MIXED FE-SP METHOD FOR NONLINEAR STRUCTURE-WATER INTERACTIONS WITH FREAK WAVES

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Summary This paper develops a mixed finite element – smoothing particle method for violent water-structure interactions involving freak waves and separation between structure and water. The structure undergoes a large rigid motion of 6 DOF with a small elastic deformation, so that its elastic displacement relative to the rigid motion can be represented in a mode summation based on FE analysis. The water is assumed inviscid - incompressible and its motion governed by nonlinear N-S equation. On the coupling interface where no FS separation happens, the equilibrium and consistence conditions are required. The numerical iteration process is suggested to solve the nonlinear FSI equations, and validation examples are shown a good agreement with available experiment results.

GOVERNING EQUATIONS

As shown in Fig. 1, a sold in a material domain Ω_s of outside boundary normal vector ν_i, on which a traction force $\hat{T}_i$ and a displacement $\hat{U}_i$ are respectively given on boundary S_T and S_U, interacts on its wet interface Σ with water domain Ω_f, of which the dynamic pressure $p=0$ on free surface Γ_f, $v_i=0$ on Γ_b and possible given velocity $\hat{v}_i$ on boundaries Γ_v or Γ_v. Three coordinate systems are defined: $o-x_1x_2x_3$ with positive x_3 in the vertical direction is fixed in the space, $O-y_1y_2y_3$, of which axis $O-y_i$ parallels axis $o-x_i$, (i = 1,2,3), is a moving system fixed at the mass centre O of solid, and $O-X_1X_2X_3$ is a material system. A material point X_j at time $t=0$ moves to a new position x_j at time t, and its motion is denoted by a summation of a translation u_i of mass centre, rigid rotation θ_i about three axes and small elastic displacement U_j of structure. Using the notations of Cartesian tensors and summation convention [1], such as Kronecker Delta δ_{ij} and permutation symbol e_{ijk}, and denoting the rigid rotation by a tensor $\beta_{ij}(\theta_k)$ with its time derivative $\dot{\beta}_{ij}=e_{imk}\dot{\theta}_m\beta_{kj}$ and partial derivative $\partial\beta_{ij}/\partial\theta_m=e_{imk}\beta_{kj}$, we can derive the dynamic equations of the system as follows.

Solid: The position and velocity of particle X_j at time t respectively are

$$x_i=u_i+\beta_{ij}X_j+\beta_{ij}U_j,\qquad \dot{x}_i=\dot{u}_i+\dot{\beta}_{ij}X_j+\dot{\beta}_{ij}U_j+\beta_{ij}\dot{U}_j=\dot{u}_i+e_{imk}\dot{\theta}_m\beta_{kj}(X_j+U_j)+\beta_{ij}\dot{U}_j. \tag{1}$$

Introducing a transformation H_{ij} with its related generalized coordinate $\tilde{U}_i$, which may be a *finite element interpolation function matrix* H_{ij} [2] *and node displacement vector*, or *a mode function matrix and generalised mode coordinate vector* [3], we denote the elastic displacement $U_i=H_{i\alpha}\tilde{U}_\alpha$ and define $\tilde{H}_{j\alpha}=\int_{\Omega_S}\rho_s H_{j\alpha}d\Omega,\ V_{jn\alpha\gamma}=\int_{\Omega_S}\rho_s H_{j\alpha}H_{n\gamma}d\Omega,\ S_{jn\alpha}=\int_{\Omega_S}\rho_s X_j H_{n\alpha}d\Omega,\ \tilde{M}_{\alpha\gamma}=V_{jj\alpha\gamma},\ (\alpha,\gamma=1,2,\cdots,N)$, so that the kinetic and potential energy of body are respectively calculated in the forms

$$\begin{gathered}
T=0.5\{M\delta_{rs}\dot{u}_r\dot{u}_s+\tilde{J}_{rs}\dot{\theta}_r\dot{\theta}_s+\tilde{M}_{\alpha\gamma}\dot{\tilde{U}}_\alpha\dot{\tilde{U}}_\gamma+2A_{rs}\dot{u}_r\dot{\theta}_s+2B_{r\alpha}\dot{u}_r\dot{\tilde{U}}_\alpha+2C_{r\alpha}\dot{\theta}_r\dot{\tilde{U}}_\alpha\},\quad A_{rs}(\beta,U)=e_{rsk}\beta_{kj}\tilde{H}_{j\alpha}\tilde{U}_\alpha,\quad B_{r\alpha}(\beta)=\beta_{rn}\tilde{H}_{n\alpha},\\
\Pi=0.5\tilde{K}_{\alpha\gamma}\tilde{U}_\alpha\tilde{U}_\gamma+gM\delta_{r3}u_r+g\delta_{i3}\beta_{ij}\tilde{H}_{j\alpha}\tilde{U}_\alpha-u_r\beta_{rl}\hat{F}_l-\tilde{U}_\alpha\tilde{F}_\alpha-\hat{\Gamma}+u_r\beta_{rl}P_l+\tilde{U}_\alpha\tilde{P}_\alpha+\Gamma_p,\quad \tilde{K}_{\alpha\gamma}=\int_{\Omega_S}H_{i\alpha,j}E_{ijkl}H_{k\gamma,l}d\Omega,\\
C_{r\gamma}(\beta,U)=e_{rki}\beta_{kj}\beta_{in}V_{jn\alpha\gamma}\tilde{U}_\alpha,\quad \tilde{J}_{rs}(\beta,U)=(J_{nn}\delta_{rs}-\beta_{rj}\beta_{sn}J_{jn})+[2\delta_{rs}S_{jj\alpha}-\beta_{rj}\beta_{sn}(S_{jn\alpha}+S_{njl})]\tilde{U}_\alpha+(\delta_{rs}V_{jj\alpha\gamma}-\beta_{rj}\beta_{sn}V_{jn\alpha\gamma})\tilde{U}_\alpha\tilde{U}_\gamma,\\
\hat{F}_l=\int_{S_T}\hat{T}_l dS,\quad \tilde{F}_\alpha=\int_{S_T}H_{l\alpha}\hat{T}_l dS,\quad \hat{\Gamma}=\int_{S_T}X_l\hat{T}_l dS,\quad P_l=\int_\Sigma p\nu_l dS,\quad \tilde{P}_\alpha=\int_\Sigma pH_{l\alpha}\nu_l dS,\quad \Gamma_p=\int_\Sigma pX_l\nu_l dS.
\end{gathered}\tag{2}$$

Here M is total mass of solid, $\tilde{M}$ is generalized mass matrix, $\hat{F}_l$, $\hat{\Gamma}$ and $\tilde{F}_r$ are resultant force, moment and generalised force, while the FSI forces involving fluid pressure p denotes by the last column in Eq. 3. We derive the dynamic equations of solid motion by using Hamilton equation [3], $d(\partial T/\partial\dot{q}_i)/dt-\partial T/\partial q_i+\partial\Pi/\partial q_i=0$, therefore, we have

$$\begin{gathered}
\begin{bmatrix} M\delta_{rs} & A_{rs} & B_{r\alpha}\\ A_{sr} & \tilde{J}_{rs} & C_{r\alpha}\\ B_{\gamma s} & C_{\gamma s} & \tilde{M}_{\gamma\alpha}\end{bmatrix}\begin{bmatrix}\ddot{u}_s\\ \ddot{\theta}_s\\ \ddot{\tilde{U}}_\alpha\end{bmatrix}+\begin{bmatrix}0 & \dot{A}_{rs} & \dot{B}_{r\alpha}\\ \dot{A}_{sr} & \dot{\tilde{J}}_{rs} & \dot{C}_{r\alpha}\\ \dot{B}_{\gamma s} & \dot{C}_{\gamma s} & 0\end{bmatrix}\begin{bmatrix}\dot{u}_s\\ \dot{\theta}_s\\ \dot{\tilde{U}}_\alpha\end{bmatrix}+\begin{bmatrix}0&0&0\\0&0&0\\0&0&\tilde{K}_{\gamma\alpha}\end{bmatrix}\begin{bmatrix}u_s\\ \theta_s\\ \tilde{U}_\alpha\end{bmatrix}\\
-\begin{bmatrix}0 & 0 & 0\\ \dot{\theta}_l\partial A_{sl}/\partial\theta_r & 0.5\dot{\theta}_l\partial\tilde{J}_{ls}/\partial\theta_r & \dot{u}_l\partial B_{ls}/\partial\theta_r+\dot{\theta}_l\partial C_{l\alpha}/\partial\theta_r\\ \dot{\theta}_l\partial A_{sl}/\partial\tilde{U}_\gamma & 0.5\dot{\theta}_l\partial\tilde{J}_{l\alpha}/\partial\tilde{U}_\gamma & \dot{\theta}_l\partial C_{l\alpha}/\partial\tilde{U}_\gamma\end{bmatrix}\begin{bmatrix}\dot{u}_s\\ \dot{\theta}_s\\ \dot{\tilde{U}}_\alpha\end{bmatrix}=\begin{bmatrix}-gM\delta_{r3}\\ -g\delta_{i3}\partial\beta_{ij}/\partial\theta_r\tilde{H}_{j\alpha}\tilde{U}_\alpha\\ -g\delta_{i3}\beta_{ij}\tilde{H}_{\gamma j}\end{bmatrix}+\begin{bmatrix}\beta_{rl}\hat{F}_l\\ u_m\partial\beta_{ml}/\partial\theta_r\hat{F}_l\\ \tilde{F}_\gamma\end{bmatrix}-\begin{bmatrix}\beta_{rl}P_l\\ u_m\partial\beta_{ml}/\partial\theta_r P_l\\ \tilde{P}_\gamma\end{bmatrix}
\end{gathered}\ . \tag{3}$$

Fluid: The water motion is governed by *Lagrangian* Navier-Stokes equation for incompressible and inviscid fluid in association with corresponding boundary conditions [4-6]

$$\dot{v}_i=g\delta_{i3}-p_{,i}/\rho_f,\ \ v_{i,i}=0,\ \text{ in }\ \Omega_f;\quad \rho_f(g\delta_{i3}+\ddot{U}_i)\nu_i=-p_{,i}\nu_i,\ \ v_i\nu_i=U_i\nu_i,\ \text{ on }\ \Sigma;\quad p=0,\ \text{ on }\ \Gamma_f;\quad v_i=\hat{v}_i,\ \text{ on }\ \Gamma_v. \tag{4}$$

PARTITIONED ITERATION SOLUTION

Equations (3) and (4) include 3 rigid translations u_s, 3 rigid rotations θ_s and N generalised coordinates $\tilde{U}_\alpha$, in which the rigid motions could be large but the elastic deformation is assumed small, so that the second order quantity of $\tilde{U}_\alpha$ are neglected. It is convenience to use a commercial FE code to obtain the natural modes and frequencies of the solid as the input data for the code designed for the method

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proposed in this paper. The partitioned iteration is suggested to solve these FSI equations [7-8], starting from initial time $t = 0$, at which the state of the system are given, to calculate the solution at time $t + \Delta t$ in the following steps. i) *Initial calculations*: FE mode dada as input to generate related matrices; initial position, velocity, acceleration and pressure of the system. ii) *Fluid SPM solver*: based on the state of solid at time t to deliver fluid variables at time $t + \Delta t$ using the projection [9] method: a) with no pressure effect to calculate an intermediate velocity $\tilde{v}_i$ and location $\tilde{x}_i$ of particles, $\tilde{v}_i = v_i^t + \Delta t g \delta_{i3}$, $\tilde{x}_i = x_i^t + \Delta t \tilde{v}_i$; b) a pressure Poisson equation is solved to obtain the pressure, $p_{,ii}^{t+\Delta t} = \rho_f \tilde{v}_{i,i} / \Delta t + \psi \rho_f (n^0 - n^t)/(n^0 \Delta t^2)$, $\psi = | n^0 - n^t / n^0 | + \eta \Delta t | v_{i,i}^t |$, where $\eta = 1, 0$ respectively for $(n^0 - n^t) v_{i,i}^t \geq 0$, and $(n^0 - n^t) v_{i,i}^t \leq 0$, to identify compressed or expanded state of fluid; c) fluid velocity and position at time $t + \Delta t$ are calculated $v_i^{t+\Delta t} = \tilde{v}_i - \Delta t p_{,i}^{t+\Delta t} / \rho_f$, $x_i^{t+\Delta t} = x_i^t + \Delta t v_i^{t+\Delta t}$. iii) *Solid solver*: based on the fluid pressure to solve solid equation by time integration iteration to obtain the state variables of the solid. iv) *Convergence check*: calculating the displacement error norm on FSI interface to see if $\int_\Sigma | U_i^{t+\Delta t} - U_i^t | dS \leq \varepsilon$ then go to next time step $t + 2\Delta t$, otherwise, to repeat process ii) and iii) until convergence reached. A calculation flow chat is shown in Fig. 2, where *n* denotes an FSI iteration number and other notations are the state variables used in the computer code, for which some SPM particular treatment techniques, such as neighbour particle searching, particle interactions with shifting and collision handling, free surface particle identification and FS surface separation flag are considered [10-13].

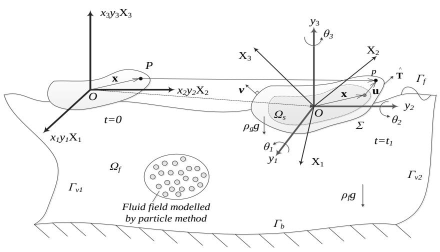

Fig.1 Scheme of FSI system

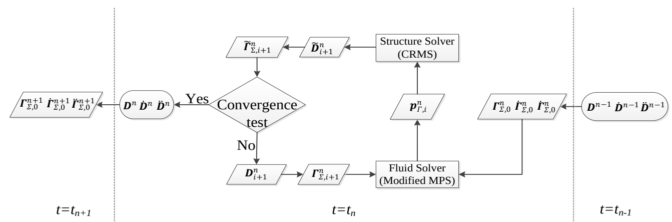

Fig. 2 Flow chart of FSI partitioned iteration process

VALIDATION AND APPLICATIONS

To validate the developed method, the numerical simulations [10-12] of rigid and elastic wedge dropping on the water surface have been completed and compared with available experiments [14-16], which are shown in Fig. 3.

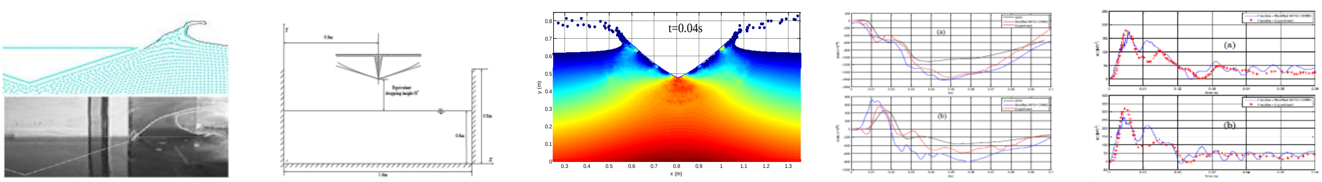

Fig. 3 Wedge dropping simulations: (i) rigid one comparison with experiment photo [14][10]; (ii) elastic test arrangement [16]; (iii) numerical picture at time 0.04s [11-12]; (iv) comparison of strains at two points from wedge tip with experiments a) 30mm, b) 120mm [15]; (v) comparison of accelerations of elastic wedge with experiments a) case 1 (4.29m/s), b) case 2 (5.57m/s) [16].

It has been shown a very good agreement with experiments, which demonstrates that the proposed method is attractive to deal with strong nonlinear FSI involving freak waves or FS separations, which is difficult to be solved by mesh based methods. This type of problems have an important background in marine engineering, such as green water impacts concerned by ship designs and aircrafts landing on the water. The developed method and its integrated computer code is still developing to tackle more engineering application cases discussed in many books, such as [17-19]. It is noticed that the system involves chaos and bifurcations that may be met in FSI numerical simulations, which requires further investigations using nonlinear dynamic approaches, such as the developed energy flow method [20].

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