0412 & 0.0992 \\ 0.1250 & 0 & 0 \\ 0 & 0.0625 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0.03125 \\ 0 \\ 0 \end{bmatrix}, C_2 = \begin{bmatrix} 0.0103 & 0.0037 & 0.0046 \end{bmatrix},$$

Vertex 3:

$$A_3 = \begin{bmatrix} 0.9941 & 0.0406 & 0.1082 \\ 0.1250 & 0 & 0 \\ 0 & 0.0625 & 0 \end{bmatrix}, B_3 = \begin{bmatrix} 0.03125 \\ 0 \\ 0 \end{bmatrix}, C_3 = \begin{bmatrix} 0.0117 & 0.0091 & 0.0089 \end{bmatrix}.$$

The 3D reference trajectory for the gantry robot used in the experimental verifications referenced above is shown in Fig. 1 with component for the axis considered in this paper shown in Fig. 2 for 200 samples (the vertical scale in both plots is in metres).

Assume that there exists an unknown actuator fault Γ but it is known that $0.7 = \underline{\Gamma} \leq \Gamma \leq \overline{\Gamma} = 1.3$. Using the constant parameter Lyapunov function and applying Theorem 1 does not give conclusive results as it is not possible to obtain decision matrices $P_1 > 0$ and $P_2 > 0$ for the LMI (25), which can be only marginally feasible. Note, however, that for a smaller uncertainty and actuator fault term Γ a solution can be found.

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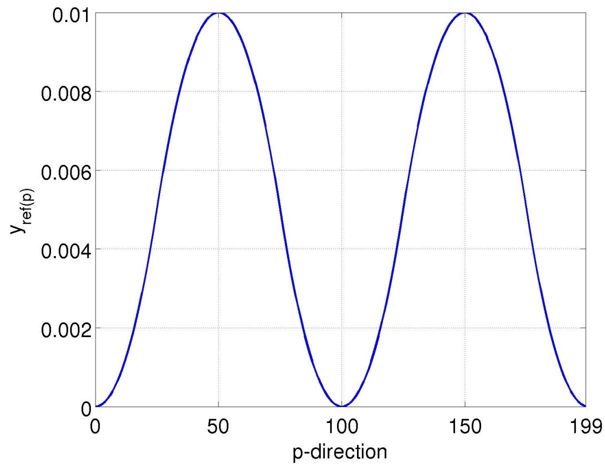

Figure 2. Z-axis reference trajectory for the gantry robot.

Using parameter dependent Lyapunov function and Theorem 3 removes this problem and the LMI of Theorem 3 is in this case feasible with the following decision matrices:

$$P_{1,1} = \begin{bmatrix} 2.1351 \times 10^{-7} & -1.3579 \times 10^{-8} & 2.8651 \times 10^{-9} \\ -1.3579 \times 10^{-8} & 5.8441 \times 10^{-9} & -1.4383 \times 10^{-9} \\ 2.8651 \times 10^{-9} & -1.4383 \times 10^{-9} & 2.0277 \times 10^{-9} \end{bmatrix}, \quad P_{2,1} = 8.5940 \times 10^{-10},$$
$$P_{1,2} = \begin{bmatrix} 2.1369 \times 10^{-7} & -1.2996 \times 10^{-8} & 3.3387 \times 10^{-9} \\ -1.2996 \times 10^{-8} & 5.5944 \times 10^{-9} & -1.7020 \times 10^{-9} \\ 3.3387 \times 10^{-9} & -1.7020 \times 10^{-9} & 1.7712 \times 10^{-9} \end{bmatrix}, \quad P_{2,2} = 8.6029 \times 10^{-10},$$
$$P_{1,3} = \begin{bmatrix} 2.1517 \times 10^{-7} & -1.4679 \times 10^{-8} & 2.3111 \times 10^{-9} \\ -1.4679 \times 10^{-8} & 5.8498 \times 10^{-9} & -1.3334 \times 10^{-9} \\ 2.3111 \times 10^{-9} & -1.3334 \times 10^{-9} & 2.2224 \times 10^{-9} \end{bmatrix}, \quad P_{2,3} = 8.5859 \times 10^{-10},$$
$$G_1 = \begin{bmatrix} 5.6875 \times 10^{-7} & -1.0835 \times 10^{-7} & -1.8945 \times 10^{-8} \\ -1.4243 \times 10^{-7} & 2.1528 \times 10^{-7} & -1.5591 \times 10^{-8} \\ -1.5804 \times 10^{-7} & -8.8637 \times 10^{-8} & 1.5881 \times 10^{-7} \end{bmatrix}, \quad G_2 = 9.5443 \times 10^{-10},$$
$$R_1 = \begin{bmatrix} -2.3451 \times 10^{-5} & 2.9069 \times 10^{-6} & 3.2565 \times 10^{-7} \end{bmatrix}, \quad R_2 = 2.3779 \times 10^{-7}.$$

The feasibility of the LMI is immediate since the decision matrices are positive definite with eigenvalues

$$\begin{aligned} \mathrm{eig}(P_{1,1}) &= \begin{bmatrix} 1.5316 \times 10^{-9} & 5.4150 \times 10^{-9} & 2.1443 \times 10^{-7} \end{bmatrix}^T, \\ \mathrm{eig}(P_{1,2}) &= \begin{bmatrix} 1.1106 \times 10^{-9} & 5.3910 \times 10^{-9} & 2.1455 \times 10^{-7} \end{bmatrix}^T, \\ \mathrm{eig}(P_{1,3}) &= \begin{bmatrix} 1.7509 \times 10^{-9} & 5.2701 \times 10^{-9} & 2.1622 \times 10^{-7} \end{bmatrix}^T. \end{aligned}$$

The control law matrices are

$$K_1 = \begin{bmatrix} -45.2421 & -11.0928 & -4.4355 \end{bmatrix}, \quad K_2 = 249.1412. \tag{56}$$

Next, two scenarios are examined by simulations.

Scenario 1: Constant Fault

The controlled process has been simulated with constant fault equal to two different values $\Gamma = 0.7$ and $\Gamma = 1.3$ for each polytope point, resulting in the controlled dynamics shown in Figs. 3–8, where SSE denotes the sum of the squared error computed over each trial. It is visible that the

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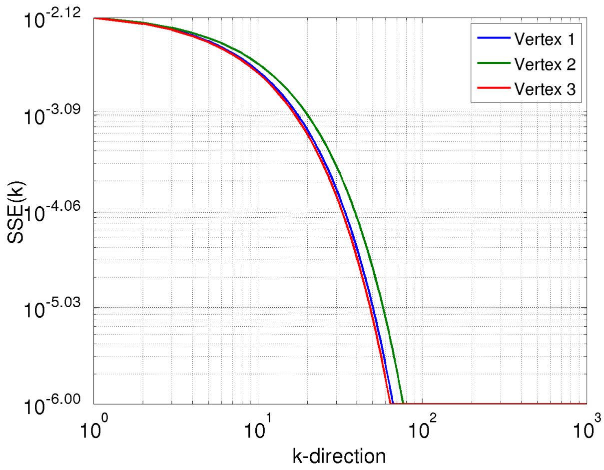

Figure 3. SSE error versus k for Vertex 1, Vertex 2 and Vertex 3 under Scenario 1, for $\Gamma = 0.7$.

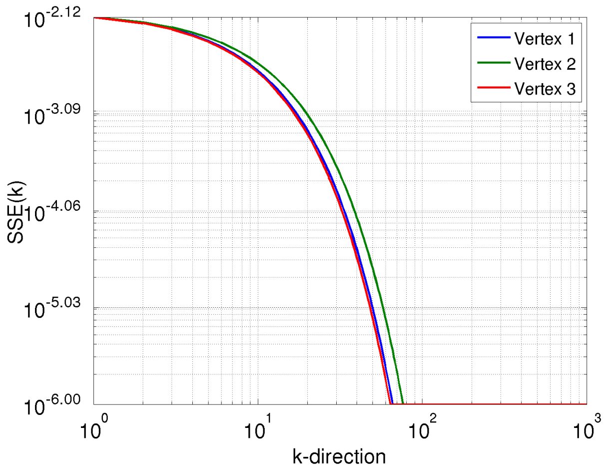

Figure 4. SSE error versus k for Vertex 1, Vertex 2 and Vertex 3 under Scenario 1, for $\Gamma = 1.3$.

ILC errors converge monotonically from trial-to-trial, in agreement with the theory, and reach very small value after approximately 40 trials and also excessive control action is not required.

Scenario 2: Trial Variant Fault

The controlled process has been simulated with $\Gamma = 0.7$ until $k = 20$ when a constant fault caused a switch to $\Gamma = 1.3$ and then at $k = 40$ a further switch to return to the previous value of Γ, which results in the controlled dynamics shown for some central point of the polytope in Figs. 9–10. Also, in this case the parameter dependent Lyapunov design the SSE error has reached a very small value after approximately 40 trials and the control signal is within bounds that would permit physical application. The switchings produce a temporary error increase, but this is rapidly damped over succeeding trials.

This example, based on the model of a physical system, has demonstrated that the parameter dependent Lyapunov design can enlarge the uncertainty range and enable robust ILC law design in the presence of actuator faults when the constant Lyapunov function approach is unable to find a solution.

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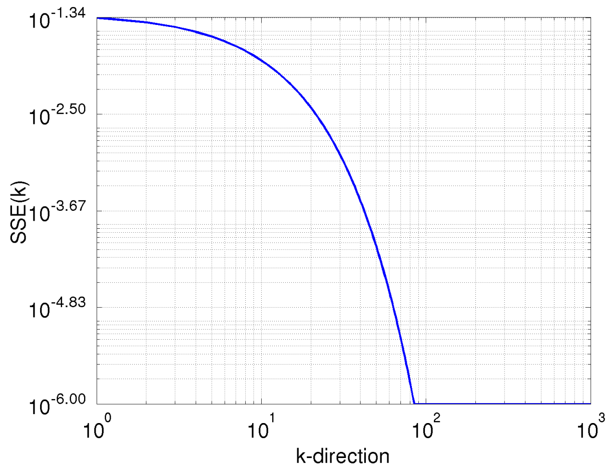
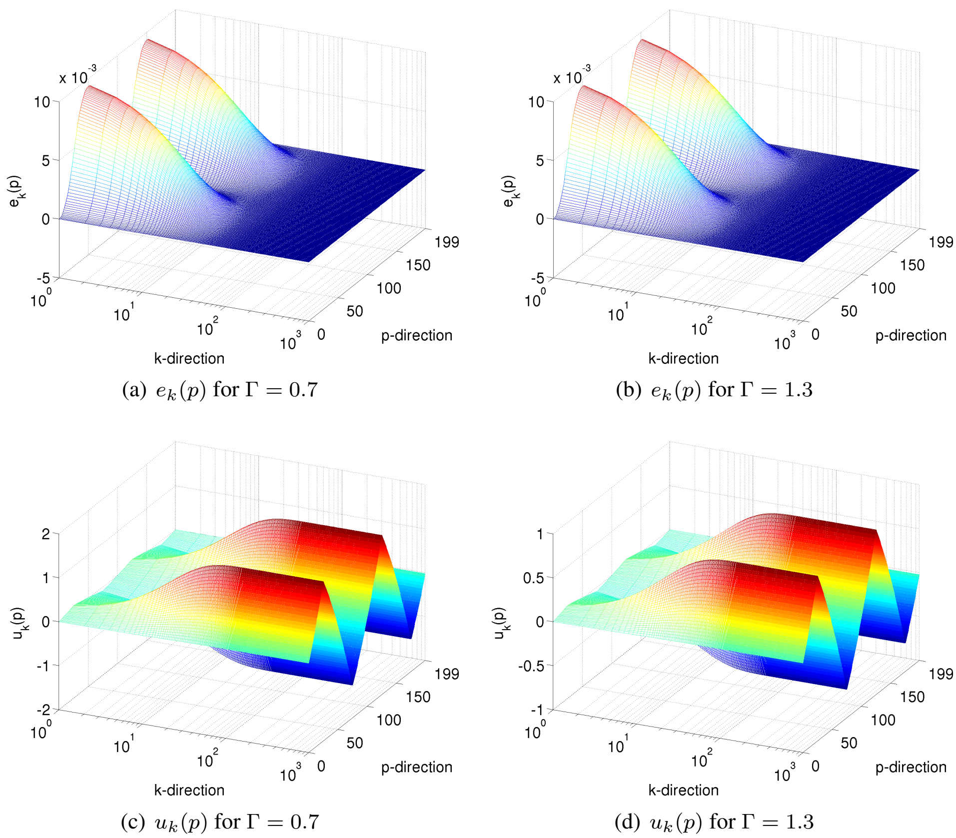


Figure 5. SSE error versus k under Scenario 1 for some central point of the polytope.



(a) $e_k(p)$ for $\Gamma = 0.7$ (b) $e_k(p)$ for $\Gamma = 1.3$

(c) $u_k(p)$ for $\Gamma = 0.7$ (d) $u_k(p)$ for $\Gamma = 1.3$

Figure 6. Error and input signal for and vertex 1 under Scenario 1.

5. CONCLUSIONS

This paper has considered robust ILC design for discrete linear systems in the presence of actuator faults. The new results have been derived by writing the ILC dynamics as a discrete linear repetitive process setting and a parameter dependent Lyapunov function employed to augment

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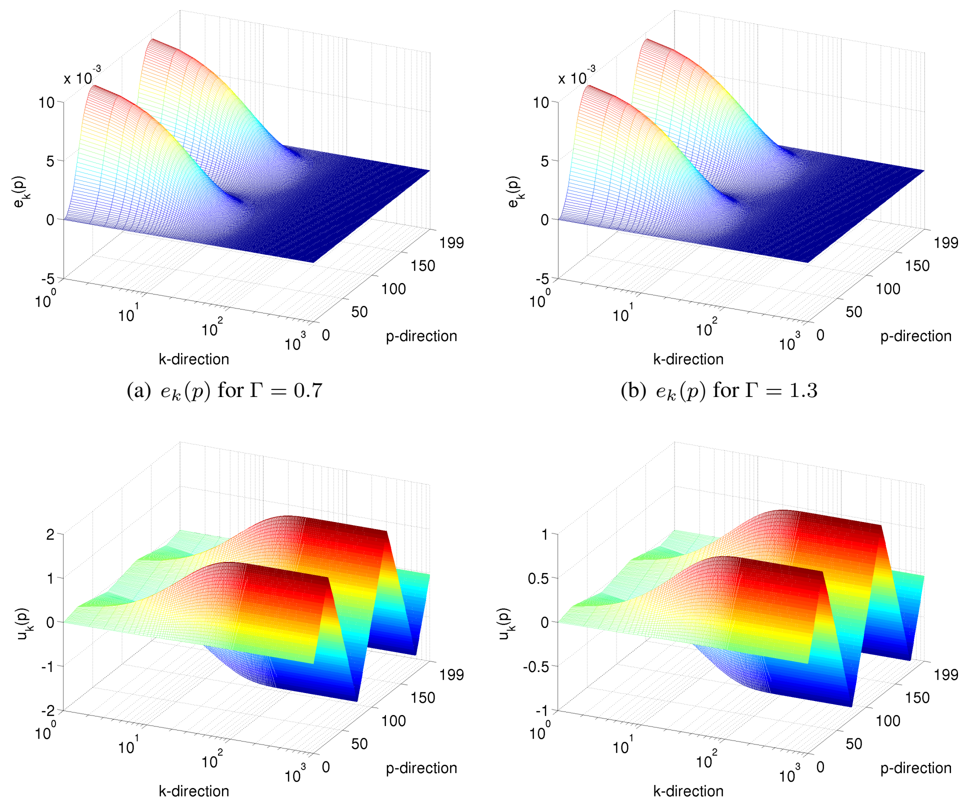

(a) $e_k(p)$ for $\Gamma = 0.7$ (b) $e_k(p)$ for $\Gamma = 1.3$

(c) $u_k(p)$ for $\Gamma = 0.7$ (d) $u_k(p)$ for $\Gamma = 1.3$

Figure 7. Error and input signal for vertex 2 under Scenario 1.

the model uncertainty range whilst preserving acceptable system performance. In particular, it has demonstrated that when the commonly used constant Lyapunov function approach fails for some required range of uncertainty, parameter dependent Lyapunov functions can allow a solution to be found. These results provide a firm basis for onward development and eventual experimental verification. The final control law (50) is composed of state feedback for stabilization of the transient dynamics along the trials and ILC phase-lead for trial-to-trial error convergence. The remaining term in the control law uses noncausal temporal information, which is the defining characteristic of ILC. The state feedback part of the ILC will require an observer if all entries in the state vector are not available for measurement. An alternative would be to seek to replace the state vectors involved with the corresponding pass profile vectors and this is one topic for future research. Another would be to expand the fault function to, for example, examine the effects of the fault detection time on the transient performance.

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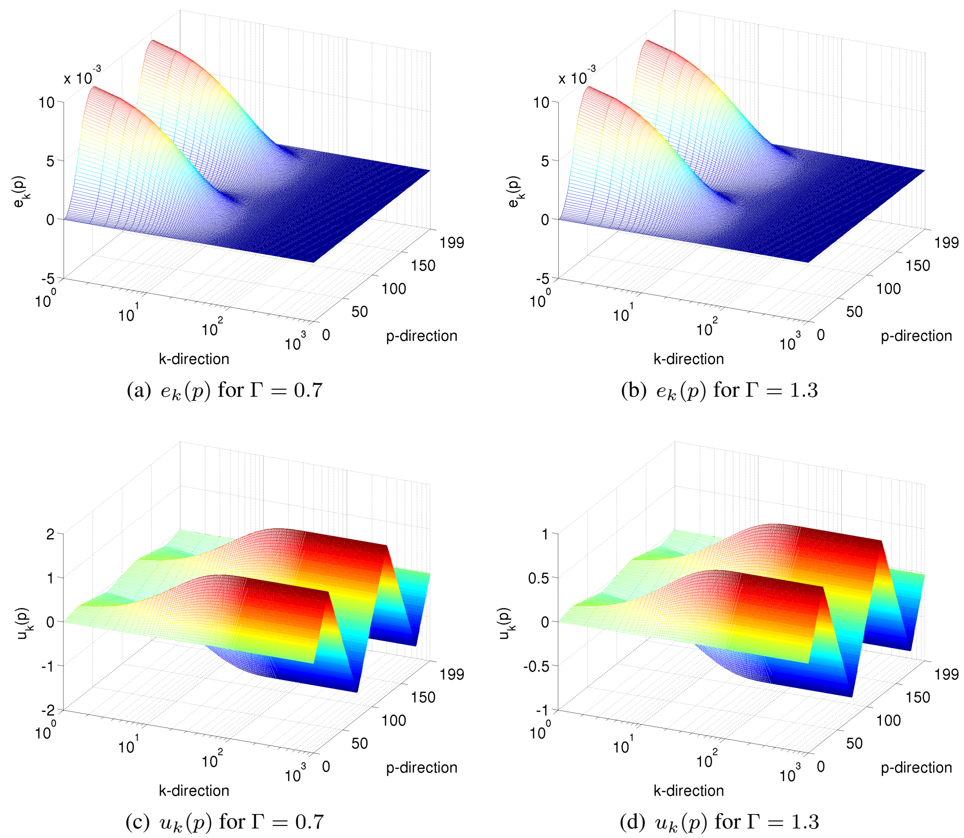

(a) $e_k(p)$ for $\Gamma = 0.7$ (b) $e_k(p)$ for $\Gamma = 1.3$

(c) $u_k(p)$ for $\Gamma = 0.7$ (d) $u_k(p)$ for $\Gamma = 1.3$

Figure 8. Error and input signal for and vertex 3 under Scenario 1.

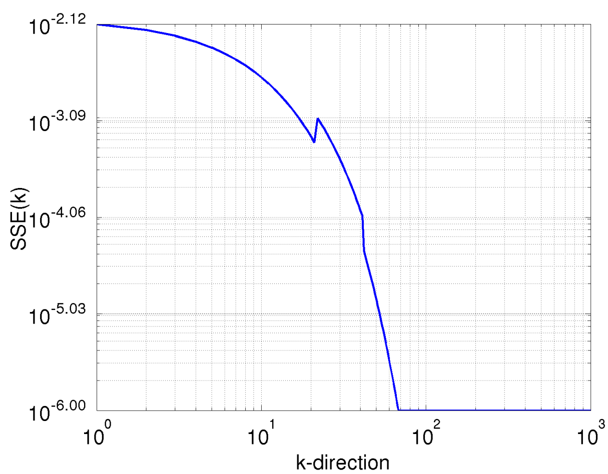

Figure 9. SSE error versus k under Scenario 2.

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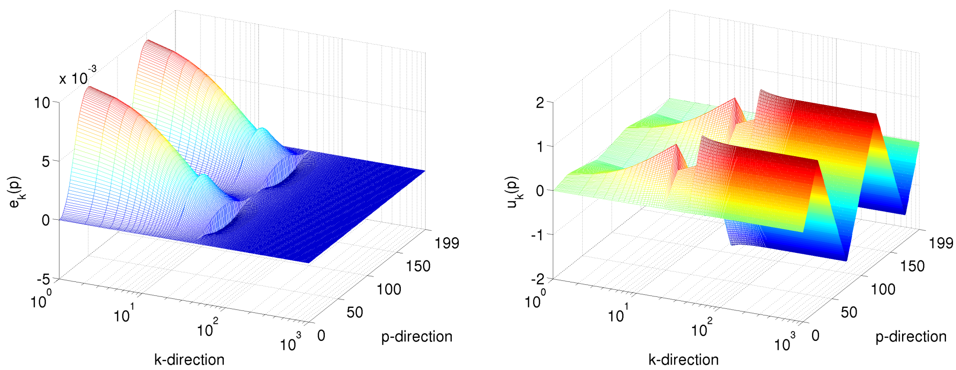

(a) $e_k(p)$ (b) $u_k(p)$

Figure 10. Error and input signal under Scenario 2.

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