UNIVERSITY OF SOUTHAMPTON

LIGHT-CONE AND SHORT DISTANCE ASPECTS OF NUCLEON WAVEFUNCTIONS

by

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ABSTRACT

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In recent years the technique of QCD sum rules has been used to obtain a number of very satisfactory results in the non-perturbative region of QCD. Here we study the first few moments of the quark distribution amplitudes of the nucleon, using the same auxiliary operators as Chernyak and Zhitnitsky. We differ from these authors in the operator product expansions we obtain for the current correlators in the Euclidean region. Nevertheless, we are able to confirm that the sum rule analysis leads to an asymmetric distribution of longitudinal momentum among the constituent quarks of the nucleon.

We investigate the implications of such an asymmetry for the rate of the decay $p \to \pi^0 e^+$ in the minimal SU(5) GUT. The calculation is performed using the chiral lagrangian formalism. We find significant enhancement (by a factor of about 6) of the proton lifetime over that predicted using a symmetric wavefunction. The effect of an asymmetric distribution of quark transverse momenta is also studied.

Finally we introduce explicit SU(3) symmetry breaking terms into the baryon number violating chiral lagrangian and demonstrate that the subsequent corrections to the decay rates of the proton are consistent with a particular choice of baryon wavefunction.

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The material of Chapter 1, which reports on a collaboration with Dr. C.T. Sachrajda, has been submitted for publication in Nuclear Physics B. Other work on topics not covered in this thesis may be found in the following publications:

I.D. King, G. Thompson and H.L. Yu, Phys. Lett. 145B (1984) 393.

I.D. King and G. Thompson, Phys. Rev. D31 (1985) 2148.

INTRODUCTION

Quantum Chromodynamics (QCD) is now firmly established as the theory of strong interactions. It gives rise to asymptotic freedom, so that at short distances the effective coupling of the strong interaction decreases to zero. This implies that it is meaningful to apply perturbation theory to hard transverse momentum processes. Use of perturbative QCD has led to many successful predictions for hard exclusive processes. For example, the proton's magnetic form factor, measured in elastic electron-proton scattering, is predicted to fall off like Q^{-4} (Q^2 is the square of the four-momentum transferred between the electron and the proton.) This dependence is confirmed by the experimental measurements. Recently, however, there has been much debate as to whether perturbation theory is applicable in the interpretation of data available at the currently attainable energies [1]. This discussion has been stimulated by the discovery that spin effects are not negligible in large angle proton-proton elastic scattering at high energies [2]. When QCD is used in conjunction with a conventional (non-relativistic) wavefunction for the proton it is found that spin effects become vanishingly small at large Q^2. Recently, Chernyak and Zhitnitsky [3] have used a QCD sum rule analysis to derive a relativistic three-quark wavefunction for the proton. It is hoped that the spin correlations inherent in this wavefunction are such that its use in a QCD calculation of proton-proton elastic scattering will eliminate the discrepancy with experiment. Already, the wavefunction of Chernyak and Zhitnitsky has been applied to many physical processes, leading to results which are in close accord with observed values. In a calculation of nucleon electromagnetic form factors, for example, it is found that for the first time the predicted signs and magnitudes of the form factors are in excellent agreement with experiment. One of the most significant properties of the new relativistic wavefunction is that the quarks are seen to play very asymmetric roles. Approximately two-thirds of the proton's longitudinal momentum (in the infinite momentum frame) is carried by one up quark whose spin is parallel to the proton's momentum. This contrasts with the completely symmetric distribution of quark momenta associated with the naive non-relativistic proton wavefunction.

In this thesis we study in detail the derivation of the nucleon wavefunction given by Chernyak and Zhitnitsky. We also present a modification of the approach used by Brodsky et al [4] to obtain a lifetime for the proton for the minimal SU(5) model. Some of the conclusions are valid for other conventional (i.e. non-supersymmetric) theories of grand unification. Allowance is made for a possible asymmetric distribution of quark momenta within the proton by using the wavefunction of Ref. 3.

The layout of the thesis is as follows: In Chapter 1 we re-examine the work of Chernyak and Zhitnitsky, who derive a novel wavefunction for the nucleon by using the QCD sum rule approach introduced by Shifman, Vainshtein and Zakharov [5]. We obtain different results from these authors for the correlators used in the QCD sum rules.

Chapter 2 begins with a discussion of a phenomenological (chiral) lagrangian relevant to proton decay [6]. The rate for the decay $p \to \pi^0 e^+$ is evaluated by using this lagrangian together with an estimate of the proton $\to$ positron annihilation amplitude. The asymmetric proton wavefunction of Chernyak and Zhitnitsky is employed in the calculation. Allowance is also made for an asymmetric dependence in the quark transverse momenta. We find significant enhancement of the proton lifetime over that predicted using a symmetric wavefunction.

In Chapter 3 we examine some of the consequences of using a symmetric quark momentum distribution in the chiral lagrangian approach to proton decay. We investigate to what extent it is consistent to use such wavefunctions in conjunction with the chiral lagrangian. Specifically, explicit SU(3) symmetry breaking terms are introduced into the baryon number violating chiral lagrangian and we try to correlate the subsequent corrections with those obtained from a refined calculation using symmetric wavefunctions.

A summary and discussion of our results is given in Chapter 4.

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CHAPTER 1

A QCD SUM RULE ANALYSIS OF PROTON DISTRIBUTION AMPLITUDES

In recent years a lot of effort has been devoted to the study of hard scattering hadronic processes. Although the investigation of inclusive scattering processes has received most attention, hard exclusive processes, such as hadronic form factors or elastic scattering at large momentum transfers, have also been studied [1,2]. Amplitudes for different processes may be related to each other by perturbation theory and the non-perturbative physics is contained in universal 'quark distribution amplitudes'. Unfortunately, only indirect information about the distribution amplitudes may be obtained from hard exclusive scattering experiments. Measurements do not enable us to extract the distribution amplitudes themselves. This is in contrast to the case of hard inclusive processes [3] such as deep inelastic lepton hadron scattering, the Drell-Yan process or the inclusive production of particles or jets with large transverse momenta, where the universal quark and gluon distribution and fragmentation functions which describe the non-perturbative physics are directly measureable in deep inelastic scattering experiments.

Several years ago, however, Shifman, Vainshtein and Zakharov (SVZ) [4] introduced the technique of QCD sum rules, which allows long distance effects to be incorporated in QCD calculations in a quantitative way. More recently, Chernyak and Zhitnitsky (CZ) have used this method to determine distribution amplitudes for both mesons [5] and nucleons [6]. We now explain the basic principles of the QCD sum rule technique.

The operator product expansion (OPE) due to Wilson [7] is assumed to hold in the physical vacuum. Both short and long distance effects are included in the OPE of a current correlator. The coefficient functions contain the short distance effects and are calculated in perturbation theory while the long range non-perturbative effects are incorporated in the vacuum expectation values of the corresponding operators. It is hoped that the OPE gives a correct description of physics at distances greater than those of the region of asymptotic freedom.

Dispersion relations are then written down for the invariant functions arising from the current correlators. The low energy behaviour of the spectral density is saturated by one or two lowest-lying resonances, while continuum states approximate the high energy region. It is usual then to apply a Borel transformation to these relations. This suppresses the contributions from the high dimension operators in the OPE, as well as increasing the effect of the lowest-lying resonance. Thus we arrive at the QCD sum rules, which are then treated to relate hadronic properties to the parameters of the QCD lagrangian and vacuum expectation values.

In recent years extensive use of QCD sum rules has led to a number of satisfactory results on the hadronic spectrum. Properties such as meson and baryon masses and couplings [8,9], electromagnetic form factors [10], magnetic moments [11] and partial hadronic widths [12] have been calculated, and in general the results are in good agreement with experiment [13].

In this chapter we apply the QCD sum rule technique to correlators containing currents with the quantum numbers of the proton. In principle this should lead to values of the first few moments of the proton distribution amplitudes, thus enabling us to deduce the distribution amplitudes (or wavefunctions) themselves at the typical hadronic mass scale $\bar{\mu} \sim 1$ GeV.

Such a calculation has been performed in Ref. 6 for the case of the nucleon. There, CZ obtain a wavefunction in which the total proton momentum is not distributed equally among the constituent quarks. About 2/3 of the proton's longitudinal momentum (in the infinite momentum frame) is carried by one u-quark with the same helicity as the proton. In this respect the new distribution amplitude differs significantly from the symmetric asymptotic form, which is exactly calculable in QCD [1]. In this chapter we re-examine the calculation of the proton's distribution amplitude. Our results differ from CZ for the correlators used in the sum rules but the values obtained for the moments of the distribution amplitude are similar. We also outline how SU(3) symmetry may be used to derive sum rules for the moments of distribution amplitudes for other baryons in the $J^P = \frac{1}{2}^+$ octet. However, the required OPE's are not computed.

Using their wavefunction, CZ argue that for the first time a calculation of nucleon electromagnetic form factors results in signs and magnitudes which are in agreement with experiment. Further impressive predictions are obtained for $J/\psi \rightarrow \bar{p}p$ and $J/\psi \rightarrow \bar{n}n$ decay widths and for the behaviour of the ep and en deep inelastic structure functions $F_2^p(x)$ and $F_2^n(x)$ in the threshold region $x \rightarrow 1$. Such asymmetric distribution amplitudes may also lead to interesting predictions for polarised hard scattering experiments and it would be interesting to see if they can explain any of the puzzling experimental results. (See for example the recent discussion in Ref. 14 and references therein).

We shall use the CZ wavefunction as input for the proton decay calculation of Chapter 2. However, a cautionary note is appropriate here. Only the lowest twist behaviour of the OPE's for the current correlators is calculated. Therefore the QCD sum rule analysis provides information about the proton wavefunction at light-like separation $x^2 \rightarrow 0$. As a result the distribution amplitude obtained should be applicable to studies of high momentum transfer processes such as the QCD calculation of nucleon form factors and the $J/\psi \rightarrow \bar{p}p$ decay rate. In contrast to these light-cone dominated processes an estimate of nucleon decay matrix elements should be sensitive to physics as $x^\mu \rightarrow 0$. Thus the two classes of process involve different aspects of the distribution amplitude. This is associated with the fact that lowest twist operators are relevant to light-cone dominated processes whereas baryon decay operators are of higher twist. In principle a proton distribution amplitude more sensitive to short distance physics could be extracted from a QCD sum rule analysis. This could be achieved by calculating the complete Wilson OPE rather than just the contributions of lowest twist. Unfortunately, the computations involved would undoubtedly be lengthy. In the absence of such a proton wavefunction we must be content to apply the information gleaned from the lowest twist contributions to the OPE in our estimation of nucleon decay matrix elements.

1.1 THE OPERATOR PRODUCT EXPANSION FOR QCD SUM RULES

In this section we introduce a practical method for calculating the coefficient functions of the OPE. This will simplify the computations of the subsequent sections. This background field method has been used in QCD sum rule applications by other authors [15], and has been developed by Govaerts et al [16]. Here, we will be relying heavily on this last work.

We begin by writing the lagrangian density of QCD:

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{i}{2} \sum_f \left[\bar{\psi}_f \gamma^\mu D_\mu \psi_f - (D_\mu \bar{\psi}_f)\gamma^\mu \psi_f\right] - \sum_f m_f \bar{\psi}_f \psi_f + \text{Gauge-fixing term} + \text{Faddeev-Popov ghost term} , \qquad (1.1.1)$$

where the sum is over quark flavours. The gluon field strength tensor $F^a_{\mu\nu}$ is defined by

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \qquad (1.1.2)$$

and the gauge covariant derivatives in the fundamental and adjoint representations of the SU(3) colour group by

$$D_\mu = \partial_\mu - i g T^a A^a_\mu \qquad (1.1.3a)$$

$$D^{ab}_\mu = \partial_\mu \delta^{ab} - g f^{abc} A^c_\mu . \qquad (1.1.3b)$$

ψ_f represents a quark field and A^a_μ a gluon field.

The quarks transform according to the fundamental representation of SU(3) and the gluons according to the adjoint representation. g and m_f are the strong coupling constant and the quark mass respectively. The T's are generators in the fundamental representation of SU(3), and satisfy the relations

$$[T^a, T^b] = i f^{abc} T^c \qquad (1.1.4a)$$

$$\mathrm{Tr}\, T^a T^b = \tfrac{1}{2} \delta^{ab} , \qquad (1.1.4b)$$

where f^{abc} are the SU(3) structure constants.

The Wilson OPE is a short distance expansion of the form

$$O_1(x)\, O_2(0) \underset{}{\overset{x^\mu \to 0}{\sim}} \sum_n C_n(x)\, O_n(0) \tag{1.1.5}$$

where O_1 and O_2 are local operators. To any finite order in x only a finite number of operators O_n contribute and they are ordered by dimension. The corresponding coefficient functions C_n may be singular as $x^2 \to 0$. The expression (1.1.5) is valid only when sandwiched between initial and final states.

Consider a correlator containing the time-ordered product of two currents J^A, J^B, which contain only light quarks (u, d or s). As indicated before, an assumption of the QCD sum rule method is that the OPE is valid for external momenta q very much larger than the quark masses; i.e.

$$i\int d^4x\; e^{iq\cdot x}\langle 0|T\, J^A(x)\, J^B(0)|0\rangle \underset{q^2\to -\infty}{=} \sum_n C_n^{AB}(q)\, \langle 0|O_n|0\rangle, \tag{1.1.6}$$

where C_n^{AB} are the Wilson coefficient functions and O_n are local Lorentz scalar and gauge invariant operators containing light quark or gluon fields. The limit $q^2 \to -\infty$ corresponds to the short distance limit of Wilson's OPE. The coefficient functions C_n^{AB} will diminish by the corresponding powers of q. The leading terms in the expansion should then give the largest contributions to the current correlator. We assume that the OPE remains valid when we neglect the contributions of operators with dimension $D \geqslant 6$. An additional reason for truncating the OPE is given by SVZ [4]. They argue that the OPE breaks down for operators of higher dimension due to instanton effects in the vacuum.

Since the magnitude of the non-perturbative interactions falls off quickly at short distances the leading contributions to the asymptotic behaviour of the coefficient functions C_n^{AB} may be obtained, by using perturbation theory, as the leading terms in series in $\alpha_s = \frac{g^2}{4\pi}$. The vacuum expectation values $\langle 0|O_n|0\rangle$, with O_n not the identity operator I, parametrise our ignorance of the non-perturbative effects.

The quarks and gluons may be pictured as propagating through the physical vacuum, interacting with long distance fluctuations of the condensates. The basic idea of the background field approach is to expand the quark and gluon fields as quantum fluctuations around classical background fields representing the vacuum fluctuations. If we then make a short distance expansion of the current correlators in the background fields we derive the OPE as a series in $\langle 0|O_n|0\rangle$ with the operators O_n as gauge-invariant functionals of those background fields only. (By definition the quantum fluctuations average to zero in the physical vacuum).

Explicitly, we set

$$\Psi_f(x) \longrightarrow \psi_f(x) + \eta_f(x) \tag{1.1.7a}$$

$$A_\mu^a(x) \longrightarrow A_\mu^a(x) + \phi_\mu^a(x) , \tag{1.1.7b}$$

where ψ_f and A_μ^a are the quark and gluon background fields satisfying the QCD equations of motion

$$i\,\gamma^\mu D_\mu \psi_f = m_f \psi_f \tag{1.1.8a}$$

$$i\,(\overline{D_\mu \psi_f})\,\gamma^\mu = -m_f \bar{\psi}_f \tag{1.1.8b}$$

$$(D^\nu G_{\mu\nu})^a = g \sum_f \bar{\psi}_f \gamma_\mu T^a \psi_f . \tag{1.1.8c}$$

$G_{\mu\nu}^a$ is the background gluon field strength and D_μ and D_μ^{ab} are background covariant derivatives. η_f and ϕ_μ^a represent the quantum fluctuations.

The colour singlet operators with zero Lorentz spin (only such operators give rise to non-zero vacuum expectation values) and dimension D not greater than six are [4]

I	(D = 0)	(1.1.9a)
$O_1 = \bar{\psi} M_1 \psi$	(D = 4)	(1.1.9b)
$O_2 = G_{\mu\nu}^a G^{a\mu\nu}$	(D = 4)	(1.1.9c)
$O_3 = \bar{\psi}\Gamma_1\psi\bar{\psi}\Gamma_2\psi$	(D = 6)	(1.1.9d)

$$O_4 \;= \bar{\psi}\sigma^{\mu\nu}T^a M_2 \psi G^a_{\mu\nu} \; (D = 6) \tag{1.1.9e}$$

$$O_5 \;= f^{abc} G^{a\mu}_{\lambda} G^{b\nu}_{\mu} G^{c\lambda}_{\nu} \quad (D = 6) \tag{1.1.9f}$$

where

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \tag{1.1.10}$$

and $$\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right] . \tag{1.1.11}$$

M_1 and M_2 are mass matrices in flavour space while the matrices Γ_1 and Γ_2 have colour, flavour and spinor indices. All other operators with $D \leqslant 6$ may be reduced to these, together with total derivatives, by using the equations of motion. The corresponding (physical) vacuum expectation values $\langle 0|O_n|0\rangle$, n = 1,2,3 parametrise the leading non-perturbative corrections, while $\langle 0|O_4|0\rangle$ and $\langle 0|O_5|0\rangle$ are estimated to lead to comparatively small adjustments in the OPE[4]. Thus in our computations we will neglect the effect of these last two matrix elements.

In our calculation of the coefficient functions we will require expressions for quark and gluon propagators. Thus we must use this formalism to develop short distance expansions for these propagators. These will describe propagation in background, or external, quark and gluon fields. Performing the substitutions (1.1.7) in the lagrangian (1.1.1) we find

$$\begin{aligned}
\mathcal{L} \rightarrow\; & \mathcal{L}(A, \psi_f, \bar{\psi}_f) + \mathcal{L}(\text{Ghost}) + \sum_f \bar{\eta}_f (i\not{D} - m_f)\eta_f \\
& + \tfrac{1}{2}\phi^a_\mu \left[\eta^{\mu\nu}(D_\lambda D^\lambda)^{ac} - (1 - 1/\alpha)(D^\mu D^\nu)^{ac} + 2g f^{abc} G^{b\mu\nu}\right]\phi^c_\nu \\
& + g \sum_f \left(\bar{\psi}_f \gamma^\mu \phi^a_\mu T^a \eta_f + \bar{\eta}_f \gamma^\mu \phi^a_\mu T^a \psi_f + \bar{\eta}_f \gamma^\mu \phi^a_\mu T^a \eta_f\right) \\
& - g^2 f^{abc} f^{adf} A^b_\mu \phi^c_\nu \phi^{d\mu} \phi^{f\nu} \\
& - g f^{abc} (\partial^\mu \phi^{a\nu}) \phi^b_\mu \phi^c_\nu \\
& - \tfrac{1}{4} g^2 f^{abc} f^{adf} \phi^b_\mu \phi^c_\nu \phi^{d\mu} \phi^{f\nu} \quad ,
\end{aligned} \tag{1.1.12}$$

where the equations of motion (1.1.8) have been used to arrive at this form. The gauge-fixing term $-\frac{1}{2\alpha}\left[(D_\mu \phi^\mu)^a\right]^2$ has been added to the lagrangian. In what follows we choose the Feynman gauge (α=1) for the quantum fluctuations.

The quark and gluon propagators, $S_f(x,y)$ and $P^{ab}(x,y)$ respectively, satisfy the defining equations

$$(i \not{D} - m_f)\, S_f(x,y) = i\, \delta^{(4)}(x-y) \tag{1.1.13a}$$

$$\left[\eta^{\mu\nu}(D_\lambda D^\lambda)^{ac} + 2g\, f^{abc} G^{b\mu\nu}\right] P^{cd}_{\nu\rho}(x,y) = i\, \eta^\mu_\rho\, \delta^{ad}\, \delta^{(4)}(x-y) \tag{1.1.13b}$$

We now exploit the residual gauge freedom in the background field A^a_μ. Short distance expansions are easiest in the commonly used Schwinger gauge [17], defined by

$$(x-x_0)^\mu A^a_\mu(x) = 0, \tag{1.1.14}$$

where x_0 is an arbitrary reference point which plays the role of a gauge parameter. This constraint breaks translation invariance but the parameter x_0 should cancel in the current correlators since the latter are gauge invariant. Thus, from the beginning, we may set x_0 equal to zero. The condition

$$x^\mu A^a_\mu(x) = 0 \tag{1.1.15}$$

may be solved to give [18]

$$\begin{aligned} A^a_\mu(x) &= \int_0^1 d\alpha\; \alpha\, x^\nu\, G^a_{\nu\mu}(\alpha x) \\ &= \sum_{k=0}^{\infty} \frac{1}{k!\,(k+2)}\, x^{\alpha_1} \cdots x^{\alpha_k} x^\nu \left(D_{\alpha_1} \cdots D_{\alpha_k} G_{\nu\mu}\right)^a(0) \end{aligned} \tag{1.1.16}$$

Use of this form will result in gauge covariant expressions for the current correlators. Solving equations (1.1.13) for the free quark and gluon propagators and adapting the results to perturbative expansions in the background fields by iteration we find

$$
\begin{aligned}
S_f(x,y) = {} & \frac{i}{2\pi^2}\,\frac{\not{t}}{(t^2-i\varepsilon)^2} - \frac{m_f}{4\pi^2}\,\frac{1}{t^2-i\varepsilon} \\
& + \frac{i m_f^2}{8\pi^2}\,\frac{\not{t}}{t^2-i\varepsilon} - \frac{g}{16\pi^2}\, G^a_{\mu\nu}(0)\, T^a\, \frac{\gamma^\mu \not{t} \gamma^\nu}{t^2-i\varepsilon} \\
& + \frac{g}{4\pi^2}\, G^a_{\mu\nu}(0)\, T^a\, \frac{x^\mu y^\nu \not{t}}{(t^2-i\varepsilon)^2} \\
& + \frac{i}{8\pi^2}\, m_f\, g\, G^a_{\mu\nu}(0)\, T^a\, \frac{x^\mu y^\nu}{t^2-i\varepsilon} \\
& - \frac{m_f g}{32\pi^2}\, G^a_{\mu\nu}(0)\, T^a\, \sigma^{\mu\nu} \ln(-\Lambda^2 t^2) \\
& + \frac{g}{48\pi^2}\, (D^\nu G_{\nu\mu})^a(0)\, T^a\, \gamma^\mu \ln(-\Lambda^2 t^2) \\
& + \frac{g}{12\pi^2}\, (D_\alpha G_{\nu\mu})^a(0)\, T^a\, \frac{x^\nu y^\mu (x^\alpha + y^\alpha) \not{t}}{(t^2-i\varepsilon)^2} \\
& - \frac{g}{144\pi^2}\, (D_\alpha G_{\nu\mu})^a(0)\, T^a\, \frac{1}{t^2-i\varepsilon} \\
& \quad \cdot \Big\{ (2x^\nu + y^\nu)\gamma^\alpha \not{t} \gamma^\mu - (x^\nu + 2y^\nu)\gamma^\mu \not{t} \gamma^\alpha \\
& \quad - 2 t^\alpha t^\mu \gamma^\nu + 3(x^\alpha + y^\alpha)\gamma^\nu \not{t} \gamma^\mu + \eta^{\alpha\nu} \not{t} \gamma^\mu \not{t} \Big\} \\
& + \frac{i g^2}{192\pi^2}\, G^a_{\mu\nu}(0)\, T^a\, G^{b\mu\nu}(0)\, T^b \left[(x\cdot y)^2 - x^2 y^2 \right] \frac{\not{t}}{(t^2-i\varepsilon)^2}
\end{aligned}
$$

+ terms of higher order in m_f

+ higher dimensional operators (1.1.17)

$$
\begin{aligned}
P^{ab}_{\mu\nu}(x,y) = {} & \frac{1}{4\pi^2}\,\frac{\eta_{\mu\nu}\delta^{ab}}{t^2-i\varepsilon} + \frac{g}{8\pi^2}\, f^{abc}\, G^c_{\mu\nu}(0) \ln(-\Lambda^2 t^2) \\
& - \frac{g}{8\pi^2}\, f^{abc}\, \eta_{\mu\nu}\, G^c_{\rho\sigma}(0)\, \frac{x^\rho y^\sigma}{t^2-i\varepsilon}
\end{aligned}
$$

+ higher dimensional operators , (1.1.18)

where t = x-y and Λ is an ultra-violet cut-off. The quark propagator has been written as a perturbative expansion in the quark mass m_f. We may make use of (1.1.17) since we will be calculating OPE's for correlators of currents containing only light quarks, i.e. $m_f \ll \bar{\mu}$, where $\bar{\mu}$ is the characteristic hadronic mass scale.

The terms in the iterative expansion of $S_f(x,y)$ containing two factors of $G^a_{\mu\nu}$ have been simplified by isolating the piece singlet in Lorentz indices. Just this piece will contribute to the coefficient function associated with the vacuum expectation value $\langle 0 | G^a_{\mu\nu} G^{a\mu\nu} | 0 \rangle$.

In general, calculations of the coefficient functions tend to be simpler when carried out in configuration space. However, we will find that some diagrams determining the corrections proportional to the matrix element $\langle 0 | \bar\psi \Gamma_1 \psi \bar\psi \Gamma_2 \psi | 0 \rangle$ are more readily computed by working in momentum space. For future reference we thus give the short distance expansions for the quark and gluon propagators in momentum space :

$$
\begin{aligned}
S_f(x,y) = \int d\!\!\!{}^-{}^4 p \; e^{-ip\cdot(x-y)} \Big\{ & \frac{i \not p}{p^2+i\varepsilon} + \frac{i m_f}{p^2+i\varepsilon} \\
& + \frac{i m_f^2 \not p}{(p^2+i\varepsilon)^2} - \frac{1}{2} g\, G^a_{\mu\nu}(0) T^a \frac{\not p \gamma^\mu \not p \gamma^\nu \not p}{(p^2+i\varepsilon)^3} \\
& + \frac{1}{2} i g\, G^a_{\mu\nu}(0)\, T^a x^\nu \frac{\not p \gamma^\mu \not p}{(p^2+i\varepsilon)^2} \\
& - \frac{1}{2} i m_f g\, G^a_{\mu\nu}(0)\, T^a \frac{\sigma^{\mu\nu}}{(p^2+i\varepsilon)^2} \\
& + i m_f g\, G^a_{\mu\nu}(0)\, T^a x^\nu \frac{p^\mu}{(p^2+i\varepsilon)^2} \\
& + \frac{2}{3} i g\, (D_\alpha G_{\nu\mu})^a(0)\, T^a \eta^{\nu\alpha} \gamma^\mu \frac{1}{(p^2+i\varepsilon)^2} \\
& - \frac{1}{3} i g\, (D_\alpha G_{\nu\mu})^a(0)\, T^a \frac{1}{(p^2+i\varepsilon)^3} \\
& \qquad \cdot \left[2\eta^{\nu\alpha} p^\mu \not p + 2 p^\alpha p^\nu \gamma^\mu + 3 p^\alpha \gamma^\nu \not p \gamma^\mu \right] \\
& - \frac{i g^2}{48} G^a_{\mu\nu}(0)\, T^a G^{b\mu\nu}(0)\, T^b \frac{1}{(p^2+i\varepsilon)^3} \\
& \qquad \cdot \left[2 x^2 p^2 \not p + \not p \not x \not p \not x \not p \right] \\
& + \text{terms of higher order in } m_f \\
& + \text{higher dimensional operators} \Big\}
\end{aligned}
\tag{1.1.19}
$$

$$P^{ab}_{\mu\nu}(x,y) = \int d^4p \; e^{-ip.(x-y)} \Big\{ \; \frac{-i\,\eta_{\mu\nu}\,\delta^{ab}}{p^2+i\varepsilon}$$

$$+ 2ig\, f^{abc}\, G^c_{\mu\nu}(0)\, \frac{1}{(p^2+i\varepsilon)^2}$$

$$+ g\, f^{abc}\,\eta_{\mu\nu}\, G^c_{\rho\sigma}(0)\, \frac{x^\rho p^\sigma}{(p^2+i\varepsilon)^2}$$

$$+ \text{ higher dimensional operators} \Big\} \, . \qquad (1.1.20)$$

The Bianchi identity

$$(D_\lambda G_{\mu\nu})^a + (D_\mu G_{\nu\lambda})^a + (D_\nu G_{\lambda\mu})^a = 0 \qquad (1.1.21)$$

has been used to express the quark propagator in this form.

1.2 PROTON DISTRIBUTION AMPLITUDES

In this section we define distribution amplitudes for the proton and derive some of their properties.

We start by writing the gauge invariant matrix element of the tri-local operator for the proton:

$$\varepsilon^{ijk} \langle 0 | \left[\exp\left(ig \int_{z_1}^{z_3} A_\mu(\sigma)\, d\sigma^\mu \right) u_\alpha(z_1) \right]^i \left[\exp\left(ig \int_{z_2}^{z_3} A_\nu(\tau)\, d\tau^\nu \right) u_\beta(z_2) \right]^j d_\gamma^k(z_3) | p \rangle , \tag{1.2.1}$$

where the exponentials are to be understood as being path ordered. Here $|p\rangle$ represents the proton state with momentum p, u and d are quark fields, i, j and k are colour indices and α, β and γ are spinor indices. It has been verified [19] that the anomalous dimensions of (1.2.1), which were originally calculated graphically [1], correspond to the anomalous dimensions of the lowest twist three-quark operators. In the light-cone gauge used in Ref 1, for which the separations between z_1, z_2 and z_3 are all light-like [20], the lowest twist part of (1.2.1) reduces to

$$\varepsilon^{ijk} \langle 0 | \, u_\alpha^i(z_1) \; u_\beta^j(z_2) \; d_\gamma^k(z_3) | p \rangle \tag{1.2.2}$$

so that in this gauge we are only interested in the three quark component of the proton's wavefunction. Using the transformation properties of the fields under proper Lorentz transformations and parity, (1.2.1) may be rewritten in terms of three invariant functions V, A and T [21] (in the infinite momentum frame $p_z \to \infty$):

$$\begin{aligned}(1.2.1) = \frac{f_N}{4} \Big\{ & (\not{p} C)_{\alpha\beta} \, (\gamma_5 N)_\gamma \; V(z_i \cdot p) \\ & + (\not{p}\gamma_5 C)_{\alpha\beta} \; N_\gamma \; A(z_i \cdot p) \\ & + i\, (\sigma_{\mu\nu} p^\nu C)_{\alpha\beta} \, (\gamma^\mu \gamma_5 N)_\gamma \; T(z_i \cdot p) \Big\} .\end{aligned} \tag{1.2.3}$$

Here C is the charge conjugation matrix, N is the proton spinor and the constant f_N is a measure of the value of the proton wavefunction at the origin. The normalisation condition used to define f_N is given below.

The three scalar wavefunctions V, A and T determine the complete wavefunction of the proton. The purpose of the ensuing QCD sum rule analysis is to calculate the lowest twist behaviour of these functions. It is convenient to define the function V in momentum space as follows:

$$V(x_1,x_2,x_3) \equiv \int V(z_1\cdot p, z_2\cdot p, z_3\cdot p) \prod_{i=1}^{3} \frac{d(z_i\cdot p)}{2\pi} \exp\left(i x_i (z_i\cdot p)\right) \tag{1.2.4}$$

Translation invariance then implies that $x_1+x_2+x_3=1$ and we deduce the inverse relation

$$V(z_1\cdot p, z_2\cdot p, z_3\cdot p) = \int_0^1 [dx]\, V(x_1,x_2,x_3) \exp\left\{-i \sum_{i=1}^{3} x_i (z_i\cdot p)\right\}, \tag{1.2.5a}$$

where

$$\int_0^1 [dx] \equiv \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \;\delta(1-\sum_{i=1}^{3} x_i)\,. \tag{1.2.5b}$$

Similar relations may be written for the A and T functions. The V, A and T functions provide information on the longitudinal momentum fractions x_i ($0 \leqslant x_i \leqslant 1$; $i = 1,2,3$) of the quarks within the proton and are called distribution amplitudes.

Determination of wavefunction moments, defined as

$$V^{(n_1,n_2,n_3)} \equiv \int_0^1 [dx]\, V(x_1,x_2,x_3)\, x_1^{n_1} x_2^{n_2} x_3^{n_3}\,, \tag{1.2.6}$$

with similar definitions for $A^{(n_1,n_2,n_3)}$ and $T^{(n_1,n_2,n_3)}$, will allow us to deduce expressions for the distribution amplitudes $V(x_i)$, $A(x_i)$ and $T(x_i)$. We hope that an estimate of the lowest moments (specifically, those with $n_1+n_2+n_3 \leqslant 2$) will suffice to extract approximate distribution amplitudes for the proton.

The distribution amplitudes are slowly varying functions of at least one renormalisation scale. For our purposes it is sufficient to have one such scale, μ^2 say, which is a measure of the separation between the quarks:

$$(z_1-z_2)^2 \sim (z_2-z_3)^2 \sim (z_1-z_3)^2 \sim 1/\mu^2\,.$$

The dependence on μ^2 is found by performing a renormalisation group calculation (See [6] and references therein). In the asymptotic limit $\mu^2 \to \infty$ the proton distribution amplitude, $\Phi_{AS}(x_i)$ say, is exactly calculable in perturbative QCD and has the totally symmetric form

$120\ x_1x_2x_3$ [1]. Using explicit forms for the (massless) spinors, together with (1.2.2), the lowest twist part of (1.2.3) may be rewritten in the form

$$|P_\uparrow\rangle = \kappa \int_0^1 [dx] \Big\{ \tfrac{1}{2}[V(x_i) - A(x_i)]\, |u_\uparrow(x_1)\, u_\downarrow(x_2)\, d_\uparrow(x_3)\rangle + \tfrac{1}{2}[V(x_i) + A(x_i)]\, |u_\downarrow(x_1)\, u_\uparrow(x_2)\, d_\uparrow(x_3)\rangle - T(x_i)\, |u_\uparrow(x_1)\, u_\uparrow(x_2)\, d_\downarrow(x_3)\rangle \Big\}, \qquad (1.2.7)$$

where the arrows denote spin projections onto the z-axis and κ is a constant. In the asymptotic limit the flavour-spin structure is known to reduce to that of the SU(6)-symmetric quark model. Thus we deduce

$$V(x_i)\Big|_{\mu^2\to\infty} = T(x_i)\Big|_{\mu^2\to\infty} \longrightarrow \Phi_{AS}(x_i) = 120\, x_1 x_2 x_3 \qquad (1.2.8a)$$

$$A(x_i)\Big|_{\mu^2\to\infty} \longrightarrow 0. \qquad (1.2.8b)$$

(The normalisation constant κ is chosen so that $\Phi_{AS}^{(0,0,0)} = 1$.) The evolution to the asymptotic form is only logarithmic so that the asymptotic result will never be useful phenomenologically. Typical results of QCD sum rule calculations of resonance properties are estimated to have an accuracy of only 10-15% [4]. Thus we feel justified in calculating the distribution amplitudes at $\mu^2 \approx 1\ \text{GeV}^2$ and neglecting the relatively small corrections resulting from the inclusion of renormalisation effects.

The identity of two u-quarks in the proton implies

$$\varepsilon^{ijk}\langle 0|\, u^i_\alpha(z_1)\, u^j_\beta(z_2)\, d^k_\gamma(z_3)\, |P\rangle = \varepsilon^{ijk}\langle 0|\, u^i_\beta(z_2)\, u^j_\alpha(z_1)\, d^k_\gamma(z_3)\, |P\rangle \qquad (1.2.9)$$

By using the symmetry of the matrices $\gamma^\mu C$ and $\sigma^{\mu\nu}C$, and the antisymmetry of $\gamma^\mu\gamma_5 C$, we deduce from (1.2.2) and (1.2.3) the symmetry properties

$$V(x_1, x_2, x_3) = V(x_2, x_1, x_3) \qquad (1.2.10a)$$

$$A(x_1, x_2, x_3) = -A(x_2, x_1, x_3) \qquad (1.2.10b)$$

$$T(x_1, x_2, x_3) = T(x_2, x_1, x_3) \qquad (1.2.10c)$$

which in turn imply the relations

$$V^{(n_1,n_2,n_3)} = V^{(n_2,n_1,n_3)} \tag{1.2.11a}$$

$$A^{(n_1,n_2,n_3)} = -A^{(n_2,n_1,n_3)} \tag{1.2.11b}$$

$$T^{(n_1,n_2,n_3)} = T^{(n_2,n_1,n_3)} . \tag{1.2.11c}$$

The proton has isospin I = ½. We must ensure that the matrix element of the orthogonal I = 3/2 combination vanishes; i.e. that

$$\varepsilon^{ijk} \langle 0 | u^i_\alpha(z_1)\, u^j_\beta(z_2)\, d^k_\gamma(z_3) + u^i_\beta(z_2)\, d^j_\gamma(z_3)\, u^k_\alpha(z_1) + d^i_\gamma(z_3)\, u^j_\alpha(z_1)\, u^k_\beta(z_2) | p \rangle = 0 . \tag{1.2.12}$$

Again using (1.2.2) and (1.2.3) this leads to the constraint

$$2\,T(x_1,x_2,x_3) = \Phi(x_1,x_3,x_2) + \Phi(x_2,x_3,x_1), \tag{1.2.13a}$$

where

$$\Phi(x_1,x_2,x_3) \equiv V(x_1,x_2,x_3) - A(x_1,x_2,x_3) . \tag{1.2.13b}$$

We conclude that there is only one independent proton distribution amplitude. CZ choose this to be the (dimensionless) function $\Phi(x_1,x_2,x_3)$. The constraint (1.2.13) may also be translated into a relation between the moments of the proton's distribution amplitudes:

$$2\,T^{(n_1,n_2,n_3)} = \Phi^{(n_1,n_3,n_2)} + \Phi^{(n_2,n_3,n_1)} . \tag{1.2.14}$$

Equation (1.2. 11b) implies that $A^{(0,0,0)} = 0$, and then from (1.2.14) we obtain $V^{(0,0,0)} = T^{(0,0,0)}$. The decay constant f_N is normalised by the choice

$$V^{(0,0,0)} = T^{(0,0,0)} = 1 . \tag{1.2.15}$$

From this it follows that $\Phi^{(0,0,0)} = 1$, which will ease comparison between $\Phi(x_i)$ and the asymptotic form $\Phi_{AS}(x_i)$.

Thus far we have considered the case of the proton. The analysis for the neutron is similar. However, since the SU(6) flavour-spin wavefunction for the neutron is obtained from that of the proton by making the substitutions $u \to d$ $d \to -u$, it follows that $\kappa \to -\kappa$ in the limit of exact isospin symmetry. It is clear that the assumption of exact isospin symmetry implies that the neutron's V, A and T functions coincide with those of the proton, thus enabling us to define a single nucleon distribution amplitude $\Phi(x_i)$.

An alternative definition of the proton distribution amplitude has been given by Brodsky and Lepage [1]. The most general form is

$$\begin{aligned}\phi_\uparrow(x_i,q^2) = \Big\{ & \tfrac{1}{\sqrt{6}}\big[d_\uparrow(1)\,u_\downarrow(2)\,u_\uparrow(3) + u_\uparrow(1)\,u_\downarrow(2)\,d_\uparrow(3) \\ & \qquad - 2\,u_\uparrow(1)\,d_\downarrow(2)\,u_\uparrow(3)\big]\,\phi^s(x_i,q^2) \\ & + \tfrac{1}{\sqrt{2}}\big[d_\uparrow(1)\,u_\downarrow(2)\,u_\uparrow(3) - u_\uparrow(1)\,u_\downarrow(2)\,d_\uparrow(3)\big]\,\phi^a(x_i,q^2)\Big\} \\ & + (1\leftrightarrow 2) + (2\leftrightarrow 3)\,,\end{aligned} \tag{1.2.16}$$

where, because of Fermi statistics, $\phi^s(x_i,q^2)$ ($\phi^a(x_i,q^2)$) are symmetric (antisymmetric) under the interchange $x_1 \leftrightarrow x_3$. The combinations in squared brackets arise from the SU(3) mixed symmetry representations for the octet states of two u-quarks and a d-quark. By isolating the terms with the d-quark in position 3 and comparing with (1.2.7) we find the relations

$$\tfrac{1}{2}\big[V(x_i) - A(x_i)\big] = \sigma\Big[\tfrac{1}{\sqrt{6}}\,\phi^s(x_1,x_2,x_3) - \tfrac{1}{\sqrt{2}}\,\phi^a(x_1,x_2,x_3)\Big] \tag{1.2.17a}$$

$$\tfrac{1}{2}\big[V(x_i) + A(x_i)\big] = \sigma\Big[\tfrac{1}{\sqrt{6}}\,\phi^s(x_2,x_1,x_3) - \tfrac{1}{\sqrt{2}}\,\phi^a(x_2,x_1,x_3)\Big] \tag{1.2.17b}$$

$$T(x_i) = \sigma\sqrt{\tfrac{2}{3}}\;\phi^s(x_1,x_3,x_2)\,, \tag{1.2.17c}$$

where σ is a constant. The constraints (1.2.10) and (1.2.13) are seen to be consistent with equations (1.2.17). This equivalent formulation proves to be useful in the proton decay analysis of Chapter 2. In the appendix at the end of this chapter both formalisms are used to deduce constraints on the V, A and T functions for the Σ^0 and Λ hyperons, where the quarks are of three different flavours.

1.3 THE CURRENT CORRELATORS

This section contains a discussion of the current correlators to be used in the QCD sum rule analysis. We study correlators of the form

$$i\int d^4x\; e^{iq\cdot x} \langle 0|\, T\left\{ F_\tau^{(n)}(x) \left(J_\sigma^{(\hat{n}_1)}(0)\right)^\dagger \right\} |0\rangle\, (\gamma^0 \not{z})_{\sigma\tau}\,, \qquad (1.3.1)$$

where $F^{(n)}$ are operators whose matrix elements $\langle 0|F_\tau^{(n)}(0)|p\rangle$ may be expressed in terms of the moments $V^{(n)}$, $A^{(n)}$ and $T^{(n)}$ of the distribution amplitudes and $J^{(\hat{n}_1)}$ are auxiliary operators or currents chosen with the aim of making the proton's contribution to (1.3.1) large.

First we turn to the problem of choosing suitable proton currents. Unlike the meson case unique quark currents (with no derivatives) do not exist. There are three independent quark currents $J^{(\hat{n})}$ with quantum numbers of the proton, and no derivatives, which give non-zero matrix elements $\langle 0|J^{(i)}|p\rangle$. The current that is chosen should satisfy two conditions [22]. In the OPE of the current correlator it is desirable that

(i) the lowest lying baryon resonance give a greater contribution than the continuum states;

(ii) the neglected non-perturbative corrections be small compared to the contributions of the retained terms.

Following CZ we select the proton current

$$J_\sigma^{(\hat{n}_1)}(x) = \varepsilon^{ijk}\Big\{\big[\big((iz\cdot D)^{\hat{n}_1} u(x)\big)^i C \not{z}\, u^j(x)\big](\gamma_5 d^k(x))_\sigma - \big[\big((iz\cdot D)^{\hat{n}_1} u(x)\big)^i C \not{z}\, d^j(x)\big](\gamma_5 u^k(x))_\sigma\Big\}, \qquad (1.3.2)$$

where z^μ is an auxiliary light-like vector introduced to help project out the leading twist component. In Ref. 6 it is claimed that use of this isospin $\frac{1}{2}$ current in the QCD sum rule analysis leads to a distribution amplitude which gives good agreement with experimental data on nucleon electromagnetic form factors and other quantities. The computations will tend to be simpler for small values of $\hat{n}_1$. Thus only the currents $J_\sigma^{(0)}$ and $J_\sigma^{(1)}$ are included in the correlators.

As mentioned earlier we hope to construct proton distribution amplitudes once the wavefunction moments have been determined in the sum rule analysis. The moments are introduced via matrix elements of suitably chosen local operators. We first consider

$$\langle 0|\, V_\tau^{(n)}(y) \equiv$$
$$\varepsilon^{ijk}\left[\left((iz\cdot D)^{n_1} u(y)\right)^i C\not{z}\left((iz\cdot D)^{n_2} u(y)\right)^j\right]\left[(iz\cdot D)^{n_3}(\gamma_5\, d(y))_\tau\right]^k |p\rangle, \tag{1.3.3}$$

where $(n) \equiv (n_1, n_2, n_3)$.

This may be rewritten as

$$(iz^\kappa)^{n_1}(iz^\lambda)^{n_2}(iz^\rho)^{n_3}\,(C\not{z})_{\alpha\beta}(\gamma_5)_{\tau\gamma}\int d^3(z_i\cdot p)\int_0^1 [dx]\,\exp\Big[i\sum_{i=1}^{3} x_i (z_i-y)\cdot p\Big]$$
$$\langle 0|\,\varepsilon^{lmn}(D_\kappa^{n_1}u_\alpha)^l(z_1)\,(D_\lambda^{n_2}u_\beta)^m(z_2)\,(D_\rho^{n_3}d_\gamma)^n(z_3)\,|p\rangle, \tag{1.3.4}$$

where α, β, γ and τ are spinor indices, and we have introduced the compact notation

$$(z\cdot D)^n = z^{\kappa_1}\ldots z^{\kappa_n} D_{\kappa_1}\ldots D_{\kappa_n} \equiv (z^\kappa)^n\,(D_\kappa)^n\,. \tag{1.3.5}$$

Because $\sum_{i=1}^{3} x_i = 1$ the y-dependence may be taken outside the integrals as a factor $e^{-ip\cdot y}$. Then, choosing the Schwinger gauge (1.1.14), for which $A_\mu^a(0) = 0$, the covariant derivatives D_μ may be replaced by partial derivatives ∂_μ. After integrating by parts to get rid of the partial derivatives, we deduce

$$\langle 0|\, V_\tau^{(n)}(y)\,|p\rangle = (z.p)^{n_1+n_2+n_3}\, e^{-ip\cdot y}\,(C\not{z})_{\alpha\beta}(\gamma_5)_{\tau\gamma}$$
$$\int_0^1 [dx]\, x_1^{n_1} x_2^{n_2} x_3^{n_3} \int d^3(z_i p)\, \exp\Big(i\sum_{i=1}^{3} x_i z_i\cdot p\Big)$$
$$\langle 0|\,\varepsilon^{lmn}\, u_\alpha^l(z_1)\, u_\beta^m(z_2)\, d_\gamma^n(z_3)\,|p\rangle. \tag{1.3.6}$$

Now we recall (1.2.2) and (1.2.3). The trace theorems for γ-matrices ensure that only the V term gives a non-zero contribution. The final form of the matrix element is

$$\langle 0|V^{(n)}_{\tau}(y)|p\rangle = -f_N\,(z\cdot p)^{n_1+n_2+n_3+1}\,N_\tau\,V^{(n)}\,e^{-ip\cdot y}. \qquad (1.3.7)$$

In the same way we introduce the moments of the $A(x_i)$,$T(x_i)$ functions:

$$\langle 0|A^{(n)}_{\tau}(y) \equiv$$
$$\varepsilon^{ijk}[((iz\cdot D)^{n_1}u(y))^i C\not{z}\gamma_5\,((iz\cdot D)^{n_2}u(y))^j][(iz\cdot D)^{n_3}d_\tau(y)]^k|p\rangle$$
$$= -f_N\,(z\cdot p)^{n_1+n_2+n_3+1}\,N_\tau\,A^{(n)}\,e^{-ip\cdot y} \qquad (1.3.8)$$

$$\langle 0|T^{(n)}_{\tau}(y) \equiv$$
$$\varepsilon^{ijk}[((iz\cdot D)^{n_1}u(y))^i\,C\,(-i\sigma_{\mu\nu})\,z^\nu\,((iz\cdot D)^{n_2}u(y))^j][(iz\cdot D)^{n_3}(\gamma^\mu\gamma_5\,d(y))_\tau]^k|p\rangle$$
$$= 2f_N\,(z\cdot p)^{n_1+n_2+n_3+1}\,N_\tau\,T^{(n)}\,e^{-ip\cdot y}. \qquad (1.3.9)$$

Thus we shall evaluate the following correlators in our investigation of wavefunction moments:

$$I^{(n)}(q,z) \equiv i\int d^4x\; e^{iq\cdot x}\,\langle 0|\,T\,(\,F^{(n)}_{\tau}(x)\,\overline{J^{(1)}_{\sigma}(0)})|0\rangle\,\not{z}_{\sigma\tau}$$
$$= (z\cdot q)^{n_1+n_2+n_3+4}\;I^{(n)}(q^2) \qquad (1.3.10)$$

$$K^{(n)}(q,z) \equiv i\int d^4x\; e^{iq\cdot x}\,\langle 0|\,T\,(\,F^{(n)}_{\tau}(x)\,\overline{J^{(0)}_{\sigma}(0)})|0\rangle\,\not{z}_{\sigma\tau}$$
$$= (z\cdot q)^{n_1+n_2+n_3+3}\;K^{(n)}(q^2). \qquad (1.3.11)$$

Here $F^{(n)}_\tau$ represents $V^{(n)}_\tau$, $A^{(n)}_\tau$ or $T^{(n)}_\tau$ and $\overline{J^{(\hat{n}_1)}} \equiv (J^{(\hat{n}_1)})^\dagger\gamma^0$. The $\not{z}$ factor ensures that the leading twist contributions survive when the traces of γ-matrices are taken. We shall compute the OPE's for $I^{(n)}(q^2)$ and $K^{(n)}(q^2)$ in the spacelike region $q^2 < 0$ and for the cases with $n_1+n_2+n_3 \leq 2$ only. We shall neglect terms proportional to the light quark masses. Effects due to the breaking of the SU(2) isospin symmetry are neglected. Thus we assume

$$\langle 0|\bar{u}\,u|0\rangle = \langle 0|\bar{d}\,d|0\rangle \equiv \langle 0|\bar{q}\,q|0\rangle \qquad (1.3.12)$$

and compute a single nucleon OPE.

Recalling that we wish to include only the leading non-perturbative corrections in the OPE (i.e. those parametrised by the vacuum expectation values $\langle 0|O_n|0\rangle$, n = 1,2,3) we write the asymptotic behaviour of the invariant structures $I^{(n)}(q^2)$ and $K^{(n)}(q^2)$ as follows:

$$
\begin{aligned}
I^{(n)}(q^2) = & -\frac{\beta_1^{(n)}}{160\pi^4}\, q^2 \ln q^2 \\
& -\frac{\beta_2^{(n)}}{48\pi^2}\, \frac{1}{q^2}\, \langle 0|\frac{\alpha_s}{\pi}\, G^a_{\mu\nu}\, G^{a\mu\nu}|0\rangle \\
& +\frac{\beta_3^{(n)}}{243\pi}\, \frac{1}{q^4}\, \langle 0|\sqrt{\alpha_s}\,\bar{q}\, q\,|0\rangle^2
\end{aligned}
\qquad (1.3.13)
$$

$$
\begin{aligned}
K^{(n)}(q^2) = & \frac{\alpha_1^{(n)}}{80\pi^4}\, q^2 \ln q^2 \\
& +\frac{\alpha_2^{(n)}}{48\pi^2}\, \frac{1}{q^2}\, \langle 0|\frac{\alpha_s}{\pi}\, G^a_{\mu\nu}\, G^{a\mu\nu}|0\rangle \\
& -\frac{\alpha_3^{(n)}}{243\pi}\, \frac{1}{q^4}\, \langle 0|\sqrt{\alpha_s}\,\bar{q}\, q\,|0\rangle^2 .
\end{aligned}
\qquad (1.3.14)
$$

The q^2 dependence of the different terms in (1.3.13) and (1.3.14) is determined (up to logarithms) by simple dimensional arguments. The next section is devoted to a calculation of the α and β coefficients.

1.4 CALCULATION OF THE COEFFICIENT FUNCTIONS OF THE OPERATOR PRODUCT EXPANSIONS

In this section we evaluate the asymptotic behaviour of the Wilson coefficient functions of the OPE's as defined by equations (1.3.13) and (1.3.14). Each of the coefficient functions may be derived as a perturbative expansion in the effective strong interaction parameter $\alpha_s(\mu^2)$. For $\mu^2 \approx 1$ GeV^2, $\alpha_s(\mu^2) \approx 0.3$–$0.4$ and hopefully is small enough to enable us to neglect all but the leading terms in the series. The coefficient functions will all be found to be ultraviolet convergent. This is a consequence of the fact that the correlators defined in Section 1.3 were constructed to extract the leading twist behaviour.

We start by considering the I-correlators. From the definition (1.3.2) of the proton current $J^{(1)}$, we find

$$\overline{J^{(1)}_\sigma(x)} = -\varepsilon^{ijk}\Big\{(\bar{d}^i(x)\gamma_5)_\sigma\,[\bar{u}^j(x)\not{z}\,C\,((i z\cdot D)\bar{u}(x))^k] - (\bar{u}^i(x)\gamma_5)_\sigma\,[\bar{d}^j(x)\not{z}\,C\,((i z\cdot D)\bar{u}(x))^k]\Big\}. \tag{1.4.1}$$

With the choice $F^{(n)}_\tau(x) = V^{(n)}_\tau(x)$ in Eq.(1.3.10), it follows that

$$\begin{aligned} I^{V(n)}(q,z) = &-\varepsilon^{ijk}\varepsilon^{lmn} z^\mu (iz^\kappa)^{n_1}(iz^\lambda)^{n_2}(iz^\rho)^{n_3}\int d^4x\, e^{iq\cdot x} \\ &\langle 0|\,T\,[(D^{n_1}_\kappa u(x))^i\, C\not{z}\,(D^{n_2}_\lambda u(x))^j]\,(D^{n_3}_\rho d_\tau(x))^k \\ &\cdot\Big\{(\bar{d}^l(0)\not{z})_\tau\,[\bar{u}^m(0)\not{z}\,C\,(D_\mu\bar{u})^n(0)] \\ &- (\bar{u}^l(0)\not{z})_\tau\,[\bar{d}^m(0)\not{z}\,C\,(D_\mu\bar{u})^n(0)]\Big\}|0\rangle, \end{aligned} \tag{1.4.2}$$

where the covariant derivatives D_μ coincide with the partial derivatives ∂_μ in the Schwinger gauge. Note that $(D^\mu\bar{u})^n(0)$ stands for $(D^\mu_x\bar{u}(x))^n\,|_{x=0}$. Similarly,

$$\begin{aligned} I^{A(n)}(q,z) = &\ \varepsilon^{ijk}\varepsilon^{lmn} z^\mu (iz^\kappa)^{n_1}(iz^\lambda)^{n_2}(iz^\rho)^{n_3}\int d^4x\, e^{iq\cdot x} \\ &\langle 0|\,T\,[(D^{n_1}_\kappa u(x))^i\, C\not{z}\gamma_5\,(D^{n_2}_\lambda u(x))^j]\,(D^{n_3}_\rho d_\tau(x))^k \\ &\cdot\Big\{(\bar{d}^l(0)\gamma_5\not{z})_\tau\,[\bar{u}^m(0)\not{z}\,C\,(D_\mu\bar{u})^n(0)] \\ &- (\bar{u}^l(0)\gamma_5\not{z})_\tau\,[\bar{d}^m(0)\not{z}\,C\,(D_\mu\bar{u})^n(0)]\Big\}|0\rangle. \end{aligned} \tag{1.4.3}$$

The constraint (1.2.14) tells us that the moments of the three distribution amplitudes V, A and T are not independent. Indeed, it is unnecessary for us to compute the I-correlators with $F^{(n)}_{\tau}(x) = T^{(n)}_{\tau}(x)$ since the values of the β coefficients for the corresponding OPE's may be deduced from an evaluation of the expressions (1.4.2) and (1.4.3).

For an evaluation of the K-correlators we need

$$\overline{J^{(0)}_{\sigma}(x)} = \varepsilon^{ijk}\{(\bar{d}^{i}(x)\gamma_5)_{\sigma}(\bar{u}^{j}(x)\not{z}\,C\,\bar{u}^{k}(x)) - (\bar{u}^{i}(x)\gamma_5)_{\sigma}(\bar{d}^{j}(x)\not{z}\,C\,\bar{u}^{k}(x))\}, \tag{1.4.4}$$

which leads to

$$\begin{aligned} K^{T(n)}(q,z) = {} & \varepsilon^{ijk}\varepsilon^{lmn}\, z^{\nu}(iz^{\kappa})^{n_1}(iz^{\lambda})^{n_2}(iz^{\rho})^{n_3}\int d^4x\, e^{iq\cdot x} \\ & \langle 0|\,T\,[(D^{n_1}_{\kappa}u(x))^{i}\,C\,\sigma_{\mu\nu}\,(D^{n_2}_{\lambda}u(x))^{j}][D^{n_3}_{\rho}(\gamma^{\mu}d(x))_{\tau}]^{k} \\ & \cdot\{(\bar{d}^{l}(0)\not{z})_{\tau}(\bar{u}^{m}(0)\not{z}\,C\,\bar{u}^{n}(0)) \\ & - (\bar{u}^{l}(0)\not{z})_{\tau}(\bar{d}^{m}(0)\not{z}\,C\,\bar{u}^{n}(0))\}|0\rangle \end{aligned} \tag{1.4.5}$$

when we choose $F^{(n)}_{\tau}(x) = T^{(n)}_{\tau}(x)$ in (1.3.11). It may be easily checked that the perturbative contributions ($\propto \alpha^{(n)}_{1}$) to the K-correlators vanish when $F^{(n)}_{\tau}(x) = A^{(n)}_{\tau}(x)$. This leads to complicated sum rules. Thus we shall restrict ourselves to a study of the $K^{T(n)}$ correlators. Such an investigation should provide a useful check on the QCD sum rule technique as the moments of the T function may also be determined by studying the I-correlators with $F^{(n)}_{\tau}(x) = V^{(n)}_{\tau}(x)$ and $A^{(n)}_{\tau}(x)$ and using the constraint (1.2.14). In principle, use of the K-correlators should enable us to reduce the errors on our estimates of the moments of the distribution amplitudes.

In accordance with the discussion of Section 1.1 we now make the expansions

$$u(x) \rightarrow u(x) + \eta_u(x) \tag{1.4.6a}$$

$$d(x) \rightarrow d(x) + \eta_d(x) \tag{1.4.6b}$$

in (1.4.2), (1.4.3) and (1.4.5), where u(x) and d(x) represent classical background fields and $\eta_u(x)$ and $\eta_d(x)$ their quantum fluctuations. By considering just those terms with quantum quark fields we calculate the coefficient functions of the identity operator I and of the gluonic operator vacuum expectation value $\langle 0| G^a_{\mu\nu} G^{a\mu\nu} |0\rangle$. The α_3 and β_3 coefficient s are computed by evaluating diagrams with some background quark fields annihilated in the vacuum.

Details of the evaluation of the α and β coefficients are now presented.

(a) Calculation of $\alpha_1^{(n)}$ and $\beta_1^{(n)}$ Coefficients

These coefficients occur in the Wilson functions associated with the identity operator in the OPE. Thus in this subsection we are determining the purely perturbative contributions to the functions $I^{(n)}(q^2)$ and $K^{(n)}(q^2)$. We compute the asymptotic behaviour of the coefficient functions, keeping only the terms independent of α_s. These terms may be represented diagrammatically as in Fig. 1.1.

We consider $I^{V(n)}(q,z)$. Retaining only the quantum quark fields after performing the substitutions (1.4.6) in (1.4.2) we use Wick's theorem for Fermi fields to obtain

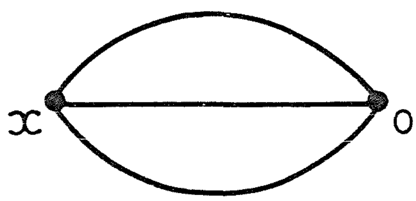
$$\begin{aligned} I^{V(n)}(q,z) = & -\varepsilon^{ijk}\varepsilon^{lmn} z^\mu (iz^\kappa)^{n_1} (iz^\lambda)^{n_2} (iz^\rho)^{n_3} \int d^4x\, e^{iq\cdot x} \\ & \langle 0| \Big\{ -\mathrm{Tr}\, C\left(D^{n_2}_{x\lambda} D^y_\mu S^T_u(x,y)\right)^{jn} C \not{z} \left(D^{n_1}_\kappa S_u(x,0)\right)^{im} \not{z} \,.\, \mathrm{Tr}\left(D^{n_3}_\rho S_d(x,0)\right)^{kl} \not{z} \\ & + \mathrm{Tr}\, C\left(D^{n_2}_{x\lambda} D^y_\mu S^T_u(x,y)\right)^{jn} C \not{z} \left(D^{n_1}_\kappa S_u(x,0)\right)^{im} \not{z} \left(D^{n_3}_\rho S_d(x,0)\right)^{kl} \not{z} \\ & + (\kappa \leftrightarrow \lambda,\ n_1 \leftrightarrow n_2) \Big\}\Big|_{y=0} |0\rangle \quad , \end{aligned} \tag{1.4.7}$$

where the trace (Tr) and transpose (T) refer to Dirac indices. This form exhibits the expected symmetry

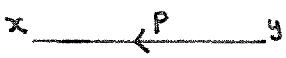
$$I^{V(n_1,n_2,n_3)}(q,z) = I^{V(n_2,n_1,n_3)}(q,z) \,. \tag{1.4.8}$$

To evaluate the $\beta_1^{V(n)}$ coefficients we neglect all g-dependent terms in the expansions of the covariant derivatives and the quark propagators. Thus the covariant derivatives reduce to

Fig. 1.1. Diagram Contributing to $\alpha_1^{(n)}$ and $\beta_1^{(n)}$



Legend for Diagrams of Chapter 1



Quark Propagator $S(x,y) = \int d^4p \; e^{-ip\cdot(x-y)} S(p)$

Gluon Propagator $P(x,y) = \int d^4p \; e^{-ip\cdot(x-y)} P(p)$

Background Quark Field

Background Gluon Field

Crosses represent annihilation of the background fields in the vacuum.

partial derivatives and the (massless) u- and d-quark propagators become the free propagators $S^{(0)}_{u,d}(x,y)$; i.e.

$$S^{im}_{u,d}(x,y) \rightarrow S^{(0)}_{u,d}(x,y)\,\delta^{im} = \frac{i}{2\pi^2}\frac{(\not x - \not y)}{[(x-y)^2 - i\varepsilon]^2}\,\delta^{im} . \qquad (1.4.9)$$

The colour factor is then identical for each term in (1.4.7) and for each (n). It is

$$\varepsilon^{ijk}\,\varepsilon^{lmn}\,\delta^{jn}\,\delta^{im}\,\delta^{kl} = 6 . \qquad (1.4.10)$$

As an explicit example we compute the perturbative contribution to $I^{V(0,0,0)}(q,z)$. Thus we calculate the asymptotic behaviour of

$$-12\, z^\mu \int d^4x\; e^{iq\cdot x}\Big\{-\mathrm{Tr}\; C(\partial^y_\mu S^{(0)}_u(x,y))^T C \not z\, S^{(0)}_u(x,0) \not z \;.\; \mathrm{Tr}\; S^{(0)}_d(x,0) \not z$$
$$+\mathrm{Tr}\; C\,(\partial^y_\mu S^{(0)}_u(x,y))^T C \not z\, S^{(0)}_u(x,0) \not z\, S^{(0)}_d(x,0) \not z\Big\}\Big|_{y=0} \qquad (1.4.11)$$

With the help of (1.4.9) and the relation

$$C\,\gamma^T_\mu\, C = \gamma_\mu \qquad (1.4.12)$$

this becomes

$$-\frac{96\,i}{\pi^6}\int d^4x\; e^{iq\cdot x}\,\frac{(x\cdot z)^4}{(x^2 - i\varepsilon)^7} . \qquad (1.4.13)$$

We have made repeated use of the fact that z^μ is a null vector, $z^2 = 0$, to derive this form.

In analysing the current correlators we encounter the more general integral [16]

$$\int d^4x\, \frac{e^{iq\cdot x}}{(x^2 - i\varepsilon)^n} \xrightarrow{q^2\to -\infty} \begin{cases} -i\,(2\pi)^2\,\dfrac{1}{q^2 + i\varepsilon} & (n=1) \qquad (1.4.14a)\\[2ex] \dfrac{i\,2^{4-2n}\,\pi^2}{(n-1)!\,(n-2)!}\,(-q^2)^{n-2}\,\ln\left(\dfrac{-q^2}{\mu^2}\right) & (n\geq 2) \qquad (1.4.14b)\end{cases}$$

(μ^2 is an ultra-violet cut-off.) By taking derivatives with respect to q_μ we may introduce factors of x^μ into the integrand as required. Since we are interested only in those terms which have a non-vanishing Borel transform (see (1.5.13)) it is unnecessary to include derivatives of the logarithm when $n \geq 3$. For the same reason all polynomials in q^2 may be omitted. As a result the perturbative contributions to $I^{(n)}(q^2)$ and $K^{(n)}(q^2)$ are cut-off independent and proportional to $q^2 \ln q^2$.

We find the perturbative contribution to $I^{V(0,0,0)}(q,z)$ to be $\frac{-1}{480\pi^4}(z.q)^4\ q^2 \ln q^2$, which corresponds to a value of 1/3 for the $\beta_1^{(0,0,0)}$ coefficient for the choice $F_\tau^{(0,0,0)}(x) = V_\tau^{(0,0,0)}(x)$ in the proton correlator.

The calculations of all the $\alpha_1^{(n)}$ and $\beta_1^{(n)}$ coefficients are carried out using the same method and the results are summarized in Tables 1.1 and 1.2.

As well as possessing the required symmetry properties

$$\beta_1^{V(n_1,n_2,n_3)} = \beta_1^{V(n_2,n_1,n_3)} \tag{1.4.15a}$$

$$\beta_1^{A(n_1,n_2,n_3)} = -\beta_1^{A(n_2,n_1,n_3)} \tag{1.4.15b}$$

$$\alpha_1^{(n_1,n_2,n_3)} = \alpha_1^{(n_2,n_1,n_3)} \tag{1.4.15c}$$

the $\alpha_1^{(n)}$ and $\beta_1^{(n)}$ coefficients are seen to satisfy other relations:

$$\alpha_1^{(0,0,0)} = \alpha_1^{(1,0,0)} + \alpha_1^{(0,1,0)} + \alpha_1^{(0,0,1)} \tag{1.4.16a}$$

$$\alpha_1^{(1,0,0)} = \alpha_1^{(2,0,0)} + \alpha_1^{(1,1,0)} + \alpha_1^{(1,0,1)} \tag{1.4.16b}$$

$$\alpha_1^{(0,0,1)} = \alpha_1^{(1,0,1)} + \alpha_1^{(0,1,1)} + \alpha_1^{(0,0,2)} \tag{1.4.16c}$$

(The $\beta_1^{(n)}$'s obey an identical set of equations.) These relations may be proved using integration by parts and are a consequence of momentum conservation. Consider, for example, Eq.(1.4.7). We see that when we retain only the leading term in the perturbative series,

$$I^{V(1,0,0)}(q,z) + I^{V(0,1,0)}(q,z) + I^{V(0,0,1)}(q,z)$$

Table 1.1 Values of $\alpha_1^{(n)}$

(n)	$\alpha_1^{(n)}$
(0,0,0)	1
(1,0,0)	1/3
(0,0,1)	1/3
(2,0,0)	1/7
(0,0,2)	1/7
(1,1,0)	2/21
(1,0,1)	2/21

Table 1.2 Values of $\beta_1^{(n)}$

$F_\tau^{(n)}(x) = V_\tau^{(n)}(x)$

(n)	$\beta_1^{V(n)}$
(0,0,0)	1/3
(1,0,0)	5/42
(0,0,1)	2/21
(2,0,0)	3/56
(0,0,2)	1/28
(1,1,0)	1/28
(1,0,1)	5/168

$F_\tau^{(n)}(x) = A_\tau^{(n)}(x)$

(n)	$\beta_1^{A(n)}$
(1,0,0)	-1/42
(2,0,0)	-1/56
(1,0,1)	-1/168

$$\rightarrow 6\, z^{\mu}\, (i\, z^{\kappa}) \int d^4x \; e^{iq\cdot x}$$

$$\Big\{ \mathrm{Tr}\; C\, (\partial^y_\mu S^{(0)}_u(x,y))^T C \not{z}\, (\partial^x_\kappa S^{(0)}_u(x,0)) \not{z} \,.\, \mathrm{Tr}\; S^{(0)}_d(x,0) \not{z}$$
$$+3 \text{ other terms}$$

$$+ \mathrm{Tr}\; C\, (\partial^x_\kappa \partial^y_\mu S^{(0)}_u(x,y))^T C \not{z}\; S^{(0)}_u(x,0) \not{z} \,.\, \mathrm{Tr}\; S^{(0)}_d(x,0) \not{z}$$
$$+3 \text{ other terms}$$

$$+ \mathrm{Tr}\; C\, (\partial^y_\mu S^{(0)}_u(x,y))^T C \not{z}\; S^{(0)}_u(x,0) \not{z} \,.\, \mathrm{Tr}\, (\partial^x_\kappa S^{(0)}_d(x,0)) \not{z}$$
$$+3 \text{ other terms} \Big\}\Big|_{y=0}$$

$$= 6\, z^{\mu}\, (i\, z^{\kappa}) \int d^4x \; e^{iq\cdot x}\, \partial^x_\kappa$$

$$\Big\{ \mathrm{Tr}\; C\, (\partial^y_\mu S^{(0)}_u(x,y))^T C \not{z}\; S^{(0)}_u(x,0) \not{z} \,.\, \mathrm{Tr}\; S^{(0)}_d(x,0) \not{z}$$
$$+ 3 \text{ other terms} \Big\}\Big|_{y=0}$$

$$= (z\cdot q)\; I^{V(0,0,0)}(q,z) \,, \tag{1.4.17}$$

which, by the definitions (1.3.10) and (1.3.13), implies the desired result

$$\beta_1^{V(1,0,0)} + \beta_1^{V(0,1,0)} + \beta_1^{V(0,0,1)} = \beta_1^{V(0,0,0)} \,. \tag{1.4.18}$$

The constraints (1.4.16) provide a useful check on our calculations.

The symmetries (1.4.15) obviously hold for all α and β coefficients. The relations (1.4.16) also generalise, though not always for the reason in the example above. Both (1.4.15) and (1.4.16) may be rewritten as relations between the corresponding moments of V, A and T; e.g. $V^{(n_1,n_2,n_3)} = V^{(n_2,n_1,n_3)}$ etc.

(b) Calculation of $\alpha_2^{(n)}$ and $\beta_2^{(n)}$ Coefficients

We now turn our attention to the calculation of the non-perturbative contributions to the OPE. Since we neglect the effects of the u- and d-quark masses the quark condensate $\langle 0|O_1|0\rangle$ does not contribute. Thus in this subsection we determine the coefficient functions of the dimension four gluon condensate $\langle 0|O_2|0\rangle$ For this case the leading terms are linear in the strong coupling parameter α_s. The diagrams corresponding to such terms are displayed in Fig 1.2.

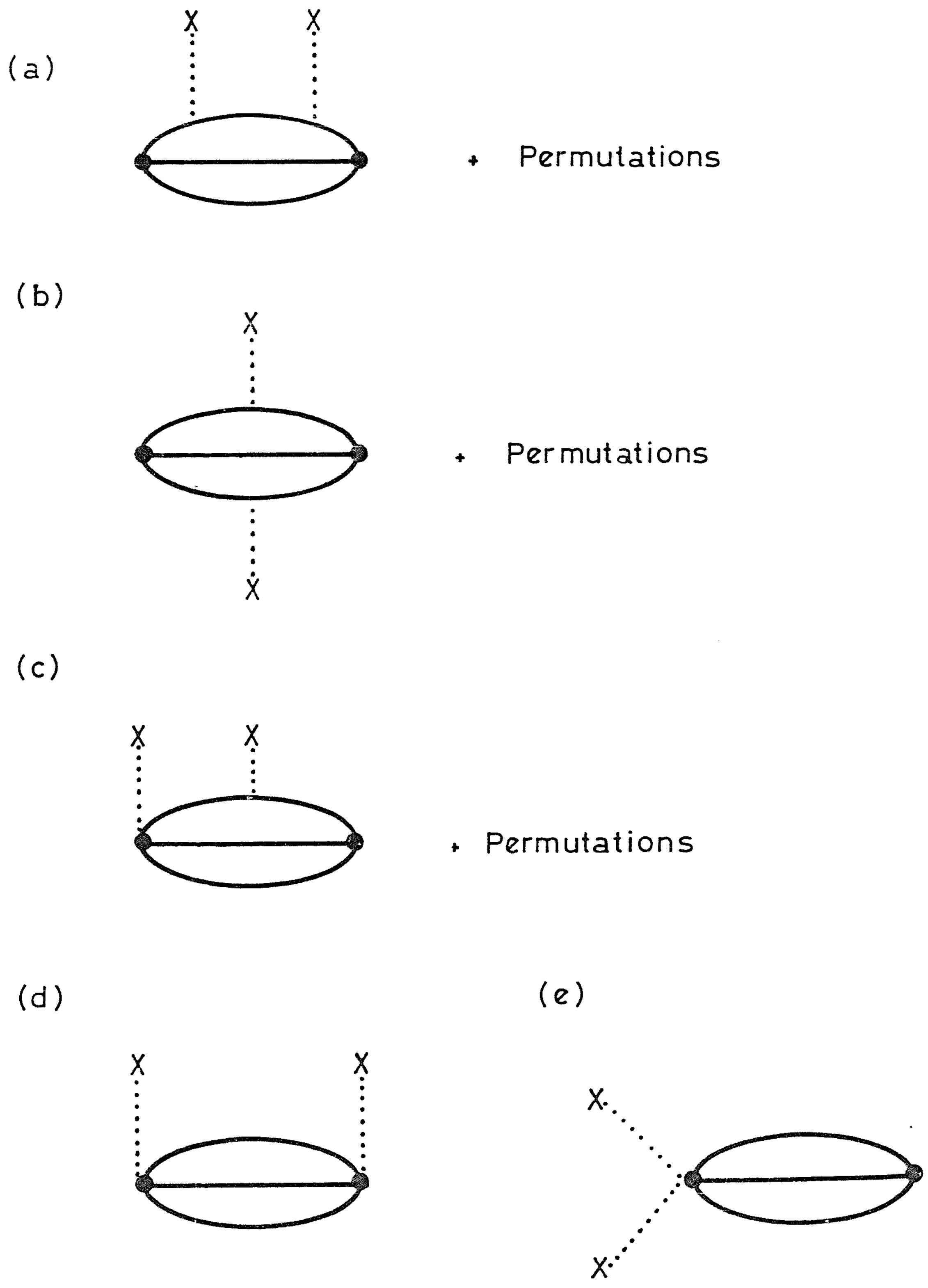
Again we consider the proton correlator with $F_{\tau}^{(n)}(x) = V_{\tau}^{(n)}(x)$. The expression (1.4.7) is our starting point. We choose the Schwinger gauge and determine the short distance expansion of $I^{V(n)}(q,z)$ using (1.1.16) and (1.1.17). All possible terms with two factors of the background gluon field strength tensor $G_{\mu\nu}^{a}$ (without derivatives) are retained. In the diagrams of Fig. 1.2 the background gluon fields originating from the vertices at x and 0 arise from the expansions of the covariant derivatives acting at these points.

It is seen that diagram (d) gives a vanishing contribution to the correlators: in the Schwinger gauge $A_{\mu}^{a}(0) = 0$ and the covariant derivatives acting at the origin reduce to partial derivatives.

A study of the short distance expansion of the fermion propagator $S_f(x,y)$ (eq. (1.1.17)) shows that the diagrams (a) also do not contribute. The '$G^2(0)$' term in the expansion vanishes as $y \to 0$ implying that with our choice of origin (background field gauge) a quark interacting twice with background gluon fields cannot propagate.

Clearly diagrams (c) will contribute only to those correlators with at least one covariant derivative acting at the vertex x; i.e. those with $\sum_{i=1}^{3} n_i \geqslant 1$; while diagram (e) is non-vanishing only for the correlators with $\sum_{i=1}^{3} n_i = 2$. As an illustration we sketch the evaluation of diagrams (b), (c) and (e) for $I^{V(0,0,2)}(q,z)$.

Fig.1.2. Diagrams Contributing to $\alpha_2^{(n)}$ and $\beta_2^{(n)}$



Diagrams (b) represent the three different possible ways in which a single factor of $G^a_{\mu\nu}(0)$ may appear in the expansions for two of the three quark propagators. For those propagators,

$$S_{u,d}(x,y) \rightarrow S^{(2)}(x,y), \qquad (1.4.19)$$

where the superscript (2) denotes the dimension of the gluon operator; i.e.

$$S^{(2)}_{u,d}(x,y) = \frac{-g}{16\pi^2} G^a_{\mu\nu}(0) T^a \frac{\gamma^\mu \not{t} \gamma^\nu}{t^2 - i\varepsilon} + \frac{g}{4\pi^2} G^a_{\mu\nu}(0) T^a \frac{x^\mu y^\nu \not{t}}{(t^2 - i\varepsilon)^2}$$

$$\equiv S^{(2)(a)}_{u,d}(x,y) + S^{(2)(b)}_{u,d}(x,y). \qquad (1.4.20)$$

Thus we calculate the $q^2 \rightarrow -\infty$ behaviour of

$$\begin{aligned}
&2\varepsilon^{ijk}\varepsilon^{lmn} z^\mu z^\lambda z^\rho \int d^4x\, e^{iq\cdot x} \\
&\langle 0|\, \mathrm{Tr}\,(\partial^y_\mu S^{c(2)}_u(y,x))^{jn} \not{z}\, (S^{(2)}_u(x,0))^{im} \not{z}\ .\ \mathrm{Tr}\,(\partial_\lambda \partial_\rho S^{(0)}_d(x,0))^{kl} \not{z} \\
&\quad + \mathrm{Tr}\,(\partial^y_\mu S^{c(2)}_u(y,x))^{jn} \not{z}\, (S^{(0)}_u(x,0))^{im} \not{z}\ .\ \mathrm{Tr}\,(\partial_\lambda \partial_\rho S^{(2)}_d(x,0))^{kl} \not{z} \\
&\quad + \mathrm{Tr}\,(\partial^y_\mu S^{c(0)}_u(y,x))^{jn} \not{z}\, (S^{(2)}_u(x,0))^{im} \not{z}\ .\ \mathrm{Tr}\,(\partial_\lambda \partial_\rho S^{(2)}_d(x,0))^{kl} \not{z} \\
&\quad - \mathrm{Tr}\,(\partial^y_\mu S^{c(2)}_u(y,x))^{jn} \not{z}\, (S^{(2)}_u(x,0))^{im} \not{z}\, (\partial_\lambda \partial_\rho S^{(0)}_d(x,0))^{kl} \not{z} \\
&\quad - \mathrm{Tr}\,(\partial^y_\mu S^{c(2)}_u(y,x))^{jn} \not{z}\, (S^{(0)}_u(x,0))^{im} \not{z}\, (\partial_\lambda \partial_\rho S^{(2)}_d(x,0))^{kl} \not{z} \\
&\quad - \mathrm{Tr}\,(\partial^y_\mu S^{c(0)}_u(y,x))^{jn} \not{z}\, (S^{(2)}_u(x,0))^{im} \not{z}\, (\partial_\lambda \partial_\rho S^{(2)}_d(x,0))^{kl} \not{z}\, |0\rangle \Big|_{y=0},
\end{aligned}$$

$$(1.4.21)$$

where we have defined

$$S^c(x,y) = C\, S^T(y,x)\, C^{-1}. \qquad (1.4.22)$$

It may be shown that

$$S^{c(0)}(y,x) = -S^{(0)}(x,y) \qquad (1.4.23a)$$

$$S^{c(2)}(y,x) = S^{(2)(a)}(x,y) - S^{(2)(b)}(x,y). \qquad (1.4.23b)$$

For diagrams (b) the fact that $z^2 = 0$ implies that all the $S^{(2)}(x,y)$ propagators in (1.4.21) may be replaced by $S^{(2)(a)}(x,y)$. The colour factor is

$$\varepsilon^{ijk}\,\varepsilon^{lmn}\,(T^a)^{jn}\,(T^b)^{im}\,\delta^{kl} = -\tfrac{1}{2}\,\delta^{ab}. \qquad (1.2.24)$$

Performing the integration using (1.4.14b) the asymptotic limit of the contribution of diagrams (b) to $I^{V(0,0,2)}(q,z)$ is found to be

$$\frac{-1}{4320\,\pi^2}\quad\frac{(z\cdot q)^6}{q^2}\;\langle 0|\,\frac{\alpha_s}{\pi}\,G^a_{\mu\nu}\,G^{a\mu\nu}\,|0\rangle,$$

where we have extracted the Lorentz and colour singlet component by making the replacement [23]

$$G^a_{\kappa\lambda}(0)\,G^b_{\rho\sigma}(0) \longrightarrow \left(\tfrac{1}{8}\,\delta^{ab}\right)\tfrac{1}{12}\,(\eta_{\kappa\rho}\,\eta_{\lambda\sigma}-\eta_{\kappa\sigma}\,\eta_{\lambda\rho})\;G^c_{\mu\nu}(0)\,G^{c\mu\nu}(0) \qquad (1.4.25)$$

Next we consider diagrams (c). This time one factor of $G^a_{\mu\nu}(0)$ must come from the expansion of the covariant derivatives. Thus, for the $I^{V(0,0,2)}$ correlator, we put

$$D_\lambda\,D_\rho \longrightarrow -\tfrac{1}{2}\,ig\left(\partial_\lambda\,x^\nu\,G^a_{\nu\rho}(0)\,T^a + x^\nu\,G^a_{\nu\lambda}(0)\,T^a\,\partial_\rho\right) \qquad (1.4.26)$$

In this case only the $S^{(2)(b)}(x,y)$ term of the interacting propagator gives a non-vanishing contribution. Using (1.4.26) and the identity

$$\varepsilon^{ijk}\,\varepsilon^{lmn}\,\delta^{jn}\,\delta^{im}\,(T^aT^b)^{kl} = \delta^{ab}, \qquad (1.4.27)$$

together with the properties of the charge conjugate propagator $S^c(x,y)$, we obtain a contribution of $\frac{-1}{8640\pi^2}\;\frac{(z\cdot q)^6}{q^2}\;\langle 0|\,\frac{\alpha_s}{\pi}\,G^a_{\mu\nu}\,G^{a\mu\nu}|0\rangle$.

Lastly we must allow for the case when both factors of $G^a_{\mu\nu}(0)$ originate in the expansions of the covariant derivatives; i.e.

$$D_\lambda\,D_\rho \longrightarrow \left(-\tfrac{1}{2}\,ig\right)^2 x^\nu\,x^\kappa\;G^a_{\nu\lambda}(0)\,T^a\,G^b_{\kappa\rho}(0)\,T^b \qquad (1.4.28)$$

for the $I^{V(0,0,2)}$ correlator. This situation is represented in diagram (e) and leads to the contribution

Table 1.3 Contributions to $\alpha_2^{(n)}$

(n)	Contribution from Diag. 1.2(b)	Contribution from Diag. 1.2(e)	$\alpha_2^{(n)}$
(0,0,0)	1/3	0	1/3
(1,0,0)	1/6	0	1/6
(0,0,1)	0	0	0
(2,0,0)	1/10	1/30	2/15
(0,0,2)	-1/30	1/30	0
(1,1,0)	1/20	-1/60	1/30
(1,0,1)	1/60	-1/60	0

Table 1.4 Contributions to $\beta_2^{(n)}$

(a) $F_\tau^{(n)}(x) = V_\tau^{(n)}(x)$

(n)	Contribution from Diag. 1.2(b)	Contribution from Diag. 1.2(c)	Contribution from Diag. 1.2(e)	$\beta_2^{V(n)}$
(0,0,0)	1/12	0	0	1/12
(1,0,0)	7/240	-1/240	0	1/40
(0,0,1)	1/40	1/120	0	1/30
(2,0,0)	1/60	-1/360	1/180	7/360
(0,0,2)	1/90	1/180	1/180	1/45
(1,1,0)	1/180	-1/360	-1/360	0
(1,0,1)	1/144	1/720	-1/360	1/180

(b) $F_\tau^{(n)} = A_\tau^{(n)}(x)$

(n)	Contribution from Diag. 1.2(b)	Contribution from Diag. 1.2(c)	Contribution from Diag. 1.2(e)	$\beta_2^{A(n)}$
(1,0,0)	-1/48	1/80	0	-1/120
(2,0,0)	-1/60	1/120	0	-1/120
(1,0,1)	-1/240	1/240	0	0

$$\frac{-1}{8640\pi^2} \quad \frac{(z\cdot q)^6}{q^2} \langle 0| \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} |0\rangle.$$

We obtain a final total of $\frac{-1}{2160\pi^2} \frac{(z\cdot q)^6}{q^2} \langle 0| \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu}|0\rangle$ for the asymptotic behaviour of the $I^{V(0,0,2)}$ correlator due to the condensate $\langle 0|O_2|0\rangle$. This corresponds to a value of 1/45 for the $\beta_2^{V(0,0,2)}$ coefficient.

The results for all the $\alpha_2^{(n)}$ and $\beta_2^{(n)}$ coefficients are displayed in Tables 1.3 and 1.4. These values have been checked by calculating the Feynman diagrams of Fig 1.2 in momentum space. It is clear that such an approach necessitates integration over two loop momenta. For these diagrams it is simpler to assume the short distance expansion (1.1.17) for the quark propagators and perform the manipulations in configuration space.

It is seen that relations analogous to (1.4.16) hold also for the $\alpha_2^{(n)}$ and $\beta_2^{(n)}$ coefficients. Indeed, they are satisfied for each individual diagram. For diagrams (b) the relations follow using the integration by parts argument of Section 1.4(a). For diagram (e), however, the fact that, for example, the contribution to the $I^{V(n)}(q,z)$ correlators satisfy

$$I^{V(1,0,1)} + I^{V(0,1,1)} + I^{V(0,0,2)} = I^{V(0,0,1)} \tag{1.4.29}$$

is a consequence of the colour factors. Specifically, the identity

$$\varepsilon^{ijk}\,\varepsilon^{lmn}\left\{2(T^a)^{jn}(T^b)^{im}\delta^{kl} + (T^aT^b)^{jn}\,\delta^{im}\,\delta^{kl}\right\} = 0 \tag{1.4.30}$$

is required. Both types of argument are needed to prove the relations between the contributions from diagrams (c).

(c) Calculation of $\alpha_3^{(n)}$ and $\beta_3^{(n)}$ Coefficients

So far we have determined the first order perturbative contributions to the functions $I^{(n)}(q^2)$ and $K^{(n)}(q^2)$ as well as the leading contributions of the dimension four gluon condensate.. The $\alpha_3^{(n)}$ and $\beta_3^{(n)}$ coefficients occur in the Wilson functions associated with the four quark operator O_3 ((1.1.9d)), which has dimension six. In our calculations we again wish to retain only the leading terms in the perturbative expansions of the coefficient functions. Contributions independent of α_s (i.e. of order $(\alpha_s)^0$) could be expected to enter via Fig. 1.3(a) while the remainder of the diagrams of Fig. 1.3 represent the possible contributions of order α_s. Again we illustrate our arguments with reference to the $I^{V(n)}(q,z)$ correlators.

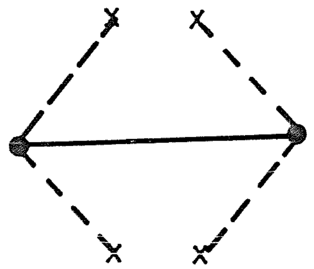
We start by considering diagram (a). As this diagram has only one quark propagator we retain only one pair of quantum quark fields after performing the substitutions (1.4.6) in the general expression (1.4.2). Since the four background quark fields will contribute to condensates of dimension $D \geqslant 6$ (see later), all covariant derivatives must reduce to ordinary derivatives. Thus we find

$$
\begin{aligned}
I^{V(n)}(q,z)\Big|_{\text{Diag.(a)}} = & -\varepsilon^{ijk}\,\varepsilon^{lmn}\, z^{\mu}\,(i z^{\kappa})^{n_1}(i z^{\lambda})^{n_2}(i z^{\nu})^{n_3} \\
& (C\not{z})_{\alpha\beta}\,(\not{z})_{\tau\gamma}\,(\not{z}\,C)_{\rho\sigma}\int d^4x\, e^{iq\cdot x} \\
\cdot\langle 0|\Big\{ & (\partial_\kappa^{n_1} u)^i_\alpha(x)\,(\partial_\lambda^{n_2} u)^j_\beta(x)\,\bar{u}^l_\rho(0)\,(\partial_\mu \bar{u})^m_\sigma(0)\,(\partial_{x\nu}^{n_3}\, S_d^{(0)}(x,0))^{kn}_{\gamma\tau} \\
& -(\partial_\kappa^{n_1} u)^i_\alpha(x)\,(\partial_\lambda^{n_2} u)^j_\beta(x)\,\bar{u}^l_\tau(0)\,(\partial_\mu \bar{u})^m_\sigma(0)\,(\partial_{x\nu}^{n_3}\, S_d^{(0)}(x,0))^{kn}_{\gamma\rho} \\
& +(\partial_\kappa^{n_1} u)^i_\alpha(x)\,(\partial_\nu^{n_3} d)^j_\gamma(x)\,\bar{d}^l_\tau(0)\,\bar{u}^m_\rho(0)\,(\partial_{x\lambda}^{n_2}\,\partial_{0\mu}\, S_u^{(0)}(x,0))^{kn}_{\beta\sigma} \\
& +(\partial_\lambda^{n_2} u)^i_\alpha(x)\,(\partial_\nu^{n_3} d)^j_\gamma(x)\,\bar{d}^l_\tau(0)\,\bar{u}^m_\rho(0)\,(\partial_{x\kappa}^{n_1}\,\partial_{0\mu}\, S_u^{(0)}(x,0))^{kn}_{\beta\sigma} \\
& +(\partial_\kappa^{n_1} u)^i_\alpha(x)\,(\partial_\nu^{n_3} d)^j_\gamma(x)\,\bar{d}^l_\tau(0)\,(\partial_\mu \bar{u})^m_\sigma(0)\,(\partial_{x\lambda}^{n_2}\, S_u^{(0)}(x,0))^{kn}_{\beta\rho} \\
& +(\partial_\lambda^{n_2} u)^i_\alpha(x)\,(\partial_\nu^{n_3} d)^j_\gamma(x)\,\bar{d}^l_\tau(0)\,(\partial_\mu \bar{u})^m_\sigma(0)\,(\partial_{x\kappa}^{n_1}\, S_u^{(0)}(x,0))^{kn}_{\beta\rho} \\
& -(\partial_\kappa^{n_1} u)^i_\alpha(x)\,(\partial_\nu^{n_3} d)^j_\gamma(x)\,\bar{d}^l_\rho(0)\,\bar{u}^m_\tau(0)\,(\partial_{x\lambda}^{n_2}\,\partial_{0\mu}\, S_u^{(0)}(x,0))^{kn}_{\beta\sigma} \\
& -(\partial_\lambda^{n_2} u)^i_\alpha(x)\,(\partial_\nu^{n_3} d)^j_\gamma(x)\,\bar{d}^l_\rho(0)\,\bar{u}^m_\tau(0)\,(\partial_{x\kappa}^{n_1}\,\partial_{0\mu}\, S_u^{(0)}(x,0))^{kn}_{\beta\sigma} \\
& -(\partial_\kappa^{n_1} u)^i_\alpha(x)\,(\partial_\nu^{n_3} d)^j_\gamma(x)\,\bar{d}^l_\rho(0)\,(\partial_\mu \bar{u})^m_\sigma(0)\,(\partial_{x\lambda}^{n_2}\, S_u^{(0)}(x,0))^{kn}_{\beta\tau} \\
& -(\partial_\lambda^{n_2} u)^i_\alpha(x)\,(\partial_\nu^{n_3} d)^j_\gamma(x)\,\bar{d}^l_\rho(0)\,(\partial_\mu \bar{u})^m_\sigma(0)\,(\partial_{x\kappa}^{n_1}\, S_u^{(0)}(x,0))^{kn}_{\beta\tau}\Big\}|0\rangle,
\end{aligned}
$$

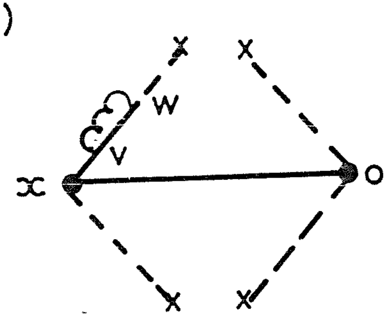
(1.4.31)

Fig.1.3 Diagrams Contributing to $\alpha_3^{(n)}$ and $\beta_3^{(n)}$

(a)

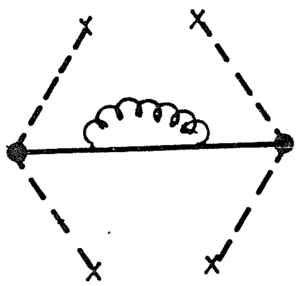


(b) (i)

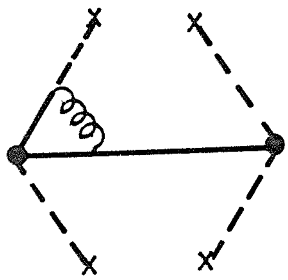


+ Permutations

(ii)

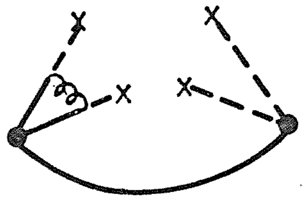


(iii)



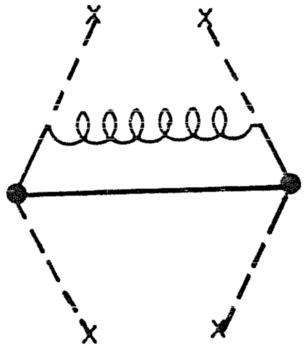
+ Permutations

(iv)



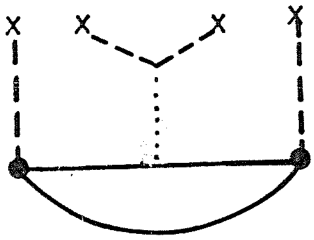
+ Permutation

(c)



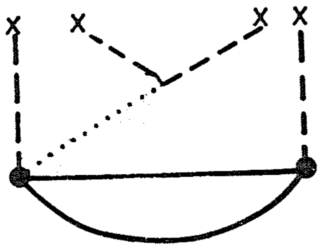
+ Permutations

(d)



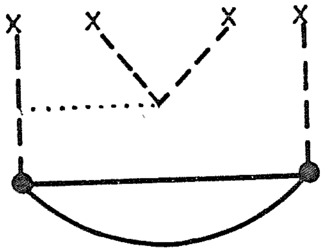
+ Permutation

(e)



+ Permutation

(f)



+ Permutation

where $\alpha,\beta,\gamma,\delta,\rho$ and σ are Dirac indices. Clearly this expression satisfies the symmetry relation

$$I^{V(n_1,n_2,n_3)}(q,z) = I^{V(n_2,n_1,n_3)}(q,z) \tag{1.4.32}$$

as expected.

Each of the terms of (1.4.31) contains a vacuum condensate of four background quark fields and their derivatives. Consider the general form

$$\langle 0|(\partial^{n_1}q^A)^i_\alpha(w)\,(\partial^{n_2}q^B)^j_\beta(x)\,(\partial^{n_3}\bar{q}^C)^k_\gamma(y)\,(\partial^{n_4}\bar{q}^D)^l_\delta(z)|0\rangle, \tag{1.4.33}$$

where A,B,C and D are quark flavour indices. We saw earlier ((1.1.16)) that the background gluon field $A^a_\mu(x)$ could be expanded in powers of covariant derivatives of the field strength tensor $G^a_{\mu\nu}(0)$ in the Schwinger gauge. In the same way [23] the gauge condition allows us to substitute covariant derivatives for oridinary derivatives in Taylor expansions of the background quark fields. Thus

$$q(x) = \sum_{k=0}^{\infty} \frac{1}{k!}\, x^{\alpha_1}\ldots x^{\alpha_k}\, D_{\alpha_1}(0)\ldots D_{\alpha_k}(0)\, q(0) \tag{1.4.34a}$$

$$\bar{q}(x) = \sum_{k=0}^{\infty} \frac{1}{k!}\, x^{\alpha_1}\ldots x^{\alpha_k}\, \bar{q}(0)\, D^\dagger_{\alpha_1}(0)\ldots D^\dagger_{\alpha_k}(0) \tag{1.4.34b}$$

where

$$D^\dagger_\mu \equiv \overleftarrow{\partial}_\mu + ig\,T^a A^a_\mu\,. \tag{1.4.35}$$

Use of these equations will result in gauge covariant expressions for products of background quark fields. It is clear that inclusion of any terms other than the leading ones of the expansions of any of the quark or anti-quark fields in (1.4.33) will lead to contributions to quark condensates of dimension D > 6. Thus the dimension six part of (1.4.33) is

$$\delta^{n_1 0}\,\delta^{n_2 0}\,\delta^{n_3 0}\,\delta^{n_4 0}\,\langle 0|\,q^{Ai}_\alpha(0)\,q^{Bj}_\beta(0)\,\bar{q}^{Ck}_\gamma(0)\,\bar{q}^{Dl}_\delta(0)|0\rangle. \tag{1.4.36}$$

To estimate this four quark vacuum expectation value we follow SVZ [4] and introduce the vacuum saturation hypothesis. In this approximation only the vacuum intermediate state is retained in each channel:

$$\begin{aligned}&\langle 0|\, q_\alpha^{Ai}(0)\; q_\beta^{Bj}(0)\; \bar{q}_\gamma^{Ck}(0)\; \bar{q}_\delta^{Dl}(0)\,|0\rangle \\ &\approx \langle 0|\, \bar{q}_\gamma^{Ck}(0)\; q_\beta^{Bj}(0)\,|0\rangle \langle 0|\, \bar{q}_\delta^{Dl}(0)\; q_\alpha^{Ai}(0)\,|0\rangle \\ &\quad - \langle 0|\, \bar{q}_\gamma^{Ck}(0)\; q_\alpha^{Ai}(0)\,|0\rangle \langle 0|\, \bar{q}_\delta^{Dl}(0)\; q_\beta^{Bj}(0)\,|0\rangle \end{aligned} \qquad (1.4.37)$$

Since only Lorentz, colour and flavour singlets may have a non-vanishing vacuum condensate we find [23]

$$\langle 0|\, \bar{q}_\alpha^{Ai}(0)\; q_\beta^{Bj}(0)\,|0\rangle = \frac{1}{12}\, \delta^{AB}\, \delta_{\alpha\beta}\, \delta^{ij} \langle 0|\, \bar{q}^A(0)\; q^A(0)\,|0\rangle. \qquad (1.4.38)$$

(No sum over A). The vacuum expectation values $\langle 0|\bar{q}^A(0)\, q^A(0)|0\rangle$ (A = u,d,s) have been estimated by several authors [24,25]. Recalling our assumption (1.3.12), for our purposes the condensate (1.4.33) may finally be written

$$\begin{aligned}&\frac{1}{144}\, \delta^{n_1 0}\, \delta^{n_2 0}\, \delta^{n_3 0}\, \delta^{n_4 0} \langle 0|\, \bar{q}\, q\,|0\rangle^2 \\ &(\delta^{BC}\delta^{AD}\, \delta_{\beta\gamma}\, \delta_{\alpha\delta}\, \delta^{jk}\, \delta^{il} - \delta^{AC}\, \delta^{BD}\, \delta_{\alpha\gamma}\, \delta_{\beta\delta}\, \delta^{ik}\, \delta^{jl}).\end{aligned} \qquad (1.4.39)$$

We now apply this approximation to our expression (1.4.31) for the contribution of diagram (a). It is clear that the action of the Kronecker delta symbols with Dirac indices is such that each term vanishes for all (n). This is because for each term at least two of the following pairs of indices are contracted:

$$(\alpha\rho),\ (\beta\rho),\ (\alpha\sigma),\ (\beta\sigma),\ (\alpha\tau),\ (\beta\tau),\ (\gamma\rho),\ (\gamma\tau).$$

The contraction of any of these pairs in the expression $(C\not{z})_{\alpha\beta}$ $(\not{z})_{\tau\gamma}$ $(\not{z}C)_{\rho\sigma}$ gives zero, either because $z^2 = 0$ or because the trace of an odd number of γ-matrices vanishes. We conclude that the leading non-zero terms in the perturbative expansions of the coefficient functions are of order α_s. To compute these terms we must calculate the rest of the diagrams of Fig. 1.3.

Consider first the diagrams (b)(i). The contributions are most readily determined by modifying each term of (1.4.31) to allow for the gluon loop. For the sample graph shown, the first term of (1.4.31) must be altered by making the replacement

$$u^i_\alpha(x) \longrightarrow \int d^4v \int d^4w \; (S^{(0)}(x,v))^{ij}_{\alpha\beta} \; ig\,(\gamma^\mu)_{\beta\gamma}\,(T^a)^{jk}\, P^{(0)ab}_{\mu\nu}(v,w) \cdot (S^{(0)}(v,w))^{kl}_{\gamma\delta}\; ig\,(\gamma^\nu)_{\delta\epsilon}\,(T^b)^{lm}\, u^m_\epsilon(w). \tag{1.4.40}$$

Since the Dirac index of only one quark field is altered the contraction of these indices still ensures the vanishing of each term of (1.4.31). A similar change occurs when the substitutions appropriate to diagrams (b) (iii) are made while the insertion of the gluon loop in the internal quark propagator of diagram (b) (ii) does not interfere with the Dirac index structure of the background quark fields at all.

For the graph illustrated in Fig. 1.3(b)(iv) the replacement to be made in the first term of (1.4.31) is

$$u^i_\alpha(x)\, u^j_\beta(x) \longrightarrow \int d^4v \int d^4w \; (S^{(0)}(x,v))^{ik}_{\alpha\gamma}\; ig\,(\gamma^\mu)_{\gamma\delta}\,(T^a)^{kl}\, u^l_\delta(v)\; P^{(0)ab}_{\mu\nu}(v,w)\,(S^{(0)}(x,w))^{jm}_{\beta\rho}\; ig\,(\gamma^\nu)_{\rho\sigma}\,(T^b)^{mn}\, u^n_\sigma(w). \tag{1.4.41}$$

This diagram too gives a vanishing contribution to the correlators. To show this, it is simplest to transfer to momentum space by using the definitions

$$S(x,y) = \int đ^4p \; e^{-ip\cdot(x-y)}\, S(p) \tag{1.4.42a}$$

$$P^{ab}_{\mu\nu}(x,y) = \int đ^4p \; e^{-ip\cdot(x-y)}\, P^{ab}_{\mu\nu}(p) \tag{1.4.42b}$$

$$u(x) = \int đ^4p \; e^{-ip\cdot x}\, u(p)\, \delta^{(4)}(p) \tag{1.4.42c}$$

(The background quark fields do not carry any momentum.)

The expression (1.4.41) becomes

$$\int d^4v \int d^4w \int đ^4p \;\; P^{(0)ab}_{\mu\nu}(p)$$
$$(S^{(0)}(p))^{ik}_{\alpha\gamma}\;\; ig\;(\gamma^\mu)_{\gamma\delta}\;(T^a)^{kl}\;\; u^l_\delta(0)$$
$$(S^{(0)}(-p))^{jm}_{\beta\rho}\;\; ig\;(\gamma^\nu)_{\rho\sigma}\;(T^b)^{mn}\;\; u^n_\sigma(0)\;. \tag{1.4.43}$$

Recalling from Section 1.1 that

$$S^{(0)}(p) = \frac{i\not{p}}{p^2+i\varepsilon} \tag{1.4.44a}$$

$$P^{(0)\,ab}_{\mu\nu}(p) = \frac{-i\,\delta^{ab}\eta_{\mu\nu}}{p^2+i\varepsilon}\,, \tag{1.4.44b}$$

we see that (1.4.43) contains as a factor the integral

$$\int d^4p \quad \frac{p^\rho p^\sigma}{(p^2+i\varepsilon)^3}$$

which vanishes in the prescription of dimensional regularisation.

Having shown that none of the diagrams (b) contributes to the Wilson coefficient functions we turn to diagrams (c). We must amend each term of (1.4.31) to account for each of the four possible Feynman graphs. Each graph corresponds to the mediation of a gluon between a chosen quark and antiquark pair. For example, one modification which must be made to the first term of (1.4.31) is given by

$$\begin{aligned} u^i_\alpha(x)\,\bar{u}^l_\rho(0) \longrightarrow \int d^4v \int d^4w \quad & P^{(0)\,ab}_{\mu\nu}(v,w) \\ & (S^{(0)}(x,v))^{ij}_{\alpha\beta}\; ig\,(\gamma^\mu)_{\beta\gamma}\,(T^a)^{jk}\,u^k_\gamma(v) \\ & \bar{u}^m_\sigma(w)\; ig\,(\gamma^\nu)_{\sigma\tau}\,(T^b)^{mn}\,(S^{(0)}(w,0))^{nl}_{\tau\rho} \end{aligned} \tag{1.4.45}$$

Once the appropriate substitutions have been carried out it is again found to be easier to complete the computations in momentum space. The final result for the contribution of diagrams (c) to the $I^{V(n)}(q,z)$ correlators is

$$\begin{aligned} I^{V(n)}(q,z)\Big|_{\text{Diag.(c)}} = \; & \frac{1}{9\pi}\;\frac{1}{q^4}\;\langle 0|\sqrt{\alpha_s}\,\bar{q}\,q\,|0\rangle^2 \\ & \cdot\Big\{ \delta_{n_1 0}\,(z\cdot q)^{n_2+n_3+4}\;\frac{(n_2+2)!\,n_3!}{(n_2+n_3+3)!} \\ & + \delta_{n_2 0}\,(z\cdot q)^{n_1+n_3+4}\;\frac{(n_1+2)!\,n_3!}{(n_1+n_3+3)!} \\ & + \delta_{n_3 0}\,(z\cdot q)^{n_1+n_2+4}\;\frac{[(n_1+2)!\,n_2! + n_1!\,(n_2+2)!]}{(n_1+n_2+3)!} \end{aligned} \tag{1.4.46}$$

Using this form it is readily confirmed that

$$I^{V(n_1+1,n_2,n_3)}\Big|_{Diag.(c)} + I^{V(n_1,n_2+1,n_3)}\Big|_{Diag.(c)}$$

$$+ I^{V(n_1,n_2,n_3+1)}\Big|_{Diag.(c)}$$

$$= (z\cdot q)\, I^{V(n)}\Big|_{Diag.(c)} \qquad (1.4.47)$$

which implies that the $\beta_3^{(n)}$ coefficients satisfy a set of relations analogous to (1.4.16).

For reference we now give the contributions of these diagrams to the proton I-correlators with $F_{\tau}^{(n)}(x) = A_{\tau}^{(n)}(x)$ and to the K-correlators with $F_{\tau}^{(n)}(x) = T_{\tau}^{(n)}(x)$:

$$I^{A(n)}(q,z)\Big|_{Diag.(c)} = \frac{1}{9\pi}\frac{1}{q^4}\langle 0|\sqrt{\alpha_s}\,\bar{q}q|0\rangle^2$$

$$\cdot\Big\{\delta_{n_1 0}\,(z\cdot q)^{n_2+n_3+4}\frac{(n_2+2)!\,n_3!}{(n_2+n_3+3)!}$$

$$-\delta_{n_2 0}\,(z\cdot q)^{n_1+n_3+4}\frac{(n_1+2)!\,n_3!}{(n_1+n_3+3)!}$$

$$+\delta_{n_3 0}\,(z\cdot q)^{n_1+n_2+4}\frac{[n_1!(n_2+2)! - (n_1+2)!\,n_2!]}{(n_1+n_2+3)!}\Big\}$$

(1.4.48)

$$K^{(n)}(q,z)\Big|_{Diag.(c)} = -\frac{2}{9\pi}\frac{1}{q^4}\langle 0|\sqrt{\alpha_s}\,\bar{q}q|0\rangle^2$$

$$\cdot\Big\{\delta_{n_1 0}\,(z\cdot q)^{n_2+n_3+3}\frac{n_3!\,(n_2+1)!}{(n_2+n_3+2)!}$$

$$+\delta_{n_2 0}\,(z\cdot q)^{n_1+n_3+3}\frac{n_3!\,(n_1+1)!}{(n_1+n_3+2)!}$$

$$+\delta_{n_3 0}\,(z\cdot q)^{n_1+n_2+3}\frac{n_1!\,n_2!}{(n_1+n_2+1)!}\Big\}. \qquad (1.4.49)$$

These contributions also display the required symmetries under $n_1 \leftrightarrow n_2$ and satisfy relations analogous to (1.4.16).

Diagrams (d), (e) and (f) differ from those already calculated in that they possess two quark propoagators (whether free or interacting) connecting the vertices at x and o. Thus, to determine the contributions of these diagrams to the $I^{V(n)}(q,z)$ correlators, we retain two pairs of quantum quark fields after making the substitutions (1.4.6) in the general expression (1.4.2). We find

$$
\begin{aligned}
I^{V(n)}(q,z)\Big|_{\text{Diags. (d), (e), (f)}} &\\
= z_\mu\, (iz^\kappa)^{n_1} (iz^\lambda)^{n_2} (iz^\nu)^{n_3}\, & \varepsilon^{ijk} \varepsilon^{\ell mn} \int d^4x\, e^{iq\cdot x} \\
\langle 0 | \Big\{ \bar{d}^\ell(0) \not{z} (D_\nu^{n_3} d)^i(x) . & \operatorname{Tr} (D_{x\kappa}^{n_1} S(x,0))^{jm} \not{z} (D_{x\lambda}^{n_2} D_o^\mu S^c(0,x))^{kn} \not{z} \\
+ (D^\mu \bar{u})^\ell(0) \not{z} (D_{x\lambda}^{n_2} S^c(0,x))^{jm} & \not{z} (D_\kappa^{n_1} u)^i(x) . \operatorname{Tr} (D_{x\nu}^{n_3} S(x,0))^{kn} \not{z} \\
+ \bar{u}^\ell(0) \not{z} (D_{x\lambda}^{n_2} D_o^\mu S^c(0,x))^{jm} & \not{z} (D_\kappa^{n_1} u)^i(x) . \operatorname{Tr} (D_{x\nu}^{n_3} S(x,0))^{kn} \not{z} \\
- \bar{d}^\ell(0) \not{z} (D_{x\lambda}^{n_2} D_o^\mu S^c(0,x))^{jm} & \not{z} (D_{x\kappa}^{n_1} S(x,0))^{kn} \not{z} (D_\nu^{n_3} d)^i(x) \\
+ (D^\mu \bar{u})^\ell(0) \not{z} (D_{x\nu}^{n_3} S^c(0,x))^{jm} & \not{z} (D_{x\lambda}^{n_2} S^c(0,x))^{kn} \not{z} (D_\kappa^{n_1} u)^i(x) \\
- \bar{u}^\ell(0) \not{z} (D_{x\nu}^{n_3} S(x,0))^{jm} & \not{z} (D_{x\lambda}^{n_2} D_o^\mu S^c(0,x))^{kn} \not{z} (D_\kappa^{n_1} u)^i(x) \\
+ (\kappa \leftrightarrow \lambda\, ,\; n_1 \leftrightarrow n_2) \Big\} | 0 \rangle . &
\end{aligned}
\tag{1.4.50}
$$

Note that these diagrams each contain a zero momentum gluon propagator. We do not attempt to evaluate such graphs by using conventional perturbation theory. Instead, graphs at a lower order in perturbation theory are computed and the diagrams in which we are interested are then produced by using the equation of motion (1.1.8c) in the lower dimensional condensates to generate the D = 6 four-quark condensates.

First let us consider the contribution of diagrams (d). Clearly we may replace all covariant derivatives in the general expression (1.4.50) by ordinary derivatives and let either of the two quark propagators of each term reduce to the free propagator $S^{(o)}$. To illustrate the procedure we study the simplest case, the $I^{V(0)}(q,z)$ correlator, and demonstrate the manipulations for the first term of (1.4.50) only. Thus

$$
\begin{aligned}
I^{V(0)}(q,z)\Big|_{\mathrm{Diag.(d)}}
&= z_\mu\, \varepsilon^{ijk}\, \varepsilon^{lmn} \int d^4x\, e^{iq\cdot x} \\
&\quad \langle 0| \Big\{ \mathrm{Tr}\Big[\big(S^{(0)}(x,0)\big)^{jm} \not{z} \big(\partial_0^\mu S^c(0,x)\big)^{kn} \not{z} \\
&\qquad + \big(S(x,0)\big)^{jm} \not{z} \big(\partial_0^\mu S^{c(0)}(0,x)\big)^{kn} \not{z} \Big] \\
&\qquad \cdot \bar{d}^l(0) \not{z}\, d^i(x) \\
&\quad + \text{other terms} \Big\} |0\rangle \,. \qquad (1.4.51)
\end{aligned}
$$

We wish to determine the contribution of this expression to the D=6 quartic quark condensate. The two background quark fields will give rise to operators of dimension $D \geqslant 3$. So we must allow a maximum dimension of three in the terms that we retain in the expansions (1.1.17) of the quark propagators. There are two ways of achieving a total operator dimension of six:

(i) Let the interacting quark propagator contribute a gluon operator of dimension two; i.e. $S(x,y) \rightarrow S^{(2)}(x,y)$ ((1.4.20)). Then the background quark fields must generate an operator of dimension four.

(ii) Let the interacting quark propagator contribute a gluon operator of dimension three; i.e. $S(x,y) \rightarrow S^{(3)}(x,y)$, where ((1.1.17))

$$S^{(3)}(x,y) = \frac{g}{48\pi^2}(D^\nu G_{\nu\mu})^a(0)\, T^a \gamma^\mu \ln(-\Lambda^2 t^2)$$
$$+\frac{g}{12\pi^2}(D_\alpha G_{\nu\mu})^a(0)\, T^a \frac{x^\nu y^\mu (x^\alpha + y^\alpha)\not{t}}{(t^2 - i\varepsilon)^2}$$
$$-\frac{g}{144\pi^2}(D_\alpha G_{\nu\mu})^a(0)\, T^a \frac{1}{t^2 - i\varepsilon}$$
$$\cdot\{(2x^\nu + y^\nu)\gamma^\alpha \not{t}\gamma^\mu - (x^\nu + 2y^\nu)\gamma^\mu \not{t}\gamma^\alpha$$
$$-2t^\alpha t^\mu \gamma^\nu + 3(x^\alpha + y^\alpha)\gamma^\nu \not{t}\gamma^\mu + \eta^{\alpha\nu}\not{t}\gamma^\mu \not{t}\}$$
$$= -S^{c(3)}(x,y). \qquad (1.4.52)$$

Here the background quark fields must give rise to an operator of dimension three.

Thus we may write

$$I^{V(0)}(q,z)\Big|_{Diag.(d)}$$
$$= -z_\mu (\not{z})_{\alpha\beta}\int d^4x\, e^{iq\cdot x}$$
$$\langle 0|\{\bar{d}^k_\alpha(0)\, d^i_\beta(x).\ \mathrm{Tr}[S^{(0)}(x,0)\not{z}\,(\partial^\mu_0 S^{c(2)}(0,x))^{ki}\not{z}$$
$$+S^{(2)}(x,0)\not{z}\,(\partial^\mu_0 S^{c(0)}(0,x))^{ki}\not{z}$$
$$+S^{(0)}(x,0)\not{z}\,(\partial^\mu_0 S^{c(3)}(0,x))^{ki}\not{z}$$
$$+S^{(3)}(x,0)\not{z}\,(\partial^\mu_0 S^{c(0)}(0,x))^{ki}\not{z}]$$
$$+\text{ other terms }\}|0\rangle. \qquad (1.4.53)$$

For case (i) the condensates to be evaluated are of the form

$$\langle 0|\bar{q}^{Ai}_\alpha(0)\; G^{ij}_{\tau\rho}(0)\; q^{Aj}_\beta(x)|0\rangle \qquad (1.4.54a)$$

$$\langle 0|(D_\sigma \bar{q}^A)^i_\alpha(0)\; G^{ij}_{\tau\rho}(0)\; q^{Aj}_\beta(x)|0\rangle \qquad (1.4.54b)$$

(A = u, d), where

$$G_{\tau\rho}(0) \equiv ig\, G^a_{\tau\rho}(0)\, T^a. \qquad (1.4.55)$$

Suppressing the colour indices and expanding the quark field using (1.4.34a), we obtain

$$\begin{aligned}
&\langle 0|\, \bar q^A_\alpha(0)\, G_{\tau\rho}(0)\, q^A_\beta(x)\, |0\rangle \\
&\qquad = \langle 0|\, \bar q^A_\alpha(0)\, G_{\tau\rho}(0)\, q^A_\beta(0)\, |0\rangle \\
&\qquad\quad + x^\sigma \langle 0|\, \bar q^A_\alpha(0)\, G_{\tau\rho}(0)\, D_\sigma(0)\, q^A_\beta(0)\, |0\rangle \\
&\qquad\quad + \text{condensates with } D \geq 7 \,. \qquad (1.4.56)
\end{aligned}$$

The first condensate on the right hand side is of dimension five but does not contribute to the correlators because of the assumption of massless quarks. The computation of the dimension six vacuum expectation value has been described in detail by Pascual and Tarrach [23]. By writing down the most general form consistent with Lorentz covariance they find

$$\begin{aligned}
&\langle 0|\, \bar q^A_\alpha(0)\, G_{\tau\rho}(0)\, D_\sigma(0)\, q^A_\beta(0)\, |0\rangle \\
&= -\frac{g^2}{96} (\sigma_{\tau\rho}\gamma_\sigma)_{\beta\alpha} \langle 0|\, \bar q^A(0)\, \gamma^\mu T^a q^A(0) \sum_{B=u,d,s} \bar q^B(0)\, \gamma_\mu T^a q^B(0)\, |0\rangle , \qquad (1.4.57)
\end{aligned}$$

where the equation of motion (1.1.8c) has been used. With the help of (1.4.37) this becomes

$$\frac{\pi}{54} (\sigma_{\tau\rho}\gamma_\sigma)_{\beta\alpha} \langle 0|\, \sqrt{\alpha_s}\, \bar q q\, |0\rangle^2 \,. \qquad (1.4.58)$$

Expanding the quark field in (1.4.54b) and integrating by parts,

$$\begin{aligned}
&\langle 0|\, (D_\sigma \bar q^A)_\alpha(0)\, G_{\tau\rho}(0)\, q^A_\beta(x)\, |0\rangle \\
&\quad = -\langle 0|\, \bar q^A_\alpha(0)\, D_\sigma(0)\, G_{\tau\rho}(0)\, q^A_\beta(0)\, |0\rangle \\
&\qquad + \text{condensates with } D \geq 7 \,. \qquad (1.4.59)
\end{aligned}$$

Pascual and Tarrach evaluate this condensate too:

$$\begin{aligned}
&\langle 0|\, \bar q^A_\alpha(0)\, D_\sigma(0)\, G_{\tau\rho}(0)\, q^A_\beta(0)\, |0\rangle \\
&= \frac{\pi}{54} \left\{ (\sigma_{\tau\rho}\gamma_\sigma)_{\beta\alpha} + 2i\, (\eta_{\sigma\tau}\gamma_\rho - \eta_{\sigma\rho}\gamma_\tau)_{\beta\alpha} \right\} \\
&\qquad\qquad \langle 0|\, \sqrt{\alpha_s}\, \bar q q\, |0\rangle^2 . \qquad (1.4.60)
\end{aligned}$$

For case (ii) the vacuum condensate $\langle 0|\bar{q}^A_\alpha(0)(D_\sigma G_{\tau\rho})(0)q^A_\beta(x)|0\rangle$ has to be computed. Again we Taylor expand the background quark field:

$$\langle 0|\bar{q}^A_\alpha(0)(D_\sigma G_{\tau\rho})(0)q^A_\beta(x)|0\rangle$$
$$= \langle 0|\bar{q}^A_\alpha(0)(D_\sigma G_{\tau\rho})(0)q^A_\beta(0)|0\rangle$$
$$+ \text{condensates with } D \geqslant 7. \qquad (1.4.61)$$

It is found that [23]

$$\langle 0|\bar{q}^A_\alpha(0)(D_\sigma G_{\tau\rho})(0)q^A_\beta(0)|0\rangle$$
$$= \frac{i\pi}{27}(\eta_{\sigma\tau}\gamma_\rho - \eta_{\sigma\rho}\gamma_\tau)_{\beta\alpha}\langle 0|\sqrt{\alpha_s}\,\bar{q}q|0\rangle^2. \qquad (1.4.62)$$

The expressions (1.4.58), (1.4.60) and (1.4.62) for the required condensates may now be used to complete the computation in the usual manner. We find the contribution of diagrams (d) to the $I^{V(0)}(q,z)$ correlator to be $\frac{2}{243\pi}\frac{(z\cdot q)^4}{q^4}\langle 0|\sqrt{\alpha_s}\,\bar{q}q|0\rangle^2$.

It is important to observe that the 'log' term of $S^{(3)}(x,y)$ does not contribute to the final result as a consequence of the fact that z^μ is a light-like vector. Thus the contribution of diagrams (d) to the $I^{V(0)}(q,z)$ correlator (and indeed to all the correlators that we study) is cut-off independent.

Next we consider the evaluation of diagrams (e). Clearly all quark propagators in the general expression (1.4.50) must be non-interacting. One covariant derivative in each term contributes gluon operators to the condensate while the rest reduce to ordinary derivatives. Thus, in the Schwinger gauge, there will be no contribution from these graphs to the correlators $I^{V(0)}(q,z)$ and $K^{(o)}(q,z)$. The condensates to be evaluated are of the form $\langle 0|\bar{q}^A_\alpha(0)A_\rho(x)q^A_\beta(x)|0\rangle$, where

$$A_\rho(x) \equiv ig\,A^a_\rho(x)\,T^a. \qquad (1.4.63)$$

Recalling the short distance expansions for the quark and gluon background fields, we obtain

$$\langle 0|\, \bar{q}^{A}_{\alpha}(0)\, A_{\rho}(x)\, q^{A}_{\beta}(x)\, |0\rangle$$

$$= \tfrac{1}{2} x^{\tau} \langle 0|\, \bar{q}^{A}_{\alpha}(0)\, G_{\tau\rho}(0)\, q^{A}_{\beta}(0)\, |0\rangle$$

$$+ \tfrac{1}{2} x^{\sigma} x^{\tau} \langle 0|\, \bar{q}^{A}_{\alpha}(0)\, G_{\tau\rho}(0)\, D_{\sigma}(0)\, q^{A}_{\beta}(0)\, |0\rangle$$

$$+ \tfrac{1}{3} x^{\sigma} x^{\tau} \langle 0|\, \bar{q}^{A}_{\alpha}(0)\, (D_{\sigma} G_{\tau\rho})(0)\, q^{A}_{\beta}(0)\, |0\rangle$$

$$+ \text{condensates with } D \geq 7. \tag{1.4.64}$$

The two dimension six condensates were encountered in (1.4.57) and (1.4.62) where their values were determined.

Finally we must include the contribution of the graphs of type (f). In this case the operator of dimension six is generated by performing Taylor expansions of the quark fields in the condensate $\langle 0|\, \bar{q}^{A}_{\alpha}(0)\, q^{A}_{\beta}(x)\, |0\rangle$.
The term of dimension six is [23]

$$\tfrac{1}{6} x^{\rho} x^{\sigma} x^{\tau} \langle 0|\, \bar{q}^{A}_{\alpha}(0)\, D_{\rho}(0)\, D_{\sigma}(0)\, D_{\tau}(0)\, q^{A}_{\beta}(0)\, |0\rangle$$

$$= \frac{i\pi}{2^{3}\, 3^{4}}\, x^{2}\, (\not{x})_{\beta\alpha} \langle 0|\, \sqrt{\alpha_s}\, \bar{q} q\, |0\rangle^{2}. \tag{1.4.65}$$

Using this result it is straightforward to complete the computation of these contributions to the correlators.

The contributions from diagrams (d), (e) and (f) have been verified by repeating the calculations in momentum space.

Diagrams with three quark propagators connecting the vertices at x and o may also give non-vanishing coefficients to the quartic quark condensate. However, such graphs are at least of order $(\alpha_s)^2$ and so represent contributions to the next-to-leading terms of the perturbative expansions of the Wilson functions.

The calculations for the K-correlators and for the I-correlators with $F^{(n)}_{\tau}(x) = A^{(n)}_{\tau}(x)$ are carried out in a similar way and the values of the α_3 and β_3 coefficients are given in Tables 1.5 and 1.6.

We observed earlier that the values of the α and β coefficients for the I-correlators with $F^{(n)}_{\tau}(x) = T^{(n)}_{\tau}(x)$ may be deduced from those for the correlators with $F^{(n)}_{\tau}(x) = V^{(n)}_{\tau}(x)$ and $A^{(n)}_{\tau}(x)$. From (1.2.14) it follows that the sum rules for the $I^{T(n_1,n_2,n_3)}$ correlators must be equivalent to those for the combination $\frac{1}{2}\left(I^{\Phi(n_1,n_3,n_2)} + I^{\Phi(n_2,n_3,n_1)}\right)$.
This is achieved when

$$\beta^{T(n_1,n_2,n_3)} = (-2) \times \frac{1}{2}\left(\beta^{\Phi(n_1,n_3,n_2)} + \beta^{\Phi(n_2,n_3,n_1)}\right). \quad (1.4.66)$$

(See Section 1.5). The β^{T} coefficients are listed in Table 1.7.

We have now completed our calculation of the invariant functions $I^{(n)}(q^2)$ and $K^{(n)}(q^2)$ for the proton. Note that the vacuum condensates of the operators O_1 and O_4 ((1.1.9)) do not appear since we have neglected the effects of the u- and d-quark masses. Apart from the higher dimensional condensates we have also neglected the 'G^3' condensate $\langle 0|O_5|0\rangle$. Like the four-quark condensate it has dimension six. However, the relevant diagrams for the lowest order contributions have two loops while those for the four-quark condensate have only one. Because of this the coefficients have a large relative suppression factor which hopefully is sufficient to make the contribution from the 'G^3' condensate negligible.

Table 1.5 Contributions to $\alpha_3^{(n)}$

(n)	Diag. 1.3(c)	Diag. 1.3(d)	Diag. 1.3(e)	Diag. 1.3(f)	$\alpha_3^{(n)}$
(0,0,0)	108	-36	0	36	108
(1,0,0)	45	0	0	12	57
(0,0,1)	18	-36	0	12	-6
(2,0,0)	63/2	2	4	12	99/2
(0,0,2)	9	-28	4	12	-3
(1,1,0)	9	2	-2	0	9
(1,0,1)	9/2	- 4	-2	0	-3/2

Table 1.6 Contributions to $\beta_3^{(n)}$

(a) $F_\tau^{(n)}(x) = V_\tau^{(n)}(x)$

(n)	Diag. 1.3(c)	Diag. 1.3(d)	Diag. 1.3(e)	Diag. 1.3(f)	$\beta_3^{V(n)}$
(0,0,0)	36	2	0	6	44
(1,0,0)	63/4	1/4	1/4	3	77/4
(0,0,1)	9/2	3/2	-1/2	0	11/2
(2,0,0)	117/10	2/5	4/5	27/10	156/10
(0,0,2)	9/5	4/5	2/5	3/5	18/5
(1,1,0)	27/10	-1/2	-1/10	3/5	27/10
(1,0,1)	27/20	7/20	-9/20	-3/10	19/20

(b) $F_\tau^{(n)}(x) = A_\tau^{(n)}(x)$

(n)	Diag. 1.3(c)	Diag. 1.3(d)	Diag. 1.3(e)	Diag. 1.3(f)	$\beta_3^{A(n)}$
(1,0,0)	-45/4	-7/4	-3/4	-3	-67/4
(2,0,0)	-99/10	-7/5	-2/5	-21/10	-138/10
(1,0,1)	-27/20	-7/20	-7/20	- 9/10	-59/20

Table 1.7 Values of $\beta^{T(n)}$ Coefficients

(n)	$\beta_1^{T(n)}$	$\beta_2^{T(n)}$	$\beta_3^{T(n)}$
(0,0,0)	-2/3	-1/6	-88
(1,0,0)	-5/21	-1/15	-83/2
(0,0,1)	-4/21	-1/30	- 5
(2,0,0)	-3/28	-1/20	-33
(0,0,2)	-1/14	-1/45	-18/5
(1,1,0)	-1/14	-1/90	-39/5
(1,0,1)	-5/84	-1/180	- 7/10

1.5 DERIVATION OF THE QCD SUM RULES

The OPE's calculated in Section 1.4 for the current correlators (1.3.10) and (1.3.11) provide the basis for our analysis. To relate the moments of the proton distribution amplitudes to the parameters of the OPE's it is necessary to postulate alternative forms for the invariant functions $I^{(n)}(q^2)$ and $K^{(n)}(q^2)$. This is done by modelling the spectral densities as follows [6]:

$$\frac{1}{\pi} \operatorname{Im} I^{(n)}(q^2) = r^{(n)} \delta(q^2 - m_N^2) + \theta(q^2 - s^{(n)}) \frac{\beta_1^{(n)}}{160\pi^4} q^2 \tag{1.5.1a}$$

$$\frac{1}{\pi} \operatorname{Im} K^{(n)}(q^2) = k^{(n)} \delta(q^2 - m_N^2) - \theta(q^2 - s^{(n)}) \frac{\alpha_1^{(n)}}{80\pi^4} q^2. \tag{1.5.1b}$$

These expressions correspond to singling out the proton's contribution to the spectral densities (m_N is the nucleon mass) and assuming that the remainder is well approximated by a continuum of states above some threshold $s^{(n)}$. The constants $r^{(n)}$ and $k^{(n)}$ may be expressed as functions of the moments $V^{(n)}$, $A^{(n)}$ and $T^{(n)}$ in the manner shown below. In Section 1.6 we shall consider the dependence of our results on the particular choices (1.5.1).

Dispersion relations are then written down for the invariant functions:

$$(\mathrm{Re})\left\{ \begin{matrix} I^{(n)}(q^2) \\ K^{(n)}(q^2) \end{matrix} \right\} = \frac{1}{\pi} \int_0^\infty ds \, \frac{1}{s - q^2} \operatorname{Im} \left\{ \begin{matrix} I^{(n)}(s) \\ K^{(n)}(s) \end{matrix} \right\}. \tag{1.5.2}$$

These allow us to extract information on the wavefunction moments by using the OPE's calculated in the last section. Using the dispersion relations it is easily checked that the forms of the continuum terms in (1.5.1) are chosen to agree with the known perturbative results. (See (1.3.13) and (1.3.14)).

We now return to the derivation of the constants $r^{(n)}$ and $k^{(n)}$. The vacuum expectation value occurring in the I-correlators may be re-written as

$$\langle 0 | T \ F_\tau^{(n)}(x) \ (J_\lambda^{(1)}(0))^\dagger | 0 \rangle$$

$$= \frac{1}{(2\pi)^3} \sum_\alpha d^4 p_\alpha \ \delta(p_\alpha^2 - m_\alpha^2) \ \theta(p_\alpha^0) \left(\theta(x^0) \, e^{-ip_\alpha \cdot x} + \theta(-x^0) \, e^{ip_\alpha \cdot x} \right)$$

$$\langle 0 | \ F_\tau^{(n)}(0) | \alpha \rangle \langle \alpha | \ (J_\lambda^{(1)}(0))^\dagger | 0 \rangle \, , \qquad (1.5.3)$$

where the sum is over all states with the quantum numbers of the proton. Retaining the contribution of the proton resonance only and using (1.3.7), (1.3.8) and (1.3.9) together with the matrix element

$$\langle 0 | J_\lambda^{(1)}(0) | p \rangle = - f_N \ (z \cdot p)^2 \ N_\lambda(p) \ \left(\tfrac{1}{2} \ \Phi^{(1,0,0)} + T^{(1,0,0)} \right), \qquad (1.5.4)$$

we find

$$I^{(n)}(q, z) \Big|_{\text{PROTON}} = \frac{i}{(2\pi)^3} \ |f_N|^2 \ \tilde{F}^{(n)} \left(\tfrac{1}{2} \ \Phi^{(1,0,0)} + T^{(1,0,0)} \right)$$

$$\cdot \int d^4 p \ \ \delta(p^2 - m_N^2) \ \ \theta(p^0) \ \ (z \cdot p)^{n_1 + n_2 + n_3 + 3}$$

$$\sum_s \bar{N}_s(p) \ \not{z} \ N_s(p)$$

$$\cdot \int d^4 x \ \ e^{iq \cdot x} \left(\theta(x^0) \ e^{-ip \cdot x} + \theta(-x^0) \ e^{ip \cdot x} \right) \qquad (1.5.5)$$

where we sum over the proton spins $s = \uparrow, \downarrow$ and $F^{(n)} = V^{(n)}, A^{(n)}$ or $-2T^{(n)}$ when $F_\tau^{(n)}(x) = V_\tau^{(n)}(x)$, $A_\tau^{(n)}(x)$ or $T_\tau^{(n)}(x)$ respectively. A Gordon decomposition leads to the simplification

$$\bar{N}_\uparrow(p) \ \not{z} \ N_\uparrow(p) = \bar{N}_\downarrow(p) \ \not{z} \ N_\downarrow(p) = 2 \, z \cdot p \, , \qquad (1.5.6)$$

where we have used the fact that the spinors satisfy the normalisation condition

$$\bar{N}_\uparrow(p)\,N_\uparrow(p) = \bar{N}_\downarrow(p)\,N_\downarrow(p) = 2m_N . \tag{1.5.7}$$

Performing the x integration we now have

$$\begin{aligned} I^{(n)}(q,z)\Big|_{\text{PROTON}} = &-4\,|f_N|^2\,\tilde{F}^{(n)}\left(\tfrac{1}{2}\,\Phi^{(1,0,0)} + T^{(1,0,0)}\right) \\ &\cdot \int d^4p\;\delta(p^2-m_N^2)\;\theta(p^0)\;(z\cdot p)^{n_1+n_2+n_3+4} \\ &\left\{ \frac{\delta^{(3)}(\underline{q}-\underline{p})}{(q^0-p^0+i\varepsilon)} - \frac{\delta^{(3)}(\underline{q}+\underline{p})}{(q^0+p^0-i\varepsilon)} \right\}. \end{aligned} \tag{1.5.8}$$

With $q^0 > 0$ the imaginary part of this expression is

$$\begin{aligned} \operatorname{Im} I^{(n)}(q,z)\Big|_{\text{PROTON}} = &\;4\,|f_N|^2\,\pi\,\tilde{F}^{(n)}\left(\tfrac{1}{2}\,\Phi^{(1,0,0)} + T^{(1,0,0)}\right) \\ &(z\cdot q)^{n_1+n_2+n_3+4}\quad \delta(q^2-m_N^2) \end{aligned} \tag{1.5.9}$$

which implies

$$\frac{1}{\pi}\operatorname{Im} I^{(n)}(q^2)\Big|_{\text{PROTON}} = 4\,|f_N|^2\,\tilde{F}^{(n)}\left(\tfrac{1}{2}\,\Phi^{(1,0,0)} + T^{(1,0,0)}\right)\delta(q^2-m_N^2). \tag{1.5.10}$$

Comparing with (1.5.1a) it now follows that

$$r^{(n)} = 4\,|f_N|^2\,\tilde{F}^{(n)}\left(\tfrac{1}{2}\,\Phi^{(1,0,0)} + T^{(1,0,0)}\right). \tag{1.5.11}$$

By similar reasoning we find

$$k^{(n)} = -12\,|f_N|^2\,T^{(n)}. \tag{1.5.12}$$

Thus in order to obtain the moments of the proton distribution amplitudes we need to determine the $r^{(n)}$'s and $k^{(n)}$'s.

By means of the dispersion relations (1.5.2), together with the spectral density models (1.5.1) and the results for the invariant functions $I^{(n)}(q^2)$ and $K^{(n)}(q^2)$, we may relate the moments of the distribution amplitudes to the strong coupling parameter and the vacuum condensates. These are QCD sum rules. We wish to use the relations to predict the properties of the proton. To enhance the contribution of the proton to the right hand side of (1.5.1) we must study low $Q^2 (= -q^2 > 0)$ values. On the other hand the OPE's are good approximations only for large Q^2. To overcome this conflict of interest it is conventional to apply the Borel transformation to both sides of the sum rules. It is defined as follows [4]:

$$\hat{B}_M \equiv \lim_{\substack{Q^2 \to \infty,\ n \to \infty \\ Q^2/n = M^2}} \frac{1}{(n-1)!} (-Q^2)^n \left(\frac{d}{dQ^2}\right)^n, \qquad (1.5.13)$$

where M^2 is fixed. By making a Borel transformation of the sum rules, the relative contribution of the lowest-lying resonance to the dispersion integral is increased while simultaneously the effect of the higher dimensional operators in the OPE is decreased. As the action of the operator $\hat{B}_M$ is to take an infinite number of derivatives all polynomials in q^2 give zero contribution. Neglecting the Q^2 dependence of the strong coupling parameter and evaluating all running quantities at the scale M^2 the sum rules now take the form

$$\begin{aligned} r^{(n)} e^{-m_N^2/M^2} = {} & \frac{\beta_1^{(n)}}{160\pi^4} M^4 \left[1 - (1 + H^{(n)}) e^{-H^{(n)}}\right] \\ & + \frac{\beta_2^{(n)}}{48\pi^2} \langle 0 | \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} | 0 \rangle \\ & + \frac{\beta_3^{(n)}}{243\pi} \frac{1}{M^2} \langle 0 | \sqrt{\alpha_s}\, \bar{q} q | 0 \rangle^2 \end{aligned} \qquad (1.5.14a)$$

$$\begin{aligned} k^{(n)} e^{-m_N^2/M^2} = {} & - \frac{\alpha_1^{(n)}}{80\pi^4} M^4 \left[1 - (1 + H^{(n)}) e^{-H^{(n)}}\right] \\ & - \frac{\alpha_2^{(n)}}{48\pi^2} \langle 0 | \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} | 0 \rangle \\ & - \frac{\alpha_3^{(n)}}{243\pi} \frac{1}{M^2} \langle 0 | \sqrt{\alpha_s}\, \bar{q} q | 0 \rangle^2, \end{aligned} \qquad (1.5.14b)$$

where

$$H^{(n)} = \frac{S^{(n)}}{M^2} \tag{1.5.15}$$

and we have used the results

$$\hat{B}_M \quad \ln q^2 \quad = -1 \tag{1.5.16a}$$

$$\hat{B}_M \quad \left(\frac{1}{Q^2}\right)^p = \frac{1}{(p-1)!} \quad \frac{1}{(M^2)^p} \quad ; \; p > 0 \quad . \tag{1.5.16b}$$

Thus from the correlators in (1.3.10) and (1.3.11) there follow sum rules from which we hope to obtain the $r^{(n)}$'s and $k^{(n)}$'s and hence the corresponding moments of the distribution amplitudes.

1.6 ANALYSIS OF THE QCD SUM RULES

In this section we attempt to use the Borel transformed sum rules (1.5.14) to determine as much as possible about the proton distribution amplitudes V, A and T. We hope that values of the constants $r^{(n)}$ and $k^{(n)}$ (and of the thresholds $s^{(n)}$) may be obtained by fitting the two sides of equations (1.5.14) in the region of M where the non-perturbative terms are less than about 40% and greater than about 10% of the perturbative contributions. It is in this region, where 1.0 $\text{GeV}^2 \lesssim M^2 \lesssim$ 1.5 GeV^2, that the sum rules are expected to be approximately valid.

As indicated earlier, the quantities appearing in the sum rules are taken to be renormalised at a point $\mu^2 \approx 1\ \text{GeV}^2$. Values for the vacuum expectation values $\langle 0|\sqrt{\alpha_s}\,\bar{q}q|0\rangle|_\mu$ and $\langle 0|\frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu}|0\rangle|_\mu$ are required. Unfortunately, at present there is no standard procedure to evaluate these condensates. The quark condensate may be estimated by using PCAC arguments and SU(6) symmetry. The value

$$\langle 0|\bar{q}\,q|0\rangle\Big|_\mu \approx -(0.25\ \text{GeV})^3 \qquad (1.6.1)$$

has been quoted by SVZ [4]. For the coupling parameter we take [3]

$$\alpha_s(\mu^2) = \frac{4\pi}{9\,\ln(\mu^2/\Lambda^2)}\,, \qquad (1.6.2)$$

where Λ is the QCD scale parameter. With Λ = 0.1 GeV this formula yields the value $\alpha_s(1\ \text{GeV}^2) \approx 0.30$. Thus we find

$$\langle 0|\sqrt{\alpha_s}\,\bar{q}\,q|0\rangle^2\Big|_\mu \approx 1.8\times 10^{-4}\ \text{GeV}^6. \qquad (1.6.3)$$

The gluon condensate has been estimated by SVZ [4] using sum rules for charmonium decays and is found to be

$$\langle 0|\frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu}|0\rangle\Big|_\mu \approx 1.2\times 10^{-2}\ \text{GeV}^4. \qquad (1.6.4)$$

We start by studying the sum rules for the K-correlators, from which we hope to determine the moments $T^{(n)}$. For the case (n) = (0,0,0) the sum rule (1.5.14b) becomes

$$-\frac{80\pi^4 k^{(0)}}{M^4} e^{-m_N^2/M^2} + e^{-H^{(0)}}(1+H^{(0)})$$

$$= 1 + \frac{0.066}{M^4} + \frac{0.196}{M^6} \qquad (1.6.5)$$

(M in GeV), where the required α coefficients have been extracted from Tables 1.1, 1.3 and 1.5 and the condensate values (1.6.3) and (1.6.4) have been used. (1.6.5) has been written so that the contributions of the non-perturbative corrections (second and third terms on the right side) may be readily compared to the perturbative term (first term on right). It is clear that for $M^2 \approx 1$ GeV^2 the corrections are of the order of 25%. Note that the $O(M^{-4})$ and $O(M^{-6})$ terms are comparable for $M^2 \approx 1$ GeV^2. This does not imply a breakdown of the expansion at such values of M^2. We must remember that we expect the coefficient of M^{-6} to be anomalously large. This is because the term in M^{-6} comes from one-loop graphs (Fig. 1.3) while the $O(M^{-4})$ contribution enters via two-loop graphs (Fig. 1.2).

To extract the 'best-fit' values of $k^{(0)}$ and $H^{(0)}$ $(= s^{(0)}/M^2)$ from this sum rule we analyse both sides of (1.6.5) in the region of M^2 where the sum of the non-perturbative corrections lies in the range 10%-40% of the perturbative contribution. (It has been checked that the results of the ensuing analysis are largely insensitive to the chosen M^2 window.) It is found that $M^2 = 1.43$ GeV^2 (0.86 GeV^2) when the non-perturbative corrections are 10% (40%). In practice the fit is achieved by minimising the sum of squared differences between the left and right sides of (1.6.5) over the required range of M^2; i.e. we minimise the integral

$$\int_{0.86}^{1.43}\left[1 + \frac{0.066}{(M^2)^2} + \frac{0.196}{(M^2)^3} + \frac{80\pi^4 k^{(0)}}{(M^2)^2} e^{-m_N^2/M^2} - e^{-s^{(0)}/M^2}\left(1 + s^{(0)}/M^2\right)\right]^2 dM^2 \qquad (1.6.6)$$

with respect to the variable parameters $k^{(0)}$ and $s^{(0)}$. The optimal values of these parameters (obtained numerically) are as follows:

$$-80\pi^4 k^{(0)} = 2.46 \ \text{GeV}^4 \qquad (1.6.7a)$$

$$s^{(0)} = 2.69 \ \text{GeV}^2 . \qquad (1.6.7b)$$

Recalling the definition (1.5.12) and the normalisation $T^{(0)} = 1$ ((1.2.15)) we find

$$|f_N| \approx 5.1 \times 10^{-3}\ \mathrm{GeV}^2, \tag{1.6.8}$$

in agreement with Ref 6. The error is dominated by the theoretical contributions which have not been included rather than by the details of the fit itself. It is of the order of 10-15% but it is not possible to control or significantly reduce the theoretical uncertainties without an enormous effort. While it is extremely difficult to check that the neglected non-perturbative corrections to the sum rules are small (condition (ii) of Section 1.3) it is easily confirmed that for this sum rule the contribution of the proton resonance is greater than that of the continuum of states (condition (i)). In fact the latter terms range from $\sim$15% to $\sim$70% of the proton contribution over the range of M^2 in which we are interested.

The value of $s^{(0)}$ in (1.6.7b) is roughly what we would expect for the continuum threshold of the spectral density. Indeed, the best-fit values of this threshold for all the well-behaved sum rules (see below) are found to lie within the range 2.5 $\mathrm{GeV}^2 \lesssim s^{(n)} \lesssim 3.0$ GeV^2.

In order to investigate the dependence of our results on the choice of model expressions for the spectral density we modify (1.5.1b) by introducing to the right side an extra narrow resonance with a mass m_R of about 1.5 GeV. This "effective resonance" contribution corresponds to the experimental spectrum in the isospin $\frac{1}{2}$ channel [6]. If we now define

$$\begin{aligned}\frac{1}{\pi}\,\mathrm{Im}\,K^{(n)}(q^2) = {} & k^{(n)}\,\delta(q^2 - m_N^2) \\ & + k_R^{(n)}\,\delta(q^2 - m_R^2) \\ & - \theta(q^2 - s^{(n)})\ \frac{\alpha_1^{(n)}}{80\pi^4}\ q^2\end{aligned} \tag{1.6.9}$$

the new expression corresponding to (1.6.5) is

$$-\frac{80\pi^4}{M^4}\left(k^{(0)}e^{-m_N^2/M^2}+k_R^{(0)}e^{-m_R^2/M^2}\right)+e^{-H^{(0)}}(1+H^{(0)})$$

$$=1+\frac{0.066}{M^4}+\frac{0.196}{M^6} \quad . \tag{1.6.10}$$

We study this sum rule by varying $s^{(0)}$ within $\approx$15% around the value (1.6.7b) and obtaining optimal values for $k^{(0)}$ and $k_R^{(0)}$. The best-fit values of $k^{(0)}$ (and hence $|f_N|^2$) are found to be relatively stable (varying only by about 4%) whereas the optimal values of $k_R^{(0)}$ range from about −0.9 (at $s^{(0)}$ = 2.3 GeV^2) to about +0.9 (at $s^{(0)}$ = 3.1 GeV^2.) (Of course from our previous analysis we know that $k_R^{(0)}$ must vanish when $s^{(0)}$ = 2.69 GeV^2.) This indicates that the sum rule is satisfied with a large contribution from the proton and is insensitive to the contribution of the effective resonance. Clearly this is a desirable feature for our analysis and gives us confidence in our estimate of the decay constant $|f_N|$.

Similar analyses are possible for the sum rules derived from the K-correlators with (n) = (1,0,0), (2,0,0) and (1,1,0). These sum rules may also be satisfied with a large contribution from the proton. For the corresponding moments we find

$$T^{(1,0,0)} \approx 0.35 - 0.40 \tag{1.6.11a}$$

$$T^{(2,0,0)} \approx 0.18 - 0.22 \tag{1.6.11b}$$

$$T^{(1,1,0)} \approx 0.09 - 0.10 \tag{1.6.11c}$$

Because the sum rules determine the optimal values of the parameters $k^{(n)} \sim T^{(n)} |f_N|^2$, and there is an uncertainty in our estimate of $|f_N|$, the corresponding uncertainties in the moments are typically about 40% larger than that for $|f_N|$.

Let us now try to understand why it is possible to satisfy the sum rules studied above. Looking again at (1.6.5) we see that the right side decreases with increasing M^2 while the second term on the left is an increasing function of M^2. For a fit, therefore, $k^{(0)}$ must be negative, implying a positive value of $|f_N|^2$, as required. When the optimal values of the fitting parameters are inserted into (1.6.5) the left and right sides assume almost identical values over the entire range of M^2 in which we are interested. This feature is reflected in the particularly small value of the integral (1.6.6):

$$(1.6.6)\Big|_{\substack{s^{(0)}=2.69\ \mathrm{GeV}^2 \\ -80\pi^4 k^{(0)}=2.46\ \mathrm{GeV}^4}} \approx 3\times10^{-5}\ \mathrm{GeV}^2. \tag{1.6.12}$$

In general, for sum rules showing similar trends when M^2 is varied, it is possible to obtain excellent fits with positive moments and thresholds in the range 2.5 $\mathrm{GeV}^2 \lesssim s^{(n)} \lesssim 3.0\ \mathrm{GeV}^2$. However, an inspection of Tables 1.1, 1.3 and 1.5 reveals that the α coefficients with (n) = (0,0,1), (0,0,2) and (1,0,1) are such that the right sides of the corresponding sum rules are increasing functions of M^2. In particular it is the negative $\alpha_3^{(n)}$ coefficients which cause this problem. One might still expect to obtain a reasonably good fit. Unfortunately, this is possible only when the continuum thresholds $s^{(n)}$ fall below about 1 GeV^2. If, instead, we try to fit the sum rules by constraining the threshold parameters $s^{(n)}$ to lie between 2.5 GeV^2 and 3.0 GeV^2, the squared difference integrals have comparatively large values of approximately $10^{-3}\ \mathrm{GeV}^2$. This indicates that the curves representing the left and right sides of the sum rules as functions of M^2 have significantly different shapes. In fact, the right sides increase with M^2 much faster than the left sides.

Despite the fact that the sum rules for the moments $T^{(0,0,1)}$, $T^{(0,0,2)}$ and $T^{(1,0,1)}$ are not satisfied very well without modifying the spectral densities we now give the results for the best fits obtained by including only the proton resonance (together with the continuum) and letting the parameters $s^{(n)}$ and $k^{(n)}$ assume their optimal values. They are

$$T^{(0,0,1)} \approx 0.16 \tag{1.6.13a}$$

$$T^{(0,0,2)} \approx 0.07 \tag{1.6.13b}$$

$$T^{(1,0,1)} \approx 0.05, \tag{1.6.13c}$$

which approximately satisfy the momentum conservation relation (see Section 1.4(a))

$$2T^{(1,0,1)} + T^{(0,0,2)} = T^{(0,0,1)} \tag{1.6.14}$$

As we have seen, the inclusion of an effective resonance at $m_R \approx$ 1.5 GeV in the spectral densities generates a contribution, proportional to $e^{-m_R^2/M^2}$, to the left side of the sum rules. This term increases more rapidly with M^2 in the range of interest and good fits are obtained for the sum rules with (n) = (0,0,1), (0,0,2) and (1,0,1). Not surprisingly, however, the fits are insensitive to the proton's contribution and the values in (1.6.13) are reduced significantly, although now it is not possible to determine these moments very precisely. If we interpret this by concluding that the values in (1.6.13) should be taken as upper limits for these moments then the momentum conservation relation

$$T^{(n_1+1,n_2,n_3)} + T^{(n_1,n_2+1,n_3)} + T^{(n_1,n_2,n_3+1)} = T^{(n_1,n_2,n_3)} \tag{1.6.15}$$

implies that $T^{(1,0,0)}$ and $T^{(2,0,0)} + T^{(1,1,0)}$ should both be increased from the values given in (1.6.11) to the following:

$$T^{(1,0,0)} \approx 0.42 \tag{1.6.16a}$$

$$T^{(2,0,0)} + T^{(1,1,0)} \approx 0.37 \tag{1.6.16b}$$

We now turn to the sum rules for the moments $V^{(n)}$ and $\Phi^{(n)}$ (= $V^{(n)} - A^{(n)}$) obtained using the current $J^{(1)}$ in the correlators. Looking at Tables 1.2, 1.4 and 1.6 it is apparent that for all (n) the $\beta_3^{V(n)}$ coefficients are of the same sign as the corresponding $\beta_1^{V(n)}$ and (non-vanishing) $\beta_2^{V(n)}$ coefficients. The same is also true for all $\beta^{\Phi(n)}$ coefficients except those for (n) = (0,1,1.) This means that when the sum rules are written in the form of (1.6.5) (so that the non-perturbative corrections may be easily compared to the perturbative contributions) the right sides, except for the case just mentioned, all decrease with increasing M^2. As we saw in our study of the K-correlators this behaviour leads to sum rules which are satisfied with a large contribution from the proton. We obtain best-fit values for the $r^{(n)}$ and $s^{(n)}$ (see (1.5.14a)) and investigate their stability when an effective resonance at about 1.5 GeV is included in the spectral density. The results are listed in Table 1.8.

The uncertainties in the moments may be reduced by ensuring that the momentum conservation relations are not violated. Indeed, we may use these constraints to help us to write down definite predictions for the moments $V^{(n)}$ and $\Phi^{(n)}$. These values are given in Table 1.9. Also included are our results for the moments $A^{(n)}$ and $T^{(n)}$ obtained from the $V^{(n)}$ and $\Phi^{(n)}$ by using the relations

$$A^{(n)} = V^{(n)} - \Phi^{(n)} \tag{1.6.17}$$

and (1.2.14.) The moments of the distribution amplitude T(x) are seen to be in fairly good agreement with those obtained from our analysis of the K-correlators. Thus the study of the K-correlators provides a useful check on the consistency of the QCD sum rule method.

Table 1.8 Estimates of Moments Extracted From Sum Rules

(n)	$V^{(n)}$	$\Phi^{(n)}$
(0,0,0)	1	1
(1,0,0)	0.34 - 0.42	0.46 - 0.59
(0,1,0)	0.34 - 0.42	0.18 - 0.21
(0,0,1)	0.22 - 0.26	0.22 - 0.26
(2,0,0)	0.18 - 0.24	0.27 - 0.37
(0,2,0)	0.18 - 0.24	0.08 - 0.09
(0,0,2)	0.10 - 0.12	0.10 - 0.12
(1,1,0)	0.08 - 0.10	0.08 - 0.10
(1,0,1)	0.06 - 0.07	0.09 - 0.11
(0,1,1)	0.06 - 0.07	Unreliable

Table 1.9 Proposed Moments of Proton Distribution Amplitudes

(n)	$V^{(n)}$	$\Phi^{(n)}$	$A^{(n)}$	$T^{(n)}$
(0,0,0)	1	1	0	1
(1,0,0)	0.38	0.55	-0.17	0.395
(0,1,0)	0.38	0.21	0.17	0.395
(0,0,1)	0.24	0.24	0	0.21
(2,0,0)	0.22	0.35	-0.13	0.235
(0,2,0)	0.22	0.09	0.13	0.235
(0,0,2)	0.12	0.12	0	0.09
(1,1,0)	0.10	0.10	0	0.10
(1,0,1)	0.06	0.10	-0.04	0.06
(0,1,1)	0.06	0.02	0.04	0.06

1.7 DISCUSSION OF RESULTS

In this section we attempt to reconcile the values obtained for some of the moments of the proton distribution amplitudes in Section 1.6 with the results of other authors. We shall mainly be concerned with comparing our work to that of Chernyak and Zhitnitsky [6] although other studies are also mentioned.

We start with a comparison of the conclusions of this chapter with those of Ref. 6. By inspection of the α- and β- coefficients we see that different results are obtained when the OPE's are computed in the spacelike region for the correlators (1.3.10) and (1.3.11). The main features of the comparison may be summarised as follows:

(i) Our evaluation of the leading perturbative contributions to the OPE's is in complete agreement with Ref. 6, i.e. the same values are obtained for the $\alpha_1^{(n)}$ and $\beta_1^{(n)}$ coefficients.

(ii) The coefficient functions of the gluon condensate differ. Note, however, that if we take the linear combinations

1. Diag. 1.2(b) - 4. Diag. 1.2(c) + 4. Diag. 1.2(e)

(1.7.1)

in Tables 1.3 and 1.4 then we get the results of CZ for the $\alpha_2^{(n)}$ and $\beta_2^{(n)}$ (on the assumption that the $\alpha_2^{(1,1,0)}$ and $\alpha_2^{(1,0,1)}$ coefficients in Table 4 of Ref. 6 should have the opposite sign so that the momentum conservation relations hold).

(iii) The coefficient functions of the quark condensate are different. We cannot see a relation analogous to (1.7.1) which would help to resolve the discrepancy. Indeed, from our calculations we cannot understand how a factor of 5 can appear in Ref. 6 in the denominator for the $\beta_3^{(n)}$ with $n_1+n_2+n_3 = 1$ and the $\alpha_3^{(n)}$ with $n_1+n_2+n_3 = 2$. Although we disagree with CZ about the magnitudes of the $\alpha_3^{(n)}$ and $\beta_3^{(n)}$ coefficients, the signs agree.

The observations on signs are particularly significant. We have seen that if $\alpha_3^{(n)}$ ($\beta_3^{(n)}$) has the same sign as $\alpha_1^{(n)}$ ($\beta_1^{(n)}$) then, for the sort of values which arise in these calculations, it is possible to satisfy the sum rule (1.5.14) in the range where the non-perturbative contribution is approximately between 10% and 40% of the perturbative one. In particular, the fact that we obtain positive values for all $\beta_3^{V(n)}$ means that the simplest form of the sum rule (i.e. using duality to model all the contributions, except that of the proton, by a continuum) is satisfied. Because the signs of the $\beta_1^{V(n)}$ and $\beta_3^{V(n)}$ coefficients are the same a reliable determination of the moments $V^{(n)}$ is possible. This is also the case for all moments of the function $\Phi(x_i)$, except that with (n) = (0,1,1.) The moments of Table 1.9 may be compared with those of the asymptotic form (1.2.8) of the distribution amplitude:

$$\Phi_{AS}^{(1,0,0)} = 0.33 \qquad (1.7.2a)$$

$$\Phi_{AS}^{(2,0,0)} = 0.14 \qquad (1.7.2b)$$

$$\Phi_{AS}^{(1,1,0)} = 0.10 \qquad (1.7.2c)$$

The asymptotic form corresponds to a completely symmetric distribution of momentum, with each quark carrying one third of the total proton longitudinal momentum. By contrast, if, following CZ, we choose $\Phi(x_i)$ to be the proton distribution amplitude (see Section 1.2) we see that the momentum is not distributed equally among the constituent quarks. About 55% of the proton's longitudinal momentum (in the $p_z \to \infty$ frame) is carried by one u-quark with the same helicity as the proton.

CZ propose the following as models for the functions $V(x_i)$ and $\Phi(x_i)$:

$$V_{CZ}(x_1,x_2,x_3) = 120\, x_1 x_2 x_3 \cdot \left[11.35\,(x_1^2+x_2^2) + 8.82\, x_3^2 - 1.68\, x_3 - 2.94\right] \qquad (1.7.3a)$$

$$\Phi_{CZ}(x_1,x_2,x_3) = 120 x_1 x_2 x_3 \cdot [18.07\, x_1^2 + 4.63\, x_2^2 + 8.82\, x_3^2 - 1.68\, x_3 - 2.94] \qquad (1.7.3b)$$

The moments of these functions are given in Table 3 of Ref. 6. We note that the predicted asymmetry among the quark momenta is greater than that implied by our results. Nevertheless, since the moments of the functions (1.7.3) lie within or close to the ranges determined by the sum rule analysis of this chapter (see Table 1.8) we feel justified in using the distribution amplitude (1.7.3b) for the proton decay calculation of Chapter 2.

Our results using $J^{(0)}$ as the correlator current are also qualitatively similar (although numerically different) to those in Ref. 6.

As was the case there, we find that some of the sum rules can be satisfied and information about the corresponding moments of the distribution amplitude T can be obtained. The values we extracted ((1.6.11)) may be compared with those of the model wavefunction proposed by CZ:

$$T(x_1,x_2,x_3) = 120 x_1 x_2 x_3 \, [13.44\,(x_1^2+x_2^2) + 4.62\, x_3^2 + 0.84\, x_3 - 3.78], \qquad (1.7.4)$$

for which $T^{(1,0,0)}$, $T^{(2,0,0)}$ and $T^{(1,1,0)}$ are 0.425, 0.26 and 0.10 respectively. We saw in Section 1.6 how the inclusion of an effective resonance in the spectral density leads to an improved fit of the sum rules for the moments $T^{(0,0,1)}$, $T^{(0,0,2)}$ and $T^{(1,0,1)}$. Assuming that this determination of the moments is reliable, momentum conservation then implies that $T^{(1,0,0)}$ and $T^{(2,0,0)} + T^{(1,1,0)}$ should both be increased to the values given in (1.6.16). These values are in reasonably good agreement with the results of Ref. 6.

Despite the encouraging results obtained using a QCD sum rule treatment of the current correlators it must be remembered that this method of analysis involves several approximations. Among the more obvious ones are:-

(i) Neglect of Higher Order Perturbative Corrections.

Perturbative corrections to hard scattering processes vary considerably, e.g. the corrections to R in e^+e^- annihilation are about 10% whereas for the Drell-Yan process they are over 100%. Recently Gorskii [26] has calculated the effects of radiative corrections to the sum rules used in determining the moments of the pion wavefunction. He concludes that the corrections are inessential for an evaluation of low moments. In the proton case, although there is no reason to expect that the corrections will be particularly large, it is possible that they will be significant.

(ii) Neglect of Operators of Higher Dimension

It is difficult to estimate what effect the inclusion of further condensates will have.

(iii) Vacuum Saturation of the Four Quark Condensate

Since the magnitude and sign of the coefficient of the four quark condensate play such an important role, one might worry whether the assumption of vacuum saturation of this condensate is correct, and to what extent this affects the present analysis.

On the other hand it should be said that such worries about the approximations would also apply to many other quantities for which nevertheless the sum rules work very effectively, leading to impressive agreement with experiment.

An independent study of the moments of the proton distribution amplitudes has been reported by Lavelle [27]. This author analyses light cone sum rules for vertex functions involving baryon-meson couplings and obtains the estimates

$$A^{(1,0,0)} = -0.18 \qquad (1.7.5a)$$

$$2V^{(2,0,0)} + V^{(0,0,2)} - 2A^{(2,0,0)} = 0.70, \qquad (1.7.5b)$$

which are to be compared with the values -0.17 and 0.82 obtained for these quantities from Table 1.9.

Estimates of the moments are also being sought by means of lattice calculations. However, at present, such studies are in the embryonic stage [28] and it may be some time before accurate results are available.

The sum rule analysis of this chapter indicates an asymmetry in the distribution of momentum among the constituent quarks of the proton at the typical hadronic scale $\bar{\mu} \sim 1$ GeV, although the asymmetry may not be as great as that predicted by the wavefunction of Ref. 6. Elsewhere in the literature [14, 29] there is increasing evidence that an asymmetric proton wavefunction may be required to correctly describe hard exclusive processes at available Q^2.

Note added:
Subsequent correspondence with Drs. Chernyak and Zhitnitsky has established agreement with the results presented above. We are grateful to these authors for pointing out a mistake in our original preprint.

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APPENDIX

COMMENTS ON DISTRIBUTION AMPLITUDES AND CORRELATORS FOR OTHER BARYONS

In Ref. 6 an attempt was made to derive a nucleon distribution amplitude. In Chapter 2 of this thesis the result is used to estimate the amplitude for the decay $p \to \pi^0 e^+$. To use the same method to estimate the decay rates of the proton into other possible decay products (as predicted by the SU(5) GUT for example) would require distribution amplitudes for the Σ^+, Σ^0 and Λ hyperons. Here we define distribution amplitudes and correlators for these $J^P = \frac{1}{2}^+$ baryons and derive some of their properties.

A treatment analogous to that given in Section 1.2 for the proton is also valid for the Σ^+ hyperon. The d-quark of the proton is replaced everywhere by the s-quark of the Σ^+. However, although the matrix element of the orthogonal SU(3) decuplet state must vanish as in (1.2.12), this is not for reasons of isospin.

Corresponding V, A and T functions may also be defined for the Σ^0 and Λ hyperons, where the quarks are of three different flavours. To deduce constraints on these functions we shall make use of the formalism introduced by Brodsky and Lepage [1]. The matrix element of the leading twist piece of the tri-local operator for Σ^0 is written

$$
\begin{aligned}
&\varepsilon^{ijk} \langle 0 | u^i_\alpha(z_1)\, d^j_\beta(z_2)\, s^k_\gamma(z_3) | \Sigma^0 \rangle \\
&= \frac{f_{\Sigma^0}}{4} \Big\{ (\not{p}\, C)_{\alpha\beta} (\gamma_5 N_{\Sigma^0})_\gamma V_{\Sigma^0}(z_i \cdot p) \\
&\qquad + (\not{p}\, \gamma_5 C)_{\alpha\beta} (N_{\Sigma^0})_\gamma A_{\Sigma^0}(z_i \cdot p) \\
&\qquad + i (\sigma_{\mu\nu} p^\nu C)_{\alpha\beta} (\gamma^\mu \gamma_5 N_{\Sigma^0})_\gamma T_{\Sigma^0}(z_i \cdot p) \Big\} .
\end{aligned}
\tag{A.1}
$$

As for the proton, this may be re-expressed as

$$|\Sigma^0_\uparrow\rangle = K_{\Sigma^0}\int_0^1 [dx]\Big\{\tfrac{1}{2}[V_{\Sigma^0}(x_i) - A_{\Sigma^0}(x_i)]\,|u_\uparrow(x_1)\,d_\downarrow(x_2)\,s_\uparrow(x_3)\rangle$$

$$+\tfrac{1}{2}[V_{\Sigma^0}(x_i)+A_{\Sigma^0}(x_i)]\,|u_\downarrow(x_1)\,d_\uparrow(x_2)\,s_\uparrow(x_3)\rangle$$

$$- T_{\Sigma^0}(x_i)\,|u_\uparrow(x_1)\,d_\uparrow(x_2)\,s_\downarrow(x_3)\rangle\Big\}\,. \qquad \text{(A.2)}$$

Similar equations hold for Λ. The requirement that the orthogonal SU(3) decuplet and singlet states give vanishing matrix elements leads to the conditions

$$\varepsilon^{ijk}\langle 0|\, u^i_\alpha(z_1)\, d^j_\beta(z_2)\, s^k_\gamma(z_3) + d^i_\beta(z_2)\, s^j_\gamma(z_3)\, u^k_\alpha(z_1)$$

$$+ s^i_\gamma(z_3)\, u^j_\alpha(z_1)\, d^k_\beta(z_2)\,|\Sigma^0,\Lambda\rangle = 0 \qquad \text{(A.3a)}$$

and

$$\varepsilon^{ijk}\langle 0|\, u^i_\alpha(z_1)\, s^j_\gamma(z_3)\, d^k_\beta(z_2) + s^i_\gamma(z_3)\, d^j_\beta(z_2)\, u^k_\alpha(z_1)$$

$$+ d^i_\beta(z_2)\, u^j_\alpha(z_1)\, s^k_\gamma(z_3)\,|\Sigma^0,\Lambda\rangle = 0 \qquad \text{(A.3b)}$$

Using (A.1) these reduce to

$$V_{\Sigma^0,\Lambda}(x_2,x_3,x_1) - A_{\Sigma^0,\Lambda}(x_2,x_3,x_1) + V_{\Sigma^0,\Lambda}(x_3,x_1,x_2) + A_{\Sigma^0,\Lambda}(x_3,x_1,x_2)$$

$$= 2\, T_{\Sigma^0,\Lambda}(x_1,x_2,x_3) \qquad \text{(A.4a)}$$

and

$$V_{\Sigma^0,\Lambda}(x_1,x_3,x_2) - A_{\Sigma^0,\Lambda}(x_1,x_3,x_2) + V_{\Sigma^0,\Lambda}(x_3,x_2,x_1) + A_{\Sigma^0,\Lambda}(x_3,x_2,x_1)$$

$$= 2\, T_{\Sigma^0,\Lambda}(x_2,x_1,x_3) \qquad \text{(A.4b)}$$

respectively. To make further progress we introduce the Brodsky and Lepage definitions of distribution amplitudes for these baryons:

$$\phi^{\uparrow}_{\Sigma^0}(x_i) = \Big\{\frac{1}{\sqrt{12}}\big[(u_\uparrow(1)\,d_\downarrow(2) + d_\uparrow(1)\,u_\downarrow(2))\,s_\uparrow(3)$$
$$+ s_\uparrow(1)\,(u_\downarrow(2)\,d_\uparrow(3) + d_\downarrow(2)\,u_\uparrow(3))$$
$$-2\,(u_\uparrow(1)\,s_\downarrow(2)\,d_\uparrow(3) + d_\uparrow(1)\,s_\downarrow(2)\,u_\uparrow(3))\big]\,\phi^{s}_{\Sigma^0}(x_i)$$
$$+\frac{1}{2}\big[(u_\uparrow(1)\,d_\downarrow(2) + d_\uparrow(1)\,u_\downarrow(2))\,s_\uparrow(3)$$
$$- s_\uparrow(1)\,(u_\downarrow(2)\,d_\uparrow(3) + d_\downarrow(2)\,u_\uparrow(3))\big]\,\phi^{a}_{\Sigma^0}(x_i)\Big\}$$
$$+ (1 \leftrightarrow 2) \quad + \quad (2 \leftrightarrow 3) \tag{A.5}$$

$$\phi^{\uparrow}_{\Lambda}(x_i) = \Big\{\frac{1}{2}\big[s_\uparrow(1)(u_\downarrow(2)\,d_\uparrow(3) - d_\downarrow(2)\,u_\uparrow(3))$$
$$+(d_\uparrow(1)\,u_\downarrow(2) - u_\uparrow(1)\,d_\downarrow(2))\,s_\uparrow(3)\big]\,\phi^{s}_{\Lambda}(x_i)$$
$$+\frac{1}{\sqrt{12}}\big[s_\uparrow(1)(u_\downarrow(2)\,d_\uparrow(3) - d_\downarrow(2)\,u_\uparrow(3))$$
$$-(d_\uparrow(1)\,u_\downarrow(2) - u_\uparrow(1)\,d_\downarrow(2))\,s_\uparrow(3)$$
$$+2\,(u_\uparrow(1)\,s_\downarrow(2)\,d_\uparrow(3) - d_\uparrow(1)\,s_\downarrow(2)\,u_\uparrow(3))\big]\,\phi^{a}_{\Lambda}(x_i)\Big\}$$
$$+ (1 \leftrightarrow 2) \quad + \quad (2 \leftrightarrow 3)\ . \tag{A.6}$$

By using the relevant pieces of these wavefunctions and (A.2) we find

$$\frac{1}{2}[V_{\Sigma^0}(x_i) - A_{\Sigma^0}(x_i)] = \sigma_{\Sigma^0}\Big[\frac{1}{\sqrt{6}}\,\phi^{s}_{\Sigma^0}(x_1,x_2,x_3) + \frac{1}{\sqrt{2}}\,\phi^{a}_{\Sigma^0}(x_1,x_2,x_3)\Big] \tag{A.7a}$$

$$\frac{1}{2}[V_{\Sigma^0}(x_i) + A_{\Sigma^0}(x_i)] = \sigma_{\Sigma^0}\Big[\frac{1}{\sqrt{6}}\,\phi^{s}_{\Sigma^0}(x_2,x_1,x_3) + \frac{1}{\sqrt{2}}\,\phi^{a}_{\Sigma^0}(x_2,x_1,x_3)\Big] \tag{A.7b}$$

$$T_{\Sigma^0}(x_i) = \sigma_{\Sigma^0}\sqrt{\frac{2}{3}}\,\phi^{s}_{\Sigma^0}(x_1,x_3,x_2) \tag{A.7c}$$

$$\frac{1}{2}[V_{\Lambda}(x_i) - A_{\Lambda}(x_i)] = \sigma_{\Lambda}\Big[-\frac{1}{\sqrt{2}}\,\phi^{s}_{\Lambda}(x_1,x_2,x_3) + \frac{1}{\sqrt{6}}\,\phi^{a}_{\Lambda}(x_1,x_2,x_3)\Big] \tag{A.8a}$$

$$\frac{1}{2}[V_{\Lambda}(x_i) + A_{\Lambda}(x_i)] = \sigma_{\Lambda}\Big[\frac{1}{\sqrt{2}}\,\phi^{s}_{\Lambda}(x_2,x_1,x_3) - \frac{1}{\sqrt{6}}\,\phi^{a}_{\Lambda}(x_2,x_1,x_3)\Big] \tag{A.8b}$$

$$T_{\Lambda}(x_i) = -\sigma_{\Lambda}\sqrt{\frac{2}{3}}\,\phi^{a}_{\Lambda}(x_1,x_3,x_2)\ . \tag{A.8c}$$

From (A.4), (A.7) and (A.8) it is now clear that the functions $V_{\Sigma^\circ,\Lambda}$, $A_{\Sigma^\circ,\Lambda}$ and $T_{\Sigma^\circ,\Lambda}$ satisfy the following constraints:

$$V_{\Sigma^\circ}(x_1,x_2,x_3) = V_{\Sigma^\circ}(x_2,x_1,x_3) \tag{A.9a}$$

$$A_{\Sigma^\circ}(x_1,x_2,x_3) = -A_{\Sigma^\circ}(x_2,x_1,x_3) \tag{A.9b}$$

$$T_{\Sigma^\circ}(x_1,x_2,x_3) = T_{\Sigma^\circ}(x_2,x_1,x_3) \tag{A.9c}$$

$$2\,T_{\Sigma^\circ}(x_1,x_2,x_3) = \Phi_{\Sigma^\circ}(x_2,x_3,x_1) + \Phi_{\Sigma^\circ}(x_1,x_3,x_2) \tag{A.9d}$$

$$V_\Lambda(x_1,x_2,x_3) = -V_\Lambda(x_2,x_1,x_3) \tag{A.10a}$$

$$A_\Lambda(x_1,x_2,x_3) = A_\Lambda(x_2,x_1,x_3) \tag{A.10b}$$

$$T_\Lambda(x_1,x_2,x_3) = -T_\Lambda(x_2,x_1,x_3) \tag{A.10c}$$

$$2\,T_\Lambda(x_1,x_2,x_3) = \Phi_\Lambda(x_2,x_3,x_1) - \Phi_\Lambda(x_1,x_3,x_2)\,. \tag{A.10d}$$

$\Phi_{\Sigma^\circ,\Lambda}$ are defined by analogy with (1.2.13b) and may be chosen as the independent distribution amplitudes of the Σ^0 and Λ hyperons. The relations (A.9) and (A.10) may readily be translated to relations among the moments of the distribution amplitudes. As a final check on our reasoning, we note that the vanishing of the asymptotic forms of A_{Σ°, V_Λ and T_Λ is confirmed by the reduction of the flavour-spin structures of $|\Sigma^\circ_\uparrow\rangle$ (equation (A.2)) and $|\Lambda_\uparrow\rangle$ to those of the SU(6) model.

Suitable hyperon currents to be included in the correlators may be obtained from (1.3.2) by use of SU(3) symmetry:

$$\begin{aligned} J^{(\hat{n}_1)}_{\Sigma^+,\sigma}(x) = \varepsilon^{ijk}\Big\{&[((iz.D)^{\hat{n}_1}u(x))^i\, C\not{z}\, u^j(x)]\,(\gamma_5\, s^k(x))_\sigma \\ &-[((iz.D)^{\hat{n}_1}u(x))^i\, C\not{z}\, s^j(x)]\,(\gamma_5\, u^k(x))_\sigma\Big\} \end{aligned} \tag{A.11}$$

$$\begin{aligned} J^{(\hat{n}_1)}_{\Sigma^\circ,\sigma}(x) = \tfrac{1}{\sqrt{2}}\,\varepsilon^{ijk}\Big\{&[((iz.D)^{\hat{n}_1}u(x))^i\, C\not{z}\, d^j(x)]\,(\gamma_5\, s^k(x))_\sigma \\ &+[((iz.D)^{\hat{n}_1}d(x))^i\, C\not{z}\, u^j(x)]\,(\gamma_5\, s^k(x))_\sigma \\ &-[((iz.D)^{\hat{n}_1}u(x))^i\, C\not{z}\, s^j(x)]\,(\gamma_5\, d^k(x))_\sigma \\ &-[((iz.D)^{\hat{n}_1}d(x))^i\, C\not{z}\, s^j(x)]\,(\gamma_5\, u^k(x))_\sigma\Big\} \end{aligned} \tag{A.12}$$

$$J^{(\hat{n}_1)}_{\Lambda,\sigma}(x) = \frac{1}{\sqrt{6}}\,\varepsilon^{ijk}\Big\{-\big[((iz\cdot D)^{\hat{n}_1}u(x))^i\, C\,\not{z}\, d^j(x)\big](\gamma_5\, s^k(x))_\sigma$$
$$+\big[((iz\cdot D)^{\hat{n}_1}u(x))^i\, C\,\not{z}\, s^j(x)\big](\gamma_5\, d^k(x))_\sigma$$
$$+\big[((iz\cdot D)^{\hat{n}_1}d(x))^i\, C\,\not{z}\, u^j(x)\big](\gamma_5\, s^k(x))_\sigma$$
$$-\big[((iz\cdot D)^{\hat{n}_1}d(x))^i\, C\,\not{z}\, s^j(x)\big](\gamma_5\, u^k(x))_\sigma$$
$$+2\big[((iz\cdot D)^{\hat{n}_1}s(x))^i\, C\,\not{z}\, u^j(x)\big](\gamma_5\, d^k(x))_\sigma$$
$$-2\big[((iz\cdot D)^{\hat{n}_1}s(x))^i\, C\,\not{z}\, d^j(x)\big](\gamma_5\, u^k(x))_\sigma\Big\}.$$

(A.13)

The current $J^{(\hat{n}_1)}_{\Sigma^0,\sigma}(x)$ has isospin I=1, and I=0 for $J^{(\hat{n}_1)}_{\Lambda,\sigma}(x)$, as required.

The current correlators for the Σ^+, Σ^0 and Λ hyperons may be defined as in (1.3.10) and (1.3.11). Definitions similar to (1.3.3), (1.3.8) and (1.3.9) are appropriate for $V^{(n)}_{\Sigma^+,\tau}(x)$, $A^{(n)}_{\Sigma^+,\tau}(x)$ and $T^{(n)}_{\Sigma^+,\tau}(x)$, with s-quarks replacing d-quarks. It is convenient to choose $V^{(n)}_{\Sigma^0,\tau}(x) = V^{(n)}_{\Lambda,\tau}(x) \equiv V^{(n)}_{0,\tau}(x)$ (with similar relations for the A and T functions). We define

$$V^{(n)}_{0,\tau}(x) = \varepsilon^{ijk}\big[((iz\cdot D)^{n_1}u(x))^i\, C\,\not{z}\,((iz\cdot D)^{n_2}d(x))^j\big]\big[(iz\cdot D)^{n_3}(\gamma_5\, s(x))_\tau\big]^k \tag{A.14a}$$

$$A^{(n)}_{0,\tau}(x) = \varepsilon^{ijk}\big[((iz\cdot D)^{n_1}u(x))^i\, C\,\not{z}\,\gamma_5\,((iz\cdot D)^{n_2}d(x))^j\big]\big[(iz\cdot D)^{n_3}s_\tau(x)\big]^k \tag{A.14b}$$

$$T^{(n)}_{0,\tau}(x) = \varepsilon^{ijk}\big[((iz\cdot D)^{n_1}u(x))^i\, C(-i\sigma_{\mu\nu})z^\nu((iz\cdot D)^{n_2}d(x))^j\big]\big[(iz\cdot D)^{n_3}(\gamma^\mu\gamma_5\, s(x))_\tau\big]^k \tag{A.14c}$$

and use equation (A.1) to determine the matrix elements:

$$\langle 0|\,V^{(n)}_{0,\tau}(x)\,|\Sigma^0(\Lambda)\rangle = -f_{\Sigma^0(\Lambda)}(z\cdot p)^{n_1+n_2+n_3+1}N_{\Sigma^0(\Lambda),\tau}\cdot V^{(n)}_{\Sigma^0(\Lambda)}\,e^{-ip.x} \tag{A.15a}$$

$$\langle 0|\,A^{(n)}_{0,\tau}(x)\,|\Sigma^0(\Lambda)\rangle = -f_{\Sigma^0(\Lambda)}(z\cdot p)^{n_1+n_2+n_3+1}N_{\Sigma^0(\Lambda),\tau}\cdot A^{(n)}_{\Sigma^0(\Lambda)}\,e^{-ip.x} \tag{A.15b}$$

$$\langle 0 | T^{(n)}_{0,\tau}(x) | \Sigma^0(\Lambda) \rangle = 2\, f_{\Sigma^0(\Lambda)}\, (z\cdot p)^{n_1+n_2+n_3+1}\, N_{\Sigma^0(\Lambda),\tau} \cdot T^{(n)}_{\Sigma^0(\Lambda)}\, e^{-ip\cdot x} \tag{A.15c}$$

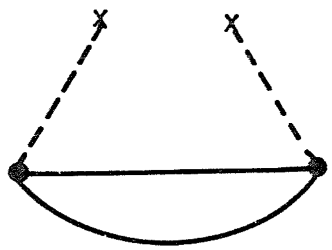
The computation of the coefficient functions of the OPE's for the various correlators for the hyperons is similar to that for the proton. In principle care must be taken to account for the mass of the strange quark by:

(i) including mass terms in the background field expansion of the strange quark propagator

(ii) including the condensate $\langle 0|O_1|0\rangle$ (and possibly $\langle 0|O_4|0\rangle$) in the OPE's with the corresponding coefficient functions obtained by calculating the diagram of Fig. 1.4.

In practice, however, all terms linear in m_s vanish because they involve traces of odd numbers of γ-matrices. If terms of order m_s^2 are retained they will enter the Borel-transformed sum rules as corrections of the form const. x $\frac{m_s^2}{M^2}$. With $m_s \approx 0.15$ GeV [25] and $M^2 \approx$ 1-2 GeV^2 we may conclude that such terms may safely be neglected in the QCD sum rule analysis provided that the multiplicative constants are not anomalously large.

Fig. 1.4 Diagram Relevant to Coefficient Function Associated with $\langle 0 | O_1 | 0 \rangle$



CHAPTER 2

A CALCULATION OF THE PROTON LIFETIME USING AN ASYMMETRIC DISTRIBUTION OF QUARK MOMENTA

In this chapter we investigate the effect of the introduction of an asymmetry to the distribution of quark momenta on the decay rate of the proton. The distribution amplitudes obtained by Chernyak and Zhitnitsky [1] are used to estimate a non-perturbative hadronic matrix element, leading to a determination of the rate for the decay $p \to \pi^0 e^+$. The method used is that of Brodsky et al [2]. The chiral lagrangian formalism of Claudson et al [3] is then employed to deduce amplitudes of other decays of the proton.

We must also bear in mind the reservations expressed in Chapter 1 on the applicability of a wavefunction sensitive to light-cone physics to a determination of nucleon decay matrix elements. Nevertheless we have no reason to expect that the asymmetry in the proton wavefunction implied by lowest twist contributions should not be retained when contributions of higher twist are included.

The distribution amplitudes govern the longitudinal momentum fractions of the quarks within the proton. As well as investigating the effect of asymmetric longitudinal momentum components on the proton lifetime, we briefly discuss the influence of the distribution of transverse quark momenta on such a calculation.

2.1 PROTON DECAY

Attempts to develop a single theory describing all non-gravitational physics led to the proposal several years ago of a number of candidate so called Grand Unified Theories (GUTS). (For a review, see Langacker [4]). The simplest of these is the minimal SU(5) GUT invented by Georgi and Glashow [5], which has the symmetry breaking sequence

$$SU(5) \xrightarrow{\sim 10^{15}\,\mathrm{GeV}} SU(3) \times SU(2) \times U(1) \xrightarrow{\sim 10^{2}\,\mathrm{GeV}} SU(3) \times U(1) \tag{2.1.1}$$

Below the unification scale $M_X \sim 10^{15}$ GeV the couplings for the gauge groups SU(3), SU(2) and U(1) evolve differently, leading to the very different observed interaction strengths.

One of the consequences of the SU(5) scheme is that the superheavy gauge bosons mediate baryon number violating transitions. This novel feature of GUTS implies that the proton is no longer predicted to be an absolutely stable particle. There have been a large number of attempts to estimate branching ratios and lifetimes for nucleon decay. Calculations with the SU(5) model indicate that the mode $p \to \pi^0 e^+$ should dominate. It is found that the proton lifetime $\tau(p \to \pi^0 e^+)$ is of the order of $10^{30 \pm 2}$ years. Despite the fact that the decay rate is so small it is quite possible that proton decay could be detected experimentally. Several major experiments have been set up throughout the world in an attempt to confirm the exciting predictions of the GUTS. (See Ref. 6 for a review.) So far, however, no events have been recorded which can definitely be attributed to nucleon decay. These negative results have led to the present experimental limit $\tau(p \to \pi^0 e^+) \gtrsim 2.5 \times 10^{32}$ years [7] which is already in conflict with the theoretical predictions of the SU(5) model [8]. Clearly some modification in either the decay rate calculation or the SU(5) scheme itself is required if the Georgi-Glashow model is to be acceptable as a GUT. Here we study the former option.

Given a model for grand unification there are several stages in the evaluation of a lifetime for the proton. Since the unification scale is so large the interactions of the superheavy gauge bosons give rise to effective four-fermion interactions at low energies. Thus, first, an effective lagrangian is deduced to describe the baryon number violating transitions. Then the effective lagrangian must be renormalised down from the unification mass to the typical hadronic scale of about 1 GeV. Finally the baryon number violating interactions at the quark level must be formulated in terms of the hadrons and the relevant decay matrix elements evaluated.

It is this last step, together with our ignorance of the unification mass M_X, which gives rise to the large uncertainty in the proton lifetime. A variety of phenomenological techniques have been used to determine the matrix element of the effective lagrangian between the hadron states. (See Falkensteiner et al [9] and references therein.) One method is to use current algebra and PCAC techniques. When the effects of SU(3) symmetry breaking are ignored these relate all nucleon $\rightarrow$ antilepton + pseudoscalar meson decay amplitudes to just two three-quark annihilation matrix elements. The results of current algebra and PCAC calculations may be derived by means of the elegant chiral lagrangian formalism. We follow the approach of Claudson et al [3] who give a phenomenological lagrangian based on chiral $SU(3)_L \times SU(3)_R$ to describe the baryon number violating interactions of baryons, leptons and pseudoscalar mesons. The pseudo-Goldstone bosons associated with the spontaneous symmetry breaking of $SU(3)_L \times SU(3)_R$ down to the vectorial subgroup $SU(3)_V$ are identified with the pseudoscalar mesons. The great predictive power of the chiral lagrangian, which allows decay rates for different modes to be related to one another, is a consequence of the small number of parameters. The chiral lagrangian is model-dependent and we perform our calculations for the minimal SU(5) GUT.

The determination of the decay rate into any particular channel requires a knowledge of non-perturbative bound-state physics of hadrons. Brodsky et al [2] express the decay rate for the mode $p \rightarrow \pi^0 e^+$ in terms of the unknown proton wavefunction (which describes the distribution of the longitudinal and transverse light-cone components of the quark momenta within the proton). They suggest a trial wavefunction symmetric in the quark momenta and find a decay rate in conflict with the experimental limit.

In this chapter we modify the calculation of Ref. 2 to allow for a possible asymmetric distribution of quark momenta within the proton.

2.2 THE CHIRAL LAGRANGIAN

Any candidate theory of the strong interaction must be approximately chiral $SU(3)_L \times SU(3)_R$ symmetric. Phenomenological lagrangians based on chiral symmetry have been used to reproduce current algebra results in the soft pseudoscalar limit [10]. Predictions for low energy hadronic processes are then made by extrapolating from this zero momentum limit to the physical region in a systematic way [11]. Such a procedure is justified for nucleon decay by Brodsky et al [2]. In this section we give the lagrangian introduced by Claudson et al [3] to describe the strong interactions of baryons and pseudoscalar mesons. Using power counting arguments it may be shown that expansions in both the number of derivatives in the interaction terms of this chiral lagrangian and in the number of loops in the Feynman diagrams to be calculated are valid for low energy processes.

The chiral lagrangian is an effective field theory based on a non-linear realisation [12] of the chiral $SU(3)_L \times SU(3)_R$ group. The chiral symmetry is spontaneously broken, leading to non-vanishing vacuum quark-antiquark condensates $\langle 0|\bar{q}\, q|0\rangle$.

The associated octet of pseudoscalar Goldstone bosons is introduced in the special unitary matrix ξ ;

$$\xi = \exp\left(\frac{i\Pi}{f_\pi}\right) , \tag{2.2.1}$$

where the pion decay constant f_π (131 MeV) sets the scale of the chiral symmetry breaking and

$$\Pi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\,\eta \end{pmatrix} . \tag{2.2.2}$$

ξ transforms non-linearly under an $SU(3)_L \times SU(3)_R$ transformation:

$$\xi \rightarrow L\,\xi\,U^\dagger = U\,\xi\,R^\dagger , \tag{2.2.3}$$

where $L(R)$ is an element of $SU(3)_L$ $(SU(3)_R)$ and the unitary 3x3 matrix U is a function of L, R and Π. The transformation becomes linear for the unbroken $SU(3)_V$ subgroup, when L=R=U.

The baryon fields are included in the matrix

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix} . \tag{2.2.4}$$

Under a chiral transformation

$$B \rightarrow UBU^+ \, . \tag{2.2.5}$$

The chiral symmetry breaking is symmetric with respect to the diagonal $SU(3)_V$ subgroup, so that the baryon fields transform linearly like an octet and the baryon spectrum displays the observed SU(3) symmetry.

We now write down (using four-component spinor notation) the most general $SU(3)_L \times SU(3)_R$ invariant lagrangian $\mathcal{L}_0$ describing the strong interactions of the pseudoscalar mesons and baryons.

$$\begin{aligned} \mathcal{L}_0 = & -\tfrac{1}{8} f_\pi^2 \, \mathrm{Tr}(\xi \partial_\mu \xi^+ - \xi^+ \partial_\mu \xi)^2 \\ & + \mathrm{Tr}\, \bar{B}(i \not\partial - m_B) B \\ & + \tfrac{1}{2} i \, \mathrm{Tr}\, \bar{B} \gamma^\mu (\xi \partial_\mu \xi^+ + \xi^+ \partial_\mu \xi) B \\ & + \tfrac{1}{2} i \, \mathrm{Tr}\, \bar{B} \gamma^\mu B \, [(\partial_\mu \xi)\xi^+ + (\partial_\mu \xi^+)\xi] \\ & - \tfrac{1}{2} i (D-F) \, \mathrm{Tr}\, \bar{B} \gamma^\mu \gamma_5 B \, [(\partial_\mu \xi)\xi^+ - (\partial_\mu \xi^+)\xi] \\ & + \tfrac{1}{2} i (D+F) \, \mathrm{Tr}\, \bar{B} \gamma^\mu \gamma_5 (\xi \partial_\mu \xi^+ - \xi^+ \partial_\mu \xi) B \\ & + \text{Terms with more derivatives.} \end{aligned} \tag{2.2.6}$$

Here m_B represents the degenerate mass of a $J^P = \frac{1}{2}^+$ baryon in the chiral limit. From measurements of semileptonic baryon decays the values D = 0.81 and F = 0.44 are obtained.

Explicit chiral symmetry breaking terms are also included, in a manner consistent with the light quark mass terms in the QCD lagrangian. As the quark mass terms transform according to $(3_L,\bar{3}_R)$ + $(\bar{3}_L,3_R)$ under $SU(3)_L \times SU(3)_R$, an $SU(3)_V$-breaking lagrangian $\mathcal{L}_1$ with the same transformation properties is added to $\mathcal{L}_o$.

$$\begin{aligned}
\mathcal{L}_1 = \; & v^3 \, \mathrm{Tr}\,\left((\xi^+)^2 m + m\,\xi^2 \right) \\
& + a_1 \, \mathrm{Tr}\, \bar{B} \left(\xi^+ m\, \xi^+ + \xi\, m\, \xi \right) B \\
& + a_2 \, \mathrm{Tr}\, \bar{B} B \left(\xi^+ m\, \xi^+ + \xi\, m\, \xi \right) \\
& + b_1 \, \mathrm{Tr}\, \bar{B} \gamma_5 \left(\xi^+ m\, \xi^+ - \xi\, m\, \xi \right) B \\
& + b_2 \, \mathrm{Tr}\, \bar{B} \gamma_5 B \left(\xi^+ m\, \xi^+ - \xi\, m\, \xi \right) \\
& \qquad + \text{Terms with derivatives} \\
& \qquad + \text{Terms with more factors of } m \quad , \qquad (2.2.7)
\end{aligned}$$

where the quark mass matrix

$$m = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \quad . \qquad (2.2.8)$$

The values of $a_1(\approx -0.45)$ and $a_2(\approx 0.88)$ are fixed by the known baryon masses. When v is chosen to be 196 MeV, the observed masses of the pseudoscalar mesons imply the usual current quark masses: m_u = 4.2 MeV, m_d = 7.5 MeV and m_s = 150 MeV. The magnitudes of the parameters b_1 and b_2 are not known but are thought to be small.

We now turn to the problem of writing a chiral lagrangian for baryon number changing interactions. First we must write down an effective lagrangian for proton decay in terms of quark and lepton fields. The most general form of the dominant part of the $\Delta B = 1$ lagrangian will include all dimension six baryon number violating operators possessing the low energy SU(3) x SU(2) x U(1) symmetry. This is because the leading effective operators with the required properties are four-fermion operators. Also, the contributions from operators of higher dimension will be suppressed by corresponding powers of the grand unification mass M_X and need not be considered. A complete list of the dimension six operators is given in Refs. 13. The effective lagrangian for proton decay is then

$$\mathcal{L}^{\Delta B=1} = \sum_{i=1}^{6} \sum_{d=1}^{2} C_d^{(i)} Q_d^{(i)} + \sum_{i=1}^{8} \sum_{d=1}^{2} \tilde{C}_d^{(i)} \tilde{Q}_d^{(i)} + \text{h.c.} \tag{2.2.9}$$

where the coefficients $C_d^{(i)}$ and $\tilde{C}_d^{(i)}$ are GUT dependent and their values must be adjusted, using renormalisation group techniques, from those at the unification mass down to the typical hadronic mass scale of approximately 1 GeV. The sums are over two lepton generations and the fourteen operators relevant for nucleon decay (six of which conserve strangeness S while the others contain one strange quark):

$\Delta S = 0$:

$$Q_d^{(1)} = \varepsilon^{ijk}(d_R^i u_R^j)(u_L^k e_{dL}) \qquad (3_L, \bar{3}_R) \tag{2.2.10a}$$

$$Q_d^{(2)} = \varepsilon^{ijk}(d_L^i u_L^j)(u_R^k e_{dR}) \qquad (\bar{3}_L, 3_R) \tag{2.2.10b}$$

$$Q_d^{(3)} = \varepsilon^{ijk}(d_L^i u_L^j)(u_L^k e_{dL}) \qquad (8_L, 1_R) \tag{2.2.10c}$$

$$Q_d^{(4)} = \varepsilon^{ijk}(d_R^i u_R^j)(u_R^k e_{dR}) \qquad (1_L, 8_R) \tag{2.2.10d}$$

$$Q_d^{(5)} = -\varepsilon^{ijk}(d_R^i u_R^j)(d_L^k \nu_{dL}) \qquad (3_L, \bar{3}_R) \tag{2.2.10e}$$

$$Q_d^{(6)} = -\varepsilon^{ijk}(d_L^i u_L^j)(d_L^k \nu_{dL}) \qquad (8_L, 1_R) \tag{2.2.10f}$$

$\Delta S = 1$:

$$\tilde{Q}_d^{(1)} = \varepsilon^{ijk}(s_R^i u_R^j)(u_L^k e_{dL}) \qquad (3_L, \bar{3}_R) \tag{2.2.11a}$$

$$\tilde{Q}_d^{(2)} = \varepsilon^{ijk}(s_L^i u_L^j)(u_R^k e_{dR}) \qquad (\bar{3}_L, 3_R) \tag{2.2.11b}$$

$$\tilde{Q}_d^{(3)} = \varepsilon^{ijk}(s_L^i u_L^j)(u_L^k e_{dL}) \qquad (8_L, 1_R) \tag{2.2.11c}$$

$$\tilde{Q}_d^{(4)} = \varepsilon^{ijk}(s_R^i u_R^j)(u_R^k e_{dR}) \qquad (1_L, 8_R) \tag{2.2.11d}$$

$$\tilde{Q}_d^{(5)} = \varepsilon^{ijk}(d_R^i u_R^j)(s_L^k \nu_{dL}) \qquad (3_L, \bar{3}_R) \tag{2.2.11e}$$

$$\tilde{Q}_d^{(6)} = \varepsilon^{ijk}(d_L^i u_L^j)(s_L^k \nu_{dL}) \qquad (8_L, 1_R) \tag{2.2.11f}$$

$$\tilde{Q}_d^{(7)} = -\varepsilon^{ijk}(s_R^i u_R^j)(d_L^k \nu_{dL}) \qquad (3_L, \bar{3}_R) \qquad (2.2.11g)$$

$$\tilde{Q}_d^{(8)} = -\varepsilon^{ijk}(s_L^i u_L^j)(d_L^k \nu_{dL}) \qquad (8_L, 1_R) \qquad (2.2.11h)$$

We use the two-component spinor notation of Abbott and Wise [14] (see Appendix 1) and list the transformation properties of the operators under $SU(3)_L \times SU(3)_R$, which are needed, together with their parity transformation properties, to rewrite $\mathcal{L}^{\Delta B=1}$ in terms of baryon and pseudoscalar meson fields. The result is

$$\begin{aligned}
\mathcal{L}^{\Delta B=1} = \alpha \sum_{d=1}^{2} \Big(& C_d^{(1)} e_{dL} \, \mathrm{Tr}\, O \xi B_L \xi \\
& + C_d^{(2)} e_{dR} \, \mathrm{Tr}\, O \xi^{+} B_R \xi^{+} \\
& - C_d^{(5)} \nu_{dL} \, \mathrm{Tr}\, O' \xi B_L \xi \\
& + \tilde{C}_d^{(1)} e_{dL} \, \mathrm{Tr}\, \tilde{O} \xi B_L \xi \\
& + \tilde{C}_d^{(2)} e_{dR} \, \mathrm{Tr}\, \tilde{O} \xi^{+} B_R \xi^{+} \\
& + \tilde{C}_d^{(5)} \nu_{dL} \, \mathrm{Tr}\, \tilde{O}'' \xi B_L \xi \\
& - \tilde{C}_d^{(7)} \nu_{dL} \, \mathrm{Tr}\, \tilde{O}' \xi B_L \xi \Big) \\
+ \beta \sum_{d=1}^{2} \Big(& C_d^{(3)} e_{dL} \, \mathrm{Tr}\, O \xi B_L \xi^{+} \\
& + C_d^{(4)} e_{dR} \, \mathrm{Tr}\, O \xi^{+} B_R \xi \\
& - C_d^{(6)} \nu_{dL} \, \mathrm{Tr}\, O' \xi B_L \xi^{+} \\
& + \tilde{C}_d^{(3)} e_{dL} \, \mathrm{Tr}\, \tilde{O} \xi B_L \xi^{+} \\
& + \tilde{C}_d^{(4)} e_{dR} \, \mathrm{Tr}\, \tilde{O} \xi^{+} B_R \xi \\
& + \tilde{C}_d^{(6)} \nu_{dL} \, \mathrm{Tr}\, \tilde{O}'' \xi B_L \xi^{+} \\
& - \tilde{C}_d^{(8)} \nu_{dL} \, \mathrm{Tr}\, \tilde{O}' \xi B_L \xi^{+} \Big)
\end{aligned}$$

+ Terms with derivatives

+ Terms with factors of m + h.c. (2.2.12)

Inclusion of the matrices

$$0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad 0' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad \tilde{0} = \begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\tilde{0}' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \tilde{0}'' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.2.13}$$

ensures that the required components of the $SU(3)_L \times SU(3)_R$ representations are projected out. Neglecting SU(3) violating effects, the α and β coefficients are given by the three-quark annihilation matrix elements [2]

$$\langle 0 | \varepsilon^{ijk} \varepsilon_{\gamma\delta} d^i_{\gamma R} u^j_{\delta R} u^k_{\varepsilon L} | p \rangle \equiv \alpha p_{\varepsilon L} \tag{2.2.14a}$$

$$\langle 0 | \varepsilon^{ijk} \varepsilon_{\gamma\delta} d^i_{\gamma L} u^j_{\delta L} u^k_{\varepsilon L} | p \rangle \equiv \beta p_{\varepsilon L} , \tag{2.2.14b}$$

where γ, δ and ε are two-component spinor indices and p represents the two-component proton spinor. We see that the amplitudes for all baryon number changing processes may be expressed in terms of the two non-perturbative parameters α and β. Indeed we shall consider only those interactions mediated by the exchange of gauge bosons. (It is assumed that the coloured Higgs particles are heavy enough for their effects to be ignored.) Then all dependence on β disappears and we obtain definite predictions for the ratios of the rates for the various nucleon decay modes.

Using the total lagrangian

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}^{\Delta B = 1} \tag{2.2.15}$$

we may evaluate the different nucleon decay rates.

Vector and axial vector mesons have recently been incorporated into the chiral lagrangian formalism by Kaymakcalan et al [15]. Some vector meson decay modes of the proton are significant when the non-relativistic quark model and the bag model are used to estimate the hadronic matrix elements. Nevertheless, throughout our work we neglect the effect of these possible decay products.

We now outline briefly how to obtain the decay rates from the lagrangian $\mathcal{L}$. The procedure is illustrated for the process $p \to \pi^o e^+$. (Decays of the proton into more than one pseudoscalar meson and an antilepton will not be considered since the decay rates corresponding to these modes are seen to be suppressed using phase space arguments.) The leading contributions to the two-body decay amplitude arise from the tree diagrams of Fig. 2.1. Only those terms in $\mathcal{L}$ which contribute to the amplitudes at lowest order in the momenta are retained.

For the pole diagram (Fig. 2.1(a)) the $pp\pi^o$ vertex is found by expanding ξ in the strong interaction lagrangian $\mathcal{L}_o + \mathcal{L}_1$ whereas the pe^+ interaction violates baryon number and is obtained from $\mathcal{L}^{\Delta B=1}$. The amplitude for this diagram is (in four-component notation)

$$\frac{\alpha(D+F)}{\sqrt{2}\, f_\pi}\, \overline{e^+}(k,r)\left(C_1^{(1)} P_L + C_1^{(2)} P_R\right)\frac{(\not k + m_p)}{(k^2 - m_p^2)}(\not p - \not k)\gamma_5\, p(p,s) \;, \qquad (2.2.16)$$

where r and s represent the spins of the positron and proton respectively and the projection operators P_L and P_R are defined as

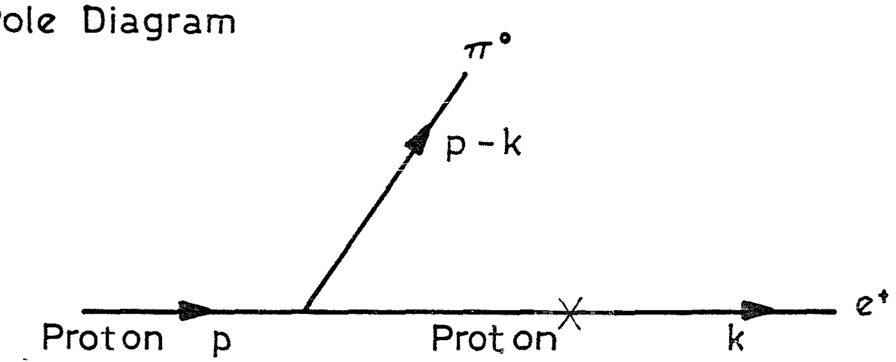
$$P_L = \frac{1-\gamma_5}{2} \quad ; \quad P_R = \frac{1+\gamma_5}{2} \quad . \qquad (2.2.17)$$

Neglecting the positron mass and using the Dirac equation for both the proton and the positron, expression (2.2.16) reduces to

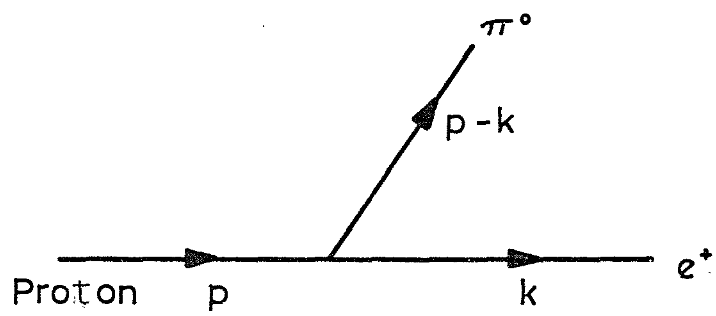
$$-\frac{\alpha(D+F)}{\sqrt{2}\, f_\pi}\, \overline{e^+}(k,r)\left(C_1^{(1)} P_L - C_1^{(2)} P_R\right) p(p,s) \quad . \qquad (2.2.18)$$

Fig. 2.1 Feynman Diagrams for $p \rightarrow \pi^{\circ} e^{+}$

(a) Pole Diagram



(b) Direct Conversion Diagram



The amplitude for the direct conversion diagram (Fig. 2.1(b)) receives a contribution from only $\mathcal{L}^{\Delta B=1}$. It is

$$-\frac{\alpha}{\sqrt{2} f_\pi} \; \overline{e^+}(k,r) \left(C_1^{(1)} P_L - C_1^{(2)} P_R \right) p(p,s) \quad . \tag{2.2.19}$$

By averaging the total amplitude over the initial proton spins and summing over the final positron spins the decay rate for the process $p \to \pi^0 e^+$ is obtained:

$$\Gamma(p \to \pi^0 e^+) = \frac{\alpha^2 m_p}{16\pi f_\pi^2} \left(C_1^{(1)2} + C_1^{(2)2} \right) (1+D+F)^2 \Delta_\pi \;, \tag{2.2.20}$$

where

$$4 \Delta_\pi = \left(1 - \frac{m_\pi^2}{m_p^2} \right)^2 . \tag{2.2.21}$$

The laboratory frame (in which the proton is at rest) is the most convenient for calculating the phase space factor.

The proton $\to$ pseudoscalar meson + antilepton decay rates are listed in Table 2.1. We have neglected all lepton masses in the calculations. For the η channel and some of the $\Delta S=1$ channels the rates are dependent on the symmetry breaking parameters b_1 and b_2 of $\mathcal{L}_1$. Following Kaymakcalan et al [15] we choose $b_1 = b_2 = 0$. Allowing for a non-vanishing Cabbibo mixing angle θ_c between the first two generations [4] the coefficients $C_d^{(i)}$ and $\tilde{C}_d^{(i)}$ take the following values in the minimal SU(5) model:

$$\begin{aligned}
C_1^{(2)} &= 4\sqrt{2}\, \tilde{G}_{SU(5)} \left(1 + \cos^2\theta_c \right) \\
\tilde{C}_2^{(2)} &= 4\sqrt{2}\, \tilde{G}_{SU(5)} \left(1 + \sin^2\theta_c \right) \\
C_1^{(1)} = \tilde{C}_2^{(1)} &= 4\sqrt{2}\, \tilde{G}_{SU(5)} \\
C_2^{(2)} = \tilde{C}_1^{(2)} &= 4\sqrt{2}\, \tilde{G}_{SU(5)} \sin\theta_c \cos\theta_c \\
C_1^{(5)} = \tilde{C}_2^{(7)} &= 4\sqrt{2}\, \tilde{G}_{SU(5)} \cos\theta_c \\
\tilde{C}_1^{(5)} &= -4\sqrt{2}\, \tilde{G}_{SU(5)} \sin\theta_c \\
\text{All other } C_d^{(i)}, \; \tilde{C}_d^{(i)} &= 0 \;,
\end{aligned} \tag{2.2.22}$$

Table 2.1 Two-Body Proton Decay Rates

Decay Mode	Intermediate Baryon(s) in Pole Diagram	Decay Rate	Branching Ratio in Minimal SU(5) (%)
$p \to \pi^o e^+$	p	$\frac{\alpha^2 m_p}{16\pi f_\pi^2}[(C_1^{(1)})^2 + C_1^{(2)})^2](1+D+F)^2\Delta_\pi$	63.0
$p \to \eta e^+$	p	$\frac{\alpha^2 m_p}{48\pi f_\pi^2}[(C_1^{(1)})^2 + (C_1^{(2)})^2](1+D-3F)^2\Delta_\eta$	0.5
$p \to K^o \mu^+$	Σ^+	$\frac{\alpha^2 m_p}{8\pi f_\pi^2}[(\tilde{C}_2^{(1)})^2 + (\tilde{C}_2^{(2)})^2](1 + \frac{m_p(D-F)}{m_\Sigma})^2 \Delta_K$	9.9
$p \to \pi^+ \bar{\nu}_e$	n	$\frac{\alpha^2 m_p}{8\pi f_\pi^2}(C_1^{(5)})^2 (1+D+F)^2 \Delta_\pi$	24.9
$p \to K^+ \bar{\nu}_\mu$	Σ^o, Λ	$\frac{\alpha^2 m_p}{8\pi f_\pi^2}\{\frac{1}{2}\tilde{C}_2^{(7)}[\frac{m_p}{m_\Sigma}(D-F) + \frac{m_p}{3m_\Lambda}(D+3F)] - \tilde{C}_2^{(5)}[1+\frac{m_p}{3m_\Lambda}(D+3F)]\}^2\Delta_K$	0.5
$p \to K^o e^+$	Σ^+	$\frac{\alpha^2 m_p}{8\pi f_\pi^2}[(\tilde{C}_1^{(1)})^2 + (\tilde{C}_1^{(2)})^2](1+\frac{m_p}{m_\Sigma}(D-F)^2\Delta_K$	0.2
$p \to \pi^o \mu^+$	p	$\frac{\alpha^2 m_p}{16\pi f_\pi^2}[(C_2^{(1)})^2 + (C_2^{(2)})^2](1+D+F)^2\Delta_\pi$	0.6
$p \to \eta \mu^+$	p	$\frac{\alpha^2 m_p}{48\pi f_\pi^2}[(C_2^{(1)})^2 + (C_2^{(2)})^2](1+D-3F)^2\Delta_\eta$	~0
$p \to K^+ \bar{\nu}_e$	Σ^o, Λ	$\frac{\alpha^2 m_p}{8\pi f_\pi^2}\{\frac{1}{2}\tilde{C}_1^{(7)}[\frac{m_p}{m_\Sigma}(D-F) + \frac{m_p}{3m_\Lambda}(D+3F) - \tilde{C}_1^{(5)}[1 + \frac{m_p}{3m_\Lambda}(D+3F)]\}^2\Delta_K$	0.4

Notation:

$$\Delta_i = \frac{1}{4}(1 - \frac{m_i^2}{m_p^2})^2 \quad ; \quad i = \pi, \eta, K.$$

where

$$\tilde{G}_{SU(5)} = \frac{A\, g_x^2}{4\sqrt{2}\, M_x^2} \quad . \tag{2.2.23}$$

g_X is the strong interaction coupling evaluated at the unification mass M_X and $A = 2.9$ [2] is the renormalisation group amplitude enhancement factor. We are thus able to deduce the branching ratios listed in Table 2.1. The value $|\cos\theta_c| = 0.9737 + 0.0025$ [16] is used in the calculation.

In passing, we note that in the derivation of the baryon number violating terms $\mathcal{L}^{\Delta B=1}$ of the chiral lagrangian it was assumed that all the baryonic three-quark annihilation matrix elements of the form $\langle 0|(q_R q_R)\, q_L|B\rangle$ had the same value as the proton amplitude $\langle 0|(d_R u_R)u_L|p\rangle$; i.e. SU(3) violating effects in the matrix elements were neglected. Consider for example the decay rates for the channels $p\to\pi^0 e^+$ and $p\to K^0\mu^+$ (Table (2.1)). The terms arising from the direct conversion diagrams may be related to the same three-quark annihilation matrix element $\langle 0|(d_R u_R)u_L|p\rangle$ in the soft pseudoscalar limit. However, whereas the terms contributed by the pole diagram (i.e. the D and F terms) in the $p\to\pi^0 e^+$ process are also related to the amplitude $\langle 0|(d_R u_R)u_L|p\rangle$ the corresponding terms for the $p\to K^0\mu^+$ decay depend on the matrix element $\langle 0|(s_R u_R)u_L|\Sigma^+\rangle$ which could differ due to SU(3) symmetry breaking. One method of accounting for such symmetry breaking effects could be to use baryon distribution amplitudes (possibly determined from QCD sum rule analyses) in a revised evaluation of the baryon number violating vertices of the pole diagrams. This would lead to new branching ratios for the various two-body proton decay modes.

2.3 AN ESTIMATE OF THE PROTON LIFETIME

In order to determine absolute values for the proton partial decay rates it is necessary to calculate the magnitude of the three-quark annihilation matrix element (2.2.14a), which we write symbolically as $\langle 0|(d_R u_R)u_L|p\rangle$. This section describes one possible procedure for obtaining an estimate for the parameter α , and hence for the proton lifetime.

Following Brodsky et al [2] we take as a model for the amplitude $\langle 0|(d_R u_R)u_L|p\rangle$ the three-quark $\rightarrow$ antilepton ($qqq \rightarrow \bar{\ell}$) annihilation following the emission of a meson. This is a natural choice in the chiral lagrangian framework, where the magnitude of the baryon $\rightarrow$ antilepton interaction of the pole diagram is proportional to the coefficient α. For example, the baryon number violating vertex of the pole diagram for the decay mode $p \rightarrow \pi^0 e^+$ may be represented as in Fig. 2.2. From $\mathcal{L}^{\Delta B=1}$ (2.2.12) this proton $\rightarrow$ positron vertex is found to be $i(C_1^{(1)} P_L + C_1^{(2)} P_R)\alpha$.

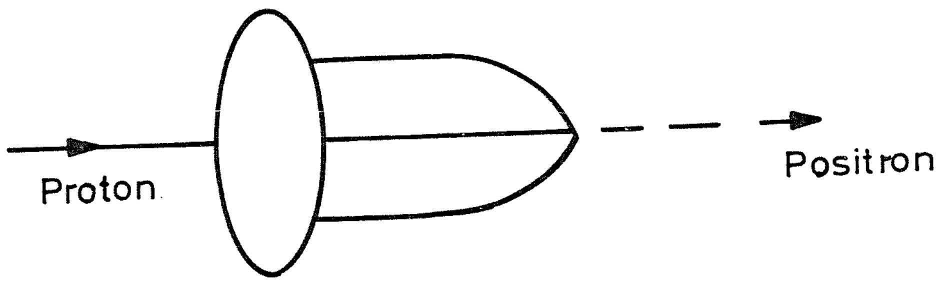
We use the light-cone formalism of Brodsky and Lepage [17] in our evaluation of the three-quark annihilation diagram of Fig. 2.2. With the definitions $k^{\pm} = k^0 \pm k^3$ the proton momentum k^μ may be re-parametrised as $(k^+, k^-, k_\perp)$. The quark light-cone momentum fractions $x_i \equiv k_i^+/k^+$, where k_i ($i = 1,2,3$) are the quark momenta, reduce to longitudinal momentum fractions in the frame where $k^3 \rightarrow \infty$, as in Chapter 1.

Because of our lack of understanding of hadron dynamics in the strong coupling regime of QCD we are unable to write down an exact three quark Fock state wavefunction $\psi_{3q}(k_i, \vec{p}, \lambda_i)$ for the proton. Here the λ_i represent the quark helicities. It is possible to perform calculations using an ansatz for the wavefunction, as is done in Refs. 2 and 18, fixing the parameters of the trial wavefunction by using experimental data. However, the valence Fock state wavefunction is closely related to the distribution amplitude [2]:

$$\phi_{3q}(x_i, \vec{p}, \lambda_i) \equiv \int (\Pi\, dk)\, \psi_{3q}(k_i, \vec{p}, \lambda_i), \qquad (2.3.1)$$

where

Fig. 2.2 The Three Quark Annihilation Diagram For The Proton ⟶ Positron Process



$$\prod dk = \prod_{i=1}^{3}\left(\frac{d^2k_{i\perp}}{16\pi^3}\right) 16\pi^3\,\delta^{(2)}\left(\sum_{i=1}^{3} k_{i\perp}\right) \quad . \tag{2.3.2}$$

(The precise relationship between ϕ_{3q} and the Brodsky and Lepage distribution amplitude (1.2.16) is given in (2.3.7).) Thus, as well as providing information on the quark light-cone momentum fractions x_i, the distribution amplitude ϕ_{3q} may be interpreted as a probability amplitude for finding the three quark valence state in the incident proton. The more complicated non-valence Fock states of the proton, which are components with extra quark-antiquark pairs and gluons, are neglected here since they do not contribute to the basic three quark annihilation matrix element. We shall choose the proton distribution amplitude $\phi(x_i,\mu \sim 1\text{GeV})$ to be that obtained from the analysis of Ref. 1 by using the definition (1.2.16) together with the relations (1.2.17).

The terms of the low energy effective lagrangian (2.2.9) relevant to the three quark annihilation diagram of Fig. 2.2 are (in four component spinor notation)

$$\begin{aligned}\mathcal{L} &= C_1^{(1)} Q_1^{(1)} + C_1^{(2)} Q_1^{(2)} \\ &= -\tfrac{1}{2}\,\varepsilon^{ijk}\left(\overline{u^c_L}^{i}\,\gamma^\mu\, u_L^j\right) \\ &\qquad \left(C_1^{(1)}\,\overline{e^+_R}\,\gamma_\mu\, d_R^k + C_1^{(2)}\,\overline{e^+_L}\,\gamma_\mu\, d_L^k\right).\end{aligned} \tag{2.3.3}$$

From the Feynman rules of Ref. 17 it follows that the amplitude S for the diagram may be written as

$$\begin{aligned}S(\lambda_p,\lambda_e) &= \int_0^\infty \left(\prod_{i=1}^{3} dk_i^+\right)\delta\left(k^+ - \sum_{i=1}^{3} k_i^+\right)\int\left(\prod dk\right) \\ &\sum_{\lambda_i} h(\lambda_i)\,\psi_{3q}(k_i,\vec{P},\lambda_i)\left\langle e^+(\lambda_e)\right| C_1^{(1)}\bar{Q}_1^{(1)} + C_1^{(2)}\bar{Q}_1^{(2)}\left|p(\lambda_i)\right\rangle,\end{aligned} \tag{2.3.4}$$

where

$$\bar{Q}_1^{(i)} \equiv \frac{Q_1^{(i)}}{\sqrt{k^+k_1^+k_2^+k_3^+}} \tag{2.3.5}$$

and λ_p and λ_e represent the proton and positron helicities respectively. The sum is over quark helicity configurations satisfying $\sum_{i=1}^{3} \lambda_i = \lambda_p$ and the coefficients $h(\lambda_i)$ are given by the proton's SU(6) flavour-spin wavefunction. The proton is assumed to be travelling along the 3-axis. Isolating those terms which have the d-quark in position three we may write the amplitude with $\lambda_p = \uparrow$ as

$$
\begin{aligned}
S(\uparrow,\lambda_e) = \frac{1}{\sqrt{6}} \int_0^\infty \Big(\prod_{i=1}^{3} dk_i^+ \Big) \delta\Big(k^+ - \sum_{i=1}^{3} k_i^+\Big) \int \big(\pi\, dk \big) \\
\Big\{ h(\uparrow,\downarrow,\uparrow)\, \psi_{3q}(k_i,\bar{p},\uparrow,\downarrow,\uparrow) \\
\langle e^+(\lambda_e) | C_1^{(1)} \bar{Q}_1^{(1)} + C_1^{(2)} \bar{Q}_1^{(2)} | u_\uparrow(1)\, u_\downarrow(2)\, d_\uparrow(3) \rangle \\
+ h(\downarrow,\uparrow,\uparrow)\, \psi_{3q}(k_i,\bar{p},\downarrow,\uparrow,\uparrow) \\
\langle e^+(\lambda_e) | C_1^{(1)} \bar{Q}_1^{(1)} + C_1^{(2)} \bar{Q}_1^{(2)} | u_\downarrow(1)\, u_\uparrow(2)\, d_\uparrow(3) \rangle \\
+ h(\uparrow,\uparrow,\downarrow)\, \psi_{3q}(k_i,\bar{p},\uparrow,\uparrow,\downarrow) \\
\langle e^+(\lambda_e) | C_1^{(1)} \bar{Q}_1^{(1)} + C_1^{(2)} \bar{Q}_1^{(2)} | u_\uparrow(1)\, u_\uparrow(2)\, d_\downarrow(3) \rangle \Big\},
\end{aligned}
\tag{2.3.6}
$$

where the functions $\psi_{3q}(k_i,\bar{\mu},\lambda_i)$ are defined by equation (2.3.1). The colour factor $1/\sqrt{6}$ ensures the correct normalisation for the flavour-spin dependence of the proton distribution amplitude (1.2.16), from which we find

$$\phi_{3q}(x_i,\bar{p},\uparrow,\downarrow,\uparrow) = \phi^s(x_1,x_2,x_3) - \sqrt{3}\,\phi^a(x_1,x_2,x_3) \tag{2.3.7a}$$

$$\phi_{3q}(x_i,\bar{p},\downarrow,\uparrow,\uparrow) = \phi^s(x_2,x_1,x_3) - \sqrt{3}\,\phi^a(x_2,x_1,x_3) \tag{2.3.7b}$$

$$\phi_{3q}(x_i,\bar{p},\uparrow,\uparrow,\downarrow) = \phi^s(x_1,x_3,x_2), \tag{2.3.7c}$$

since

$$h(\uparrow,\downarrow,\uparrow) = h(\downarrow,\uparrow,\uparrow) = -\tfrac{1}{2}\, h(\uparrow,\uparrow,\downarrow) = \frac{1}{\sqrt{6}}\,. \tag{2.3.8}$$

It is not yet possible to perform the integrations over the transverse momenta $k_{i\perp}$ and thus express the amplitude S in terms of the distribution amplitude. We must first evaluate the matrix elements of the baryon number violating operators $Q_1^{(1)}$ and $Q_1^{(2)}$ to determine their momentum dependence. To simplify the computation of the matrix elements we first apply the Fierz transformation (A.1.8) to the operators $Q_1^{(1)}$ and $Q_1^{(2)}$:

$$Q_1^{(1)} = -\tfrac{1}{2}\,\varepsilon^{ijk}\left(\overline{u^c}^{i}(1)\,\gamma^\mu P_L\, u^j(2)\right)\left(\overline{e^+}\,\gamma_\mu P_R\, d^k(3)\right) \qquad (2.3.9a)$$

$$= -\,\varepsilon^{ijk}\left(\overline{u^c}^{i}(1)\, P_R\, d^j(3)\right)\left(\overline{e^+}\, P_L\, u^k(2)\right) \qquad (2.3.9b)$$

$$Q_1^{(2)} = -\tfrac{1}{2}\,\varepsilon^{ijk}\left(\overline{u^c}^{i}(1)\,\gamma^\mu P_L\, u^j(2)\right)\left(\overline{e^+}\,\gamma_\mu P_L\, d^k(3)\right)$$

$$= -\,\varepsilon^{ijk}\left(\overline{u^c}^{i}(2)\, P_L\, d^j(3)\right)\left(\overline{e^+}\, P_R\, u^k(1)\right) \quad .$$

We choose to evaluate the matrix elements using the helicity spinors of Brodsky and Lepage [17] (Appendix 2). The results are listed in Tables 2.2 and 2.3. All matrix elements include a colour factor of 6 which arises as a result of the anticommutative property of Fermi fields. We now use these values to compute $S(\uparrow,\uparrow)$. The operator $Q_1^{(2)}$ does not contribute to this amplitude when the mass of the positron is neglected. We find

$$S(\uparrow,\uparrow) = C_1^{(1)} \int_0^1 [dx]\,\frac{1}{x_1 x_2 x_3} \int (\pi\, dk) \Big\{ \psi_{3q}(k_i, \bar{P}, \uparrow,\downarrow,\uparrow)\; k_{2\perp}^{-}\,(x_1 k_{3\perp}^{+} - x_3 k_{1\perp}^{+})$$
$$+\, \psi_{3q}(k_i, \bar{P}, \downarrow,\uparrow,\uparrow)\; M_u^2\, x_3$$
$$+\, 2\,\psi_{3q}(k_i, \bar{P}, \uparrow,\uparrow,\downarrow)\; M_u M_d\, x_1 \Big\} \; , \qquad (2.3.10)$$

where we have used the fact that, in the frame in which we are working, the positron does not carry any transverse momentum. The masses M_u and M_d are those appearing in the Dirac equation for the quarks:

$$(\not{p} - M_q)\, q(p) = 0 \; . \qquad (2.3.11)$$

Thus they are constituent quark masses and not to be confused with the current masses of the last section. The definition (2.3.1) now allows us to express the second and third terms as functions of the distribution amplitude. However, we need to introduce some sort of approximation to perform the integration over transverse momenta in the first term. (Note that the contribution of this term vanishes for a wavefunction $\psi_{3q}(k_i, \bar{P}, \uparrow,\downarrow,\uparrow)$ symmetric under the interchange $k_1 \leftrightarrow k_3$.) As is usual in the literature we assume a symmetric transverse momentum dependence for the proton wavefunction. This implies

Table 2.2 Matrix Elements of the Operator $\bar{Q}_1^{(1)}$

For all momenta p, $p_\perp^\pm = p^1 \pm ip^2$

λ_1	λ_2	λ_3	λ_e	$\bar{Q}_1^{(1)} = \frac{-6}{\sqrt{K^+K_1^+K_2^+K_3^+}} \overline{u^c_{\lambda_1}}(K_1)P_R u_{\lambda_3}(K_3)\ \overline{v^c_{\lambda_e}}(K)P_L u_{\lambda_2}(K_2)$
↑	↑	↓	↑	$-\frac{6M_2M_3}{K_2^+\ K_3^+}$
↑	↓	↑	↑	$\frac{-6}{K^+K_1^+K_2^+K_3^+}(K_1^+K_{3\perp}^+ - K_3^+K_{1\perp}^+)\ (K_2^+K_\perp^- - K^+K_{2\perp}^-)$
↓	↑	↑	↑	$\frac{6M_1M_2}{K_1^+\ K_2^+}$
↑	↓	↓	↑	$\frac{-6M_3}{K^+K_2^+K_3}(K_2^+K_\perp^- - K^+K_{2\perp}^-)$
↓	↑	↓	↑	0
↓	↓	↑	↑	$\frac{6M_1}{K^+K_1^+K_2^+}(K_2^+K_\perp^- - K^+K_{2\perp}^-)$
λ_1	λ_2	λ_3	↓	0 (for all λ_i, neglecting positron mass)

Table 2.3 Matrix Elements of the Operator $\bar{Q}_1^{(2)}$

λ_1	λ_2	λ_3	λ_e	$\bar{Q}_1^{(2)} = \frac{-6}{\sqrt{K^+K_1^+K_2^+K_3^+}} \overline{u^c_{\lambda_2}}(K_2)P_L u_{\lambda_3}(K_3)\overline{v^c_{\lambda}}(K)P_R u_{\lambda_1}(K_1)$
↑	↑	↓	↓	$\frac{6M_2}{K^+K_1^+K_2^+}(K^+K_{1\perp}^+ - K_1^+K_\perp^+)$
↑	↓	↑	↓	$\frac{-6M_3}{K^+\ K_1^+\ K_3^+}(K^+K_{1\perp}^+ - K_1^+K_\perp^+)$
↓	↑	↑	↓	0
↑	↓	↓	↓	$\frac{-6}{K^+K_1^+K_2^+K_3^+}(K^+K_{1\perp}^+ - K_1^+K_\perp^+)(K_3^+K_{2\perp}^- - K_2^+K_{3\perp}^-)$
↓	↑	↓	↓	$\frac{6M_1M_2}{K_1^+\ K_2^+}$
↓	↓	↑	↓	$-\ \frac{6M_1M_3}{K_1^+\ K_3^+}$
λ_1	λ_2	λ_3	↑	0 (for all λ_i, neglecting positron mass)

$$\int(\pi dk)\,\psi_{3q}(k_i,\bar{p},\uparrow,\downarrow,\uparrow)\,k_{1\perp}^{+}\,k_{2\perp}^{-} = \int(\pi dk)\,\psi_{3q}(k_i,\bar{p},\uparrow,\downarrow,\uparrow)\,k_{3\perp}^{+}\,k_{2\perp}^{-}$$

$$= -\tfrac{1}{2}\int(\pi dk)\,\psi_{3q}(k_i,\bar{p},\uparrow,\downarrow,\uparrow)\,k_{1\perp}^{2} \quad . \tag{2.3.12}$$

This last form we approximate by

$$-\tfrac{1}{2}\,\langle k_{i\perp}^{2}\rangle\int(\pi dk)\,\psi_{3q}(k_i,\bar{p},\uparrow,\downarrow,\uparrow) \quad , \tag{2.3.13}$$

where $\langle k_{i\perp}^{2}\rangle^{1/2} \approx$ 300 MeV (i =1,2,3) [19] is the root mean square transverse quark momentum in the nucleon. Expression (2.3.10) for the amplitude $S(\uparrow,\uparrow)$ now becomes

$$\begin{aligned} S(\uparrow,\uparrow) = C_1^{(1)}\int_0^1 [dx]\,\frac{1}{x_1x_2x_3} & \Big\{-\tfrac{1}{2}\,\langle k_{i\perp}^{2}\rangle\,(x_1-x_3)\;\phi_{3q}(x_i,\bar{p},\uparrow,\downarrow,\uparrow) \\ & + M_u^2\,x_3\;\phi_{3q}(x_i,\bar{p},\downarrow,\uparrow,\uparrow) \\ & + 2\,M_u M_d\,x_1\;\phi_{3q}(x_i,\bar{p},\uparrow,\uparrow,\downarrow)\Big\} \, . \end{aligned} \tag{2.3.14}$$

Further simplification occurs when the symmetry properties of the expressions (2.3.7) are recalled. After some manipulation we find

$$\begin{aligned} S(\uparrow,\uparrow) = C_1^{(1)}\int_0^1 [dx]\,\frac{1}{x_1x_2} & \Big\{(M_u^2+2M_uM_d)\,\phi^{s}(x_1,x_2,x_3) \\ & -\sqrt{3}\,(M_u^2+\langle k_{i\perp}^{2}\rangle)\,\phi^{a}(x_1,x_2,x_3)\Big\} \, . \end{aligned} \tag{2.3.15}$$

Because of the uncertainty introduced by the quantity $\langle k_{i\perp}^{2}\rangle$ we cannot justify a treatment of SU(2) symmetry breaking effects by retaining distinct u- and d-quark masses. Thus we write $\hat{M}$ for the masses of these quarks. However, the form (2.3.15) will be useful in the calculations of Chapter 3. The distribution amplitudes V, A and T obtained by Chernyak and Zhitnitsky [1] correspond to the expressions

$$\begin{aligned} \phi^{s}(x_1,x_2,x_3) = 15\sqrt{2}\,|f_N|\,x_1x_2x_3\,[&13.45\,(x_1^2+x_3^2)+4.63\,x_2^2 \\ &+0.84\,x_2-3.78] \end{aligned} \tag{2.3.16a}$$

$$\phi^{a}(x_1,x_2,x_3) = -15\sqrt{\tfrac{2}{3}}\,|f_N|\,x_1x_2x_3\,[4.63\,(x_1^2-x_3^2)+0.84\,(x_1-x_3)] . \tag{2.3.16b}$$

(Here we have made use of (1.2.17). The normalisation has been fixed by enforcing agreement with the electromagnetic form factor calculation of Ref.1.) The integrations over the light-cone momentum fractions x_i in (2.3.15) are performed using the formula

$$\int_0^1 [dx]\, x_1^{n_1} x_2^{n_2} x_3^{n_3} = \frac{n_1!\, n_2!\, n_3!}{(n_1+n_2+n_3+2)!} , \tag{2.3.17}$$

valid for non-negative integers n_i (i = 1,2,3). The result is

$$S(\uparrow,\uparrow) = 4.02\,|f_N|\, C_1^{(1)} \left\{ 2(M_u^2 + 2M_u M_d) - (M_u^2 + \langle k_{i\perp}^2 \rangle) \right\}. \tag{2.3.18}$$

Consider now the amplitude $S(\uparrow,\downarrow)$. From (2.3.4) and Tables 2.2 and 2.3 we deduce

$$\begin{aligned} S(\uparrow,\downarrow) = -C_1^{(2)} \int_0^1 [dx] \frac{1}{x_1 x_2 x_3} \int (\pi\, dk)\, k_{1\perp}^{+} \\ \{ 2M_u\, x_3\, \psi_{3q}(k_i, \bar{P}, \uparrow, \uparrow, \downarrow) \\ + M_d\, x_2\, \psi_{3q}(k_i, \bar{P}, \uparrow, \downarrow, \uparrow) \} . \end{aligned} \tag{2.3.19}$$

If we assume

$$\int (\pi\, dk)\, k_{1\perp}^{+}\, \psi_{3q}(k_i, \bar{P}, \lambda_i) \approx \langle k_{1\perp}^{+} \rangle \int (\pi\, dk)\, \psi_{3q}(k_i, \bar{P}, \lambda_i) \tag{2.3.20}$$

then the amplitude vanishes since the mean transverse quark momentum is zero. (Alternatively, in the literature it is common to propose an ansatz for the proton wavefunction which is a function of the squares of the quark transverse momenta and/or scalar products with one another [2,18]. In this case, after a suitable change of variables, if necessary, the amplitude $S(\uparrow,\downarrow)$ vanishes upon integration of an odd function over all momenta).

Using similar reasoning and manipulations we find

$$S(\downarrow,\uparrow) = S(\uparrow,\downarrow) = 0 \tag{2.3.21}$$

$$S(\downarrow,\downarrow) = \frac{C_1^{(2)}}{C_1^{(1)}}\, S(\uparrow,\uparrow) . \tag{2.3.22}$$

These results are the expected ones since, when the coefficients multiplying the baryon number violating operators are isolated, the amplitude for the quark annihilation diagram of Fig. 2.2 must be unaltered when all helicities are reversed. We obtain the desired structure for the proton-positron vertex:

$$i\left(S(\uparrow,\uparrow)\, P_L + S(\downarrow,\downarrow)\, P_R \right) = \frac{i\, S(\uparrow,\uparrow)}{C_1^{(1)}} \left(C_1^{(1)} P_L + C_1^{(2)} P_R \right). \tag{2.3.23}$$

Note however that while the vertex as calculated using the baryon number violating chiral lagrangian $\mathcal{L}^{\Delta B=1}$ (2.2.12) has dimensions of mass the above expression has dimensions of $(\text{mass})^2$. This is because in the evaluation of the matrix elements leading to the amplitudes $S(\uparrow,\uparrow)$ and $S(\downarrow,\downarrow)$ we have included spinors for the incoming and outgoing particles whereas the chiral lagrangian vertex $i\alpha(C_1^{(1)} P_L + C_1^{(2)} P_R)$ has not yet been sandwiched between proton and positron spinors. Before using the result (2.3.23) to compute the pole diagram, therefore, it must be normalised by the matrix elements of the projection operators P_L and P_R between proton and positron spinors. Since

$$\overline{v^c}_\uparrow(k)\, P_L\, p_\uparrow(k) = \overline{v^c}_\downarrow(k)\, P_R\, p_\downarrow(k) = m_p \tag{2.3.24}$$

we deduce that the appropriate normalisation factor is $1/m_p$. Thus we conclude that

$$\alpha = \frac{S(\uparrow,\uparrow)}{C_1^{(1)} m_p}. \tag{2.3.25}$$

With the values $\langle k_{\perp}^2\rangle = (300\ \text{MeV})^2$ and $m_p = 3\hat{M} = 938$ MeV we find

$$\alpha = 0.009\ \text{GeV}^3. \tag{2.3.26}$$

The decay rates and branching ratios for the various two-body decays of a proton into a pseudoscalar meson and an antilepton have already been listed in Table 2.1. The value of the parameter α just found allows us to obtain a numerical estimate for the proton lifetime. Choosing to work with the decay mode $p \to \pi^0 e^+$ we find the rate of decay into this channel to be ($\alpha_X \equiv g_X^2/4\pi = 0.0242$)

$$\Gamma(p \to \pi^0 e^+) = \frac{4.0 \times 10^{-4}}{M_X^4}\ \text{GeV}. \qquad (M_X \text{ in GeV}) \tag{2.3.27}$$

This corresponds to a proton lifetime of

$$\tau(p \to \pi^0 e^+) = 5 \times 10^{31}\ \text{years} \times \left(\frac{M_X}{10^{15}\ \text{GeV}}\right)^4. \tag{2.3.28}$$

With the experimental lower limit $\tau(p \to \pi^o e^+) > 2.5 \times 10^{32}$ years [7] this result leads to the inequality

$$M_x > 1.5 \times 10^{15} \text{ GeV}. \tag{2.3.29}$$

These conclusions are to be compared with those of Brodsky et al [2], who use the same method of calculation. They adopt the following ansatz for the proton wavefunction:

$$\psi_{3q}(x_i, k_{i\perp}, \bar{p}, \lambda_i) = B \exp\left[-b^2 \sum_{i=1}^{3} \left(\frac{k_{i\perp}^2 + \hat{M}^2}{x_i}\right)\right]. \tag{2.3.30}$$

The parameters B and b^2 are constrained by using various pieces of experimental information (including nucleon magnetic form factor and J/ψ decay data). The proton distribution amplitude is a completely symmetric function of the light-cone momentum fractions. The parameter α is found to be

$$\alpha = \frac{3\hat{M}^2}{m_p} \int_0^1 [dx] \frac{1}{x_1 x_2} \int (\pi\, dk)\, \psi_{3q}(x_i, k_{i\perp}, \bar{p}, \lambda_i)$$
$$= 0.022 \text{ GeV}^3. \tag{2.3.31}$$

Since the proton lifetime τ varies as α^{-2} we see that by performing the calculation with the distribution amplitude (2.3.16) obtained by Chernyak and Zhitnitsky we obtain an enhancement factor for τ of about 6 when we compare with the result of Ref.2. Note that in (2.3.15) the contribution of the antisymmetric piece (under $x_1 \leftrightarrow x_3$) of the distribution amplitude $\phi^a(x_1, x_2, x_3)$ tends to cancel that of the symmetric function $\phi^s(x_1, x_2, x_3)$, leading to a reduction in the magnitude of the amplitude $S(\uparrow,\uparrow)$. We may argue that a similar analysis will give rise to such a suppression in the decay rate of the proton for all grand unified models consistent with the chiral lagrangian $\mathcal{L}^{\Delta B=1}$ (2.2.9), i.e. for all conventional (non-supersymmetric) GUTS.

Finally, we note that we may perform a similar evaluation of the three quark annihilation diagram of Fig 2.2 using as a trial wavefunction a factorisable form suggested by Isgur and Llewellyn Smith [19]:

$$\psi_{3q}(x_i, k_{i\perp}, \bar{p}, \lambda_i) = C\,(x_1 x_2 x_3)^{\nu} \exp\left[\frac{-(k_{\rho\perp}^2 + k_{\lambda\perp}^2)}{2\,\delta^2}\right]. \tag{2.3.32}$$

Here $k_\rho = \frac{1}{\sqrt{2}}(k_1-k_2)$ and $k_\lambda = \frac{1}{\sqrt{6}}(k_1+k_2-2k_3)$ so that the exponent is a completely symmetric function of the quark transverse momenta. The value $\delta = 0.32$ GeV is thought to best fit the low energy properties of the proton. The normalisation constant C is fixed, by using experimental data on the rate of the decay $J/\psi \rightarrow p\bar{p}$. When the parameter ν is varied in the range 1.0 - 1.5 the calculation of the three quark annihilation matrix elements gives

$$\alpha \sim 10^{-3} - 10^{-4} \text{ GeV}^3, \tag{2.3.33}$$

implying a suppression in the decay rate of the proton by a factor of the order of 10^4 when compared with the calculation of Brodsky et al. However, this model gives very poor agreement with nucleon magnetic form factor data and thus is unreliable for predicting non-perturbative bound state physics.

2.4 THE EFFECT OF AN ASYMMETRIC TRANSVERSE MOMENTUM DISTRIBUTION

In this section we briefly discuss a modification to the proton decay calculation of Section 2.3 to allow for an asymmetric distribution of quark transverse momenta in the proton wavefunction. The analysis of Chapter 1 offered no information on the transverse momentum dependence of the wavefunction. Here we postulate a form for this dependence while ensuring that we retain the distribution amplitude (2.3.16).

To keep the calculations as simple as possible we propose a wavefunction which factorises into the product of two functions; the longitudinal momentum dependence is restricted to the function $\chi(x_i,\bar{p},\lambda_i)$ while the transverse momentum distribution is given by the function $\eta(k_{i\perp},\bar{p})$. Thus we write

$$\psi_{3q}(x_i,k_{i\perp},\bar{p},\lambda_i) = \chi(x_i,\bar{p},\lambda_i)\;\eta(k_{i\perp},\bar{p})\,. \qquad (2.4.1)$$

We see immediately that if we impose the constraints

$$\chi(x_i,\bar{p},\lambda_i) \equiv \phi_{3q}(x_i,\bar{p},\lambda_i) \qquad (2.4.2)$$

$$\int(\pi\, dk)\;\eta(k_{i\perp},\bar{p}) = 1 \qquad (2.4.3)$$

then the corresponding distribution amplitude will coincide with (2.3.16). The form of the function $\eta(k_{i\perp},\bar{p})$ is further restricted by ensuring that the root mean square quark transverse momentum $\langle k_{i\perp}^2\rangle^{1/2}$ is approximately 300 MeV and that the amplitude falls off for increasing transverse momenta. These conditions may be satisfied by choosing appropriate decaying exponential or inverse power law functions, for example. We opt for the former.

The question then arises as to how we should parametrise the asymmetry in the exponent. The simplest way would appear to be to allow only terms quadratic in the momenta and to add to the completely symmetric quantity

$$c_1\left(k_{1\perp}^2 + k_{2\perp}^2 + k_{3\perp}^2\right) + c_2\left(k_{1\perp}\cdot k_{2\perp} + k_{2\perp}\cdot k_{3\perp} + k_{3\perp}\cdot k_{1\perp}\right) \tag{2.4.4}$$

terms of the form

$$a_{12}\,(k_{1\perp} - k_{2\perp})^2 + a_{23}\,(k_{2\perp} - k_{3\perp})^2 + a_{13}\,(k_{1\perp} - k_{3\perp})^2 . \tag{2.4.5}$$

It turns out that such a combination is unnecessarily complicated - only two independent parameters are required to control the asymmetry. If we perform the usual calculation of the three quark annihilation diagram (Fig. 2.2) we find that for the amplitude to be invariant under inversion of all helicities we must impose the condition

$$a_{13} = a_{23} \quad . \tag{2.4.6}$$

Thus in the exponent it is forbidden to have a non-zero coefficient of the quantity

$$(k_{1\perp} - k_{3\perp})^2 - (k_{2\perp} - k_{3\perp})^2 = (k_{1\perp} + k_{2\perp} - 2k_{3\perp}).(k_{1\perp} - k_{2\perp}) . \tag{2.4.7}$$

It is natural then to parametrise the asymmetric terms in the following way:

$$\frac{\rho}{2}\,(k_{1\perp} - k_{2\perp})^2 + \frac{\lambda}{6}\,(k_{1\perp} + k_{2\perp} - 2k_{3\perp})^2 , \tag{2.4.8}$$

and to modify the symmetric transverse momentum distribution (2.3.32) of Ref. 19. Therefore we choose

$$\eta(k_{i\perp}, \bar{P}) = \frac{3(1+\rho)(1+\lambda)(16\pi^2)^2}{4\delta^4} \exp\left\{\frac{-[(1+\rho)k_{\rho\perp}^2 + (1+\lambda)k_{\lambda\perp}^2]}{2\delta^2}\right\} , \tag{2.4.9}$$

where the normalisation has been fixed by satisfying Eq. (2.4.3).

We note in passing that the matrix element involved in the calculation of the decay rate ratio $\Gamma(J/\psi \to p\bar{p})/\Gamma(J/\psi \to \text{all})$ depends only on the proton distribution amplitude [1, 18]. Thus it is unaffected by our choice of transverse momentum distribution. Similar reasoning is valid for predictions of the nucleon electromagnetic form factor [1]. (Use of the Chernyak and Zhitnitsky distribution amplitudes (2.3.16) leads to predictions for both these quantities which are in good agreement with experiment).

We now turn our attention to the evaluation of the three quark annihilation matrix element using the wavefunction defined by Eqs. (2.4.1), (2.4.2) and (2.4.9). In contrast to the two calculations just mentioned the parameter α does depend on the asymmetry coefficients ρ and λ. Recalling Eq. (2.3.10) for the amplitude $S(\uparrow,\uparrow)$ this dependence arises through the terms with the factors

$$I_{12} \equiv \int (\pi\, dk)\, \psi_{3q}(x_i, k_{i\perp}, \vec{p}, \uparrow, \downarrow, \uparrow)\, k^{+}_{1\perp}\, k^{-}_{2\perp} \tag{2.4.10a}$$

$$I_{13} \equiv \int (\pi\, dk)\, \psi_{3q}(x_i, k_{i\perp}, \vec{p}, \uparrow, \downarrow, \uparrow)\, k^{+}_{3\perp}\, k^{-}_{2\perp}\,. \tag{2.4.10b}$$

Now that we have a simple mathematical form for the transverse momentum distribution these integrals may be evaluated without the need to revert to an approximation. We find

$$I_{12} = \frac{-[3(1+\lambda)-(1+\rho)]\,\delta^2}{3\,(1+\rho)(1+\lambda)} \tag{2.4.11a}$$

$$I_{23} = \frac{-2\,\delta^2}{3\,(1+\lambda)}\,. \tag{2.4.11b}$$

Eq. (2.3.15) is modified to

$$\begin{aligned} S(\uparrow,\uparrow) = C_1^{(1)} \int_0^1 [dx]\, \frac{1}{x_1 x_2} & \Big\{ \Big[M_u^2 + 2M_u M_d + \frac{(\lambda-\rho)\,\delta^2}{(1+\lambda)(1+\rho)} \Big] \phi^s(x_1,x_2,x_3) \\ & - \sqrt{3}\Big[M_u^2 + \frac{(3(1+\lambda)+(1+\rho))\,\delta^2}{3(1+\lambda)(1+\rho)} \Big] \phi^a(x_1,x_2,x_3) \Big\} \end{aligned} \tag{2.4.12}$$

while we still have

$$S(\uparrow,\downarrow) = S(\downarrow,\uparrow) = 0. \tag{2.4.13}$$

Performing the integrations over the light-cone momentum fractions in (2.4.12) we deduce that

$$\alpha \equiv 0.011 - \frac{(32-22\lambda+54\rho)\times 10^{-4}}{(1+\lambda)(1+\rho)}\ \mathrm{GeV}^3. \tag{2.4.14}$$

What restrictions may be placed on the values of the parameters ρ and λ ? Clearly we require

$$\rho > -1 \quad , \quad \lambda > -1 \tag{2.4.15}$$

to ensure that the magnitude of the wavefunction falls off as the transverse momenta increase. Further information may be gained from a determination of the probability P_{3q} for finding the valence three quark Fock state in the proton. This quantity is defined as follows:

$$P_{3q} = \int_0^1 [dx] \int (\pi\, dk) \left| \psi_{3q}(x_i, k_{i\perp}, \bar{p}, \lambda_i) \right|^2 . \tag{2.4.16}$$

A calculation of P_{3q} requires a knowledge of the proton wavefunction rather than the distribution amplitude. Thus, unless we assume an explicit form for the transverse momentum dependence, the constraint $0 \leqslant P_{3q} \leqslant 1$ cannot be checked for the distribution amplitude (2.3.16). With the factorised wavefunction defined above, we find

$$\begin{aligned} P_{3q} &= \int_0^1 [dx] \Big\{ \tfrac{1}{6} [\phi^s(x_1,x_2,x_3)]^2 + \tfrac{1}{2} [\phi^a(x_1,x_2,x_3)]^2 \\ &\qquad + \tfrac{1}{6} [\phi^s(x_2,x_1,x_3)]^2 + \tfrac{1}{2} [\phi^a(x_2,x_1,x_3)]^2 \\ &\qquad + \tfrac{2}{3} [\phi^s(x_1,x_3,x_2)]^2 \Big\} \cdot \int (\pi\, dk) \left| \eta(k_{i\perp}, \bar{p}) \right|^2 \\ &= 2\cdot 23 \,(1+\rho)(1+\lambda) \quad , \end{aligned} \tag{2.4.17}$$

which implies the inequality

$$(1+\rho)(1+\lambda) \leqslant 0\cdot 45 \ . \tag{2.4.18}$$

(We see that a negative probability is already excluded by the conditions (2.4.15)).

Similarly a direct evaluation of the root mean square quark transverse momentum $\langle k_{i\perp}^2 \rangle^{\frac{1}{2}}$ is possible:

$$\begin{aligned} \langle k_{i\perp}^2 \rangle^{\frac{1}{2}} &\equiv \left(\int_0^1 [dx] \int (\pi\, dk)\, k_{i\perp}^2 \left| \psi_{3q}(x_i, k_{i\perp}, \bar{p}, \lambda_i) \right|^2 \right)^{\frac{1}{2}} \\ &= 195 \, [3(1+\lambda) + (1+\rho)]^{\frac{1}{2}} \ \text{MeV} . \end{aligned} \tag{2.4.19}$$

Let us restrict the value of $\langle k_{i\perp}^2\rangle^{\frac{1}{2}}$ to the interval 300 MeV $< \langle k_{i\perp}^2\rangle^{\frac{1}{2}} <$ 400 MeV [19]. Recalling the constraints (2.4.15) and (2.4.18) we see that if such a factorised wavefunction is to describe the long distance bound state physics of the proton then the coefficients controlling the asymmetry of the quark transverse momenta should be confined to the regions of (ρ,λ) parameter space indicated in Fig. 2.3.

Note that the special case $\rho = \lambda \neq 0$ corresponds to a symmetric transverse momentum distribution with an appropriate rescaling of the parameter δ.

For the physically acceptable values of the asymmetry coefficients ρ and λ we find that the magnitude of the parameter α is bounded from below:

$$|\alpha| > 0.002 \ \ \mathrm{GeV}^3. \tag{2.4.20}$$

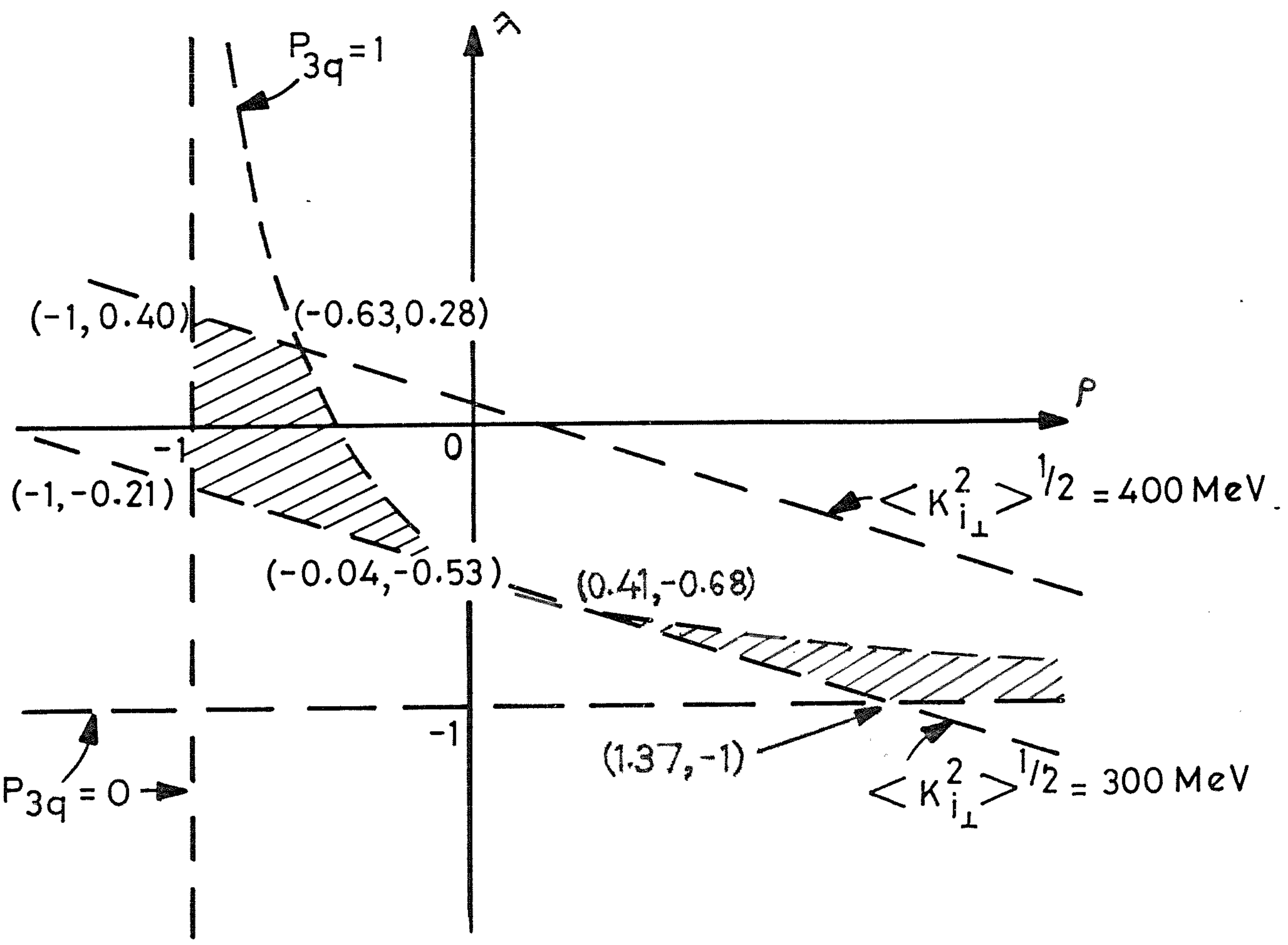
Thus we obtain an upper limit on the proton lifetime of

$$\tau(p \to \pi^\circ e^+) \leq 1.0 \times 10^{33} \ \text{years} \times \left(\frac{M_X}{10^{15}\,\mathrm{GeV}}\right)^4 \tag{2.4.21}$$

using this model.

We see that the physically acceptable values of ρ and λ in this model allow further suppression of the decay rate of the proton beyond that discovered in Section 2.3. Indeed, if we relax our constraint on $\langle k_{i\perp}^2\rangle^{\frac{1}{2}}$, so that the lower limit of the permittedrange is less than 297 MeV, then the shaded regions of Fig. 2.3 merge and we do not obtain an upper limit for the proton lifetime. However, should the true value of P_{3q} be less than $\sim$0.5 then, assuming 300 MeV $< \langle k_{i\perp}^2\rangle^{\frac{1}{2}}$ $<$ 400 MeV, we must conclude that no extra enhancement of the proton lifetime is predicted using this model for the distribution of the quark transverse momenta.

Fig. 2.3 Region of Physically Acceptable Values of The Asymmetry Coefficients ρ and λ



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APPENDIX 1 TWO COMPONENT SPINOR NOTATION

There are two types of two component spinor, called left-handed and right-handed in the literature. They transform under the group SL(2,$\mathbb{C}$) (the group of 2x2 complex unimodular matrices) according to representations which are complex conjugates. Left-handed fields are labelled by an undotted index and right-handed fields by a dotted index. Indices may be raised or lowered using the SL(2,$\mathbb{C}$) invariant antisymmetric tensors $\varepsilon_{\alpha\beta} = \varepsilon^{\alpha\beta} = \varepsilon_{\dot\alpha\dot\beta} = \varepsilon^{\dot\alpha\dot\beta}$.

The relationship with the four component spinor notation is readily seen when the chiral representation is used for the γ-matrices:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix} \tag{A.1.1}$$

$$\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} , \tag{A.1.2}$$

where

$$\sigma^\mu = (1, \sigma^i) = (\sigma^\mu)_{\alpha\dot\alpha} \tag{A.1.3a}$$

$$\bar\sigma^\mu = (1, -\sigma^i) = (\bar\sigma^\mu)^{\dot\alpha\alpha} \tag{A.1.3b}$$

and the σ^i (i = 1,2,3) are the Pauli spin matrices. The projection operators (2.2.17) become

$$P_L = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} , \quad P_R = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} . \tag{A.1.4}$$

A four component spinor may be built using two of the two component spinors:

$$\psi = \begin{pmatrix} L_\alpha \\ R^{\dot\alpha} \end{pmatrix} , \quad \bar\psi = \begin{pmatrix} \bar R^\alpha & \bar L_{\dot\alpha} \end{pmatrix} . \tag{A.1.5}$$

The charge conjugate spinor may also be constructed:

$$\psi^c = \begin{pmatrix} \bar R_\alpha \\ \bar L^{\dot\alpha} \end{pmatrix} , \quad \overline{\psi^c} = \begin{pmatrix} L^\alpha & R_{\dot\alpha} \end{pmatrix} . \tag{A.1.6}$$

For two component spinors A, B, C and D it may be shown that

$$A_L \, \sigma^\mu B_R = - B_R \, \bar{\sigma}^\mu A_L \qquad (A.1.7)$$

$$(A_R B_R)(C_L D_L) = -\tfrac{1}{2} \, (A_R \, \bar{\sigma}_\mu D_L)(C_L \, \sigma^\mu B_R) \, . \qquad (A.1.8)$$

The latter identity is a Fierz transformation.

As an example of how to transcribe from two component to four component notation consider the baryon number violating operator $Q_d^{(1)}$ (2.2.10a). The two component form is

$$Q_d^{(1)} = \varepsilon^{ijk} \, (d_R^i \, u_R^j)(u_L^k \, e_{dL}) \, . \qquad (A.1.9)$$

Performing the Fierz transformation (A.1.8) this becomes

$$\begin{aligned} Q_d^{(1)} &= -\tfrac{1}{2} \, \varepsilon^{ijk} \, (d_R^k \, \bar{\sigma}_\mu \, e_{dL})(u_L^j \, \sigma^\mu \, u_R^i) \qquad (A.1.10) \\ &= -\tfrac{1}{2} \, \varepsilon^{ijk} \, (u_R^i \, \bar{\sigma}^\mu \, u_L^j)(e_{dL} \, \sigma_\mu \, d_R^k) \, , \end{aligned}$$

by (A.1.7). Using Eqs.(A.1.1), (A.1.4), (A.1.5) and (A.1.6) we deduce the equivalent four component form:

$$Q_d^{(1)} = -\tfrac{1}{2} \, \varepsilon^{ijk} \, (\overline{u_L^c}^{\,i} \, \gamma^\mu \, u_L^j)(\overline{e^+_{dR}} \, \gamma_\mu \, d_R^k) \, . \qquad (A.1.11)$$

APPENDIX 2 DIRAC SPINORS IN THE HELICITY FORMALISM

Four component spinors for particles and antiparticles may be constructed from eigenstates of the projection operators

$$\Lambda_+ \equiv \tfrac{1}{2}\left(1 - \gamma^3\gamma^0\right) \tag{A.2.1a}$$

$$\Lambda_- \equiv \tfrac{1}{2}\left(1 + \gamma^3\gamma^0\right) \tag{A.2.1b}$$

The Dirac spinors for a particle with momentum p and mass M are

$$\left.\begin{array}{l} u_\uparrow(p) \\ u_\downarrow(p) \end{array}\right\} = \frac{1}{\sqrt{p^+}}\left(p^+ 1 + M\gamma^0 + p^1\gamma^0\gamma^1 + p^2\gamma^0\gamma^2\right)\left\{\begin{array}{ll} \chi(\uparrow) & \text{(A.2.2a)} \\ \chi(\downarrow) & \text{(A.2.2b)} \end{array}\right.$$

while those for an antiparticle are

$$\left.\begin{array}{l} v_\uparrow(p) \\ v_\downarrow(p) \end{array}\right\} = \frac{1}{\sqrt{p^+}}\left(p^+ 1 - M\gamma^0 + p^1\gamma^0\gamma^1 + p^2\gamma^0\gamma^2\right)\left\{\begin{array}{ll} \chi(\downarrow) & \text{(A.2.3a)} \\ \chi(\uparrow), & \text{(A.2.3b)} \end{array}\right.$$

where $\chi(\uparrow) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ and $\chi(\downarrow) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$ are

eigenstates of Λ_+. The spinors are helicity eigenstates in the $p^3 \to \infty$ frame.

With the Dirac representation for the γ-matrices,

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \tag{A.2.4}$$

we find the following explicit forms for the spinors:

$$u_\uparrow(p) = v^c_\uparrow(p) = \frac{1}{\sqrt{2p^+}}\begin{pmatrix} p^+ + M \\ p_\perp^+ \\ p^+ - M \\ p_\perp^+ \end{pmatrix} \tag{A.2.5a}$$

$$u_{\downarrow}(p) = v^c_{\downarrow}(p) = \frac{1}{\sqrt{2p^+}}\begin{pmatrix} -p_{\perp}^- \\ p^+ + M \\ p_{\perp}^- \\ -p^+ + M \end{pmatrix} \tag{A.2.5b}$$

$$v_{\uparrow}(p) = u^c_{\uparrow}(p) = \frac{1}{\sqrt{2p^+}}\begin{pmatrix} -p_{\perp}^- \\ p^+ - M \\ p_{\perp}^- \\ -p^+ - M \end{pmatrix} \tag{A.2.5c}$$

$$v_{\downarrow}(p) = u^c_{\downarrow}(p) = \frac{1}{\sqrt{2p^+}}\begin{pmatrix} p^+ - M \\ p_{\perp}^+ \\ p^+ + M \\ p_{\perp}^+ \end{pmatrix} \tag{A.2.5d}$$

$$\bar{u}_{\uparrow}(p) = \overline{v^c}_{\uparrow}(p) = \frac{1}{\sqrt{2p^+}}\left(p^+ + M \quad p_{\perp}^- \quad -p^+ + M \quad -p_{\perp}^-\right) \tag{A.2.6a}$$

$$\bar{u}_{\downarrow}(p) = \overline{v^c}_{\downarrow}(p) = \frac{1}{\sqrt{2p^+}}\left(-p_{\perp}^+ \quad p^+ + M \quad -p_{\perp}^+ \quad p^+ - M\right) \tag{A.2.6b}$$

$$\bar{v}_{\uparrow}(p) = \overline{u^c}_{\uparrow}(p) = \frac{1}{\sqrt{2p^+}}\left(-p_{\perp}^+ \quad p^+ - M \quad -p_{\perp}^+ \quad p^+ + M\right) \tag{A.2.6c}$$

$$\bar{v}_{\downarrow}(p) = \overline{u^c}_{\downarrow}(p) = \frac{1}{\sqrt{2p^+}}\left(p^+ - M \quad p_{\perp}^- \quad -p^+ - M \quad -p_{\perp}^-\right) \tag{A.2.6d}$$

The notation $p_{\perp}^{\pm} = p^1 \pm i p^2$ has been used.

CHAPTER 3

THE CHIRAL LAGRANGIAN APPROACH TO PROTON DECAY WITH A SYMMETRIC BARYON WAVEFUNCTION

In the last chapter we estimated the proton lifetime by using a lagrangian based on chiral $SU(3)_L \times SU(3)_R$ symmetry and evaluating the three-quark annihilation diagram with an asymmetric proton wavefunction. The computation was a refinement of that carried out by Brodsky and co-workers [1] who performed the calculation with a wavefunction which was symmetric in the quark momenta. Here we consider again the work of Ref. 1 and ask to what extent it is consistent to use a symmetric baryon wavefunction when calculating with the chiral lagrangian of Section 2.2. To this end we evaluate, using the wavefunction (2.3.30) and its analogues for other $J^P = \frac{1}{2}^+$ baryons, all nine baryon $\to$ antilepton annihilation amplitudes which occur in the pole diagrams for the various two-body decay modes of the proton. This problem is tackled in two stages:

(i) We use one wavefunction (with equal quark masses) for the proton, neutron and the Σ^+, Σ^0 and Λ hyperons and neglect quark mass differences in the matrix elements of the baryon number violating operators. We expect the baryon $\to$ antilepton amplitudes to be in proportion to the corresponding vertices calculated from the chiral lagrangian $\mathcal{L}^{\Delta B=1}$. In other words, when SU(3) symmetry breaking effects are neglected, comparison of the two sets of amplitudes should imply a unique value for the parameter α. This is what we mean by consistency.

(ii) We adjust both sets of calculations to allow for the effects of explicit SU(3) symmetry breaking. Mass terms are introduced into the baryon number violating lagrangian $\mathcal{L}^{\Delta B=1}$ and we look for consistency with the calculation of the three-quark annihilation amplitudes when we allow quark mass differences in the operator matrix elements. An obvious difficulty must be overcome before we may compare the results of the two sets of calculations: the quark masses in the chiral lagrangian are current masses whereas those occurring in the exponent of the baryon wavefunctions are constituent masses. An application of chiral perturbation theory [2], in which baryon masses are expressed as expansions in the short-distance masses of the light quarks, is found to be helpful in relating the two formalisms. Once we have a consistent framework for dealing with the effects of SU(3) symmetry breaking it becomes possible to estimate quantitatively the resulting corrections to the two-body decay rates of the proton.

Note that we could not hope to extract a single value for the parameter α if we were to use asymmetric baryon distribution amplitudes and no other SU(3) breaking effects. Asymmetries in the quark momenta could in principle arise from derivative terms in a chiral lagrangian formulated in terms of quark fields. However, this cannot be achieved using the chiral lagrangian of Section 2.2. It is constructed in the hadron basis and so any derivatives will act on the baryon as a whole, giving no information on the distribution of momentum among the constituent quarks.

Throughout, the calculations are based on a determination of the baryon $\rightarrow$ antilepton annihilation amplitude. Therefore we are interested only in the consequences of adding mass terms to the baryon number violating piece ($\mathcal{L}^{\Delta B=1}$) of the chiral lagrangian. We do not account for SU(3) symmetry breaking effects arising from the terms of $\mathcal{L}_1$((2.2.7)).

We restrict ourselves to an analysis of the minimal SU(5) GUT by retaining just the non-zero coefficients of Eq. (2.2.22).

3.1 INCLUSION OF LINEAR MASS TERMS IN THE CHIRAL LAGRANGIAN

In this section we introduce explicit quark mass terms into the baryon number violating chiral lagrangian $\mathcal{L}^{\Delta B=1}$. Such terms additonally break the chiral $SU(3)_L \times SU(3)_R$ symmetry.

We must ensure that when we include factors of the quark mass matrix m we do not destroy the $SU(3)_L \times SU(3)_R$ transformation properties of the various terms in $\mathcal{L}^{\Delta B=1}$. In general this implies that powers of m must not be inserted adjacent to the matrix B of baryon fields. This restriction arises because of the complicated non-linear transformation (2.2.5) of B under $SU(3)_L \times SU(3)_R$. To illustrate, consider the operator $Q_d^{(1)}$ (2.2.10a). In the hadron basis the corresponding term in the lagrangian $\mathcal{L}^{\Delta B=1}$ is (see Eq. (2.2.12))

$$\alpha \sum_{d=1}^{2} C_d^{(1)} \; e_{dL} \; \mathrm{Tr}\; O\xi \; B_L \; \xi \quad + \quad \mathrm{h.c.} \tag{3.1.1}$$

The combined operation of premultiplying by the matrix O and taking the trace ensures that only the relevant component, in this case the component in the first row and third column, of the $SU(3)_L \times SU(3)_R$ representation is projected out. The term (3.1.1) transforms according to $(3_L, \bar{3}_R)$ since

$$\xi \; B \; \xi \;\; \rightarrow \;\; L \; \xi \; B \; \xi \; R^{+} \; . \tag{3.1.2}$$

We may add to the chiral lagrangian the terms

$$\alpha \sum_{d=1}^{2} (K_d^{(1)} \; e_{dL} \; \mathrm{Tr}\; Om\xi B_L \xi \; + \; L_d^{(1)} \; e_{dL} \; \mathrm{Tr}\; mO\xi B_L \xi) \; + \; \mathrm{h.c.} \tag{3.1.3}$$

since these do not alter the chiral transformation properties. $K_d^{(1)}$ and $L_d^{(1)}$ are model-dependent constants. Thus the term in $\mathcal{L}^{\Delta B=1}$ associated with the baryon number violating operator $Q_d^{(1)}$ becomes

$$\alpha \sum_{d=1}^{2} T_d^{(1)} \; e_{dL} \; \mathrm{Tr} O\xi B_L \xi \quad + \; \mathrm{h.c.}, \tag{3.1.4}$$

where the constant $T_d^{(1)}$ is given by

$$T_d^{(1)} \; = \; C_d^{(1)} \; + m_u \; K_d^{(1)} \; + m_s \; L_d^{(1)} \; . \tag{3.1.5a}$$

We observe that the inclusion of linear mass terms in $\mathcal{L}^{\Delta B=1}$ has the effect of adjusting only the coefficient multiplying the operator.

A similar analysis is valid for the other operators listed in Section 2.2. For those transforming like $(3_L, \bar{3}_R)$ or $(\bar{3}_L, 3_R)$ the modifications to the coefficients are

$$C_d^{(2)} \to T_d^{(2)} = C_d^{(2)} + m_u K_d^{(2)} + m_s L_d^{(2)} \tag{3.1.5b}$$

$$C_d^{(5)} \to T_d^{(5)} = C_d^{(5)} + m_d K_d^{(5)} + m_s L_d^{(5)} \tag{3.1.5c}$$

$$\tilde{C}_d^{(1)} \to \tilde{T}_d^{(1)} = \tilde{C}_d^{(1)} + m_u \tilde{K}_d^{(1)} + m_d \tilde{L}_d^{(1)} \tag{3.1.5d}$$

$$\tilde{C}_d^{(2)} \to \tilde{T}_d^{(2)} = \tilde{C}_d^{(2)} + m_u \tilde{K}_d^{(2)} + m_d \tilde{L}_d^{(2)} \tag{3.1.5e}$$

$$\tilde{C}_d^{(5)} \to \tilde{T}_d^{(5)} = \tilde{C}_d^{(5)} + m_s \tilde{K}_d^{(5)} + m_s \tilde{L}_d^{(5)} \tag{3.1.5f}$$

$$\tilde{C}_d^{(7)} \to \tilde{T}_d^{(7)} = \tilde{C}_d^{(7)} + \hat{m} \tilde{K}_d^{(7)} + \hat{m} \tilde{L}_d^{(7)} , \tag{3.1.5g}$$

where $\hat{m} \equiv \frac{1}{2}(m_u+m_d)$. This last substitution deserves some comment. The operator $\tilde{Q}_d^{(7)}$ is of interest since it contributes to the $\Sigma^o \to \bar{\nu}_\mu$ and $\Lambda \to \bar{\nu}_\mu$ annihilation amplitudes. The relevant term in the chiral lagrangian is

$$\sum_{d=1}^{2} \tilde{C}_d^{(7)} \tilde{Q}_d^{(7)} = \sum_{d=1}^{2} \tilde{C}_d^{(7)} \varepsilon^{ijk} (\bar{u}^{c\,i} P_R s^j)(\overline{\nu_d^c} P_L d^k), \tag{3.1.6}$$

which becomes

$$-\alpha \sum_{d=1}^{2} \tilde{C}_d^{(7)} \nu_{dL} \ \mathrm{Tr}\ \tilde{O}' \xi B_L \xi \tag{3.1.7}$$

when reformulated in terms of hadron fields. A naive analysis based on the arguments given above for the operator $Q_d^{(1)}$ leads to the substitution

$$\tilde{C}_d^{(7)} \to \tilde{C}_d^{(7)} + m_d \tilde{K}_d^{(7)} + m_d \tilde{L}_d^{(7)} \tag{3.1.8}$$

when linear mass terms are included. However, the operator $\tilde{Q}_d^{(7)}$ has no definite isospin. It may be rewritten as a linear combination of isospin one and isospin zero components:

$$\sum_{d=1}^{2} \tilde{C}_d^{(7)} \tilde{Q}_d^{(7)} = \frac{1}{2} \sum_{d=1}^{2} \tilde{C}_d^{(7)} \varepsilon^{ijk} \Big\{ [(\overline{u^{ci}} P_R s^j)(\overline{\nu_d^c} P_L d^k) + (\overline{d^{ck}} P_R s^j)(\overline{\nu_d^c} P_L u^i)]$$

$$+ [(\overline{u^{ci}} P_R s^j)(\overline{\nu_d^{ci}} P_L d^k) - (\overline{d^{ck}} P_R s^j)(\overline{\nu_d^c} P_L u^i)] \Big\}. \qquad (3.1.9)$$

In a more careful treatment only the first term (which has isospin I=1) should contribute to the $\Sigma^0 \to \bar{\nu}_\mu$ amplitude, while the $\Lambda \to \bar{\nu}_\mu$ vertex should be derived from the second (I=0) term. In the hadron basis expression (3.1.9) becomes

$$\frac{1}{2} \alpha \sum_{d=1}^{2} \tilde{C}_d^{(7)} \Big\{ [-\nu_{dL} \operatorname{Tr} \tilde{O}\xi B_L \xi + \nu_{dL} \operatorname{Tr} \tilde{O}''' \xi B_L \xi]$$

$$+ [-\nu_{dL} \operatorname{Tr} \tilde{O}' \xi B_L \xi - \nu_{dL} \operatorname{Tr} \tilde{O}''' \xi B_L \xi] \Big\}, \qquad (3.1.10)$$

where the projection matrix $\tilde{O}'''$ is defined as

$$\tilde{O}''' = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \qquad (3.1.11)$$

We now deduce that the introduction of linear mass terms is effected by the replacement (3.1.5g). (A similar treatment of the operator $\tilde{Q}_d^{(5)}$ is unnecessary since it has no I=1 component).

The modified expressions for the baryon$\rightarrow$antilepton vertices required for the two-body decays of the proton are displayed in Table 3.1.

We shall investigate the effects of linear mass terms only. Terms of higher order in the current quark masses may be incorporated by extending the above procedure - the general expression occurring in $\mathcal{L}^{\Delta B=1}$, corresponding to the operator $Q_d^{(1)}$, is

$$\alpha \sum_{d=1}^{2} \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} N_d^{(1)}(n_1,n_2)\, e_{dL} \operatorname{Tr} m^{n_1} O\, m^{n_2} \xi B_L \xi + \text{h.c.}, \qquad (3.1.12)$$

where the coefficients $N_d^{(1)}$ are dependent on the non-negative integers n_1 and n_2. As we shall discover later, the inclusion of linear mass terms leads to relatively small corrections and so we have reason to believe that higher order terms may be safely neglected.

Table 3.1 Baryon → Antilepton Vertices from the Chiral Lagrangian When Linear Mass Terms are Included

Baryon → Antilepton Interaction in Pole Diagram	Relevant Decay Mode (s)	Vertex from $\mathcal{L}^{\Delta B=1}$
$p \to e^+$	$p \to \pi^o e^+$, $p \to \eta e^+$	$i\alpha(T_1^{(1)}P_L + T_1^{(2)}P_R)$
$\Sigma^+ \to \mu^+$	$p \to K^o \mu^+$	$-i\alpha(\tilde{T}_2^{(1)}P_L + \tilde{T}_2^{(2)}P_R)$
$n \to \bar{\nu}_e$	$p \to \pi^+ \bar{\nu}_e$	$-i\alpha\ T_1^{(5)}P_L$
$\Sigma^o \to \bar{\nu}_\mu$	$p \to K^+ \bar{\nu}_\mu$	$-\frac{i\alpha}{\sqrt{2}}\tilde{T}_2^{(7)}P_L$
$\Lambda \to \bar{\nu}_\mu$	$p \to K^+ \bar{\nu}_\mu$	$\frac{i\alpha}{\sqrt{6}}\tilde{T}_2^{(7)}P_L$
$\Sigma^+ \to e^+$	$p \to K^o e^+$	$-i\alpha\ \tilde{T}_1^{(2)}P_R$
$p \to \mu^+$	$p \to \pi^o \mu^+$, $p \to \eta\mu^+$	$i\alpha\ T_2^{(2)}P_R$
$\Sigma^o \to \bar{\nu}_e$	$p \to K^+ \bar{\nu}_e$	0
$\Lambda \to \bar{\nu}_e$	$p \to K^+ \bar{\nu}_e$	$-\frac{2i\alpha}{\sqrt{6}}\tilde{T}_1^{(5)}P_L$

Since the inclusion of linear mass terms is accounted for by the replacement of the 'C' coefficients by the corresponding 'T' coefficients we deduce that only the overall amplitudes and decay rates for each mode are altered. (See Table 2.1). In particular, the relative contributions of the two tree diagrams of Fig. 2.1 to the decay rates are not affected. In this respect we differ from the work of Campbell et al [3].

Consider the example of the decay $p \to K^o \mu^+$. We find the contributions of the direct conversion diagram and the pole diagram to the decay amplitude to be in the ratio $1 : \frac{m_p}{m_\Sigma}(D-F)$, even when the corrections due to mass terms are incorporated. Campbell and co-workers write this ratio as $1 : \lambda\,(D-F)$ and argue that the SU(3) symmetry breaking factor λ varies when the effects of mass terms are included. They write the mass terms associated with the operator $Q_d^{(1)}$ in the form

$$\alpha \sum_{d=1}^{2} \sum_{n_1} \sum_{\substack{n_2 \\ n_1+n_2 \geqslant 1}} N_d^{(1)}(n_1,n_2)\, e_{dL}\, \mathrm{Tr}\, m^{n_1}\, O\, m^{n_2}\, B_L + \text{h.c.} \tag{3.1.13}$$

Factors of the matrix ξ of pseudoscalar fields, which are required to preserve the $SU(3)_L \times SU(3)_R$ transformation properties, have been omitted. Terms such as (3.1.13) contribute to the pole diagram but not to the direct conversion diagram and this leads to a change in the value of λ. We agree that the ratio of contributions from the two diagrams may change as a consequence of a discrepancy between the values of the matrix elements $\langle 0|(d_R u_R)\, u_L|p\rangle$ and $\langle 0|(s_R u_R)\, u_L|\Sigma^+\rangle$. However, we believe that a treatment of mass corrections within the chiral lagrangian framework does not provide information on SU(3) symmetry breaking in the three-quark annihilation matrix elements.

Nevertheless the conclusions of Ref. 3 may remain valid. The authors were attempting to see if the inclusion of SU(3) breaking effects in the chiral lagrangian formalism enhanced the rate for the decay $p \to K^o\mu^+$ relative to that for $p \to \pi^o e^+$. We find

$$\frac{\Gamma(p\to K^o\mu^+)}{\Gamma(p\to\pi^o e^+)} = 2\left\{\frac{(\tilde{T}_2^{(1)})^2 + (\tilde{T}_2^{(2)})^2}{(T_1^{(1)})^2 + (T_1^{(2)})^2}\right\}\left(\frac{1 + \frac{m_p}{m_\Sigma}(D-F)}{1 + D + F}\right)^2 \frac{\Delta_K}{\Delta_\pi}\,. \tag{3.1.14}$$

Since the coefficients $\tilde{T}_2^{(1)}$ and $\tilde{T}_2^{(2)}$ are expanded in powers of the u- and d-quark masses only we expect the corrections to be of the order of 1%. Although $T_1^{(1)}$ and $T_1^{(2)}$ depend on m_s the deviations from $C_1^{(1)}$ and $C_1^{(2)}$ should not be greater than 20%. Thus it seems likely that the mode $p \to \pi^o e^+$ will remain dominant in the minimal SU(5) model when the effects of SU(3) symmetry breaking are included. This conjecture is substantiated in Section 3.5.

At first sight it appears that nothing has been gained by writing down mass terms for $\mathcal{L}^{\Delta B=1}$. Many new unknown parameters (the 'K' and 'L' coefficients) have been introduced. Only if we are able to determine these coefficients will any firm predictions on the effects of SU(3) symmetry breaking be possible. In the following sections we examine the possibility of using a refined calculation of the three-quark annihilation diagram to resolve this problem.

3.2 SU(3) SYMMETRY BREAKING IN THE THREE - QUARK ANNIHILATION DIAGRAMS

Here we briefly describe the evaluation of the baryon $\rightarrow$ antilepton interactions occurring in the pole diagrams of the two-body decays of the proton. The assumptions and approximations used are discussed.

As before we follow Brodsky and co-workers in using as our model the three-quark annihilation diagram of Fig. 2.2. We choose the valence three-quark wavefunction for the $J^P = \frac{1}{2}^+$ baryons to take the form of the proton wavefunction of Ref. 1. The effects of SU(3) symmetry breaking are included by allowing differences in the (constituent) quark masses occurring in the matrix elements of the various baryon number violating operators. However, for ease of computation, such differences are excluded from the baryon wavefunctions. This also ensures a symmetric distribution of momentum among the quarks. We write the degenerate baryon wavefunction as

$$\psi^B_{3q}\ (x_i,\ k_{i\perp},\bar{\mu},\lambda_i) \ = \ B_B \exp\left[-b_B^2 \sum_{i=1}^{3}\left(\frac{k_{i\perp}^2+M_q^2}{x_i}\right)\right] . \qquad (3.2.1)$$

M_q ($\approx$ 350 MeV) represents the typical constituent quark mass of a baryon in the $J^P = \frac{1}{2}^+$ octet. The parameters B_B and b_B may be estimated from experimental data but this is inessential for the forthcoming analysis. Performing the integrations over the quark transverse momenta we find

$$\phi^B_{3q}\ (x_i,\bar{\mu},\lambda_i) \ = \ D_B\ x_1x_2x_3 \exp\left[-b_B^2M_q^2\ (\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3})\right] , \qquad (3.2.2)$$

where

$$D_B \ = \ \frac{B_B\ \pi^2}{(16\pi^3)^2\ b_B^4} . \qquad (3.2.3)$$

The evaluation of the three-quark annihilation diagrams using this distribution amplitude is straightforward. The procedure has been outlined in Section 2.3. In the notation used previously,

$$\phi_B^s\,(x_1,x_2,x_3) \quad = \quad \phi_{3q}^B\,(x_i,\bar{\mu},\lambda_i) \tag{3.2.4a}$$

$$\phi_B^a\,(x_1,x_2,x_3) \quad = \quad 0 \quad . \tag{3.2.4b}$$

It follows that the corresponding flavour-spin structures are simply those given by SU(6) symmetry. Once again we must split the operators $\tilde{Q}_{\alpha}^{(5)}$ and $\tilde{Q}_{\alpha}^{(7)}$ into components of definite isospin before evaluating the hadronic matrix elements. Tables 2.2 and 2.3 are sufficiently general to allow the matrix elements of all the baryon number violating operators to be readily deduced. We note that the use of a wavefunction symmetric in quark momenta implies a cancellation of the contributions of terms with explicit transverse momentum dependence to the amplitudes $S(\uparrow,\uparrow)$ and $S(\downarrow,\downarrow)$. Thus these quantities may be expressed directly in terms of the baryon distribution amplitudes without the need to use an approximation such as that in Section 2.3. (See (2.3.13)).

The various three-quark annihilation amplitudes are listed in Table 3.2. They are linear in I_B, where

$$I_B \quad \equiv \int_0^1 [dx] \quad \frac{1}{x_1 x_2} \quad \phi_{3q}^B\,(x_i,\bar{\mu},\lambda_i) \quad , \tag{3.2.5}$$

and, when correctly normalised, inversely proportional to the masses of the intermediate baryons of the pole diagrams.

Now let us compare the entries of Tables 3.1 and 3.2 for the various baryon $\rightarrow$ antilepton amplitudes. When the effects of SU(3) symmetry breaking are neglected we are able to deduce a unique value for the magnitude of the parameter α :

$$|\alpha| \quad = \quad \frac{3\,M_q^2\,I_B}{m_B} \quad , \tag{3.2.6}$$

thereby confirming the consistency, in this limit, of using a symmetric baryon wavefunction in conjunction with the chiral lagrangian approach to proton decay.

We now turn our attention to establishing a link between the patterns of SU(3) symmetry breaking in the baryon $\rightarrow$ antilepton

Table 3.2 Three-Quark Annihilation Amplitudes Calculated Using Symmetric Baryon Wavefunctions

Baryon → Antilepton Interaction in Pole Diagram	Three-Quark Annihilation Amplitude
$p \rightarrow e^+$	$\frac{I_B}{m_p}(M_u^2 + 2M_uM_d)\ i(C_1^{(1)}P_L + C_1^{(2)}P_R)$
$\Sigma^+ \rightarrow \mu^+$	$\frac{I_B}{m_{\Sigma^+}}(M_u^2 + 2M_uM_s)\ i(\tilde{C}_2^{(1)}P_L + \tilde{C}_2^{(2)}P_R)$
$n \rightarrow \bar{\nu}_e$	$\frac{I_B}{m_n}(M_d^2 + 2M_uM_d)\ i\ C_1^{(5)}P_L$
$\Sigma^o \rightarrow \bar{\nu}_\mu$	$-\frac{I_B}{\sqrt{2}m_{\Sigma^o}}(M_uM_d + M_uM_s + M_dM_s)\ i\ \tilde{C}_2^{(7)}P_L$
$\Lambda \rightarrow \bar{\nu}_\mu$	$-\sqrt{\frac{3}{2}}\ \frac{I_B}{m_\Lambda}\ M_uM_d\ i\ \tilde{C}_2^{(7)}P_L$
$\Sigma^+ \rightarrow e^+$	$\frac{I_B}{m_{\Sigma^+}}(M_u^2 + 2M_uM_s)\ i\ \tilde{C}_1^{(2)}P_R$
$p \rightarrow \mu^+$	$\frac{I_B}{m_p}(M_u^2 + 2M_uM_d)\ i\ C_2^{(2)}P_R$
$\Sigma^o \rightarrow \bar{\nu}_e$	0
$\Lambda \rightarrow \bar{\nu}_e$	$\sqrt{\frac{3}{2}}\ \frac{I_B}{m_\Lambda}(M_uM_s + M_dM_s)\ i\ \tilde{C}_1^{(5)}P_L$

vertices predicted by the chiral lagrangian and by the light-cone formalism calculation of the three-quark annihilation amplitudes. Note that these latter amplitudes are proportional to products of pairs of constituent quark masses whereas the modifications to the chiral lagrangian are linearly dependent on current masses. A consistent method of relating these two sets of masses will have to be used if we are to make a meaningful comparison of SU(3) breaking in the two formalisms. First, however, we obtain relations among the unknown constants introduced by linear mass terms into $\mathcal{L}^{\Delta B=1}$ by examining a chiral lagrangian formulated in terms of quark fields [4].

3.3 A CHIRAL LAGRANGIAN INVOLVING QUARK FIELDS

Weinberg [5] and Manohar and Georgi [6] have used the chiral lagrangian formalism to write down an effective field theory describing strong interactions between quarks, gluons and pseudoscalar Goldstone bosons. This has been extended to include baryon number violating processes by Chadha et al [4], who perform a nucleon decay calculation for the minimal SU(5) GUT. As with the chiral lagrangian formulated in terms of baryon fields the chiral quark theory is based on an $SU(3)_L \times SU(3)_R$ flavour symmetry which is spontaneously broken down to an $SU(3)_V$ symmetry, introducing an octet of pseudoscalar Goldstone bosons. The scale of the chiral symmetry breaking, $\Lambda_{\chi SB}$, has been estimated, using both experimental data [7] and theoretical arguments [6], to be of the order of 1 GeV, and is larger than the confinement scale $\Lambda_{QCD} \sim 100 - 300$ MeV. The effective lagrangian in the intermediate region involves fundamental quark and gluon fields together with the pseudoscalar octet. The advantages and problems of the chiral quark approach are discussed in Ref. 6.

The nonlinear realisation of the Goldstone bosons of the spontaneously broken $SU(3)_L \times SU(3)_R$ group is chosen to be that of Section 2.2. (See Eqs. (2.2.1), (2.2.2) and (2.2.3).) The quark fields are introduced as a flavour triplet ψ of Dirac fermions:

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix} . \tag{3.3.1}$$

Under a chiral transformation

$$\psi \rightarrow U\psi \; . \tag{3.3.2}$$

The effective lagrangian between the chiral symmetry breaking and confining scales and invariant under $SU(3)_L \times SU(3)_R$ transformations may be written as

$$\begin{aligned} \mathcal{L}_0^{'CQ'} = {} & \bar{\psi}\,(i\,\not{D} + \not{V})\psi + g_A\,\bar{\psi}\,\not{A}\,\gamma_5\psi - \mathcal{M}\bar{\psi}\psi \\ & + \frac{1}{8}\,f_\pi^2\,\mathrm{Tr}(\partial_\mu\Sigma^+)(\partial^\mu\Sigma) - \frac{1}{2}\,\mathrm{Tr}\,F_{\mu\nu}\,F^{\mu\nu} \\ & + \text{Terms with more derivatives,} \end{aligned} \tag{3.3.3}$$

where D_μ is the QCD covariant derivative (1.1.3a) and

$$V_\mu \equiv \frac{1}{2}(\xi^+ \partial_\mu \xi + \xi \partial_\mu \xi^+) \qquad (3.3.4a)$$

$$A_\mu \equiv \frac{-i}{2}(\xi^+ \partial_\mu \xi - \xi \partial_\mu \xi^+) \quad . \qquad (3.3.4b)$$

The parameters g_A and $\mathcal{M}$ (which represents a contribution to constituent quark masses due to chiral symmetry breaking) take the approximate values 0.75 and 350 MeV respectively.

A chiral symmetry breaking lagrangian with explicit current quark mass terms may be added by analogy with the chiral lagrangian of Ref. 8. However, such terms tend not to contribute significantly to physical processes and we neglect their effects here.

The baryon number violating operators consistent with SU(3) x SU(2) x SU(1) symmetry, $Q^{(i)}_\alpha$ (i = 1, ..,6) and $\tilde{Q}^{(i)}_\alpha$ (i = 1, ...,8), have been listed in Section 2.2. These operators involve the quark fields of the QCD lagrangian. They must be matched across the boundary at the chiral symmetry breaking scale $\Lambda_{\chi SB}$ to all possible operators in the effective theory which have the same chiral transformation properties. Since we consider only those interactions mediated by heavy gauge bosons we wish to construct operators involving quarks and pseudoscalar mesons from the effective theory which transform according to $(\bar{3}_L, 3_R)$ and $(3_L, \bar{3}_R)$. Such operators are given in Ref. 4:

$$D_{ad} = \frac{1}{2}\varepsilon^{ijk}\varepsilon_{efg}(\xi^+)_{ab}(\xi^+)_{gd}[\overline{\psi^{ci}_e}(\alpha_1+\alpha_2\gamma_5)\psi^j_f][\overline{\ell^c}P_R\psi^k_b] \qquad (3.3.5a)$$

$$E_{ad} = \frac{1}{2}\varepsilon^{ijk}\varepsilon_{efg}(\xi)_{ab}(\xi)_{gd}[\overline{\psi^{ci}_e}(\alpha_1-\alpha_2\gamma_5)\psi^j_f][\overline{\ell^c}P_L\psi^k_b], \qquad (3.3.5b)$$

where a,b,d,e,f and g are indices in SU(3) flavour space. The parity operation has been used to leave α_1 and α_2 as the only arbitrary parameters. Using the identity

$$\varepsilon_{abd} N_{ae} N_{bf} N_{dg} = (\det N)\,\varepsilon_{efg} \quad , \qquad (3.3.6)$$

which holds for any 3x3 matrix N, and the unitary property of the matrix U we find

$$D \to R\, D\, L^{+} \tag{3.3.7a}$$
$$E \to L\, E\, R^{+} \tag{3.3.7b}$$

under an $SU(3)_L \times SU(3)_R$ transformation. With the help of the projection operators (2.2.13) the effective baryon number violating lagrangian may be deduced from (2.2.9) via the matching conditions

$$Q_d^{(1)} \to \mathrm{Tr}(O\, E) = E_{13} \tag{3.3.8a}$$
$$Q_d^{(2)} \to \mathrm{Tr}(O\, D) = D_{13} \tag{3.3.8b}$$
$$Q_d^{(5)} \to -\mathrm{Tr}(O' E) = -E_{23} \tag{3.3.8c}$$
$$\tilde{Q}_d^{(1)} \to \mathrm{Tr}(\tilde{O}\, E) = -E_{12} \tag{3.3.8d}$$
$$\tilde{Q}_d^{(2)} \to \mathrm{Tr}(\tilde{O}\, D) = -D_{12} \tag{3.3.8e}$$
$$\tilde{Q}_d^{(5)} \to \mathrm{Tr}(\tilde{O}'' E) = E_{33} \tag{3.3.8f}$$
$$\tilde{Q}_d^{(7)} \to \mathrm{Tr}(\tilde{O}' E) = -E_{22}\ . \tag{3.3.8g}$$

We omit terms with derivatives acting on the quark fields. Such terms are suppressed by powers of $E_q/\Lambda_{\chi SB}$, where E_q represents the typical energy of a constituent quark, and may thus be safely neglected.

The rates for the nucleon $\to$ pseudoscalar meson + antilepton ($N \to P + \bar{\ell}$) decays may be calculated from the diagrams of Fig. 3.1 by using the effective lagrangian.

Now we address the problem of introducing explicit mass terms to the chiral lagrangian $\mathcal{L}^{\Delta B=1}$. The procedure is similar to that of Section 3.1 - powers of the current quark mass matrix m are inserted so as to preserve the chiral transformation properties of the baryon number violating operators. Consider the example of the operator D_{ad}. Because of the nonlinear chiral transformation properties of the effective quark fields, factors of m must not be inserted in such a way that their matrix indices are contracted with the flavour indices of the triplets ψ and $\bar{\psi}^c$. For example, this excludes the term

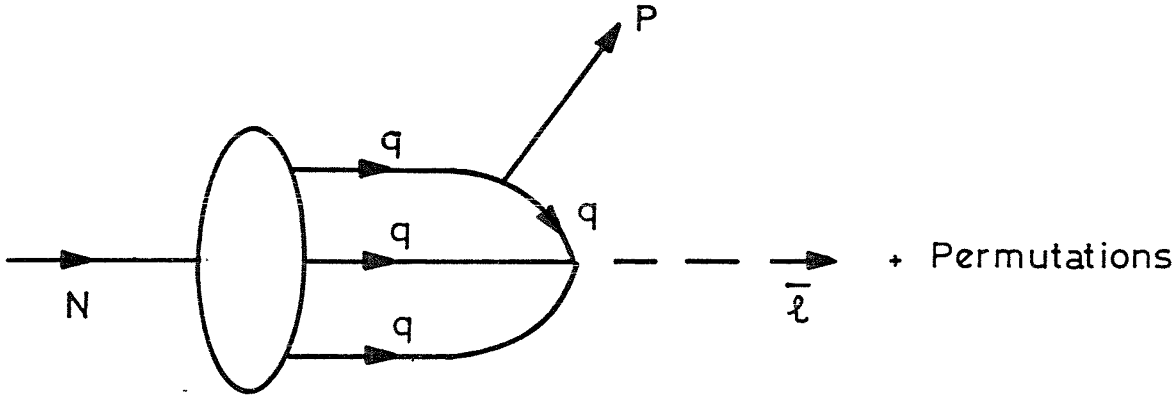
$$\frac{1}{2}\, \varepsilon^{ijk}\, \varepsilon_{efg}\, (\xi^{+}m)_{ab}(\xi^{+})_{gd}[\overline{\psi_e^{ci}}\, (\alpha_1' + \alpha_2'\gamma_5)\psi_f^j][\bar{\ell}^c P_R \psi_b^k]\ . \tag{3.3.9}$$

Also, the term

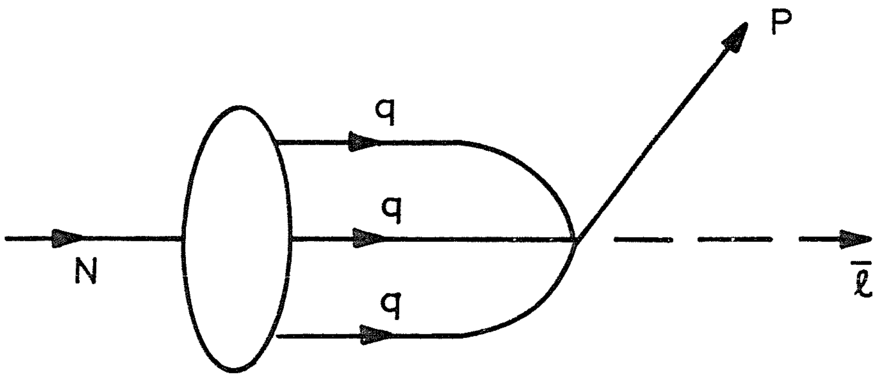
$$\frac{1}{2}\, \varepsilon^{ijk}\, \varepsilon_{efg}\, (\xi^{+})_{ab}\, (m\xi^{+})_{gd}[\overline{\psi_e^{ci}}(\alpha_1'' + \alpha_2''\gamma_5)\psi_f^j][\bar{\ell}^c P_R \psi_b^k] \tag{3.3.10}$$

Fig. 3.1 Diagrams for The Decay $N \to P + \bar{\ell}$ in The Chiral Quark Formalism

(a) Pole Diagram



(b) Non - Pole Diagram



is disallowed since the identity (3.3.6) may not be used. The permitted linear mass terms are

$$\frac{1}{2}\,\varepsilon^{ijk}\,\varepsilon_{efg}\,(m\xi^{+})_{ab}\,(\xi^{+})_{gd}\,[\overline{\psi^{ci}_{e}}\,(\tilde{\beta}_{1}+\tilde{\beta}_{2}\gamma_{5})\psi^{j}_{f}][\overline{\ell^{c}}P_{R}\psi^{k}_{b}]$$
$$+\frac{1}{2}\,\varepsilon^{ijk}\,\varepsilon_{efg}(\xi^{+})_{ab}(\xi^{+}m)_{gd}[\overline{\psi^{ci}_{e}}\,(\tilde{\gamma}_{1}+\tilde{\gamma}_{2}\gamma_{5})\psi^{j}_{f}][\overline{\ell^{c}}P_{R}\psi^{k}_{b}]. \quad (3.3.11)$$

An analogous treatment is applicable to the operator E_{ad}.

Considering each baryon number violating operator in turn we find that the inclusion of (linear) mass terms is effected by appropriate redefinitions of the parameters α_1 and α_2 :

$$Q_d^{(1)},\ Q_d^{(2)}\ :\ \alpha_i \rightarrow \delta_i \equiv \alpha_i + m_u\tilde{\beta}_i + m_s\tilde{\gamma}_i \quad (3.3.12a)$$

$$Q_d^{(5)}\ :\ \alpha_i \rightarrow \varepsilon_i \equiv \alpha_i + m_d\tilde{\beta}_i + m_s\tilde{\gamma}_i \quad (3.3.12b)$$

$$\tilde{Q}_d^{(1)},\ \tilde{Q}_d^{(2)}\ :\ \alpha_i \rightarrow \kappa_i \equiv \alpha_i + m_u\tilde{\beta}_i + m_d\tilde{\gamma}_i \quad (3.3.12c)$$

$$\tilde{Q}_d^{(5)}\ :\ \alpha_i \rightarrow \lambda_i \equiv \alpha_i + m_s\tilde{\beta}_i + m_s\tilde{\gamma}_i \quad (3.3.12d)$$

$$\tilde{Q}_d^{(7)}\ :\ \alpha_i \rightarrow \mu_i \equiv \alpha_i + \hat{m}\tilde{\beta}_i + \hat{m}\,\tilde{\gamma}_i\ ;\ i=1,2 \quad (3.3.12e)$$

Once again the replacement (3.3.12e) follows from a careful treatment of the isospin components of the operator $\tilde{Q}_d^{(7)}$. The requirement of isospin conservation implies that the quarks in the intermediate state of the pole diagram must have an isospin I=1 or I=0.

If we compare the above mass corrections to those appropriate to the chiral lagrangian formulated in terms of baryon fields (Eqs. (3.1.5)) we find linear combinations of the same current quark masses for each operator. Moreover, we observe that in the chiral quark formalism all linear mass corrections are expressed in terms of just four arbitrary parameters, $\tilde{\beta}_i$ (i=1,2) and $\tilde{\gamma}_i$ (i=1,2). This is a manifestation of the greater predictive power of the chiral quark lagrangian noted by Manohar and Georgi [6]. Since our aim was to reduce the number of unknowns introduced into the chiral lagrangian which had baryons as fundamental fields we see that progress may be made if we demand that the mass corrections be added consistently in the two formalisms. Before we can write down definite relations between the various 'K' and 'L' coefficients of (3.1.5) we must be able to link the values of the parameters $\beta_1 (\equiv \frac{\tilde{\beta}_1}{\alpha_1})$ and $\gamma_1 (\equiv \frac{\tilde{\gamma}_1}{\alpha_1})$ to their respective partners β_2 and γ_2. With this in mind we briefly consider the evaluation of the proton decay rate with the chiral quark lagrangian.

Chadha and co-workers [4] use the nucleon wavefunction of the non-relativistic quark model to perform their calculations. For our purposes the choice of wavefunction and the computation of the diagrams of Fig. 3.1 are not crucial. The important thing to note is that, independent of the method of calculation, the decay amplitudes are linear in the coefficients α_1 and α_2. (For the moment we neglect the effects of mass terms.) For the decay modes involving η and K^o, the contributions to the amplitudes from the pole and non-pole diagrams are different functions of both α_1 and α_2. (See Ref. 4.). Hence the ratios of these contributions are dependent on the quantity $r \equiv \frac{\alpha_1}{\alpha_2}$. Such a dependence does not arise in the chiral lagrangian of Section 2.2 where the ratios of the pole and non-pole contributions to the amplitudes are determined by the short distance GUT dynamics [1]. We observed in Section 3.1 that when mass terms are included in this lagrangian the relative contributions of the two tree diagrams remain unaffected. If we now impose the same condition on the amplitudes calculated with the chiral quark lagrangian we deduce the relations

$$r = \frac{\delta_1}{\delta_2} = \frac{\kappa_1}{\kappa_2} \tag{3.3.13}$$

These equations are satisfied when

$$\beta_1 = \beta_2 \equiv k \tag{3.3.14a}$$

and

$$\gamma_1 = \gamma_2 \equiv \ell \tag{3.3.14b}$$

So for the mass corrections to be added consistently in the two formalisms, the following constraints must hold:

$$\frac{K_d^{(1)}}{C_d^{(1)}} = \frac{K_d^{(2)}}{C_d^{(2)}} = \frac{K_d^{(5)}}{C_d^{(5)}} = \frac{\tilde{K}_d^{(1)}}{\tilde{C}_d^{(1)}} = \frac{\tilde{K}_d^{(2)}}{\tilde{C}_d^{(2)}} = \frac{\tilde{K}_d^{(5)}}{\tilde{C}_d^{(5)}} = \frac{\tilde{K}_d^{(7)}}{\tilde{C}_d^{(7)}} = k \tag{3.3.15a}$$

$$\frac{L_d^{(1)}}{C_d^{(1)}} = \frac{L_d^{(2)}}{C_d^{(2)}} = \frac{L_d^{(5)}}{C_d^{(5)}} = \frac{\tilde{L}_d^{(1)}}{\tilde{C}_d^{(1)}} = \frac{\tilde{L}_d^{(2)}}{\tilde{C}_d^{(5)}} = \frac{\tilde{L}_d^{(5)}}{\tilde{C}_d^{(5)}} = \frac{\tilde{L}_d^{(7)}}{\tilde{C}_d^{(7)}} = \ell \tag{3.3.15b}$$

In this way we conclude that the effects of explicit SU(3) symmetry breaking due to the inclusion of linear quark mass terms in the baryon number violating effective lagrangian are controlled by just two arbitrary parameters; k and ℓ. In Table 3.3 the expressions for the baryon $\rightarrow$ antilepton vertices calculated from the chiral lagrangian are rewritten to make this dependence explicit. Note in particular that the inclusion of mass terms ammounts to just a redefinition of the parameter α appropriate to the vertex in question so that the γ-matrix structures of the vertices are not changed. This result has an important consequence - it allows a direct comparison of these values of the vertices with those obtained by evaluating the three quark annihilation amplitudes (see Table 3.2).

Table 3.3 Baryon $\rightarrow$ Antilepton Vertices from the Chiral Lagrangian, Exhibiting the Linear Dependence on Quark Masses

Baryon $\rightarrow$ Antilepton Interaction in Pole Diagram	Vertex from $\mathcal{L}^{\Delta B=1}$
$p \rightarrow e^+$	$i\alpha(1+m_u k+m_s \ell)(C_1^{(1)} P_L + C_1^{(2)} P_R)$
$\Sigma^+ \rightarrow \mu^+$	$-i\alpha(1+m_u k+m_d \ell)(\tilde{C}_2^{(1)} P_L + \tilde{C}_2^{(2)} P_R)$
$n \rightarrow \bar{\nu}_e$	$-i\alpha(1+m_d k+m_s \ell)\ C_1^{(5)} P_L$
$\Sigma^o \rightarrow \bar{\nu}_\mu$	$\frac{-i\alpha}{\sqrt{2}}(1+\hat{m}k+\hat{m}\ell)\ \tilde{C}_2^{(7)}\ P_L$
$\Lambda \rightarrow \bar{\nu}_\mu$	$\frac{i\alpha}{\sqrt{6}}\ (1+\hat{m}k+\hat{m}\ell)\ \tilde{C}_2^{(7)}\ P_L$
$\Sigma^+ \rightarrow e^+$	$-i\alpha(1+m_u k+m_d \ell)\ \tilde{C}_1^{(2)} P_R$
$p \rightarrow \mu^+$	$i\alpha(1+m_u k+m_s \ell)\ C_2^{(2)} P_R$
$\Sigma^o \rightarrow \bar{\nu}_e$	0
$\Lambda \rightarrow \bar{\nu}_e$	$\frac{-2i\alpha}{\sqrt{6}}\ (1+m_s k+m_s \ell)\ \tilde{C}_1^{(5)}\ P_L$

3.4 CHIRAL PERTURBATION THEORY

As we remarked earlier, the main stumbling block in any attempt to link the evaluation of the three quark annihilation amplitudes to the corresponding vertices obtained from the chiral lagrangian is that we require a consistent method for relating constituent (M_q, $q = u,d,s$) and current (m_q) quark masses.

One such procedure is provided by a simple rule of thumb [2]. The two sets of quark masses are seen to differ approximately by a common constant:

$$M_q = m_q + M, \tag{3.4.1}$$

where $M(\approx 300$ MeV) is of the order of the typical scale of the strong interaction. However, an inspection of the expressions for the various baryon number violating vertices presented in Tables 3.2 and 3.3 shows that we require a more general set of relations. For example, we see that the proton $\rightarrow$ positron vertex is dependent on the current u- and s-quark masses whereas the corresponding three quark annihilation amplitude is a function of M_u and M_d only.

A procedure by which hadron masses may be related to current quark masses has been given by Gasser and Leutwyler [2]. They develop a technique in which hadronic energy levels are expanded about the chiral limit $m_u = m_d = m_s = 0$. This expansion in light quark masses is known as chiral perturbation theory. The derivation is outlined below.

As the masses of the u-, d- and s-quarks are small on the scale of the strong interactions it is to be expected that the $SU(3)_L \times SU(3)_R$ symmetry of the QCD lagrangian with massless quarks should be approximately valid in the real world. The deviations from this chiral symmetry may be investigated by treating the quark mass terms as perturbations of massless QCD. Gasser and Leutwyler consider the expansion of any hadronic energy level in powers of the light quark masses. Using the results of first order perturbation theory they deduce the following result for the mass m_n of the hadron n:

$$m_n^2 \;=\; A_n + m_u\, B_n^u + m_d\, B_n^d + m_s\, B_n^s + \ldots , \tag{3.4.2}$$

where A_n represents the square of the hadron mass in the unperturbed system (massless QCD) and the B_n^q coefficients denote the matrix elements of the operators $\bar{q}q$ in the chirally symmetric state; i.e.

$$B_n^q \;=\; \langle n|\,\bar{q}q\,|\,n\,\rangle . \tag{3.4.3}$$

The terms omitted in the expansion (3.4.2) are of two types. First there are the terms in the series obtained from higher orders in the perturbation theory. However, there are also nonanalytic terms of order $(\text{mass})^{3/2}$ [2] to be added to the naive perturbation expansion. These arise when massless Goldstone bosons, which cause infrared divergences in the chiral perturbation theory, are present in the unperturbed system. It was partly because of the complications caused by these latter corrections that we confined our attention to the effects of linear quark mass terms in the baryon number violating effective lagrangian. Gasser and Leutwyler [2] study the effects of the corrections of order $(\text{mass})^{3/2}$ and $(\text{mass})^2$. They conclude that they tend to cancel one another so that the overall correction to the first order formula (3.4.2) is small.

Four unknown coefficients have been introduced in the linear expansion (3.4.2). This procedure may be applied to all hadrons but will lack predictive power unless the A_n and B_n^q parameters for different particles may be inter-related. This is achieved by using the SU(3) flavour symmetry of the QCD lagrangian in the chiral limit. This implies, for example, that

$$B_p^u = B_n^d = B_{\Sigma^+}^u = B_{\Sigma^-}^d = B_{\Xi^0}^s = B_{\Xi^-}^s \tag{3.4.4a}$$

$$B_p^d = B_n^u = B_{\Sigma^+}^s = B_{\Sigma^-}^s = B_{\Xi^0}^u = B_{\Xi^-}^d \tag{3.4.4b}$$

$$B_p^s = B_n^s = B_{\Sigma^+}^d = B_{\Sigma^-}^u = B_{\Xi^0}^d = B_{\Xi^-}^u , \tag{3.4.4c}$$

while the constants A_n must be degenerate within each multiplet of SU(3). With the definition

$$B_p^q \;=\; B^q \tag{3.4.5}$$

the linear mass formulae for the $J^P = \frac{1}{2}^+$ baryon octet are

$$m_p^2 = A + m_u B^u + m_d B^d + m_s B^s + .. \tag{3.4.6a}$$

$$m_n^2 = A + m_u B^d + m_d B^u + m_s B^s + .. \tag{3.4.6b}$$

$$m_{\Sigma^+}^2 = A + m_u B^u + m_d B^s + m_s B^d + .. \tag{3.4.6c}$$

$$m_{\Sigma^-}^2 = A + m_u B^s + m_d B^u + m_s B^d + .. \tag{3.4.6d}$$

$$m_{\Xi^o}^2 = A + m_u B^d + m_d B^s + m_s B^u + .. \tag{3.4.6e}$$

$$m_{\Xi^-}^2 = A + m_u B^s + m_d B^d + m_s B^u + .. \tag{3.4.6f}$$

$$m_{\Sigma^o}^2 = A + \hat{m}(B^u+B^s) + m_s B^d + .. \tag{3.4.6g}$$

$$m_\Lambda^2 = A + \tfrac{1}{3}\hat{m}(B^u+4B^d+B^s) + \tfrac{1}{3} m_s(2B^u-B^d+2B^s) + .. \; . \tag{3.4.6h}$$

Since we include the effects of isospin breaking there is mixing between the unperturbed Σ^o and Λ states. Consequently, the squared masses of these hyperons receive extra corrections of order $(m_u-m_d)^2$, which we neglect.

If we retain only linear quark mass terms, equivalent perturbation expansions may be given for the masses of the baryons. Since $A \neq 0$, we may write

$$m_p = a + m_u b^u + m_d b^d + m_s b^s + .. \tag{3.4.7a}$$

$$m_n = a + m_u b^d + m_d b^u + m_s b^s + .. \tag{3.4.7b}$$

$$m_{\Sigma^+} = a + m_u b^u + m_d b^s + m_s b^d + .. \tag{3.4.7c}$$

$$m_{\Sigma^-} = a + m_u b^s + m_d b^u + m_s b^d + .. \tag{3.4.7d}$$

$$m_{\Xi^o} = a + m_u b^d + m_d b^s + m_s b^u + .. \tag{3.4.7e}$$

$$m_{\Xi^-} = a + m_u b^s + m_d b^d + m_s b^u + .. \tag{3.4.7f}$$

$$m_{\Sigma^o} = a + \hat{m}(b^u+b^s) + m_s b^d + .. \tag{3.4.7g}$$

$$m_\Lambda = a + \tfrac{1}{3}\hat{m}(b^u+4b^d+b^s) + \tfrac{1}{3} m_s(2b^u - b^d + 2b^s) + .., \tag{3.4.7h}$$

with $a = A^{\frac{1}{2}}$ and $b^q = \frac{1}{2}A^{-\frac{1}{2}}B^q$. The two sets of formulae differ by terms of order $(\text{mass})^2$ and so for our purposes they are equally valid. They both satisfy the Gell-Mann - Okubo relation

$$\tfrac{1}{2}(m_\Sigma + 3m_\Lambda) = m_N + m_\Xi, \tag{3.4.8}$$

where

$$m_\Sigma \equiv m_{\Sigma^o} = \tfrac{1}{2}(m_{\Sigma^+} + m_{\Sigma^-}) \tag{3.4.9a}$$

$$m_N \equiv \tfrac{1}{2}(m_p + m_n) \tag{3.4.9b}$$

$$m_\Xi = \tfrac{1}{2}(m_{\Xi^o} + m_{\Xi^-}), \tag{3.4.9c}$$

and the Coleman-Glashow formulae

$$(m_p - m_n) + (m_{\Xi^o} - m_{\Xi^-}) = (m_{\Sigma^+} - m_{\Sigma^-}) \tag{3.4.10a}$$

$$(m_{\Sigma^+} - m_p) + (m_{\Xi^-} - m_{\Sigma^-}) = (m_{\Xi^o} - m_n) \tag{3.4.10b}$$

$$(m_{\Sigma^-} - m_\Lambda) + (m_{\Xi^o} - m_{\Sigma^+}) = (m_{\Xi^-} - m_p) \tag{3.4.10c}$$

which are all well approximated by the physical masses.

Before we may use these results to correlate the mass corrections to the baryon $\rightarrow$ antilepton annihilation amplitudes we must be able to express the masses of the baryons in terms of their constituent quark masses. Such relations are given by the additive rule:

$$m_p = 2M_u + M_d \tag{3.4.11a}$$

$$m_n = M_u + 2M_d \tag{3.4.11b}$$

$$m_{\Sigma^+} = 2M_u + M_s \tag{3.4.11c}$$

$$m_{\Sigma^-} = 2M_d + M_s \tag{3.4.11d}$$

$$m_{\Xi^0} = M_u + 2M_s \tag{3.4.11e}$$

$$m_{\Xi^-} = M_d + 2M_s \tag{3.4.11f}$$

$$m_{\Sigma^0} = m_\Lambda = M_u + M_d + M_s \quad . \tag{3.4.11g}$$

These formulae are consistent with the perturbation expansions (3.4.7) provided

$$M_u = \frac{1}{3}\,[a + m_u(2b^u - b^d) + (m_d + m_s)b^s + \,..] \tag{3.4.12a}$$

$$M_d = \frac{1}{3}\,[a + m_d(2b^u - b^d) + (m_u + m_s)b^s + \,..] \tag{3.4.12b}$$

$$M_s = \frac{1}{3}\,[a + m_s(2b^u - b^d) + (m_u + m_d)b^s + \,..] \tag{3.4.12c}$$

and

$$b^u + b^s = 2b^d. \tag{3.4.13}$$

These results provide a direct link between constituent and current quark masses.

3.5 THE CONSISTENCY OF THE CHIRAL LAGRANGIAN WITH THE PROTON DECAY CALCULATION

Having expressed the constituent light quark masses as perturbation expansions in the current masses we are now in a position to study the consistency of the chiral lagrangian formalism with the calculation of the three-quark annihilation amplitudes using symmetric baryon wavefunctions.

Using the results of Section 3.4 it is interesting to note that the combinations of constituent quark masses in which we are interested, namely those of Table 3.2, are simply related to the relevant baryon masses. We find

$$M_u^2 + 2M_uM_d = \frac{1}{3}m_p^2 \tag{3.5.1a}$$

$$M_u^2 + 2M_uM_s = \frac{1}{3}m_{\Sigma^+}^2 \tag{3.5.1b}$$

$$M_d^2 + 2M_uM_d = \frac{1}{3}m_n^2 \tag{3.5.1c}$$

$$M_uM_d + M_uM_s + M_dM_s = \frac{1}{3}m_{\Sigma^0}^2 = \frac{1}{3}m_\Lambda^2 \quad . \tag{3.5.1d}$$

These equations hold only when terms quadratic and of higher order in the current masses are neglected. It now follows that the three quark annihilation amplitudes are proportional to the masses of the intermediate baryons of the pole diagrams. (But see below for a comment on the $\Lambda \to \bar{\nu}_\mu$ and $\Lambda \to \bar{\nu}_e$ vertices).

An examination of the expressions for the vertices involving the proton, neutron and Σ^+ and Σ^0 hyperons in Tables 3.2 and 3.3 leads to the following conditions for consistency:

$$|\alpha| = \frac{a\, I_B}{3} \tag{3.5.2}$$

$$k = \frac{b^u}{a} \tag{3.5.3a}$$

$$0 = b^d \tag{3.5.3b}$$

$$\ell = \frac{b^s}{a} \quad . \tag{3.5.3c}$$

The same constraints are required to equate the sums of the amplitudes for the $\Lambda \to \bar{\nu}_\mu$ and $\Lambda \to \bar{\nu}_e$ vertices. It should not surprise us that the individual amplitudes may not be related using the conditions (3.5.2) and (3.5.3) since only the sum depends on m_Λ in the way that the amplitudes for all the other vertices depend on the corresponding baryon masses.

With the vanishing of the coefficient b^d it follows from Eq. (3.4.13) that

$$b^u + b^s = 0 \ . \tag{3.5.4}$$

We now wish to determine the corrections to the branching ratios for the decays of the proton into a pseudoscalar meson and an antilepton which result from inclusion of linear mass terms in the baryon number violating chiral lagrangian. To achieve this we must obtain values for the parameters k and ℓ, which in turn are determined from the coefficients a, b^u and b^s.

By inspecting Eqs. (3.4.7) and (3.5.1) in the chiral limit we deduce that

$$a = 3M_q = m_B \ . \tag{3.5.5}$$

Thus the single value of the parameter $|\alpha|$ which implies consistency for all vertices coincides with that obtained when the effects of SU(3) symmetry breaking are neglected. (See Eq. (3.2.6)).

Using the baryon number conserving part of the chiral lagrangian with fundamental baryon fields $(\mathcal{L}_0+\mathcal{L}_1)$ it is possible to express the baryon masses in terms of the parameters a_1 and a_2 of $\mathcal{L}_1$. The results are

$$m_p = m_B - 2(m_u a_1 + m_s a_2) + \ldots \tag{3.5.6a}$$

$$m_n = m_B - 2(m_d a_1 + m_s a_2) + \ldots \tag{3.5.6b}$$

$$m_{\Sigma^+} = m_B - 2(m_u a_1 + m_d a_2) + \ldots \tag{3.5.6c}$$

$$m_{\Sigma^-} = m_B - 2(m_d a_1 + m_u a_2) + .. \quad (3.5.6d)$$

$$m_{\Xi^0} = m_B - 2(m_s a_1 + m_d a_2) + .. \quad (3.5.6e)$$

$$m_{\Xi^-} = m_B - 2(m_s a_1 + m_u a_2) + .. \quad (3.5.6f)$$

$$m_{\Sigma^0} = m_B - 2\hat{m}(a_1+a_2) + .. \quad (3.5.6g)$$

$$m_{\Lambda} = m_B - \frac{2}{3}(\hat{m} + 2m_s)(a_1+a_2) + .. \quad (3.5.6h)$$

As we mentioned in Section 2.1, with the values $a_1 \approx -0.45$ and $a_2 \approx 0.88$, these formulae give reasonable predictions for the baryon masses. The value $m_B = 1197$ MeV is also required. The relations (3.5.6) are entirely consistent with the chiral perturbation results (3.4.7) provided

$$b^u = -2a_1 \quad (3.5.7a)$$

$$b^d = 0 \quad (3.5.7b)$$

$$b^s = -2a_2 \quad (3.5.7c)$$

It is encouraging to see that the baryon number conserving chiral lagrangian also predicts that the parameter b^d should vanish. However, although the coefficients a_1 and a_2 have opposite signs, they do not sum to zero. It appears that the consistency of the different evaluations of the baryon $\rightarrow$ antilepton vertices imposes an extra constraint ($b^u = -b^s \equiv b$) which is not realised when linear current quark mass formulae are used to interpret the physical baryon masses. However, the condition (3.5.4) has its origins in the use of the additive rule to relate the baryon masses to the masses of the constituent quarks. Suppose that, instead of solving Eqs. (3.4.7) and (3.4.11) to express the constituent quark masses as series expansions in the current masses, we directly interpret the functions of constituent masses occurring in Table 3.2 as being proportional to the squares of the baryon masses. We then arrive at Eqs. (3.5.1) without any constraints on the unknowns b^u, b^d and b^s. There is little motivation for this. Nevertheless, a pattern for the constituent mass combinations of Table 3.2 does emerge. For example, $M_u^2 + 2M_uM_d$ is the sum of the products of all constituent quark mass pairs of the proton. If this quantity is taken as being proportional to m_p^2, the correct constant of proportionality follows from a study of the chiral limit. Similar arguments are valid for the other three-quark annihilation amplitudes although, as before, we must sum the amplitudes for the $\Lambda \to \bar{\nu}_\mu$ and $\Lambda \to \bar{\nu}_e$ vertices.

For the purposes of calculating corrections to the branching ratios of the two-body decays of the proton we use the knowledge gained from a study of the baryon number conserving chiral lagrangian. The adjustments are then controlled by two parameters (b^u and b^s) instead of one(b). With this extra freedom the coefficients may be chosen so that the chiral perturbation expansions give baryon masses which are in closer agreement with experimental values. From the best-fit results

$$a = 1197 \text{ MeV} \tag{3.5.8a}$$

$$b^u \approx 0.90 \tag{3.5.8b}$$

$$b^s \approx -1.76 \tag{3.5.8c}$$

we find the coefficients

$$k \approx 0.8 \times 10^{-3} \text{ MeV}^{-1} \tag{3.5.9a}$$

$$l \approx -1.5 \times 10^{-3} \text{ MeV}^{-1} \quad . \tag{3.5.9b}$$

The above values of the b^q parameters contrast with those which may be deduced from the work of Weinberg [9]. He argues that the value m_s = 150 MeV gives a reasonable fit to the observed SU(3) mass splittings provided that

$$\langle p|\ \bar{s}\ s|p\rangle \approx 0 \tag{3.5.10a}$$

$$\langle \Sigma|\ \bar{s}\ s|\Sigma\rangle \approx \frac{1}{2}\langle \Xi|\ \bar{s}\ s|\Xi\rangle. \tag{3.5.10b}$$

In our notation these assumptions are equivalent to the constraints

$$b^s \approx 0 \tag{3.5.11a}$$

$$b^d \approx \tfrac{1}{2} b^u . \qquad (3.5.11b)$$

When we recall that the matrix elements of the $\bar{q}q$ operators entered as coefficients in the chiral perturbation expansion we may conclude that the conditions (3.5.10) in some sense amount to an identification of constituent and current quarks.

Using the values (3.5.9) together with those for the current quark masses quoted in Section 2.2 we have performed a quantitative investigation of the effects of explicit SU(3) symmetry breaking on the two-body decay rates of the proton in the minimal SU(5) GUT. The results are presented in Table 3.4.

The corrections to the amplitudes obtained in the chiral limit are expected to be of the form $1 + m_q/\mu$, where $\mu(\approx 0.5\text{–}1.0$ GeV) is of the order of the characteristic scale of QCD [10]. With the values (3.5.9) for the parameters k and ℓ, the relative magnitudes of the corrections to the decay amplitudes are in close accord with this rule of thumb. The SU(2) x SU(2) subgroup of the full chiral symmetry is exact when $m_u = m_d = 0$. As these masses are tiny, we expect the resulting deviations from the soft pseudoscalar limit to be small. From Table 3.4 we are able to confirm that these corrections are of the order of 1%, while those due to SU(3) breaking are roughly 10-20%. The fact that the adjustments to the amplitudes are significantly smaller than the uncorrected results is a vindication of the validity of the expansion in powers of current quark masses.

A striking feature of our results is that for all decay modes the inclusion of SU(3) symmetry breaking effects leads to a suppression of the decay rates. This suppression is significant only for those channels in which there are non-vanishing coefficients of m_s in the chiral perturbation expansion. In particular we find an enhancement factor of approximately 1.6 for the proton lifetime $\tau\ (p \to \pi^o e^+)$. Nevertheless, the mode $p \to \pi^o e^+$ remains dominant, and relatively large rates are still predicted for the decays $p \to \pi^+ \bar{\nu}_e$ and $p \to K^o \mu^+$. Only the branching ratio for the mode $p \to K^o \mu^+$ is increased significantly.

Table 3.4 Two-Body Proton Decay Rates and Branching Ratios With SU(3) Symmetry Breaking Effects Included.

Decay Mode	Enhancement Factor for Decay Rate in Minimal SU(5)	Branching Ratio in Minimal SU(5) (%)
$p \to \pi^o e^+$	$(1 + m_u k + m_s \ell)^2 = 0.61$	59.1
$p \to \eta e^+$	$(1 + m_u k + m_s \ell)^2 = 0.61$	0.5
$p \to K^o \mu^+$	$(1 + m_u k + m_d \ell)^2 = 0.98$	14.9
$p \to \pi^+ \bar{\nu}_e$	$(1 + m_d k + m_s \ell)^2 = 0.61$	23.3
$p \to K^+ \bar{\nu}_\mu$	$(1 + \hat{m}k + \hat{m}\ell)^2 = 0.99$	0.8
$p \to K^o e^+$	$(1 + m_u k + m_d \ell)^2 = 0.98$	0.3
$p \to \pi^o \mu^+$	$(1 + m_u k + m_s \ell)^2 = 0.61$	0.6
$p \to \eta \mu^+$	$(1 + m_u k + m_s \ell)^2 = 0.61$	~ 0
$p \to K^+ \bar{\nu}_e$	$(1 + m_s k + m_s \ell)^2 = 0.80$	0.5

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CHAPTER 4

SUMMARY AND CONCLUSIONS

In Chapter 1 an attempt was made to obtain values of the first few moments of the proton distribution amplitudes by applying the technique of QCD sum rules to the operator product expansions of suitably chosen current correlators. Only the lowest twist contributions to the OPE's were included. For most of the sum rules good fits were found to be possible and reliable estimates of the corresponding moments were obtained. The results are displayed in Table 1.9. The moments clearly indicate an asymmetric distribution of longitudinal momentum among the constituent quarks of the proton. The largest part of the proton longitudinal momentum (in the infinite momentum frame) is carried by a u-quark with the same helicity as the proton.

Our results are to be compared with those of Chernyak and Zhitnitsky [1], who analysed the same current correlators. These authors were also able to extract accurate moment values from almost all of their sum rules, and hence deduce explicit expressions for the distribution amplitudes. Their results predict a greater asymmetry in the distribution of quark momenta than that implied by the moments of Table 1.9. Our disagreement with Ref 1 is not one of interpretation in the treatment of the sum rules. We differ about the results of a well defined calculation, that of the Wilson coefficient functions of the OPE's.

The two most obvious sources of uncertainty in our analysis are the omission of non-leading perturbative corrections to the coefficient functions and the neglect of the contributions of higher dimensional operators to the OPE's. Higher order perturbative corrections have been included by Gorskii [2] in the QCD sum rule analysis of meson distribution amplitudes originally performed by Chernyak and Zhitnitsky [3]. In Ref. 2 it is concluded that the resulting corrections to the moments of the pion wavefunction are small. It is to be hoped that a similar calculation for the nucleon, as well as an estimate of the contributions of operators of higher dimension, will be feasible.

Perhaps lattice gauge theory offers the best hope of an accurate determination of the moments of hadronic wavefunctions. Preliminary results for the pion give a much larger value of the second moment [4] than that obtained using the technique of QCD sum rules. Predictions are not yet available for the moments of nucleon wavefunctions. Hopefully improvements in the techniques of lattice gauge theory and increased computational power will lead to more accurate calculations of hadronic parameters in the near future.

Chapter 2 dealt with the application of the distribution amplitude obtained by Chernyak and Zhitnitsky to a calculation of the proton decay rate. While the details of the analysis of Ref. 1 may be disputed there is increasing evidence [5] that an asymmetric distribution of quark momenta may be required to give a correct description of hadronic physics, particularly hard exclusive scattering processes. It must also be remembered that only the lowest twist contributions were included in the OPE's of the current correlators so that the sum rules were sensitive to light-cone physics rather than the short distance physics appropriate to a calculation of nucleon decay matrix elements.

The rate for the decay $p \to \pi^0 e^+$ was evaluated by estimating the proton $\to$ positron three-quark annihilation amplitude and using the chiral lagrangian formalism. The main conclusion of Chapter 2 was that use of the Chernyak and Zhitnitsky distribution amplitude led to an enhancement of the proton lifetime by a factor of about 6 over that obtained using a completely symmetric wavefunction. The contribution of the antisymmetric component of the distribution amplitude to the decay amplitude was of opposite sign to that of the symmetric component, indicating that a qualitatively similar result may arise using other asymmetric wavefunctions. The result obtained (for the minimal SU(5) GUT) was

$$\tau(p \to \pi^0 e^+) = 5 \times 10^{31} \text{ years} \times \left(\frac{M_X}{10^{15}\,\text{GeV}}\right)^4 .$$

This implies that

$$M_X > 1.5 \times 10^{15} \text{ GeV},$$

on the basis of the experimental lower limit $\tau\ (p \to \pi^o e^+) > 2.5 \times 10^{32}$ years established by the Irvine-Michigan-Brookhaven (IMB) group [6]. Calculation gives [7]

$$M_X = (1 \text{ to } 2) \times 10^{15} \times \Lambda_{\overline{MS}} ,$$

where the QCD scale parameter (in the $\overline{MS}$ scheme) $\Lambda_{\overline{MS}}$ is found from experiments and lattice QCD calculations to lie in the range 100 to 400 MeV. This is clearly in conflict with the limit $\Lambda_{\overline{MS}} \gtrsim 1$ GeV inferred from (4.2) and (4.3). While the enhancement of the proton lifetime due to the use of an asymmetric distribution amplitude is insufficient to resolve the discrepancy with experiment it is possible that other GUTS may not be excluded.

A model was also chosen for the distribution of the transverse momentum of the quarks within the proton. It was found that further suppression of the predicted decay rate was possible only when the probability of finding the valence three-quark Fock state in the proton was greater than about 0.5.

In Chapter 3, explicit SU(3) symmetry breaking terms were added to the baryon number violating chiral lagrangian, and the subsequent corrections to the decay rates of the proton into its various decay products were determined. Using chiral perturbation theory, by which combinations of current quark masses may be related to constituent quark masses, it was demonstrated that these corrections were consistent with those obtained by including the effects of SU(3) symmetry breaking in the baryon wavefunctions used by Brodsky et al [8].

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