Supplemental material: Optical resonance shifts in the fluorescence of thermal and cold atomic gases

S. D. Jenkins and J. Ruostekoski
Mathematical Sciences, University of Southampton, Southampton SO17 1BJ, United Kingdom

J. Javanainen
Department of Physics, University of Connecticut, Storrs, Connecticut 06269-3046

R. Bourgain, S. Jennewein, Y. R. P. Sortais, and A. Browaeys
Laboratoire Charles Fabry, Institut d'Optique, CNRS, Univ Paris Sud,
2 Avenue Augustin Fresnel, 91127 Palaiseau cedex, France

(Dated: May 5, 2016)

In this supplement we compare the spectral intensity scattered from an inhomogeneously broadened sample of ^{87}Rb atoms to the Voigt profile fit to the spectral intensity. In addition, we show the scatter plots of the distribution of collective eigenmode resonance linewidths and line shifts.

Voigt profiles

At the very low temperatures of the experiments the atoms are stationary to first approximation, but we wish to retain the option to discuss moving atoms. In principle one could allow the atoms to move either ballistically or as a result of the trapping forces, dipole forces between the atoms, and even collisions of other origin, and integrate the equations of the polarization amplitudes. The drawback is a greatly complicated coding. Instead, we resort to a simple model of inhomogeneous broadening: We add to the detuning of the $j^{\rm th}$ atom a random quantity ζ_j that has a Gaussian distribution with the root-mean-square width $\Delta\omega$,

$$f(\zeta) = \frac{1}{\sqrt{2\pi}\Delta\omega} e^{-\frac{\zeta^2}{2\Delta\omega^2}} \,. \tag{1}$$

For instance, with $\Delta\omega = ku$ and $u = \sqrt{k_B T/m}$ being the thermal velocity, this would be the standard model for the Doppler broadening of a resonance line of an atom with mass m at the temperature T. Instead of the usual Lorentzian resonance line of width γ, we then expect a resonance line shape that is the convolution of a Lorentzian and a Gaussian, commonly known as the Voigt profile:

$$\begin{aligned} V(\Delta,\gamma,\Delta\omega) &= \frac{1}{\sqrt{2\pi}\Delta\omega}\int d\zeta \frac{\gamma^2}{(\Delta+\zeta)^2+\gamma^2} e^{-\frac{\zeta^2}{2\Delta\omega^2}} \\ &= \sqrt{\frac{\pi}{2}}\frac{\gamma}{\Delta\omega}\Re\left[e^{\frac{(\gamma-i\Delta)^2}{2\Delta\omega^2}}\operatorname{erfc}\left(\frac{\gamma-i\Delta}{\sqrt{2}\Delta\omega}\right)\right], \end{aligned} \tag{2}$$

where erfc stands for the complement of the error function. This function is built in to, say, Mathematica. It is routine in spectroscopy, and here as well, to fit a function of the form $A\,V(\Delta-\delta,\gamma,\Delta\omega)$ to a resonance line, with any of A, δ, γ or $\Delta\omega$ regarded as variable parameters as needed (an example curve shown in Fig. 1). The standard spectroscopic fitting then gives an estimate of the shift δ of the resonance from the original atomic resonance at ω_0.

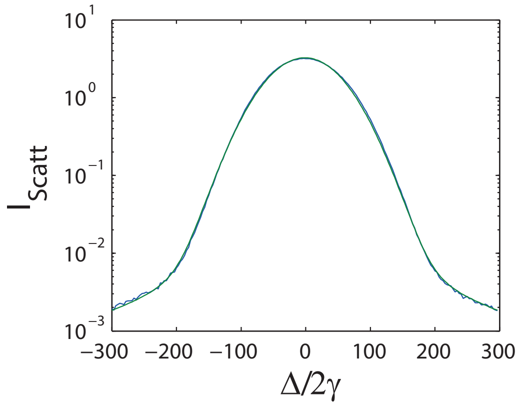

FIG. 1. The light scattered from an inhomogeneously broadened (100γ) sample of 450 ^{87}Rb atoms as a function of incident field detuning Δ. The peak density is $\rho = 2.4\times10^{14}\,\mathrm{cm}^{-3}$. The blue line is the scattered intensity calculated from Monte-Carlo simulations, while the green line is the fit to a Voigt profile. Here $\Delta = \omega - \omega_0'$ where ω_0' is the frequency at which a single ^{87}Rb atom scatters the most incident light in the presence of a magnetic field.

Scatter plots of eigenmodes

In the main section we illustrated the effect of collective modes by displaying the distribution of the resonance linewidths of the collective modes for both thermal and cold atomic ensembles. One can also see how collective modes differ in cold and thermal gasses by examining the joint distributions of δ_n and $\log_{10} \upsilon_n$. Scatter plots of δ_n and $\log_{10} \upsilon_n$ obtained from five realizations of

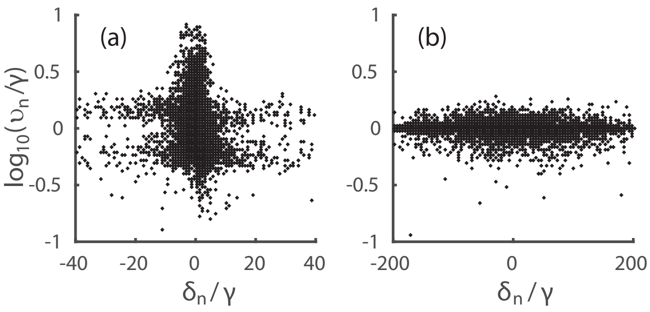

FIG. 2. Representation in the complex plane of the eigenvalues associated with the collective modes for a particular realization of the atomic positions. Each mode is represented by a point, with the coordinates being the mode resonance frequencies δ_n and its decay rates υ_n. (a) Homogeneously and (b) inhomogeneously broadened (with a root-mean-square Gaussian width of 100γ) samples of 450 ^{87}Rb atoms.

atomic positions in samples of 450 atoms are illustrated in Fig. 2. In the cold gas, many of the mode resonance frequencies are shifted from the single-atom resonance, while collective decay rates span several orders of magnitude. The distribution of mode resonance frequencies in the inhomogeneously broadened sample, on the other hand, mostly reflects the Gaussian distributions of single-atom resonance frequencies. Furthermore, the collective mode decay rates closely match single-atom decay rates, indicating that the atoms respond to light nearly independently.