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The Effect of Particle Elongation on the Strength of Granular Materials

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ABSTRACT

It has long been recognised that the macroscopic mechanical behaviour of a granular material depends on particle shape. However, a systematic investigation into particle shape is lacking. Particle shape is commonly split into the independent categories of form, angularity and roughness. The form of a particle can be quantified using the Longest (L), Intermediate (I) and Shortest (S) dimension of an equivalent scalene ellipsoid; two independent parameters of particle form are defined, termed platyness and elongation.

We use DEM simulations with the Potential Particle Method to investigate the effect of particle form on the friction angle of a granular material at critical state. It is found that deviation of particle form from that of a sphere leads to higher angles of friction at critical state. It is argued that, to some extent, the higher critical state strength exhibited by non-spherical particles is due to form suppressing particle rotation and leading to increased interparticle sliding, a mechanism that in comparison requires more energy to be expended.

Key Words: *Form ; Granular Materials ; DEM ; Ballast ; Shape*

1. Introduction

Particle shape is generally assumed to consist of 3 different independent properties: Form, Angularity and Roughness. Particle form describes the general shape, angularity describes the relative sharpness of angles on the surface and roughness describes the microscopic undulations on particles surface. Particle Form is normally quantified using the longest (L), intermediate (I) and shortest (S) dimensions of the particle, although there is no consensus as to which method is best [1]. Particle form can be characterised by two parameters, Elongation (ζ) and Platyness (α) presented in [4, 5]. Then we use DEM Models made up of particles of a single form and impose triaxial compression conditions to study the effect particle elongation has on the critical state strength. The particles were modelled in an in-house DEM code [2] that uses the concept of potential particles to model arbitrary ellipsoids.

2. Form

In this study we assume form is represented by the particle's longest (L), intermediate (I) and shortest (S) dimension. If we consider L, I and S to be coordinates in a three dimensional space, any particle can be represented by a vector $\mathbf{f}$ linking the origin of the axes to point (L, I, S). Clearly the shape (form) of the particle is represented by the direction of $\mathbf{f}$, whereas the magnitude of $\mathbf{f}$ merely quantifies the size of the particle. By considering the intersection F of $\mathbf{f}$ with the $L+I+S-1=0$ "deviatoric" plane, which is normal to the spherical axis $L=I=S$ (Figure 1), the form of each particle is uniquely defined by the two in-plane coordinates of F in a frame of reference centred at the intersection P of the spherical axis. In this way particle form is essentially quantified as the deviation of a particle's shape from that of a sphere. These two independent parameters of form are given by Equations 1 and are referred to as *platyness*

(α) and *elongation* (ζ) respectively. All possible scalene ellipsoids plot, on the α-ζ plane, within the triangle shown in Figure 1.

$$\alpha = \frac{2(I - S)}{L + I + S}, \qquad \zeta = \frac{L - I}{L + I + S} \tag{1}$$

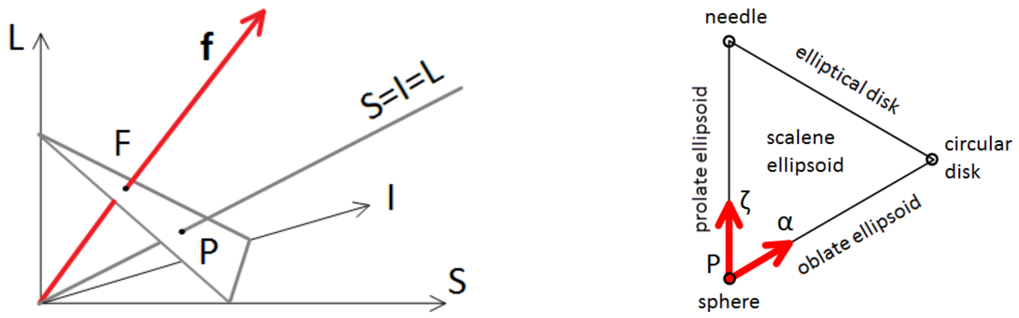


Figure 1: Left: L, I and S space; Right:Elongation and Platyness space with description of forms

Here we focus on the effect of particle elongation by systematically varying ζ, starting from a sphere and conidering increasingly prolate ellipsoid ($L \geqslant I = S$) by increasing the longest radius. Platyness α is kept at 0. The range of forms used is shown in table 1.

Form	Elongation	L : I : S
Form 1	0.000	1.00 : 1.00 : 1.00
Form 2	0.104	1.35 : 1.00 : 1.00
Form 3	0.200	1.75 : 1.00 : 1.00
Form 4	0.305	2.30 : 1.00 : 1.00
Form 5	0.400	3.00 : 1.00 : 1.00

Table 1: Range of forms tested

3. Method

Models were created using particles of a single form and a particle size distribution (PSD) representative of that of railway ballast. To determine the PSD, 5 different sizes between the maximum and minimum gradation curves for railway ballast, keeping I equal to the respective sieve size. To help constrain the models and ensure comparability of results, the total volume of solids contained within each model was kept as close to $0.2m^3$ as possible. Table 2 shows the different model parameters that where used, the material properties where that of a typical granite ballast.

Properties	Value
Particle Density	2700Kg/m^3
Interparticle Friction Angle	30 degrees
Particle Bulk Modulus	50 GPa
Particle Poison's Ratio	0.3

Table 2: Material Properties

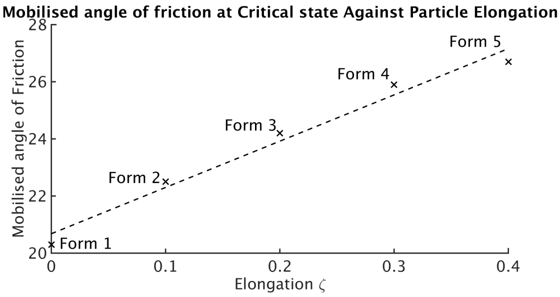
To create a model, a number of particles were randomly dispersed within 3D space to a target initial void ratio of 2.0. The particles themselves were given random orientation to remove any bias in the initial conditions that could affect results. The model was then subjected to isotropic compression using periodic boundaries and zero gravity forces. Once a void ratio of 0.65 was reached, isotropic stress of $100kPa$ was applied to the boundaries and the model was allowed to reach equilibrium. At that point triaxial compression conditions where applied, with stress controlled lateral boundaries at $100kPa$ and a strain controlled top boundary moving downwards at a constant velocity.

4. Results

The strength of a granular assembly is given by the mobilised angle of friction. When a model reaches a critical state the mobilised angle of friction is independent of its initial void ratio. Figure 2 shows how the strength changes as particle form becomes more elongated. It can be seen that spheres have the lowest strength and a linear relationship exists between between increases in elongation and increases in strength.

Quantifying rotations and rotation increments in 3D can be done using quaternions. Here we focus on differences of the amount of rotation between relatively closely spaced time-steps, so that the axis of rotation can be considered unchanged, and divide rotation by the corresponding vertical strain increment to produce a rate of rotation with strain.

Figure 2 plots the median rate of particle rotation against the proportion of sliding contacts for the different particle forms at critical state. Spherical particles rotate much more compared to elongated particles; the rate of rotation decreases with increased elongation. Also, as particle elongation increases there is a reduction in the proportion of sliding contacts at critical state. An inverse relationship is seen between rate of rotation and the proportion of sliding contacts at critical state. Therefore to some extent the higher critical state strength exhibited by elongated particles is due to elongation suppressing particle rotation and leading to increased interparticle sliding, a mechanism that in comparison requires more energy to be expended.



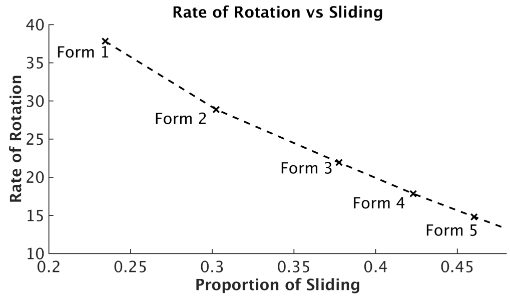


Figure 2: Left: Critical state angle of friction against particle elongation; Right: Median Rate of rotation at critical state against the proportion of contacts sliding

The orientation of elongated particle (prolate ellipsoid), due to symmetry, can be described by a single vector of the direction of the L axis. The average particle orientation with respect to the global model axes can be quantified by a fabric tensor given by Equation 2, [3]. This describes the average orientation of a set of n unit vectors V^k. For randomly oriented vectors the diagonal of G_{ij} will be equal to 0.33 and off-diagonal values will be zero, indicating lack of a preferential direction.

$$G_{ij} = \frac{1}{n}\sum_{k=1}^{n} V_i^k V_j^k \tag{2}$$

Figure 3 shows how the vertical component (G_{33}) of the fabric tensor, corresponding to the L axis of each particle varies with respect to vertical strain. This shows that whilst particles start off with a random orientation, for elongated particles as the model is strained there is a prevalence for the particles to orientate them selves with the L axis perpendicular to the major principal stress. As more particles orientate themselves "flat" there is a layering of particles, leading to a greater number of contacts in the vertical direction. This would provides an efficient skeleton that transfers vertical load, leading to an increase in strength.

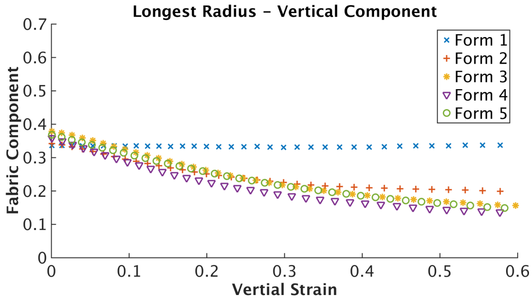


Figure 3: Vertical component of the fabric tensor of L axis

5. Conclusions

In this paper we present a method of characterising form using a scalene ellipsoid and used DEM to investigate the effect of changing particle elongation on critical state strength. It was found that as particle elongation increases there is a similar increase in critical state strength.

This increase in strength accompanied by a reduction in the amount of rotation and an increase in the proportion of sliding contacts. As the model is strained the preference for particles to orientate their longest axis perpendicular to the major principal stress allows for a more efficient way to transfer forces vertically, leading to a stronger assembly.

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