UNIVERSITY OF SOUTHAMPTON

PHOTONIC CRYSTALS IN PLANAR WAVEGUIDES

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In this thesis the properties of waveguide modes in photonic crystal planar waveguides are considered. These are waveguides that have been etched with multi-dimensional gratings to create new wavelength dispersive and spatially dispersive behaviours. Analytical models have been developed for the modes in one and two-dimensional photonic crystal waveguides. These describe many of the rich phenomena that may be observed. Weak two-dimensional photonic crystal planar waveguides have been fabricated and their properties have been measured with a specially developed conical prism coupling technique.

This thesis demonstrates the advantages of combining photonic crystals with planar waveguides. While future lithographic systems will have sufficient resolution to incorporate photonic crystal regions in integrated optical devices, it has been shown that the waveguide geometry increases the actual grating period required for optical band gaps and so lessens technological difficulties. It is also shown that there are stationary modes which could act as microresonators and that ranges of modes can be suppressed in multimode waveguides. Prism coupling has demonstrated the strong dispersive and frequency selective behaviour of weak photonic crystal waveguides.

The future application of this work to efficient, broadband, nonlinear wavelength conversion is proposed.
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1. INTRODUCTION TO PHOTONIC CRYSTALS

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1. Introduction to Photonic Crystals

A photonic crystal (PC) is an artificial composite material with a periodic modulation of the dielectric constant that strongly perturbs optical modes causing photonic band gaps (PBG) to occur. The simplest PC is a periodic dielectric multilayer stack that reflects light at normal incidence with a wavelength satisfying the Bragg condition. There is a range of frequencies and directions around the Bragg condition where propagation of light into or within the material is forbidden. This is called a photonic band gap, by analogy with the solid state physics of crystals where permitted electron energies exist in bands that are separated by band gaps.

Extending the analogy with electronic band gaps, Yablonovitch proposed a material multiply periodic in three dimensions with overlapping PBGs in all directions, termed a full PBG. A PC with a full PBG at microwave frequencies was subsequently demonstrated.

The theoretical analysis of photonic crystals involves the solution of Maxwell’s equations in a periodic medium. With linear materials there is no interaction between photons, which means that very accurate theoretical predictions can be made. Another aspect of Maxwell’s equations is that there is no fundamental length scale, so that a photonic crystal, with a given dielectric constant, designed to operate at microwave frequencies will exhibit the same band structure as a scaled down photonic crystal operating at optical frequencies.

One of the main applications of PC materials will be to form optical cavities. Simply surrounding the cavity region with a PC can do this. Light produced in the cavity cannot propagate into the PC and hence is trapped. The smallest cavity is produced when a single unit cell of the PC is altered. In a 3-D PC the resulting defect will localise light in a volume of order of the optical wavelength cubed. The ultimate cavity would be a microcavity supporting only one single frequency mode. A radiating atom placed in such a cavity would emit only into this mode and in the absence of other radiation mechanisms the quantum efficiency of light emission would be unity.

In this thesis we will consider one and two-dimensional photonic crystals. These are materials that are periodic in one or two dimensions and spatially invariant in the others. These are much easier to fabricate and hence are likely to lead to the first real applications. The feature sizes required are of the order of the optical wavelength divided by the
refractive index and are within the capabilities of UV and X-ray lithography systems currently under development\(^4\).

One and two-dimensional PCs cannot provide confinement in the directions in which they are spatially invariant so, to produce resonant modes that are completely confined, they are formed into waveguiding structures. The waveguide can be designed to support very few guided modes, by making it thin, and this eases the task of the PC in suppressing modes. A planar waveguide mode can also have a large fraction of its wavevector momentum directed out of the plane of propagation, this means that the in-plane wavevector component is smaller and that a larger grating period is required to reflect it. This is an added contribution to the ease of fabrication of optical 2-D PC waveguides.

1.1 Synopsis

In the rest of this chapter the history and background of photonic crystals and photonic band gaps will be described. In chapter 2 we develop several analytical models for the modes in 1-D and 2-D photonic crystal waveguides. These give valuable insights into the behaviour of Bloch waves, which are the natural modes of periodic structures. The method for fabricating a 2-D photonic crystal waveguide is described in chapter 3. This involves the interferometric exposure of gratings in photoresist, which are then transferred to planar tantalum pentoxide waveguides by ion beam etching. A novel prism coupling method was developed to measure the anisotropic properties of the PC waveguide and this is described in chapter 4. The results of prism coupling into 2-D PC waveguides are presented in chapter 5. Conclusions from this work are discussed in chapter 6 and directions for future work are considered.

1.2 Background

There have been many recent publications on photonic band gaps including books\(^1\, ^5\) and special issues of the Journal of the Optical Society of America\(^6\) and the Journal of Modern Optics\(^7\). Joannopoulos\(^8\), Yablonovitch\(^9\) and Russell\(^10\) have reviewed general progress and Pendry\(^11\) and Haus\(^12\) have reviewed theoretical progress. Introductory articles on the related topics of photon localisation\(^13\) and optical microcavities\(^14\) are also recommended. The people working on photonic band gaps come from a wide variety of fields each bringing its own flavour of interpretation and experience. Those from classical optics are
familiar with multilayer dielectric mirrors, birefringent crystals and optical cavities and generally prefer to think about the reflection and refraction of optical rays. There are those from electronic band theory that deal with photonic wave states, band gaps and defect states that act as donors or acceptors. Finally there are those from quantum optics who concern themselves with spontaneous emission, Rabi splitting and strong coupling between single atomic transition states and single photonic cavity states in optical microcavities. There is therefore a rich mixture of interpretation and analogy in describing the properties of photonic crystals.

1.3 One-dimensional Photonic Crystals

There are many conventional applications of electromagnetic gratings, including periodic waveguides\(^{15}\), distributed feedback lasers\(^{16}\), holography\(^{17}\), acousto-optic\(^{18}\), electro-optic\(^{19}\) and x-ray diffraction\(^{20}\), where the gratings consist of weak perturbations about a mean refractive index. The PBG’s in these gratings will be very weak and incomplete, but the wide range of functions and high efficiency of some of these devices offer a tantalizing peek into the future applications of strongly modulated PC’s. The gratings can perform many functions including phase matching, wave coupling, wavefront conversion and wavelength dispersion. The use of holographic design methods can combine many of these functions into a single compact grating device. The theoretical treatment of these weak gratings is based on a perturbation approach that leads to coupled mode equations\(^{16}\) for a finite set of plane waves in the grating.

Not all conventional periodic structures have weak gratings; an application where large PBG’s are present is that of multilayer dielectric stacks which have been used for many years for both high reflection and antireflection coatings on optical components. Molecular beam epitaxy technology enables the manufacture of high quality multilayer coatings with very accurate layer thicknesses. This technology has also been used to grow distributed Bragg reflectors for vertical cavity surface emitting lasers in semiconductor materials\(^{21,22}\).

Strongly modulated gratings cause many multiple reflections of plane waves, which makes the coupled two-wave analysis invalid. Instead we use a translation matrix method\(^{23}\), which allows us to derive the natural modes of the grating which are called Floquet-Bloch modes\(^{24}\). This method is discussed more fully in chapter 2 where it is used to find the modes in a 1-D PC waveguide that is supported on a low index substrate. A similar system
has been modelled by Fan et al\textsuperscript{25} and has been fabricated by Villeneuve et al\textsuperscript{26}. Their system is a channel waveguide suspended in air with a periodic array of holes etched throughout its length. They have considered regular gratings which show complete PBG’s in the guided modes and gratings with a $\lambda/4$ phase shift half way along the waveguide so that a strong Fabry-Perot-like resonance is created. This high-Q resonant mode appears as a defect state within the PBG.

Experiments on weak singly periodic surface relief gratings on planar waveguides have demonstrated many interesting aspects of Bloch modes\textsuperscript{27-29}. While these gratings consist of relatively weak refractive index pertubations, nevertheless they display anomolous and intriguing propagation effects close to their Bragg conditions. By prism coupling it is possible to excite the Bloch modes as rays and observe their propagation. The Bloch rays demonstrate regular beam effects such as refraction and interference but, in addition to this, novel effects such as negative refraction, beam shaping and polarisation dependent beam splitting can be demonstrated. All of these effects are readily explained by use of the wavevector diagram, a useful tool for predicting Bloch wave behaviour, which will be explained in the next chapter.

\subsection*{1.4 Two-dimensional Photonic Crystals}

Zengerle\textsuperscript{29} has investigated weak multiply periodic surface relief gratings on planar waveguides. A square lattice was formed by two sequential interferometric grating exposures with a 90$^\circ$ rotation of the waveguide in between. The grating depth was 75nm in a tantalum pentoxide ($n=2.12$) waveguide of thickness 160nm and consisted of islands of thick waveguide surrounded by thin waveguide. Zengerle was able to measure the detail of the wavevector diagram and then go on to use this to predict ray propagation phenomena. This thesis covers the extension of this work to hexagonal lattices and the fabrication and testing of such structures is described in chapters 3 to 5.

There are two main numerical methods for calculating the band structure of two and three-dimensional PC’s. The first is the k-space or plane-wave expansion method\textsuperscript{30}, based on a Fourier expansion of both the periodic index distribution and the field. The resulting infinite set of algebraic equations is truncated to finite size before the eigenvalues are found. The second, real-space, method\textsuperscript{31} involves discretization of the field and index distributions on a grid of points within the unit cell. These points are grouped into sub-cells, within which the
fields are related by transfer matrices. The subcells are small enough to preclude numerical
instabilities caused by exponentially growing modes. A numerically stable scattering matrix
relates the fields of adjacent subcells.

Much experimental and theoretical work only considers in-plane propagation; this is
propagation normal to the cylinders that form the 2-D PC. In this case the Bloch modes
decouple into tranverse electric (TE) and tranverse magnetic (TM) polarisations. The
biggest in-plane band gap arises with a hexagonal lattice of low index holes in a high index
background\textsuperscript{32,33}. Such structures have been fabricated in a wide variety of materials.
Robertson et al\textsuperscript{34} have measured band structure at microwave frequencies. Complete
PBG’s for individual polarisations in the near infrared have been found by transmission
measurements in microchannel glass arrays\textsuperscript{35,36} and GaAs structures\textsuperscript{37}. A complete PBG at
5\(\mu\text{m}\) has been measured in macroporous silicon\textsuperscript{38} with a lattice constant of 2.3\(\mu\text{m}\) and a
hole depth of 75\(\mu\text{m}\). This large aspect ratio was enabled by a special highly directional
electrochemical etching process. A full photonic bandgap in the visible\textsuperscript{39} has been
demonstrated for surface plasmon polariton modes. These modes are nonradiative TM
modes that propagate at the planar interface between a metal and a dielectric. This means
that a 2-D PBG is sufficient to completely prohibit propagation. The lattice period used
was 458nm and this produced a full PBG for vacuum wavelengths from 620nm to 649nm.

There have been few studies of 2-D photonic crystals that consider propagation out of
plane\textsuperscript{40}. While working on this thesis I have been involved in the fabrication of a photonic
crystal optical fibre\textsuperscript{41}. This is an optical fibre, made out of silica, which has a microscopic
hexagonal array of holes running along its full length. Even though the refractive index
contrast in this structure is only 1.45:1 (fused silica:air) there are full 2-D photonic band
gaps\textsuperscript{42}, for all states of polarisation, for fixed (non zero) values of the longitudinal
component of the wavevector (more commonly known as the propagation constant, \(\beta\).) To
guide light in the fibre a defect has to be placed in the middle of the hexagonal array, this
can take the form of an oversized hole or a missing hole. A fibre with a solid defect has
exhibited novel single mode operation over a spectral range of at least 458nm to 1550nm\textsuperscript{41}
and an effective index model shows that it can be single mode at any wavelength\textsuperscript{43}. By
considering out of plane propagation it is also possible to design 2D PC’s that produce
PBG’s that are to some extent 3-D\textsuperscript{44} in that they will reflect all light incident at any angle on
an external planar interface within a certain frequency range.
Another system where the consideration of out of plane propagation is essential is in thin film 2D PC waveguides\textsuperscript{45} as mentioned earlier. These will be considered more closely in the next chapter where we develop intuitive analytical methods for finding the guided Bloch modes.

### 1.5 Three-dimensional Photonic Crystals

The first 3D PC possessing a complete PBG was fabricated by Yablonovitch et al\textsuperscript{3} at microwave frequencies and is known as “Yablonovite”. The fabrication technique involved covering a slab of dielectric with a mask consisting of a triangular array of holes. Each hole is drilled through three times at an angle $35.26^\circ$ away from the normal and spread $120^\circ$ in azimuth. The relative size of the photonic band gap, i.e. the gap frequency divided by the mid-gap frequency, was found to be around 21%, which agrees well with theory\textsuperscript{46}. Cheng et al\textsuperscript{47} have tried to recreate Yablonovite at 1.5\,\mu m operating wavelengths by using electron-beam lithography to define the air channels but they were only able to create the first few layers of the structure. Several other 3D PC designs offer complete 3D PBG’s\textsuperscript{48-50}. Of these, the woodpile structure\textsuperscript{49} is the smallest 3D PC with an experimentally demonstrated\textsuperscript{51} 3D PBG at wavelengths approaching 600\,\mu m. This was achieved by stacking thin micro-machined silicon wafers. Another new approach is 3D metallodielectric PC’s\textsuperscript{52} that involve the creation of a periodic lattice of isolated metallic regions within a dielectric host. These have been shown, theoretically\textsuperscript{53}, to have enormous omnidirectional bandgaps approaching 80%.

### 1.6 Point Defects In Photonic Crystals - Microcavities

As mentioned earlier, a defect in a PBG material will create a cavity that can trap light. The coupled-wave theory of such cavities in weakly modulated one-dimensional PBG materials is well understood and has been used for many years in the design of distributed feedback lasers\textsuperscript{54}. In strongly modulated structures a matrix transfer method has to be used as developed by Yeh\textsuperscript{23}. Stanley et al. have analyzed a defect in a distributed Bragg reflector structure\textsuperscript{55}. The structure is based on a classic Bragg mirror with alternating $\lambda/4$ layers of GaAs and AlAs. Varying the width of one of the central GaAs layers creates the defect. When the width is $\lambda/4$ the structure functions as a standard DBR and there is a large stop band. As the thickness of the defect is varied a defect state, seen as a narrow transmission
peak, tracks from the high energy stop band edge, across the stop band, to the low energy stop band edge. When the layer is \( \lambda/2 \) the Fabry-Perot condition is satisfied and it can be shown that at this point the spatial extent of the field and the spectral width are both minimised. This strong localisation leads to interesting quantum electro-dynamical effects when radiative atoms are introduced into the impurity layer. Many people consider the rate of spontaneous emission to be a material constant, in fact this is not the case, it is proportional to the vacuum field intensity. When the atom is placed in a high \( Q \) microcavity the vacuum field intensity is enhanced and therefore the lifetime of the atom is decreased.

The enhancement of spontaneous emission from GaAs quantum wells in microcavities has been demonstrated by Yokoyama et al\(^6\). The spontaneous emission intensity along the cavity axis was increased by a factor of 3.6. There was also a decrease of the radiative decay time from 1ns to 0.6ns. Similar results have also been obtained for an \( \text{Er}^{3+} \) doped silica layer enclosed by silicon/silica DBR mirrors\(^22\). The large field enhancements at the centre of semiconductor microcavities can produce strong exciton-photon coupling if the linewidth of the exciton is close to that of the resonant microcavity mode. This leads to Rabi splitting effects, which can be seen as a twin peaked structure in the absorption spectrum of the device close to the resonant frequency\(^57\).

Point defects have also been modelled in 2-D PC's\(^8\). Joannopoulos has shown that as the size of a defect changes then the symmetry of the electric field patterns change and the defect modes can be assigned an orbital angular momentum. This momentum plays a part when considering the selection rules for atomic transitions and it may be possible to engineer the PC to allow transitions that are normally forbidden and to forbid transitions that are normally allowed. This technique could prove useful in the development of novel lasers and light emitting diodes.

Yablonovitch\(^58\) has demonstrated the effect of point defects in three-dimensional PBG materials at microwave wavelengths. When dielectric material is added to a unit cell a defect mode is created near the upper PBG edge. This defect state can be likened to a donor mode in a semiconductor. Similarly when dielectric material is removed an acceptor-like state appears. These acceptor-like states exhibit a very high quality factor, \( Q \), and are particularly well suited to act as laser microresonator cavities. By coupling all the spontaneous emission into a single mode it should be possible to make a thresholdless laser. Yamamoto et al.\(^59\) have proposed semiconductor microlaser structures and De Martini et
al. have demonstrated a thresholdless (pump threshold energy = 50 pJ) microlaser in an active layer of dye enclosed by metallic or multilayer mirrors.\(^6\)

### 1.7 Line Defects In Photonic Crystals – PC Waveguides

If we introduce a line defect into a PC we can guide light from one location to another. One example of this is the photonic crystal fibre.\(^4\) Joannopoulos et al\(^1\) have modelled line defects in two-dimensional photonic crystals. Introducing single rows of defects forms PC waveguides that confine light propagating in plane. The in-plane confinement is provided by the PBG whereas in a regular waveguide the light would be confined by total internal reflection (TIR). The PBG confines light over all angles so that very tight waveguide bends are feasible. 2-D simulations of in-plane propagation\(^6\) indicate that 98% of the power is transmitted through 90° bends in 2D PC. It is hoped that this result will apply equally well to 3D PC.

### References


2. THEORETICAL MODELLING OF PHOTONIC CRYSTAL WAVEGUIDE MODES

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2. Theoretical Modelling of Photonic Crystal Waveguide Modes

2.1 Introduction

In this chapter the theory of one and two-dimensional photonic bandgap waveguides is considered. We develop several analytical models to build up an intuitive appreciation of the behaviour of modes in the periodic structures. Prior analyses on Bragg diffraction in periodic waveguides (e.g. fibre gratings or distributed feedback lasers) start with the assumption that the refractive index modulation is weaker than the index step that forms the waveguide. This allows one to construct a theory based on the coupling of power between a pair of guided modes satisfying a Bragg condition, the essential approximation being that the “strongly” guided modes are resistant to the weaker periodic perturbation. In this chapter, driven by the PBG requirement for large index modulation, we tackle the case where this is no longer a good approximation. Rather than building coupled mode equations from the guided modes of a film of the same average index, we construct the guided modes of the fully etched layer from the Bloch waves of the periodic medium out of which the layer is constructed. The resulting guided Bloch modes contain all the salient features of propagation in the periodic layer, including the photonic band structure, dispersion and group velocity.

Before considering a waveguide grating we will first discover separately the basic properties of a simple homogenous planar waveguide and a one-dimensional grating of infinite extent.

2.2 Basic Waveguide Properties

In order to understand the behaviour of light in waveguides it is necessary to know about the properties of the guided modes. In a uniform dielectric planar slab waveguide the natural modes are composed of plane waves. Cartesian axes are oriented with y along the waveguide layer and z normal to the waveguide layer (Figure 2.1). The modes separate into TM and TE polarisations with $E_x = H_y = H_z = 0$ and $H_x = E_y = E_z = 0$ respectively. In each case, all field components can be expressed in terms of the surviving x component (denoted here by $F$), which itself satisfies a Helmholtz equation in the waveguide:
\[ \nabla^2 F + k_0^2 n^2 F = 0 \quad (2.1) \]

where \( k_0 \) is the free space wavevector and \( n \) is the index of refraction. The waveguide modes can be found by using the zigzag ray model\(^3\). A ray of light bounces to and fro in a waveguide and is reflected at the boundaries by total internal reflection (Figure 2.1). The ray in the waveguide can be represented by the function

\[ F_{wg} = F_{wg} \exp \left( \pm \beta_{wg} z + k_y y \right) \quad (2.2) \]

where \( \beta_{wg} = \sqrt{\left( k_0 n_{wg} \right)^2 - k_y^2} \). At the boundaries the ray couples to evanescent waves which decay into the cover and substrate. These are given by

\[ F_{co} = F_{co} \exp(-\beta_{co} (z - h/2) + jk_y y) \quad (z \geq h/2) \]
\[ F_{ss} = F_{ss} \exp(+\beta_{ss} (z + h/2) + jk_y y) \quad (z \leq -h/2) \quad (2.3) \]

where \( \beta_{co} = \sqrt{k_y^2 - (k_0 n_{co})^2} \), \( \beta_{ss} = \sqrt{k_y^2 - (k_0 n_{ss})^2} \) and \( h \) is the thickness of the waveguide layer. The guided modes are found by requiring continuity of the tangential components of \( E \) and \( H \) at the interfaces, which can be expressed as:

\[ F_{wg} = F_j, \quad \xi_{wg} \frac{dF_{wg}}{dz} = \xi_j \frac{dF_j}{dz} \quad (2.4) \]

where \( j = co \) at \( z = h/2 \) and \( j = ss \) at \( z = -h/2 \). The polarisation dependence is included in the polarisation parameter, \( \xi_j \), which is given by:

\[ \xi_j = 1 \quad (TE) \quad \text{or} \quad 1 / n_j^2 \quad (TM). \quad (2.5) \]

The solution to these equations yields the following dispersion relation:

\[ \tan(\beta_{wg} h + m\pi) = \frac{\beta_{wg} \left( \xi_{ss} \beta_{ss} + \xi_{co} \beta_{co} \right)}{\beta_{wg}^2 - \xi_{ss} \beta_{ss} \xi_{co} \beta_{co}} \quad (2.6) \]

where \( m \) is an integer.
It is useful to define an effective index, \( n_{\text{eff}} = \frac{k_y}{k_0} \), which is a normalised representation of the mode propagation constant, \( k_y \). Figure 2.2 shows a plot of solutions to (2.6) for a tantalum pentoxide waveguide (\( n_{\text{wg}} = 2.12 \)) with a silica substrate (\( n_{\text{ss}} = 1.458 \)) and an air cover (\( n_{\text{co}} = 1 \)). The figure shows the first three sets of modes corresponding to \( m = 0, 1, 2 \). The effective index values are bounded by the waveguide and substrate index values. The range of effective index values is a very important design criterion for the selection of a prism to couple light into the waveguide and will be considered further in the next chapter.

To simplify the observation of mode coupling in a periodically patterned waveguide it is desirable to have just one mode of each polarisation; it can be seen from Figure 2.2 that such a ‘single mode’ waveguide requires a waveguide thickness less than approximately 0.38 times the vacuum wavelength.

In the next section we will consider the basic properties of grating structures, with grating periods, \( \Lambda = 280 \text{nm} \), for which it is convenient to define an effective normalised frequency, \( \nu_{\text{eff}} = \frac{k_y n_{\text{eff}}}{\Lambda} \). When the normalised frequencies for TE and TM modes are compared for typical experimental conditions a simple relationship is found. Figure 2.3 shows the normalised frequency as a function of wavelength for a typical waveguide thickness of 270nm and a titanium sapphire laser tuning range of 775nm to 930nm. The normalised frequencies obey the simple relation.

Figure 2.2 - Modal dispersion in a planar waveguide. (\( n_{\text{wg}} = 2.12 , \quad n_{\text{ss}} = 1.45 , \quad n_{\text{co}} = 1 \))
where $\delta_\nu \approx 0.32$ in this case. This relationship will be very useful in the rest of the chapter when we will be constructing wavevector diagrams for multiply periodic structures.

**2.3 Basic One-dimensional Photonic Band Gap Properties**

In this section we find the dispersion relations for the natural modes of the microstructured material that forms our waveguide layer. In this case the material is an infinite periodic stack of alternating layers of high and low refractive index material and so the natural modes are Bloch waves. The dispersion relations can be plotted in many forms and one of the most useful is the wavevector diagram, which reveals the group velocity direction of the Bloch modes. Finally we will simplify the waveguide problem by reducing the representation of each Bloch wave to two partial waves.
2.3.1 Bloch Waves of an Infinite Periodic Stack

Our starting point is the standard translation matrix technique for a dielectric stack formed from alternating layers of high and low refractive index\(^4,5\). We use this to obtain the dispersion relation for the Bloch waves. Here we present the main steps in the analysis; the details can be found in reference 2.

The dielectric stack consists of alternating layers of refractive index \(n_1\) and \(n_2\) and widths \(h_1\) and \(h_2\), the stack period being \(\Lambda = (h_1 + h_2)\). Cartesian axes are oriented with \(y\) normal to the layer boundaries and \(z\) along the layers (Figure 2.4). No field variation with \(x\) is allowed, which allows separation of the fields into transverse magnetic (TM) and transverse electric (TE) states, with respectively \(E_x = H_y = H_z = 0\) and \(H_x = E_y = E_z = 0\). In each case, all field components can be written in terms of the surviving \(x\)-component, \(f\), which may be expressed in the \(j\)th layer (\(j = 1,2\)) of the \(N\)-th period as:

\[
f_j^N (y) = a_j^N \cos[p_j (y - y_j^N)] + b_j^N \frac{\sin[p_j (y - y_j^N)]}{\xi_j p_j \Lambda}
\]  

(2.8)

where \(a_j^N\) and \(b_j^N\) are constants to be determined, \(y_j^N\) is the value of \(y\) at the centre of the \(j\)-th layer of the \(N\)-th period and \(p_j\) is the wavevector component of the field normal to the interface within each medium:

\[
p_j = \sqrt{k^2 n_j^2 - \beta^2}
\]  

(2.9)

where \(\beta\) is the propagation constant in the \(z\) direction and \(k\) is the vacuum wavevector.

The TE and TM cases are selected via the parameter \(\xi_j\):

\[
\xi_j = 1 \quad (\text{TE}) \quad \text{or} \quad \xi_j = 1 / n_j^2 \quad (\text{TM}).
\]  

(2.10)
The field throughout the stack is completely specified by a two-component state
vector consisting of the constants $a_j^N$ and $b_j^N$. The state vector in one layer is related
to the state vector in the corresponding layer in the previous period by operation with
a $2 \times 2$ translation matrix, $M$:

$$
\begin{pmatrix}
  a_{j+1}^N \\
  b_{j+1}^N
\end{pmatrix}
= M
\begin{pmatrix}
  a_j^N \\
  b_j^N
\end{pmatrix}
= \begin{pmatrix} A & B \\ C & A \end{pmatrix}
\begin{pmatrix}
  a_j^N \\
  b_j^N
\end{pmatrix}
$$

(2.11)

See appendix A for the elements of $M$, and for the elements of the matrix $M_{12}$ relating
the state vector in layer $j = 1$ to the state vector in the neighbouring layer $j = 2$. The
eigenvalues and eigenvectors of $M$ are given simply by:

$$
\lambda_{\pm} = A \pm (BC)^{1/2}, \quad f_{\pm} = \begin{pmatrix} +B^{1/2} \\ \pm C^{1/2} \end{pmatrix}
$$

(2.12)

where $BC = A^2 - 1$ and $|M| = 1$, i.e., $M$ is unimodular. This implies that the product of
the eigenvalues is unity and thus, without loss of generality, that:

$$
\lambda_{\pm} = \exp(\pm j k_y \Lambda), \quad k_y = \frac{\arccos A}{\Lambda}
$$

(2.13)

where $k_y$ is the Bloch wavevector. For a given polarisation state at fixed optical
frequency and $\beta$, the complete field in the structure is expressible as a superposition of
two Bloch waves with field distributions:

$$
f_{\pm}(y) \exp[j \beta z] = B_{\pm}(y) \exp[j(\beta z \pm k_y y)]
$$

(2.14)

where the function $B_{\pm}(y)$ is periodic with period $\Lambda$.

2.3.2 The Wavevector Diagram

The wavevector diagram is a plot of the loci of real wavevectors at fixed optical
frequency in the multilayer stack. It is extremely useful for establishing a clear
graphical understanding of the boundary conditions on either side of the periodic
layer. First the following set of normalised parameters is adopted:

$$
n_{av} = \left( n_1 h_1 + n_2 h_2 \right) / \Lambda
$$

(2.15)

$$
\nu = k n_{av} \Lambda, \quad n_R = n_2 / n_1, \quad \tau = h_2 / \Lambda
$$
where $n_{av}$ is the weighted average index, $\nu$ is the normalised frequency, $n_R$ the index ratio and $\tau$ the relative layer thickness. A series of wavevector diagrams, plotted for a multilayer structure consisting of alternating layers of air and silicon ($n_R = 3.45$) with $\tau = 0.8$, is given in Figure 2.5. For a normalised frequency $\nu = 2$ and TE propagation, the mode index of the Bloch waves is approximately isotropic and equal to the average index, $n_{av}$. The circles repeat in the $y$ direction at intervals of $2\pi/\Lambda$ as a consequence of Bloch's theorem. The TM wavevector diagram on the other hand is elliptical, demonstrating the birefringence of the periodic structure. At a normalised frequency of $\nu = 3$ a momentum gap appears within a certain range of $\beta$ values. In this gap the Bloch waves are evanescent, i.e., if the stack is infinite in extent they cannot exist. The group velocity of the travelling Bloch waves is given by:

$$v_g = \nabla_\kappa \omega(\kappa)$$

(2.16)

Which indicates that $v_g$ is oriented normal to the curves in wavevector space, pointing in the direction of increasing frequency. The points where the momentum gap is narrowest occur at $k_y \Lambda / \pi = 1$, and will be referred to as the symmetric points; at these points the group velocity points exactly along the layers. When the normalised frequency is increased to 4, ellipse-like shapes appear in the momentum gaps. These give rise to an additional pair of symmetric points. We shall refer to the Bloch waves on the ‘ellipse’ as the fast Bloch waves and those on the outer branches as slow Bloch waves, a naming convention that relates to the phase velocity along the layers.
Figure 2.5 - Series of wavevector diagrams for TE and TM cases at three different normalised frequencies $\nu$ in a multilayer stack with $n_1 = 1$, $n_2 = 3.45$ and $\tau = 0.8$. The solid lines represent the modes in the multilayer stack. The dashed lines are the modes in a homogeneous substrate ($n_a = 1.57$).
2.3.3 Symmetrical Points on the Frequency Versus $\beta$ Diagram

On this diagram (Figure 2.6), the positions of the momentum gap edges are plotted as a function of frequency for the same structure as treated in Figure 2.5. In the regions of the diagram that are not shaded $k_y$ is real and the corresponding Bloch waves propagate freely in the structure. Below the $k_y = 0$ line the Bloch waves are cut-off. In the shaded regions between the $k_y = \pi / \Lambda$ lines there are stop bands, $k_y$ is complex and the Bloch waves are evanescent. In the TM case the gap width shrinks to zero at $\beta\Lambda / \pi \approx 0.35$, which occurs when the rays in each layer are incident on the interfaces at Brewster’s angle.

![Figure 2.6](image)

*Figure 2.6* - Frequency versus $\beta$ diagram for the multilayer stack of Figure 2.5 ($n_1 = 1$, $n_2 = 3.45$ and $\tau = 0.8$).

2.4 One-dimensional Photonic Crystal Waveguides

The one-dimensional photonic crystal waveguide structure is shown schematically in Figure 2.7(a). The waveguide layer consists of a strip of the multilayer stack from Figure 2.4, which is now constrained to a thickness $h$ in the $z$ direction. The multilayer stack is supported by a silica substrate and is covered by air. The guided Bloch modes are found by matching $k_y$ at the substrate and air interfaces and then satisfying a resonance condition. At each interface four Bloch waves are matched to two evanescent plane waves, i.e. an incident wave of one type scatters partially into a wave
of the other type at the interface. However only the waves that satisfy the resonance condition will propagate along the waveguide.

![Waveguide Diagram](image)

**Figure 2.7** - (a) A fast guided Bloch mode. The fast Bloch rays couple to slow Bloch rays and evanescent modes at the interfaces. When the fast Bloch rays satisfy the waveguide resonance condition a stable fast guided Bloch mode is created. (b) A slow guided Bloch mode. (c) TE wavevector diagram detail at symmetric points for $\nu = 4$. The boundary condition requires that $k_y$ is constant across the boundary. The construction line shows the four Bloch waves that couple together at the boundary.

### 2.4.1 Boundary Conditions

We can use the wavevector diagram to determine if the Bloch rays are reflected at the waveguide boundaries. At fixed optical frequency, boundary conditions specify that the
components of wave momentum along a planar interface must be conserved as the
interface is crossed. By superimposing the wavevector diagrams for the periodic layer
and the substrate, fulfilment of this condition is easy to visualise graphically. In our
case the diagram for the substrate is simply a circle:

\[ k_{y}^2 + \beta^2 = k^2 n_s^2 = \left( \frac{n_s V}{n_v \Lambda} \right)^2 \]  

(2.17)

which is shown (with dotted curves) on Figure 2.5. To treat phase matching at an
interface in the \((x, y)\) plane, like one of the waveguide boundaries, a vertical
construction line is drawn on the \((\beta, k_y)\) diagram (Figure 2.7(c)). For \(k_y \Lambda / \pi > 0.68\)
this line does not intersect the substrate circle, so that total internal reflection occurs
and the Bloch waves are trapped in the periodic layer. For \(k_y \Lambda / \pi < 0.68\) the line
intersects the substrate circle, and the Bloch waves radiate from the periodic layer into
the substrate. The construction line indicates the four Bloch rays that couple together
at the boundary.

2.4.2 Two-Wave Approximation For The Bloch Waves

To reduce the complexity of the problem, we now take the Fourier transform of the
periodic part \(B_\pm(y)\) of the Bloch wave fields, and extract the amplitudes of the two
dominant partial waves in the plane-wave expansion. These are then matched to fields
with upward (+y) and downward (-y) progressing phase velocities (evanescent in the z-
direction) in the substrate and cover regions. All the higher order partial plane waves
are ignored; as we shall show, the accuracy of this approximation is such that the
solutions compare favourably with the results of a numerical finite-difference analysis.
Each Bloch wave can be expanded in terms of an infinite set of partial plane waves
whose wavevectors are related by Floquet's theorem:

\[ k_n = \beta \hat{z} + (k_y + nK)\hat{y} \]  

(2.18)

where \(K = 2\pi / \Lambda\) is the grating vector. This permits us to express the exact solutions
from the translation matrix analysis, \(B_\pm(y)\) in (2.14), in the general form:

\[ B_\pm(y) = \sum_n S_n^\pm \exp(-jnK_y) \]  

(2.19)
where the $S^*_n$ are the complex plane wave amplitudes, whose values are easily found by performing Fourier analysis, yielding:

![Diagram](image_url)

**Figure 2.8** - Percentage root mean square errors in (a) field amplitude and (b) phase.
\[ S^\pm_\alpha = \frac{1}{\Lambda} \int_{-\Lambda/2}^{\Lambda/2} B_\alpha(y) \exp(jnK_y) \, dy \] \hspace{1cm} (2.20)

Retaining the two dominant partial waves, the Bloch wave fields \( b_\pm(y,z) \) are given approximately by:

\[ b_\pm(y,z) = \exp[-j(\beta z \pm k_y y)] \left( S^\pm_\alpha \exp(jK_y) \right) \] \hspace{1cm} (2.21)

where, as before, the choice of + or − determines the group of Bloch waves that progresses (or evanesces) in the +y or −y directions.

The percentage errors in amplitude and phase introduced by this approximation are plotted in Figure 2.8 for normalised frequencies of 3 and 4. For slow Bloch waves the amplitude error is less than 3% and the phase error less than 0.5% over the parameter range of interest. For fast Bloch waves the worst case is at the top of the frequency range and gives an amplitude error between 6 and 7% and a phase error less than 0.8%.

### 2.4.3 Dispersion Relation

We are now in a position to obtain the dispersion relation of the guided Bloch modes. The interfaces are considered to be parallel and separated by a distance \( h \). The boundary conditions require that all wavevector components in the \( y \)-direction be continuous across the interfaces. The most general case (of four participating Bloch waves) is illustrated in Figure 2.7(c), the arrows indicating the directions of the group velocities in the layer. The upward (U) and downward (D) partial waves in each of the four Bloch waves (labelled by \( f \) (fast) and \( s \) (slow) for \( \beta > 0 \), and \( \bar{f} \) (fast) and \( \bar{s} \) (slow) for \( \beta < 0 \)) are now matched to the upward and downward evanescent waves in the cover (co) and substrate (ss). The surviving \( x \)-components \( b_f, b_{\bar{f}}, b_s, b_{\bar{s}} \) of the Bloch wave fields (from (2.21), taking without loss of generality the + sign and replacing the subscripts ± with \( f \) or \( s \)) are:

\[ \frac{b_f}{V_f \exp(j\beta_f z)} = \frac{b_{\bar{f}}}{V_{\bar{f}} \exp(j\beta_{\bar{f}} z)} = \exp(-j k_y y) \left( F_U + F_D \exp(jK_y) \right) \] \hspace{1cm} (2.22)

where \( f \) and \( F \) are simply replaced by \( s \) and \( S \) for the slow Bloch waves. The \( F_U \) and \( F_D \) are the renamed upward and downward partial wave amplitudes (identical for \( \beta = \pm|\beta| \)) from (2.21), \( \beta_f \) is the value of \( \beta \) on the inner (fast) stop-band branch, \( \beta_{\bar{s}} \) is the
value of $\beta$ on the outer (slow) stop-band branch and $V_j$ are the Bloch wave amplitudes (to be determined). If the ‘ellipse’ is not present, then $\beta_f$ is pure imaginary.

The evanescent fields in the cover ($E_{co}, z \geq h/2$) and substrate ($E_{ss}, z \leq -h/2$) regions are given by:

$$E_{co} \exp(j k_y y) = U_{co} \exp\left[-\sqrt{k_y^2 - k^2 n_{co}^2}(z - h/2)\right]$$

$$+ D_{co} \exp\left[-\sqrt{(k_y - K)^2 - k^2 n_{co}^2}(z - h/2)\right] \exp(jK y)$$

$$E_{ss} \exp(j k_y y) = U_{ss} \exp\left[\sqrt{k_y^2 - k^2 n_{ss}^2}(z + h/2)\right]$$

$$+ D_{ss} \exp\left[(k_y - K)^2 - k^2 n_{ss}^2(z + h/2)\right] \exp(jK y)$$

where $h$ is the layer width, $U_{co}, U_{ss}, D_{co},$ and $D_{ss}$ being the upward ($+y$) and downward ($-y$) progressing wave amplitudes in the cover and substrate. Requiring continuity of the $x$-components and derivatives $^6$ of the upward and downward fields at the substrate and cover interfaces yields eight boundary conditions, which are most conveniently written in the form of a matrix equation:

$$
\begin{pmatrix}
F_U e^{\phi} & F_U e^{-\phi} & S_U e^{-\sigma} & S_U e^{+\sigma} & -1 & 0 & 0 & 0 \\
F_D e^{+\phi} & F_D e^{-\phi} & S_D e^{-\sigma} & S_D e^{+\sigma} & 0 & -1 & 0 & 0 \\
F_U e^{-\phi} & F_U e^{+\phi} & S_U e^{+\sigma} & S_U e^{-\sigma} & 0 & 0 & -1 & 0 \\
F_D e^{-\phi} & F_D e^{+\phi} & S_D e^{+\sigma} & S_D e^{-\sigma} & 0 & 0 & 0 & -1 \\
\beta_f F_U e^{+\phi} & -\beta_f F_U e^{-\phi} & -\beta_s S_U e^{-\sigma} & \beta_s S_U e^{+\sigma} & -j p_{ssU} & 0 & 0 & 0 \\
\beta_f F_D e^{+\phi} & -\beta_f F_D e^{-\phi} & -\beta_s S_D e^{-\sigma} & \beta_s S_D e^{+\sigma} & 0 & -j p_{ssD} & 0 & 0 \\
\beta_f F_U e^{-\phi} & -\beta_f F_U e^{+\phi} & -\beta_s S_U e^{+\sigma} & \beta_s S_U e^{-\sigma} & 0 & 0 & j p_{coU} & 0 \\
\beta_f F_D e^{-\phi} & -\beta_f F_D e^{+\phi} & -\beta_s S_D e^{+\sigma} & \beta_s S_D e^{-\sigma} & 0 & 0 & 0 & j p_{coD}
\end{pmatrix}
\begin{pmatrix}
V_t \\
V_t^* \\
V_s \\
V_s^* \\
U_{ss} \\
D_{ss} \\
U_{co} \\
D_{co}
\end{pmatrix} = 0
$$

where:

$$\phi = j \beta_f h/2, \quad \sigma = j \beta_s h/2.$$  

$$p_{ju} = \xi_j \sqrt{k_y^2 - k^2 n_j^2}, \quad p_{jd} = \xi_j \sqrt{(k_y - K)^2 - k^2 n_j^2},$$

$$\xi_j = 1 \text{ (TE)} \quad \text{or} \quad \xi_j = n_{av}^2 / n_j^2 \text{ (TM)}$$
are the definitions of the various parameters and \( j = \text{co or ss} \). Real values of \( k_y \) for which the determinant of this matrix is zero yield the guided Bloch modes of the periodic layer. At the symmetric points \( (k_y \Lambda / \pi = \pm 1) \), the upward and downward partial waves have equal and opposite wavevectors, which means that the conditions for the upward and downward waves are identical. Since the fast and slow guided Bloch modes are orthogonal at this point and can be considered separately, the problem reduces to a much simpler \( 4 \times 4 \) matrix yielding the following dispersion equation for the guided modes:

\[
\tan(\beta_q h + m\pi) = \frac{\beta_q (p_{\text{ssU}} + p_{\text{coU}})}{\beta_q^2 - p_{\text{ssU}} p_{\text{coU}}} = \frac{\beta_q (p_{\text{ssD}} + p_{\text{coD}})}{\beta_q^2 - p_{\text{ssD}} p_{\text{coD}}} \tag{2.26}
\]

where \( q = f \) or \( s \) (for the fast or slow Bloch waves) and \( m \) is an integer. This is very similar to the standard dispersion relation for an asymmetric slab waveguide (2.6).

### 2.4.4 Guided Bloch modes at symmetric points

Figure 2.9 shows plots of normalised frequency \( \nu \) versus \( h/\lambda \) for a silicon structure on a glass substrate \( (n_1 = 1, \ n_2 = 3.45, \ n_{ss} = 1.57, \ n_{co} = 1, \ \tau = 0.8, \text{yielding } n_{av} = 2.96) \). Since they reside at the symmetrical points on the wavevector diagram, these guided Bloch modes have zero group velocity in the direction parallel to the substrate and are fully confined within the layer. For small values of \( h \) the modes are widely spaced in frequency. The upper set of curves (dashed line style) is for the fast modes. These disappear at normalised frequencies below \( \nu = 3.2996 \), which corresponds to the disappearance of the ‘ellipse’ on the wavevector diagram. The lower set of curves (full line style) is for the slow modes. The fast and slow modes occur in pairs, each pair straddling the corresponding mode that would occur in a homogeneous slab waveguide of the same average index.
Figure 2.9 - Normalised frequency $\nu$ versus $h/\Lambda$ for the guided Bloch modes at the symmetrical points in a silicon structure on a glass substrate ($n_1 = 1$, $n_2 = 3.45$, $n_{ss} = 1.57$, $n_{co} = 1$ and $\tau = 0.8$). The solid lines represent slow modes and the dashed lines represent fast modes. The fast mode cutoff is at $\nu = 3.2996$. In the TM case (b) the modes switch from fast to slow at $\nu = 3.435$. This is a result of the definition of fast and slow modes and the crossing of the $k_y = \pi/\Lambda$ lines in Figure 2.6 where the stop band closes.
2.4.5 Behaviour away from symmetric points

Away from the symmetric points, the guided Bloch modes are described by full solutions of (2.24). Plots of $k_y \Lambda / \pi$ versus normalised layer thickness $h/\Lambda$ are presented in Figure 2.10 for TE and TM modes at a normalised frequency $\nu = 3$. At this frequency only slow modes exist, there being no ‘ellipse’ on the wavevector diagram, rendering the fast modes evanescent. Note that for small enough layer thickness only one mode is available over the whole range of $k_y$. The guided modes cut off when $k_y \Lambda / \pi$ intersects with the substrate circle; this condition is indicated by the horizontal line near the base of the figures.

Figure 2.11 is a repeat of Figure 2.10 for a normalised frequency $\nu = 4$. The set of near-vertical curves corresponds to slow modes, and the second set of curves corresponds to fast modes. As the modes move away from the symmetrical point, increasingly strong anti-crossing occurs at the intersection points of the curves. This is due to coupling between fast and slow Bloch waves at the interfaces. When, for example, a fast Bloch wave collides with the cover or substrate interface, it is split by total internal reflection into a mixture of a strong fast and a weaker slow Bloch wave.

![Figure 2.10](image-url) - Plots of $k_y \Lambda / \pi$ versus normalised layer thickness $h/\Lambda$ for (a) TE and (b) TM modes in a structure with $n_1 = 1$, $n_2 = 3.45$, $n_\infty = 1.57$, $n_{co} = 1$ and $\tau = 0.8$ at a normalised frequency $\nu = 3$. The fast modes are evanescent. The slow modes cut-off when the value of $k_y \Lambda / \pi$ intersects with the substrate circle.
2.4.6 Brillouin diagram

Figure 2.12 shows plots of $\nu$ versus $k_y/\Lambda$ for TE modes for a structure of thickness $1.5\lambda$ ($n_1 = 1$, $n_2 = 3.45$, $n_{ss} = 1.57$, $n_{co} = 1$ and $\tau = 0.8$). The shaded regions to the upper left and right occur when $k_y/\Lambda \leq \nu n_{ss}/\pi n_{av}$, i.e., when the substrate circle is touched and there is radiation into the substrate. The lowest curve with the points c and f marked on it corresponds to the zero order slow Bloch wave mode. At a, b, c, d & e the group velocity of the Bloch wave is zero in the $y$ direction. The vanishingly small group velocity at the symmetric points will result in an enhancement in the interaction between the electromagnetic guided mode and an incorporated dipole of the same frequency. An excitation of finite length will contain a range of wavevectors and a spectrum of guided Bloch modes will be excited. The guided Bloch modes that do not lie precisely on the symmetrical points will cause energy to leak away and reduce the lifetime of the resonance\(^2\). The other two curves on the diagram represent the first order slow mode and the zero order fast mode. Between the points d and e the modes display an anticrossing behaviour. At points a, g and h the mode consists solely of the first order slow mode; at b there is the zero order fast mode; finally at d and e there is a mixture of the two modes. This is confirmed by the field microstructure in the next section. An intriguing feature of these plots is the reduction in the number of
guides the frequency rises. This is the reverse of the behaviour in normal waveguides, where higher frequencies imply a larger number of modes, and is caused by the encroachment of the momentum gap within the permitted range of \( \beta \) values. The modes in a homogeneous waveguide with the same average index are shown as dashed lines on the diagram. In the periodic structure the zero order mode is suppressed between points b and c, and the first order mode is suppressed above a. In thicker layers this mode-suppression effect is even more dramatic.7

2.4.7 Microstructure of the fields

Figure 2.13 shows the electric field intensity distribution of the TE guided Bloch modes at points a, b, d and e on Figure 2.12. Figure 2.13(a) shows the first order slow mode at point a. This is characteristically concentrated in the high index regions and has a double lobed structure in the z direction. The zero order fast mode (point b on Figure 2.12) is shown in Figure 2.13(b); it is guided predominantly in the air gaps. This
very unusual behaviour arises because the field pattern is below the resolution limit of free waves in the cover and substrate regions. The modes at points a and b have zero group velocity along the waveguide, as confirmed by the 100% visibility of the modal fringe pattern - no power can flow through regions where the fields are zero. It is intriguing that four other points of zero group velocity occur, at anticrossing points on either side of the symmetrical point (e.g., d and e). The field intensity patterns of the modes at these points are given in Figure 2.13(c) and Figure 2.13(d). It turns out the anticrossing is caused by the simultaneous resonance of the zero order fast and the first order slow modes, which are then coupled strongly together at the upper and lower boundaries (see Figure 2.7), creating a stop-band in $\beta$. Since they travel in opposite directions along the guide, a kind of “tug-of-war” results between the fast and slow modes, giving rise to zero group velocity at the anticrossing point. The overall modal field distributions of these “mixed” modes are superpositions of fast and slow modes, whose relative phase is such that constructive interference occurs near the substrate in both cases (d and e).
2.4.8 Comparison with numerical analysis

In order to confirm the accuracy of our simple analytical model, a numerical calculation was performed based on the method of Pendry and MacKinnon\textsuperscript{8}. Our analytical model could be extended by generalisation of (2.24) to include the contributions of higher order Fourier components and evanescent (imaginary $\beta$) solutions in the expansion of the field within the grating layer. In practice it is more

![Field Intensity Distributions](image)

**Figure 2.13** - Field intensity distributions of selected TE guided Bloch modes at points a, b, d and e in Figure 2.12. The substrate is below the horizontal dashed line and three periods of the high index waveguide layer are shown. (a) a first order slow mode, (b) a zero order fast mode, (c) a zero group velocity slow ‘mixed’ mode and (d) a zero group velocity fast ‘mixed’ mode.
efficient to recast the equations to relate the Fourier components in the cover to
those in the substrate by means of a transfer matrix. The elements of this transfer
matrix could be calculated by use of the dispersion relation (2.13), the Fourier
decomposition (2.19) and the matching conditions at the substrate and cover
boundaries. In practice, due to its availability and flexibility, a finite difference
algorithm, initiated by Pendry and MacKinnon, was used to calculate the transfer
matrix. This works by discretising the fields on a real space mesh and has the added
advantage of being able to describe more complex grating geometries, such as V-
grooves and two dimensional periodicity. These structures could be considered in
future work.

The condition for appearance of a guided mode is that there must be no unphysical
exponentially diverging modes in the substrate and cover. This results in a
determinantal equation, based on the transfer matrix, which is numerically solved.
Table 2.1 shows a comparison of the resonant frequencies calculated at a number of
points indicated on Figure 2.12. The results for both methods correspond very well,
close to the symmetric point the error is less than 0.5% and it increases to 3.16%
when $k_y \Lambda / \pi$ is reduced to 0.5.

<table>
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<th>point</th>
<th>$k_y \Lambda / \pi$</th>
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<th>analytic frequency $\nu$</th>
<th>% error</th>
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<td>3.77557</td>
<td>0.33</td>
</tr>
<tr>
<td>b</td>
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<td>3.62286</td>
<td>3.61706</td>
<td>0.16</td>
</tr>
<tr>
<td>c</td>
<td>1</td>
<td>2.9912</td>
<td>2.98968</td>
<td>0.05</td>
</tr>
<tr>
<td>d</td>
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<td>3.71371</td>
<td>0.39</td>
</tr>
<tr>
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<td>3.71143</td>
<td>3.70417</td>
<td>0.20</td>
</tr>
<tr>
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<td>2.89861</td>
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</tr>
<tr>
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<td>3.16</td>
</tr>
</tbody>
</table>

Table 2.1 - Comparison of numerical and analytic models
2.5 Two-dimensional Photonic Band Gap Waveguides

In this section we consider a 2D PC waveguide consisting of a hexagonal array of air holes in a high index waveguiding layer on a homogeneous low index substrate (Figure 2.14). An analytical method is developed, based on coupled mode equations, which is valid for structures with small coupling constants. This is used in chapter 5 to interpret experimental measurements on 2D PC waveguides. Strongly modulated PC waveguides are treated by numerically modelling the out of plane propagation in an infinitely long PC and then applying a simple transverse resonance condition to find the waveguide modes in a thin PC layer.

![Figure 2.14 - A two dimensional photonic crystal waveguide consisting of a high index waveguide layer embedded with a hexagonal array of holes on a homogeneous low index substrate.](image)

2.5.1 Average index approximation - infinitesimal coupling

The simplest approximation to the wavevector diagram for a multiply periodic material is to assume that there is infinitesimal coupling between the partial waves of the Bloch modes so that the loci of all the partial waves are average index circles. Circles with radii of \( k_0 n_{\text{eff}}^\text{TE} \) and \( k_0 n_{\text{eff}}^\text{TM} \) are centred on each of the reciprocal lattice points, \( \mathbf{r} \), where

\[
\mathbf{r} = K (l \hat{\mathbf{y}} + m \frac{(\sqrt{3} \hat{\mathbf{x}} + \hat{\mathbf{y}})}{2} + n \frac{(\sqrt{3} \hat{\mathbf{x}} - \hat{\mathbf{y}})}{2})
\]  

\( l, m, n \) are integers and \( n_{\text{eff}}^j \) is the effective refractive index of the \( j \) polarised waveguide mode. The normalised frequencies and effective refractive indices of the TE and TM modes are related by (2.7), so that

\[
\nu = k_0 n_{\text{eff}}^\text{TE} \Lambda = k_0 n_{\text{eff}}^\text{TM} \Lambda + \delta_{\nu}
\]
If we consider a 270nm thick tantalum pentoxide waveguide on a silica substrate, as in Figure 2.3, then $\delta = 0.32$. Figure 2.15 shows how the wavevector diagram evolves with frequency. At a low frequency (Figure 2.15(a), $\nu_{TE} = 3$), the circles are so small that they don’t intersect at all, here the wavelength is much greater than the grating period so the wave only sees the average index of the grating. As the frequency is increased the circles overlap more and there are lots of intersections. At each intersection a pair of plane waves, corresponding to the average index circles, are coupled together. At $\nu_{TE} = 3$ (Figure 2.15(b)) the intersections are grouped around the triple points and so with the introduction of coupling the detail around these points will be very interesting. There are several special normalised frequencies between 3 and 4 where three circles intersect and couple together at the triple point. For example at $\nu_{TE} = 4$ (Figure 2.15(c)), the TM plane waves all couple together at the triple points. Close to these special frequencies there will be small confined shapes at the triple points which can be thought of as representing modes with very small effective index values centred on the triple points. The small index values mean that the modes have small effective group velocities and so are easily deflected by varying the local waveguide properties. As the normalised frequency is increased further the intersections become more evenly spaced across the diagram (Figure 2.15(d)).
2.5.2 Coupled wave approximation - weak coupling

In this section we revert to a coupled wave method in order to calculate the detail of the wavevector diagram close to the triple points. For a periodic structure each point on the wavevector diagram in the first Brillouin zone is associated with a unique travelling Bloch wave. The full wavevector diagram is obtained by tiling the first Brillouin zone.

**Figure 2.15** - Wavevector diagram based on average index circles for a tantalum pentoxide hexagonal PC waveguide of thickness 270nm. The TE (TM) modes are indicated by the solid (dashed) lines. (a) \( \nu_{TE} = 3 \), the circles do not overlap, at this frequency the light sees a homogeneous average index. (b) \( \nu_{TE} = 3.5 \), the TE circles overlap to form small triangles at the “triple” points. (c) \( \nu_{TE} = 4 \), the TM circles overlap to form small triangles at the “triple” points. (d) \( \nu_{TE} = 4.4 \), the TE circle intersections are equally spaced across the diagram.

**2.5.2 Coupled wave approximation - weak coupling**

In this section we revert to a coupled wave method in order to calculate the detail of the wavevector diagram close to the triple points. For a periodic structure each point on the wavevector diagram in the first Brillouin zone is associated with a unique travelling Bloch wave. The full wavevector diagram is obtained by tiling the first Brillouin zone.
Brillouin zone to cover all wavevector space. Each point in the higher Brillouin zones is associated with a particular point in the first order Brillouin zones and it is the combination of each set of associated points that forms the Bloch wave. Viewed in a different way, each Bloch wave is composed of an infinite set of plane waves, where one plane wave comes from each Brillouin zone, and these plane waves are linked by Bloch’s theorem.

One approach to obtaining the Bloch wave function is to expand the fields in terms of an infinite sum of partial plane waves, with one for each Brillouin zone. This is then truncated appropriately and the resulting eigenvalue problem is solved using standard techniques. We now make a drastic reduction in the number of partial waves in order to obtain illustrative analytical expressions for the dispersion relations.

The Bloch wave function can be expressed in terms of 6 partial waves, one for each of the average index circles crossing the region of interest. The partial waves are expanded around the triple point and their wavevectors are given by:

\[ \mathbf{k}_i = \mathbf{\delta} + \mathbf{t}_i \mathbf{K} / \sqrt{3} \]  

(2.29)

where \( \mathbf{K} \) is the grating wavevector, \( \mathbf{\delta} \) is the deviation of the partial wavevectors from the triple point and 

\[ \hat{\mathbf{t}}_1 = \hat{x}, \quad \hat{\mathbf{t}}_2 = (-\hat{x} + \hat{y}\sqrt{3}) / 2, \quad \hat{\mathbf{t}}_3 = (-\hat{x} - \hat{y}\sqrt{3}) / 2 \]  

(2.30)

are unit vectors in three triple point directions. Substituting a trial solution consisting of six plane waves with amplitudes \( V_{1TE}, V_{2TE}, V_{3TE}, V_{1TM}, V_{2TM} \) and \( V_{3TM} \) and wavevectors (2.29) into Maxwell’s equations, and neglecting higher order terms in \( \mathbf{\delta} \), leads to a 6×6 matrix whose determinant must equal zero for non-trivial solutions of the following:

\[
\begin{pmatrix}
\gamma_{TE1} & \kappa_{TTE} & \kappa_{TTE} & \kappa_{TTE} & \kappa_{TTE} & \kappa_{TTE} \\
\kappa_{TTE} & \gamma_{TE2} & \kappa_{TTE} & \kappa_{TTE} & \kappa_{TTE} & \kappa_{TTE} \\
\kappa_{TTE} & \kappa_{TTE} & \gamma_{TE3} & \kappa_{TTE} & \kappa_{TTE} & \kappa_{TTE} \\
\kappa_{TMTE} & \kappa_{TMTE} & \kappa_{TMTE} & \gamma_{TM1} & \kappa_{TMTM} & \kappa_{TMTM} \\
\kappa_{TMTE} & \kappa_{TMTE} & \kappa_{TMTE} & \kappa_{TMTM} & \gamma_{TM2} & \kappa_{TMTM} \\
\kappa_{TMTE} & \kappa_{TMTE} & \kappa_{TMTE} & \kappa_{TMTM} & \kappa_{TMTM} & \gamma_{TM3}
\end{pmatrix}
\begin{pmatrix}
V_{1TE} \\
V_{2TE} \\
V_{3TE} \\
V_{1TM} \\
V_{2TM} \\
V_{3TM}
\end{pmatrix}
= 0
\]  

(2.31)

where \( \kappa_{jk} \) is the coupling constant between modes with polarisations \( j \) and \( k \),

\[
\gamma_{jk} = \theta_j / 2 - \mathbf{\delta} \cdot \hat{\mathbf{t}}_k
\]  

(2.32)
and the dephasing parameter, \( \theta_j \), is defined by

\[
\theta_j = 2(n_{\text{eff}}^j k_0 - K / \sqrt{3})
\]  

(2.33)

where \( n_{\text{eff}}^j \) is the effective refractive index of the \( j \) polarised waveguide mode. The coupling constants can be determined experimentally and typical values\(^\text{10}\) are given in Table 2.2, where the normalised coupling constant, \( \kappa_N = \sqrt{2} \kappa_{jk} / K \).

<table>
<thead>
<tr>
<th>Polarisation Coupling</th>
<th>Normalised coupling constant, ( \kappa_N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE TE</td>
<td>( \leq 0.0015 )</td>
</tr>
<tr>
<td>TE TM</td>
<td>0.0100</td>
</tr>
<tr>
<td>TM TE</td>
<td>0.0105</td>
</tr>
<tr>
<td>TM TM</td>
<td>0.0075</td>
</tr>
</tbody>
</table>

Table 2.2 - Typical coupling constants from reference 10.

The matrix determinant in (2.31) has an analytical solution that is readily found and evaluated using a modern symbolic mathematical computer program such as Mathematica. Equation (2.7) can be used to substitute for \( n_{\text{eff}}^{TM} \), so that plots can be made for selected values of \( n_{\text{eff}}^{TE} \). Figure 2.16 shows the detail at the triple points for a range of normalised frequencies. The central triangular feature in Figure 2.16(a) consists solely of TE modes and is a detailed view of a triple point from Figure 2.15(b). Note that the TE-TE gaps are much smaller than the TE-TM gaps as dictated by Table 2.2. At \( \nu = 3.7 \) (Figure 2.16(c)) the central triangle consists of TE modes and is isolated by relatively large gaps because of the interaction of three sets of modes (TE-TE-TM) at each of its apexes. Figure 2.16(f) shows the detail at a triple point of the plot in Figure 2.15(c), for which the central triangle consists solely of TM modes.

The group velocity of the modes is given in the usual way by\(^9\):

\[
\mathbf{v}_g = \nabla_{\delta} \omega(\delta) = \frac{\partial \omega}{\partial \delta} = \frac{d \omega}{d \theta} \frac{\partial \theta}{\partial \delta} = \frac{c}{2n} \frac{\partial \theta}{\partial \delta}
\]  

(2.34)

where \( \omega = c k \) is the angular frequency and \( \theta \) is the in-plane angle. Note that the group velocity is inversely proportional to the curvature and so at all the tight curves in Figure 2.16 there are modes that have a reduced group velocity.
Figure 2.16 - Wavevector diagram detail at triple point.
This can be seen more clearly by calculating the Brillouin (or band) diagram around the triple point. The inset to Figure 2.17 shows the first order Brillouin zone with the symmetry points labelled in the usual way. The band diagram shows the modes along part of the path Γ-J-X. This is equivalent to following the $\delta x$-axis on Figure 2.16. At the J point there are several turning points in the bands that correspond to zero group velocity standing wave modes. These occur as the triangular shapes, such as that in the centre of Figure 2.16(a), become vanishingly small. An important consequence of this slowing down of light is that it has more time to interact with matter - dipoles, nonlinearities and scattering centres. Apparently weak perturbations in refractive index can result in strong scattering and non-linear effects are enhanced.

2.5.3 Full vector analysis - strong coupling

A real-space method can be employed to calculate the full three-dimensional wavevector diagram for a 2-D PC at a given optical frequency. This diagram is a map of all the Bloch modes existing in the PC for propagation in any direction. We can
therefore use this information with a simple waveguide resonance condition to qualitatively predict waveguide modes in 2-D PC waveguides.

To facilitate the visualisation of these Bloch modes it is convenient to plot a “$\beta$ band diagram”\(^{11}\) that resembles the more commonly used frequency band diagram. The $\beta$ band diagram is plotted with the $z$ component of wavevector, $\beta$, on the vertical axis and the in-plane wavevector, $k_p$, on the $x$-axis. $k_p$ is the linear distance on the wavevector diagram from the origin, $\Gamma$, along a standard trajectory ($\Gamma$-$X$-$J$-$\Gamma$) around the Brillouin zone, where $X$ and $J$ are the usual high symmetry points.

For total internal reflection to occur at the waveguide boundaries the magnitude of the in-plane wavevector in the PC must exceed that in the substrate. We therefore plot the wavevector dispersion surface for the substrate on the same diagram as the PC. The substrate material is uniform and is thus described by a simple hemispherical surface:

$$\beta_s = \sqrt{k_x^2 n_s^2 - k_y^2 - k_z^2}$$

(2.35)

The in-plane wavevector $k$ is converted to $k_p$ by simple geometry\(^{11}\) and the resulting circles for a silica substrate ($n_s=1.46$) are shown with dashed lines on Figure 2.18. The shaded regions in the figure show where the magnitude of the in-plane wavevector of the PC Bloch modes is less than that of the substrate modes so that the Bloch modes couple out into the substrate. The solid lines on Figure 2.18 correspond to the Bloch modes of a 2-D PC consisting of a hexagonal array of air holes ($n_1=1$) in a silicon waveguide ($n_2=3.64$). The volume fraction of air, $\sigma$, is 0.8 and the $\beta$ band diagram is plotted for three different optical frequencies, $\nu/n_{av} = 1.4, 1.8$ and 2.2.

The Bloch modes in the unshaded regions of the diagram are trapped in the PC waveguide by total internal reflection. To find a stable guided Bloch mode we have to impose a transverse resonance condition on the trapped modes:

$$\beta h - \Phi / 2 = m\pi$$

(2.36)
where $\Phi$ is the sum of the phase changes upon total internal reflection and $h$ is the waveguide thickness. The spacing, $\Delta \beta$, between successive modes is therefore (ignoring changes in $\Phi$) roughly $\pi/h$. We can use this fact to illustrate qualititively the effect of the waveguide in selecting the guided Bloch modes. We ignore the phase change upon reflection, $\Phi$, and draw a sequence of horizontal construction lines spaced by $\Delta \beta \Lambda = \pi \Lambda / h$ on the $\beta$ band diagram for a waveguide thickness of $7\Lambda$ (Figure 2.18). Each

![Figure 2.18](image-url)
construction line corresponds to a plane, which is normal to the $z$ direction, in the full three-dimensional wavevector diagram. The intersection between each plane and the PC wavevector dispersion surface locates a set of guided Bloch modes. Figure 2.20 shows a set of guided Bloch modes for $v/n_{av} = 2.2$ and $\beta \Lambda = 1.76$, which corresponds to $m = 4$ for a waveguide thickness of $7\Lambda$ (dashed line AA on Figure 2.18 (c)). The points on the diagram represent where discrete numerical solutions have been found, the actual wavevector diagram consists of smooth continuous curves. Each intersection between the wavevector diagram and the $\Gamma$-J-X-$\Gamma$ trajectory on Figure 2.20 corresponds to an intersection between the dashed line AA and the $\beta$ band diagram for the hexagonal PC on Figure 2.18 (c). From symmetry, each intersection between the transverse resonances (dashed horizontal lines) and the PC dispersion curves (solid lines) corresponds to a set of symmetrically related guided Bloch modes. The number of intersections can be counted to see how effective the PC is at suppressing guided modes.

At $v/n_{av} = 1.4$ (Figure 2.18 (a)) the frequency is low so that the PC acts as a uniform effective index medium. The solid quarter circles (ellipses) indicate TE (TM) like guided Bloch modes in a similar manner to Figure 2.5. There are two sets of TE like modes and three sets of TM like modes (neglecting the in-plane, $\beta = 0, m = 0$, solutions). At $v/n_{av} = 1.8$ (Figure 2.18 (b)) the PC is still acting as a uniform effective index medium and there are now three sets of TE like and four sets of TM like guided Bloch modes. At $v/n_{av} = 2.2$ (Figure 2.18 (c)) the PC forces the three lowest order modes to cut-off and there is strong coupling between the other modes which causes local $\beta$ band gaps to appear. A uniform waveguide of the same average index would support six orders of guided modes whereas the PC waveguide only supports three orders of guided Bloch modes. This number could be decreased further if there was a full in-plane $\beta$ band gap which coincided with one or more of the horizontal construction lines, it would then suppress the corresponding orders of guided Bloch modes. If the normalised frequency is increased further the average index circle will cross the Brillouin zone at the X point and the unshaded region will reduce to a thin strip around the J point. This will mean that local $\beta$ band gaps around the J point will be able to completely suppress guided modes. More interestingly the waveguide thickness could be “tuned” so that one of the construction lines intersects the
wavevector dispersion surface just at the band edge of one of the gaps at the J point. This would correspond to a stationary resonance with zero in-plane group velocity, like those considered in the previous section for a 1-D PC, and may be suitable as a micro resonator mode for a vertical cavity surface emitting laser.

Figure 2.19 shows a schematic design of a photonic crystal waveguide laser. The waveguide contains a hexagonal photonic crystal designed so that the light produced by the quantum wells is emitted into a stationary resonance. The in-plane group velocity of the emitted light is zero so that the energy does not spread in the waveguide beyond the excited area. There is only non-zero group velocity in the z direction. If a circular area is excited, either optically or electrically, then a circular "top hat" output beam is emitted in the z direction. The "top hat" profile is ideal for pumping other lasers such as OPOs. The mode volume can be as large as required as long as sufficient excitation energy can be supplied. This contrasts with microresonator designs which have mode volumes of the order of the wavelength cubed.

Figure 2.19 - Schematic of a photonic crystal waveguide laser. The waveguide contains a hexagonal photonic crystal designed so that the light produced by the quantum wells is emitted into a stationary resonance. The in-plane group velocity of the emitted light is zero so that the energy does not spread in the waveguide beyond the excited area. There is only non-zero group velocity in the z direction. If a circular area is excited, either optically or electrically, then a circular "top hat" output beam is emitted in the z-direction.
When the substrate index circle crosses the J point then no guided Bloch modes exist. This is because for every guided plane wave mode outside the first Brillouin zone there is a mode, related by Bloch’s theorem, inside the first Brillouin zone that is coupled to a substrate mode. The substrate index circle crosses the J point when

\[ kn_{ss} > K/2 \quad \text{or} \quad \Lambda > \lambda/(2n_{ss}) \]  

(2.37)

If propagation into air is required then this simplifies to

\[ k > K/2 \quad \text{or} \quad \Lambda > \lambda/2 \]  

(2.38)

So to improve the performance of an LED emitting at 850nm would require a hexagonal PC with a lattice period greater than 425nm i.e. the lattice period can be chosen to be several microns to ease fabrication difficulties. This result relies on strong coupling between the constituent plane waves of each Bloch wave into the first Brillouin zone. It also assumes that there will be substantial coupling between the plane waves in the first Brillouin zone and the radiation modes in the air. Any potential gains from using this method would have to be balanced against the loss of gain volume created by the holes and losses associated with surface states on the much increased surface area.

**Figure 2.20** – (a) Numerically calculated transverse wavevector diagram at \( \beta \Lambda = 1.76 \) for (b) a 2-D PC consisting of a hexagonal array of air holes \( (n_1=1) \) in a silicon waveguide \( (n_2=3.64) \). The volume fraction of air, \( \sigma_1 \), is 0.8. Figure (a) is taken from reference 11.
References


6. For the TM case the boundary condition, at the interface between the periodic structure and the cover or substrate, requires continuity of \((1/n^2(y))(dH_z/dz)\). In the present analysis it is assumed that, for this boundary condition, \(n(y)\) is constant and equal to the average index, \(n_{av}\). A more accurate approach would involve finding Fourier components of \(1/n^2(y)\) and incorporating these into the analysis. However, when the results from both methods are compared, the error is very small, validating the initial assumptions.


3. FABRICATION OF PHOTONIC CRYSTAL WAVEGUIDES

3.1 Introduction

3.2 Photonic Crystal Waveguide Design

3.3 Planar Waveguides

3.4 Photoresist Gratings
   3.4.1 Photoresist Spinning
   3.4.2 Grating Pattern Exposure
   3.4.3 Creating Hexagonal Patterns
   3.4.4 Transitions

3.5 Photoresist Developing

3.6 Checking the Photoresist Pattern

3.7 Ion Beam Etching

3.8 Checking the Etched Waveguide Pattern
3. Fabrication of Photonic Crystal Waveguides

3.1 Introduction

In this chapter we describe the method for designing and fabricating hexagonal photonic crystal structures in planar waveguides. The method closely follows that used by Zengerle\(^1\) to fabricate square arrays of holes in planar waveguides. The fabrication procedure is summarised in Figure 3.1. A planar tantalum pentoxide waveguide is RF sputtered onto a fused silica substrate. A thin layer of photoresist is spin coated onto the waveguide and it is exposed with a hexagonal intensity pattern with an interferometer. Areas of smooth waveguide are defined by exposure with a mask. The mask is raised from the photoresist so that the transitions between the smooth waveguide and the photonic crystal waveguide are smooth. The pattern is developed and then etched through into the waveguide by ion beam etching.

The most difficult step in the process is exposing and developing the photoresist pattern. It will be shown that the grating period required is 280nm and so the only practical way of making large area gratings is by interferometric exposure. Conventional devices are fabricated by photolithography for which the state of the art microlithography equipment\(^2\), using deep-UV lasers and new photoresist formulations, can create features as small as 0.17\(\mu\)m. It is likely that in the near future this feature size will be reduced further and that it will be possible to create photonic crystal devices with this technology. This will enable the deliberate inclusion of defects and non-uniform unit cells in the photonic crystal material so as to create novel resonators, waveguides and polarising devices. The alternative to photolithography is direct write e-beam lithography. In this technique the pattern is written into the photoresist by raster scanning an electron beam. Hexagonal arrays have been successfully fabricated by this technique\(^3\), but the process is slow and expensive and only limited areas of order one millimetre square can be fabricated without stitching errors.
Two beams from an argon ion laser (wavelength 454nm) interfere at an angle of 53.6° to produce a grating with a period, \( \lambda \), of 282nm.

The photoresist is exposed through an opaque mask and a glass spacer so that the edges are blurred by diffraction and smooth transitions are formed.

Ideally the waveguide should be bare at the photoresist grating minima to enable precise control of the grating depth.

The etching rates of the photoresist and the waveguide are similar so the surface profile is transferred into the waveguide by the etching process.

The remaining photoresist is cleaned away by washing in acetone or by ashing in a high temperature oxygen atmosphere.

Figure 3.1 – Photonic crystal waveguide fabrication sequence.
3.2 Photonic Crystal Waveguide Design

The main parameters of the design to be chosen are the thickness of the waveguide, h, and the grating period, Λ. A tunable laser (λ = 770 to 910nm) is used to test the waveguides so that a fixed centre wavelength of 850nm can be chosen for the design and any approximations can be accommodated by the wide tuning range. Tantalum pentoxide waveguides are used because of the high refractive index (n=2.12) and the availability of a tantalum sputtering target. Fused silica is used for the substrate because of its low thermal expansion, low refractive index and low loss properties. The thickness of the waveguide is chosen to be 270nm so that the tantalum pentoxide waveguide is well within the single mode regime as described in section 2.2.

The grating period is chosen so that the triple points (figure 2.15(b) and point J on figure 2.19(a)) occur at the design wavelength of 850nm. Simple geometry gives

\[ \Lambda = \frac{\lambda}{2n_{\text{eff}} \cos 30^\circ} \]  

where \( n_{\text{eff}} \) is the effective index of the waveguide mode, which can be approximated by the value in a homogeneous waveguide. For the TM\(_0\) mode, \( n_{\text{eff}} = 1.74 \), so that \( \Lambda = 282\text{nm} \approx 280\text{nm} \).

3.3 Planar Waveguides

The first step in the fabrication sequence is to create a planar waveguide. High quality planar waveguides are fabricated by RF sputter deposition on fused silica substrates. Before sputtering the fused silica substrates have to be thoroughly cleaned and dried in the following sequence:

(i) Degrease by scrubbing with cotton buds soaked in acetone, methanol and then isopropyl alcohol

(ii) Clean in ultrasonic bath at 66°C with Microclean detergent for 60 minutes

(iii) Rinse in deionised water for 10 minutes

(iv) Clean in ultrasonic bath at 66°C with deionised water for 60 mins

(v) Blow dry with nitrogen and then dry for 30 minutes in an oven at 120°C.
The cleanliness of the surface can be checked by observing the edge of the water film as the surface is dried from one corner. If the surface is clean the water film edge is straight and evenly coloured thickness fringes can be seen. Any contamination on the surface causes distortions in the water film edge.

Tantalum pentoxide layers are deposited by reactive RF plasma sputtering. In this process a radio frequency plasma discharge is created in a mixed argon oxygen gas atmosphere between two electrodes. At one electrode there is a tantalum target and at the other there is a substrate. Ions are created in the plasma and accelerate towards the tantalum target causing tantalum atoms to be ejected. These tantalum atoms are readily oxidised in the highly reactive dissociated oxygen atmosphere. The energetic tantalum oxide molecules drift around the chamber and eventually condense on the cooled substrate. This is the main mechanism by which tantalum pentoxide films are grown but it should be noted that there are many other competing processes in the gas discharge which can lead to sputtering of the film and ion implantation of impurities into the film. The gas pressures used are 10mT argon and 1mT oxygen with a discharge power of 500W. The tantalum target (diameter 15cm) is cleaned immediately prior to film deposition by a 10 minute pre-sputtering run with a shutter positioned in front of the sample (50×50mm²). At the start of a film deposition the power is turned down to minimum and the shutter is opened. The power is then gradually increased to the desired level over a one minute period. This improves the waveguide loss by allowing the slow formation of a high quality interface between the substrate and the waveguide film. For the above conditions the deposition rate of tantalum pentoxide is approximately 1.6 nm/minute, therefore a typical run time would be 180 minutes to produce a 290nm thick film.

The film thickness is checked by simultaneously depositing onto a microscope slide with strips of photoresist on it. After deposition the photoresist is lifted off by soaking in acetone and the film thickness is determined by measuring the step height between the tantalum pentoxide layer and the bare substrate with an α-step stylus profiler. This gives the thickness to within 5nm. If the 50×50mm sample is placed in the centre of the sample holder then the film thickness is uniform across the sample to within 5nm.

A more accurate method of measuring the waveguide thickness is to prism couple directly into the layer but this often creates scratches on the waveguide. Prism coupling
is considered in detail in the next chapter. Prism coupling also allows the waveguide loss to be measured by observing the exponential decay of a ray travelling across the waveguide. The waveguide loss is approximately 2.6 dB/cm \( \pm 20\% \) (see section 5.2).

3.4 Photoresist Gratings

3.4.1 Photoresist Spinning

For a grating of reasonable strength we need only etch about 20nm into the surface of the waveguide. To achieve this it is essential that the photoresist mask is fully developed, leaving the waveguide surface bare, in the high intensity regions of the interference pattern produced by the interferometer. For good quality grating reproduction the photoresist thickness should be no more than the grating period which is 280nm. Thin photoresist layers can be produced by diluting a standard Shipley Microposit S1400-17 photoresist with thinner in a 10:3 ratio. In order to improve adhesion of the film it is necessary to clean the waveguide thoroughly in the same way as the silica substrate is cleaned and then leave it in a primer atmosphere for 15 minutes. The waveguide is transferred directly from the primer atmosphere to a spinner. Several drops of photoresist are applied to the centre of the waveguide, which is spun for 30 seconds at 4500 rpm. The photoresist is then softbaked for 30 minutes at 90°C this dries the solvent from the film without significantly affecting its photosensitivity or solubility. To measure the photoresist film thickness a scratch is made in the film with the blunt edge of a pair of tweezers so that the waveguide below is exposed and the thickness can be measured with the \( \alpha \)-step stylus profiler. The photoresist thickness is uniform across the sample to within 5nm, except at the very edges of the sample where thickness fringes can be seen within a few mm of the edges. The thickness fringes are visible because the optical thickness (thickness, 280nm, times refractive index, 1.64) of the photoresist layer is comparable to the wavelength of visible light. Small changes in thickness can be seen as colour changes and this is a very quick and useful diagnostic of the photoresist film quality.
3.4.2 Grating Pattern Exposure

The grating pattern is exposed in the photoresist layer by a fibre interferometer (Figure 3.2). Light from an argon ion laser (Spectra Physics 2025) at 454nm is launched into a special, germanium free, single mode fibre. The light is split in a 50:50 tapered fibre coupler and emerges from two cleaved bare fibre ends, which point, at equal angles, towards the sample where the beams interfere to form a grating pattern. For maximum fringe visibility the path lengths in the two arms of the interferometer must be equal. This is achieved by cleaving the fibres in the two arms to the same lengths to within 1mm.

The grating period, $\Lambda$, is determined by

$$\Lambda = \frac{\lambda_0}{2 \sin \theta_i} \quad (3.2)$$

where $\theta_i$ is the incident angle of the light from the two bare fibre ends on the sample and $\lambda_0$ is the argon ion laser wavelength (454nm). A grating period of 282nm requires an incident angle, $\theta_i$, of 53.6°.

**Figure 3.2** - Fibre Interferometer. Light from an argon ion laser goes through a 50:50 fibre coupler and produces interference fringes at the sample. The turntable is used to set the incident angle, $\theta_i$, which determines the grating period, $\Lambda$. Macroscopic fringes are produced by the beam splitter, which can be used to stabilise the interferometer by controlling the piezo fibre stretcher.
Good uniformity in the fringe patterns requires that the distances from the fibre ends to the sample be equal. Initial alignment, to within 1mm, can be achieved by placing an aluminium mirror and a plano-convex lens of the appropriate focal length in the sample holder which is mounted on a horizontal turntable. The turntable is rotated so that the mirror is normal to the incident angle, $\theta_I$. When the cleaved fibre end is positioned normal to the mirror at the focal length of the lens a tightly focused spot is seen on the fibre end.

To enable stabilisation of the interferometer a 50:50 beam splitter cube is placed at the top edge of the sample. The beam splitter cube is orientated so that it forms an image consisting of a superposition of the two cleaved fibre ends. If the difference in the path lengths of the arms of the interferometer is an exact multiple of $\pi$ then one of the beam splitter outputs will be zero intensity and the other will be maximum intensity. If the path length difference is then gradually increased by another $\pi$ then the power will slowly transfer from one output of the beam splitter to the other. By detecting the power at the beam splitter outputs and using this signal to control a piezo-electric fibre stretcher in one of the interferometer arms it is possible to stabilise the interferometer such that the fringe positions are locked.

The beam splitter in combination with the 50:50 fibre coupler can also be thought of as a Mach Zender interferometer. This aids in alignment as, when the images of the fibre tips are not perfectly overlapping, a fine fringe pattern is observed at the beam splitter outputs. By fine adjustment of the fibre ends and the beam splitter orientation it is possible to produce large fringes with spacing comparable to the size of the beam splitter.

The beamsplitter outputs are detected by two silicon photodiodes, which are amplified, subtracted and fed to a PID controller that outputs a high voltage to a piezo electric fibre stretcher. The photodiode outputs can also be used to optimise the interferometer performance. The most important property is the fringe visibility which can be measured by periodically modulating the high voltage on the fibre stretcher and measuring the maximum, $V_{\text{max}}$, and minimum, $V_{\text{min}}$, output voltages from one of the photodiodes. The visibility, $\xi$, is given by
\[ \xi = \frac{V_{\text{max}} - V_{\text{min}}}{V_{\text{max}}}. \] \hspace{1cm} (3.3)

Table 3.1 shows the change in the fringe visibility as (a) the aperture of the argon ion laser is changed and (b) as the power of the argon ion laser is changed. Table 3.1(a) shows that as the laser aperture is decreased the fringe visibility increases with the optimum aperture value being 5. This is because the aperture suppresses higher order transverse modes so that more power goes into the TM\(_{00}\) mode. At aperture 4 the TM\(_{00}\) mode becomes slightly suppressed. Table 3.1(b) shows that as the laser output power increases, the fringe visibility decreases.

<table>
<thead>
<tr>
<th>Laser Aperture</th>
<th>Laser Tube Current (A)</th>
<th>(V_{\text{max}}) (mV)</th>
<th>(V_{\text{min}}) (mV)</th>
<th>Fringe Visibility, (\xi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>36</td>
<td>385</td>
<td>95</td>
<td>0.753</td>
</tr>
<tr>
<td>7</td>
<td>38</td>
<td>460</td>
<td>120</td>
<td>0.739</td>
</tr>
<tr>
<td>6</td>
<td>40</td>
<td>760</td>
<td>180</td>
<td>0.763</td>
</tr>
<tr>
<td>5</td>
<td>42</td>
<td>920</td>
<td>200</td>
<td>0.783</td>
</tr>
<tr>
<td>4</td>
<td>54</td>
<td>880</td>
<td>240</td>
<td>0.727</td>
</tr>
</tbody>
</table>

Table 3.1 (a) - Fringe visibility as a function of argon ion laser aperture for a fixed laser output power of 200mW

<table>
<thead>
<tr>
<th>Laser Output Power (mW)</th>
<th>Laser Tube Current (A)</th>
<th>(V_{\text{max}}) (mV)</th>
<th>(V_{\text{min}}) (mV)</th>
<th>Fringe Visibility, (\xi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>125</td>
<td>32</td>
<td>620</td>
<td>100</td>
<td>0.839</td>
</tr>
<tr>
<td>150</td>
<td>35</td>
<td>720</td>
<td>120</td>
<td>0.833</td>
</tr>
<tr>
<td>175</td>
<td>38</td>
<td>840</td>
<td>160</td>
<td>0.810</td>
</tr>
<tr>
<td>200</td>
<td>42</td>
<td>920</td>
<td>200</td>
<td>0.783</td>
</tr>
<tr>
<td>225</td>
<td>46.5</td>
<td>1020</td>
<td>260</td>
<td>0.745</td>
</tr>
<tr>
<td>250</td>
<td>54</td>
<td>1240</td>
<td>330</td>
<td>0.734</td>
</tr>
</tbody>
</table>

Table 3.1 (b) - Fringe visibility as a function of argon ion laser output power for a fixed laser aperture of 5.
power is increased the fringe visibility decreases. The optimum laser tube current is 35A with an output power of 150mW. It is important that the output coupler of the laser is cleaned regularly as the laser efficiency drops after several tens of hours of use. To achieve maximum fringe visibility it is also necessary to ensure that the polarisations of the two interfering beams are parallel. Unfortunately the fibre used in the interferometer is not polarisation preserving and so the direction of polarisation at the fibre output is very sensitive to any pressure or bending along the length of the fibre. The desired fibre output polarisation is electric field vertical (normal to the plane of Figure 3.2). The output polarisation is changed by placing small steel weights along the fibre length and adjusting their position. The polarisation is monitored with a Glan Thomson polariser, set to pass horizontally polarised light, and a photodiode. The output polarisation is optimum when the light passed by the polariser is minimised. The polarisation stability is monitored by checking the fringe visibility before and after each exposure, and it is found to be stable for a few hours.

The grating exposure time in the interferometer is determined by the power density at the sample. For a laser output power of 150mW the power coming from each fibre end, p, is 6.3mW. The beam diameter at the sample, d, is 190mm for a fibre end to sample spacing of 50cm. The average power density at the sample is \( \frac{8p}{\pi d^2} = 444 \text{mW/m}^2 \). The threshold exposure energy for the photoresist is typically 30 mJ/cm\(^2\), so the exposure time should be around 10 minutes. In practice, exposure times of 30 minutes are required to produce strong gratings. This long exposure time means that it is essential to actively stabilise the interferometer for which the phase typically drifts by \( \pi \) in about 5 minutes. To improve the stability further the whole of the interferometer is enclosed in an air tight perspex box. The exposure time is controlled by a mechanical shutter placed in front of the sample. When the interferometer has been stabilised the shutter is opened to begin the exposure. At the end of the exposure time the laser beam is blocked.

Reflections from the bottom face of the substrate will cause spurious light intensity variations and macroscopic gratings to appear. These reflections are virtually eliminated by putting a black lens tissue soaked in index matching fluid on the back of the sample.
The grating produced by the fibre interferometer is not perfectly uniform across the sample. The light from the bare fibre ends has a Gaussian intensity distribution and it also has a phase distribution across the sample. With the fibre ends 50cm from the sample, the Gaussian distribution means that the intensity drops by 10% from the centre to the edge of the sample. This means that gratings at the centre of the sample will be slightly more exposed. The fibre ends act as point sources of light so that there is a phase distribution across the sample. The effect of the phase distribution is to cause a chirp in the grating period across the sample. Figure 3.3 shows the calculated grating period across the sample normalised to the grating period at the centre. At the edge of the sample the grating period increases by 0.13% compared to the centre.

![Figure 3.3](image-url)  

**Figure 3.3** – Grating period, $\Lambda$, across the sample normalised to the value at the centre of the sample. The change in grating period occurs because point sources of light are used in the interferometer.
3.4.3 Creating Hexagonal Patterns

Many groups are investigating two-dimensional photonic crystals consisting of a hexagonal array of air holes. This pattern can be generated by a superposition of three gratings orientated at 120° to each other. The photoresist is exposed three times in the interferometer with the sample being rotated by 120° after each exposure. The grating formed by a single exposure is shown in Figure 3.4(a). A hexagonal array of holes is

Figure 3.4 – Simulated grating interferometer intensity patterns. White corresponds to maximum intensity and black is zero intensity. (a) Single grating exposure. (b),(c) and (d) Three sequential grating exposures where φ is the relative phase between the three gratings.
formed when three gratings are combined exactly in phase as shown in Figure 3.4(b). The white spots correspond to regions of maximum light intensity recorded in the photoresist. When the photoresist is developed these high intensity spots will appear as holes in the photoresist layer, therefore by considering the unexposed regions as a high index lattice this pattern can be recognised as a graphite crystal structure.

In the real interferometer the grating period and phase of each grating varies across the area of the sample, so that the interference pattern between the three gratings will also vary. When the relative phase, $\phi$, between the three gratings is $\pi$ then the interference pattern formed is the inverse of that formed when the gratings are in phase. Figure 3.4(c) shows the pattern when the gratings are $\pi$ out of phase. Now the unexposed

![Figure 3.5](image.png) - Variation of relative phase, $\phi$, across the central part of the sample arising from the point sources of light in the interferometer. Black represents $\pi/2$ and white is $-\pi/2$. 
regions form a hexagonal array of high index spots and so this is a hexagonal crystal structure. Figure 3.4(d) shows the pattern when the gratings are $\pi/2$ out of phase. This pattern is a mixture of the hexagonal and graphite structures with a hexagonal array of holes and an interspersed hexagonal array of high index spots. By further calculation it can be shown that as the relative phase increases the crystal structure will gradually transform from graphite to hexagonal and back again.

Figure 3.5 shows how the relative phase of the three gratings varies across the sample. In the figure, 50% gray corresponds to zero relative phase and black (white) corresponds to $\pi/2$ ($-\pi/2$). These relative phases do not relate directly to those in Figure 3.4 as the pattern at the centre will depend on the exact position of the sample as it is rotated for each exposure and so there is some arbitrary constant phase to add which will vary from sample to sample. It can be seen that within 3mm of the centre of the sample the pattern is constant. Further out the pattern periodically varies between the hexagonal and graphite structures with the period getting shorter further from the centre. For the purposes of the experiment regions of interest on the sample are defined by a mask (Figure 3.6). The areas under the black mask are protected from further exposure and the rest of the sample is overexposed so that smooth waveguide regions are formed. In the central region the pattern will be constant whereas, in the top right region, the structure varies with a period of approximately 0.33mm as shown in Figure 3.7.

![Figure 3.6](image)

*Figure 3.6 – Mask used to define photonic crystal regions. The black areas are protected from further exposure and the rest of the sample is overexposed so that smooth waveguide regions are formed.*
The problem of varying relative phase can be overcome by constructing an interferometer with three point sources that interfere at the sample. This is not practical with the power available from the argon ion laser and it would be difficult to stabilise the fringes.

An alternative is to make just two grating exposures at $120^\circ$ to each other. The predicted pattern from such an exposure is shown in Figure 3.8. The hexagonal array of holes is present as required and the pattern is independent of the relative phases of the two superimposed gratings. Unfortunately the holes are rectangular in shape and the grating is not perfectly hexagonally symmetric, but the intensity peaks are

**Figure 3.7** – Magnified view of the relative phase in the top right corner of the sample, centred on the black region of the mask in Figure 3.6. There is a macroscopic variation of the grating pattern with a period around 0.33mm.

The problem of varying relative phase can be overcome by constructing an interferometer with three point sources that interfere at the sample. This is not practical with the power available from the argon ion laser and it would be difficult to stabilise the fringes.

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approximately circular. This means that by careful exposure control and development, circular holes are possible.

3.4.4 Transitions

Practical use of photonic bandgap waveguides will inevitably involve combining them with regular planar waveguides. To couple light efficiently from a regular waveguide into a photonic band gap waveguide an adiabatic transition is required. Transitions in photoresist can be made by exposing the sample in a mask aligner with a mask (Figure 3.6) separated some height above the sample. The edges of the mask are blurred by diffraction of the collimated light source producing smooth transitions between the

Figure 3.8 - Simulated grating interferometer intensity pattern with two sequential grating exposures. The sample is rotated through 120° between exposures.

3.4.4 Transitions

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exposed and unexposed regions. The mask exposure is performed in a mask aligner with an exposure time of 0.5 seconds. A 1mm glass slide is used as a spacer between the mask and the photoresist layer (Figure 3.1(c)) and this produces a 36μm long transition with a depth of 280nm. This length is equivalent to approximately 100 wavelengths in the waveguide and should be sufficient for a smooth transition. The ideal photoresist profile is shown schematically in Figure 3.1(d).

### 3.5 Photoresist Developing

The photoresist pattern is developed by immersion and gentle agitation in Microposit developer for 30 seconds. The sample is then immediately transferred to a beaker of fresh deionised water for a minute and then washed with a deionised water spray gun for a few minutes. The sample is blown dry with nitrogen. The photoresist must not be baked as it will melt and flow.

### 3.6 Checking the Photoresist Pattern

The reproducibility of grating exposure intensities is limited by polarisation drift in the fibre interferometer. Each time an exposure is made the interferometer must be adjusted to optimise the polarisation of the interfering beams. This means that there is a 10% uncertainty in the exposure dose that each sample receives. Thus the grating quality must be assessed before the sample is etched. If the grating quality is poor the sample is cleaned and the patterning process is repeated.

It is difficult to check the quality of the photoresist pattern without destroying it. The simplest test is to view the colours diffracted from the sample when it is illuminated with a white filament lamp. The sample is viewed along the direction of the grating vector at an angle of around 80° to the surface normal with the white light source next to the viewer’s eye and pointing towards the sample. When the sample is orientated correctly, and a strong grating is present, brightly coloured blue-green light is reflected back. The three gratings can be checked by rotating the sample by 120° between viewings. The triple exposure method produces gratings that are equally strong in the three directions. The double exposure method produces two strong gratings but the grating in the third direction is much weaker.
There are several standard methods for direct characterisation of samples with small feature sizes but these are quite destructive. The surface profile can be measured with an atomic force microscope (AFM) but unfortunately this requires that the sample be diced into pieces no larger than 25mm × 25mm so that it fits into the AFM. The AFM stylus can also scratch the soft photoresist surface. Sacrificial samples characterised in this way show that the final grating depth is typically around 30nm in photoresist layers with a thickness from 80 to 150nm. This poor visibility means that, to produce photoresist gratings with bare substrate at the intensity maxima, the developed photoresist layer thickness should be below 30nm. It is very difficult to maintain such thin layers in the development stage of the process. In practice, trial and error was used to achieve the thinnest layers possible without the layer disappearing completely. This was achieved by adjusting the exposure times so that the central shape on the mask (Figure 3.6) was extremely faint or non existent when developed. The exposure intensity at the outer shapes is approximately 10% less so that, when the central shape disappears, the thickness of the outer shapes will be very small.

3.7 Ion Beam Etching

The photoresist patterns are transferred to the waveguide layer by an etching process. This could be chemical, sputter, reactive sputter, ion beam or reactive ion beam etching. Wet chemical etching suffers from poor directionality, which causes undesirable effects such as undercutting and is unsuitable for high aspect ratio etching. Sputter etching is achieved by ionising atoms in a glow discharge and accelerating them towards a target with a high voltage electrical potential. When the ionised atoms hit the target they knock atoms off and these can escape from the surface or be redeposited. The problem with sputter etching is that the immediate environment around the target must support the glow discharge plasma and this makes the target prone to redeposition and electrical charging. The preferred method is ion beam etching where a parallel beam of ions is generated by an ion beam source separated from the target. The ion beam passes through a throttle to a low pressure chamber where the sample is placed, this minimises redeposition of target material. The beam is also neutralised, which minimises expansion of the beam by coulomb repulsion and prevents charging up of insulating targets. The parallel ion beam allows the control of
the incident beam direction and this can be used to prevent trenching whereby trenches form at the edges of etch pits due to the reflection of ions from the side walls of the pit. Ion beam etching also provides the ability to control beam current and energy independently of the target material properties. Reactive sputter etching and reactive ion beam etching involve the addition of a highly reactive gas, usually CHF₃, to the discharge. The etch rates in reactive etching are usually much higher and there is generally more differentiation between the etch rates of different types of materials. It was decided to use ion beam etching in an argon atmosphere with the sample rotating and tilted at 40°. The etch rate for photoresist (tantalum pentoxide) was measured to be 28.1nm/minute (25.0nm/minute). Therefore, etch times of between 30 seconds and 2 minutes can be used to producing grating depths between 12.5nm and 50nm.

3.8 Checking the Etched Waveguide Pattern

After etching the remaining photoresist is removed by soaking the sample in acetone. The grating quality can be checked by visual inspection as before. The gratings can be more fully characterised by prism coupling methods which are described in the next chapter.

References


4. EXPERIMENTAL METHOD FOR PRISM COUPLING INTO PHOTONIC CRYSTAL WAVEGUIDES

4.1 Introduction

4.2 Prism Coupling

4.3 Prism Design

4.4 Optimum Coupling

4.5 Experimental Arrangement

4.6 Choice of light source

4.7 Alignment

4.8 Observation of dark m-lines
4. Experimental Method for Prism Coupling into Photonic Crystal Waveguides

4.1 Introduction

The design and construction of a prism-coupling apparatus suitable for probing photonic crystal waveguides is considered in this chapter. The prism coupling technique has been widely studied and used in various experiments in integrated optics\textsuperscript{1-6}. In particular we consider a technique employing aluminium spacers\textsuperscript{5} that enable non-contact operation at the base of the prism while providing optimum excitation of the waveguide modes. We also employ a novel conical glass prism that enables rotation of the waveguide without changing the coupling conditions thus enabling consistent measurement of properties in all directions.

4.2 Prism Coupling

The coupling of a laser beam into a planar dielectric waveguide is governed by the incident angle, $\theta_i$, at the prism base (Figure 4.1). This angle determines the phase velocity in the y direction of the incident wave in the prism (refractive index $n_p$) and in the air gap. Strong coupling of light into the waveguide only occurs when we choose $\theta_i$ so that the phase velocity parallel to the interface at the prism base matches the phase velocity of one of the characteristic modes of the waveguide. This is a simple

![Figure 4.1](image-url)
phase matching condition and is equivalent to requiring the wavevector components parallel to the interface to be equal in the two materials. If two or more waveguide modes can be excited, by varying the incident angle and polarisation, then the thickness and effective refractive index of the waveguide can be calculated from the well known waveguide dispersion equation (2.6).

4.3 *Prism Design*

The two most popular prism shapes are the half prism (Figure 4.1) and the symmetrical prism (Figure 4.2). The prism shape determines the way that a waveguide mode is observed. With the half prism a streak of light is seen when a mode is excited, this is the easiest way to see modes in low-loss single-mode waveguides. The mode is visible due to scattering at cosmetic defects in the waveguide. With the symmetrical prism it is difficult to see modes directly, as light is coupled back from the waveguide into the prism further along from the coupling spot. However, coupling into a waveguide mode can be viewed indirectly by looking for a dark $m$-line in the spot reflected from the prism base. The dark $m$-line appears because the light is incident on the prism at a range of angles of which only a small fraction is actually coupled into the waveguide mode. The bright spot reflected from the prism base is observed on a screen and a dark line is seen corresponding to the missing light that is coupled into the waveguide at the prism base.
Besides the prism shape, the most important parameters of a prism are its refractive index, $n_p$, and the prism angle, $\theta_p$, they determine the range of effective refractive indices, $n_{\text{eff}}$, that can be measured with the prism. The effective index, $n_{\text{eff}}$, is related to the angle of incidence on the entrance face of the prism, $\alpha$, by

$$n_{\text{eff}} = \sin \alpha \cos \theta_p + (n_p^2 - \sin^2 \alpha)^{1/2} \sin \theta_i$$

(4.1)

In a film of index, $n_1$, deposited on a substrate of index, $n_0$, the effective indices, $n_{\text{eff}}$, of all the guided modes will lie in the interval $n_0 < n_{\text{eff}} < n_1$ as shown in chapter 2. Figure 4.3 shows the effective index, $n_{\text{eff}}$, versus the prism angle, $\theta_p$, over the full range of incident angles, $\alpha$, for a Ga:La:S prism ($n_p = 2.45$). $\alpha$ is the incident angle. The horizontal lines are the refractive indices of tantalum pentoxide ($n_{\text{eff}}=2.12$) and silica ($n_{\text{eff}}=1.457$). These are the limits of $n_{\text{eff}}$ for modes in a tantalum pentoxide waveguide on a silica substrate.

![Figure 4.3](image)

**Figure 4.3** - Effective index, $n_{\text{eff}}$, vs. prism angle, $\theta_p$, for a Ga:La:S prism ($n_p = 2.45$). $\alpha$ is the incident angle. The horizontal lines are the refractive indices of tantalum pentoxide ($n_{\text{eff}}=2.12$) and silica ($n_{\text{eff}}=1.457$). These are the limits of $n_{\text{eff}}$ for modes in a tantalum pentoxide waveguide on a silica substrate.

Incident angles, $\alpha$, for a Ga:La:S (chalcogenide glass, $n_p = 2.45$) prism. The upper and lower limits of $n_{\text{eff}}$ are shown for a tantalum pentoxide waveguide ($n_{\text{eff}} = 2.12$) and a fused silica substrate ($n_{\text{eff}} = 1.457$). The figure shows that the prism angle, $\theta_p$, should be about 45°, but exactly 45° is unsuitable as internal reflections in the prism can interfere with the main beam. In practice we choose $\theta_p$ to be 40°. Another popular prism material is TiO₂ (rutile, $n_o = 2.304$, $n_e = 2.539$) which is a uniaxial birefringent crystal. The optical axis of a rutile prism should be orientated normal to the cross
section shown in Figure 4.1 so that a constant value of \( n_p \) can be used for each of the input polarisations. For TM (TE) polarisation the ordinary (extraordinary) refractive index, \( n_o \) (\( n_e \)), is used. These prisms are usually cut with \( \theta_p \) around 40° so they are suitable for the lower range of \( n_{\text{eff}} \) values.

### 4.4 Optimum Coupling

In the experimental observation of guided modes a linearly polarised, TEM\(_{00}\), laser beam is focused into the prism so that the beam waist coincides with the prism base. The point where the beam strikes the prism base is the coupling spot. It is at this point that the effective refractive index and thickness of the waveguide are determined. In the case of the symmetric prism the coupling spot is preferably near the centre of the prism base and in a half prism it must be close to the corner. The optical system should be adjusted so that the coupling spot remains practically stationary on the prism base as the prism is rotated.

The amount of light that is coupled into the waveguide is determined by the air gap profile at the coupling spot. Ulrich\(^6\) has shown that the optimum gap profile is an exponentially increasing taper. In practice, local surface roughness or contamination near to the coupling spot determines the gap profile. As the clamping pressure between the prism and waveguide is increased, the air gap width decreases and the coupling strength increases. If the coupling strength is too high the modes can be shifted and broadened. Therefore for accurate measurement of modal properties the clamping pressure must be reduced as far as possible while still being able to observe the mode.

The periodically patterned waveguides that we wish to probe have sub-micron features and are likely to be very fragile, so it is undesirable to clamp hard prisms directly onto them. To overcome this problem a symmetrical prism with aluminium spacers was fabricated (Figure 4.4). This allows non-contact probing of the periodic structure at the coupling spot. It also turns out that this configuration produces an air gap profile that is close to optimum efficiency for coupling into the waveguide\(^5\).
The properties of the periodic waveguides are highly anisotropic so it is necessary to measure them consistently over many different directions in the waveguide. If a regular prism is used then this requires releasing the clamping pressure and lifting the prism away from the waveguide after each measurement so that the sample can be rotated without damage. To enable measurement in different directions without changing the coupling conditions the prism needs to be rotationally symmetric about an axis normal to the plane of the waveguide. A hemispherical prism shape fulfils this criterion and in addition the input beam is able to enter nearly normal to the cylindrical surface in all directions thus extending the range of accessible effective index values, $n_{eff}$. In practice it is very difficult to polish a good quality hemisphere from the high index materials required. In addition to this, the accuracy of the hemispherical coupler is critical on lateral displacements of the input beam, which must point exactly to the centre of the prism\(^1\). A better solution is a conical prism, which is formed by rotating the symmetrical prism in Figure 4.4 about the $z$-axis. To maintain cylindrical symmetry the prism material must be isotropic, so rutile is unsuitable. Ga:La:S is a high index glass which is suitable due to its amorphous structure. A Ga:La:S prism was polished by Crystran Ltd, U.K. A photo of the prism is shown in Figure 4.5, note the horizontal scratches on the prism, which are due to the polishing technique required to maintain the prism angle and the straightness of the prism side. There are also some microscopic inhomogeneities in the prism material, which can be seen as bright spots at the top of the prism.

**Figure 4.4** – Conical Ga:La:S prism with aluminium spacers on the base. The prism straddles the patterned region allowing non-contact probing.
Aluminium spacers were evaporated onto the perimeter of the base of the prism through a brass mask. The brass mask was made by coating a piece of 15μm × 25 mm × 25 mm brass shim with photoresist and exposing it in a mask aligner with a specially designed lithographic mask. The photoresist pattern was developed and then etched through the brass in a ferric chloride bath. The brass mask was clamped against the prism base and a ~1μm aluminium film was evaporated onto the perimeter of the prism base using an Edwards coater.

4.5 Experimental Arrangement

The general arrangement for the observation of coupling and the measurement of coupling angles is shown schematically in Figure 4.6. The waveguide is clamped against the base of the prism by a spring-loaded anvil. The anvil consists of an 8mm diameter steel rod with a rounded end of radius 4mm and applies pressure behind the waveguide at the required coupling spot. The pressure is varied with a micrometer.
Figure 4.6 - Schematic of experimental prism coupling arrangement. The waveguide sample is clamped against the prism base by a spring loaded anvil. The x-y-z stages allow precise position of the incident beam on the prism. The electronic rotation stage enables precise adjustment and measurement of the incident angle. The manual rotation stage enables the waveguide to be rotated without changing the coupling conditions, allowing different in plane directions to be probed.
screw, which has a travel of 10mm and compresses a music wire spring with a spring constant of 8.92N/mm. The conical prism and anvil are mounted concentrically on a manual turntable (10 minutes of arc accuracy in reading the in-plane angle, $\phi$), which is mounted, so that the axis of rotation is horizontal, on x-y-z translation stages. The stages are mounted on a precision electronically controlled turntable (.001° resolution in reading the incident angle, $\alpha$), which is mounted, with the axis of rotation vertical, on an optical table. A laser is mounted on the optical table so that it points at the intersection of the turntable axes. An optical rail is positioned along the beam so that lenses and polarising elements can be used to control the beam.

The rutile half prism can also be mounted in the same apparatus, in which case the manual turntable is locked so that the c-axis of the rutile prism is parallel to the z-axis of Figure 4.6. This enables an easy initial evaluation of the waveguide properties and so offers a guide as to where the dark $m$-lines may be observed when using the conical prism.

### 4.6 Choice of light source

Two different lasers are used to probe the waveguides. A 10mW HeNe laser is used for initial alignment and waveguide characterisation. The HeNe emits a single, visible wavelength, TEM$_{00}$ beam at 633nm. The rutile half prism is used to measure the zero order mode coupling angles for TE and TM polarisation, thus enabling the calculation of the waveguide thickness and index. The waveguide loss can be quantitatively assessed by observing the decay in the brightness of rays as they traverse the waveguide.

The other light source is a titanium sapphire (Ti:Sapph) laser which is pumped by a multi-line argon ion laser. The Ti:Sapph laser is operated at the lowest available power, just above threshold, where the output power is around 30mW for 2.5W of argon ion pump power. Ti:Sapph has a broad gain curve stretching from 650nm to 1100nm. The mirror set used with the Ti:Sapph allows lasing over a tuning range of 770nm to 930nm with a typical linewidth of 1.7nm. The wide tuning range of the Ti:Sapph relaxes the need to fabricate a precise grating period and allows wavelength-dependent characterisation of the grating properties. The unfortunate problem with the Ti:Sapph is that its output is in the near infra red region of the spectrum and so is
practically invisible to the naked eye. The Ti:Sapph output is observed with a handheld infrared viewer or a vidicon video camera.

4.7 Alignment

The rutile half prism is initially aligned by observing the reflection from the entrance face. A microscope slide and a 1mm pinhole are placed in the beam path and the prism is manipulated until a reflected spot is observed on the screen (Figure 4.7). Reflected light can only return through the pinhole when the reflection is normal to the entrance face to the prism. At this point the reading on the electronic turntable is reset to zero so that it corresponds with the incident angle, $\alpha$. This method allows the zero point to be located reproducibly with an error of less than 0.01°. The prism angle can be measured directly by sequential reflection measurements from the prism base and entrance face.

Alignment of the conical prism is much more difficult as the prism axis must be aligned exactly with the manual turntable axis. The prism axis is centred on the turntable axis by direct alignment of the anvil point to the circular prism base. The axes are made parallel by mounting a planar reference sample against the prism base. The y translation stage is moved so that the incident light reflects directly from the waveguide surface. The normal to the waveguide surface is located, as before, by centring the reflected spot on the pinhole. If the prism axis is not aligned with the turntable axis then, as the manual turntable is rotated, the reflected spot will move away from the pinhole. Fine

![Diagram](image-url)
adjustment of the prism orientation, with respect to the manual turntable, was obtained by including spring washers in the four bolts that were used to clamp the prism support to the turntable face. Thus allowing exact alignment of the prism and turntable axes.

The prism angle can be determined by sequential reflection measurements from the waveguide surface and the entrance face of the prism, which is curved so that the reflected spot is elongated in the x direction with an aspect ratio of approximately 100:1. The consistency of the prism angle can be checked by rotating the manual rotation stage.

4.8 Observation of dark m-lines

Small strips of wavevector space can be viewed directly by using the conical prism. The light incident on the entrance face is focused to a point at the coupling spot by the curved prism side (Figure 4.8). The focusing only occurs in one direction and acts to increase the range of directions, $\Delta\phi$, probed in the plane of the waveguide. In the other direction, corresponding to the plane of the incident angle, $\theta_i$, the beam is not focused and so the light is concentrated in a small angle, $\Delta\alpha$, determined by the divergence of the incident beam (Figure 4.8). The angle $\Delta\alpha$ corresponds to a wavevector range, $\Delta k$, on the wavevector diagram (Figure 4.9(a)). The two ranges of angle in the incident beam mean that it will illuminate the shaded region on the wavevector diagram.

The light reflected from the prism base and transmitted through the exit face of the prism is viewed on a white screen placed about 75cm away from the prism. The reflected spot (Figure 4.9(b)) is elongated in the x direction due to the focusing effect of the curved prism side. Each position along the x direction of the spot corresponds to a particular angle, $\phi$, in the plane of the waveguide. The range of angles, $\Delta\phi$, is determined by the size of the spot incident on the entrance side of the prism and can be varied from approximately 30 arc minutes to 600 arc minutes. Each position along the z direction of the spot corresponds to a particular value of k, the in-plane wavevector. If the coupling spot is not at the centre of the prism then spherical aberrations occur and the elongated spot on the screen transforms from a vertical line to a C-shaped curve. This aberration can be prevented by translating the prism in the y and z directions so that the elongated spot is always a vertical line.
When a waveguide mode is excited, a dark region becomes visible in the spot viewed on the screen (Figure 4.9(b)). For a uniform waveguide the dark region is a thin vertical line, known as a dark $m$-line. The extent of the dark region in the $x$-$z$ plane corresponds to the range of $n_{\text{eff}}$ coupled into the mode. When the spot size is large, reflections from the scratches on the prism side can also be seen as dark vertical lines and it is hard to distinguish between these and the dark $m$-lines. The distinguishing feature of a dark $m$-line is that its angular position rotates at approximately half the rate of the dark scratch lines when the prism is rotated. Therefore a good way to find dark $m$-lines is to start with a large spot and look for a slow moving dark line on the screen as the prism is rotated.

**Figure 4.8** - Conical prism coupling geometry. The incident light excites a range of wavevectors on the wavevector diagram (Figure 4.9(a)) determined by the angles $\Delta \phi$ and $\Delta \alpha$. The transmitted light forms an elongated spot on the screen (Figure 4.9(b)).
References


Figure 4.9 - (a) Wavevector diagram for modes in a homogenous planar waveguide. The shaded region is illuminated by the light incident on the prism. (b) The light transmitted through the prism forms an elongated spot on the screen corresponding to the shaded region in (a). A dark m-line is seen where light couples into the waveguide modes.
5. RESULTS FROM PRISM COUPLING INTO 2-D PHOTONIC CRYSTAL WAVEGUIDES

5.1 Introduction

5.2 Basic Waveguide Properties

5.3 Observation of Band Gaps

5.4 Grating Periods and Angles

5.5 Triple Points

5.6 Ray Coupling
5. Results From Prism Coupling Into 2-D Photonic Crystal Waveguides

5.1 Introduction

In this chapter the results are presented for prism coupling into a periodically etched waveguide sample. In the first few sections the grating properties are measured by coupling directly from the conical prism base into the periodic parts of the waveguides and observing dark $m$-lines. In the final section ray propagation in periodic waveguides is observed. A divergent beam is prism coupled into a smooth region of the waveguide so that it propagates through a transition region and into the periodic structure.

5.2 Basic Waveguide Properties

From the range of samples fabricated the best samples are 29/10A and 29/10B. These are selected by visual inspection of diffracted white light. The important details of the sample fabrication are listed below; other parameters are described in chapter 3.

- Tantalum Pentoxide waveguide: Sputter for 180 minutes.
- $\alpha$-step measurement: 29/10A: $280\pm8$nm, 29/10B: $293\pm7$nm.
- 0.5 second mask exposure.
  - 29/10A: $2 \times 18$ minute grating exposures, $120^\circ$ rotation between exposures.
  - 29/10B: $3 \times 12$ minute grating exposures, $120^\circ$ rotation between exposures.
- RIBE for 2 minutes.

The first measurement to be made on the fabricated samples is the waveguide thickness and refractive index. This is deduced by prism coupling with a rutile half prism and a He:Ne laser ($\lambda = 632.8$nm) into a smooth waveguide region. The prism angle is $40.057^\circ\pm0.013^\circ$ and its refractive index is given by the following Sellmeier equations:\(^1\):

\[
\begin{align*}
    n_o &= \sqrt{5.913 + 0.2441/(\lambda^2 - 0.0803)} \\
    n_e &= \sqrt{7.917 + 0.3322/(\lambda^2 - 0.0843)}
\end{align*}
\]

(5.1)
where $\lambda$ is in $\mu$m. For a smooth region of sample 29/10A, the TE and TM coupling angles are measured as $6.95^\circ \pm 0.005^\circ$ and $15.48^\circ \pm 0.005^\circ$ which correspond to a waveguide refractive index of $2.095 \pm 0.1\%$ and a thickness of $272.6\text{nm} \pm 0.1\%$.

The waveguide loss can be estimated by observation of the guided streak of light. The light undergoes an exponential decay of the form

$$\frac{P_x}{P_0} = 10^{-\alpha x/10}$$

(5.2)

where $P_x$ is the power at point $x$, $P_0$ is the power at $x = 0$ and $\alpha$ is the loss measured in dB/cm when $x$ is measured in cm. The streak is measured with a vidicon camera and the intensity data is fit to the function in (5.2). The loss in the plain waveguide is $2.6 \text{dB/cm} \pm 20\%$.

The measurements are repeated with the conical Ga:La:S prism (prism angle = $40.33^\circ \pm 0.01^\circ$) and the TE and TM coupling angles are $28.05^\circ \pm 0.005^\circ$ and $19.7^\circ \pm 0.005^\circ$. Given that the waveguide refractive index and thickness are already known, these measurements can be used to calculate the refractive index of the conical prism. The resulting value for $n_p$ is $2.476 \pm 0.2\%$ which lies within the range of values obtained elsewhere\textsuperscript{2}. By fitting the Ga:La:S refractive index data in reference 2 to a Sellmeier equation the wavelength values in Table 5.1 are obtained.

Taking into account these data and the prism measurement, the refractive index of the Ga:La:S prism can be estimated to be $2.415 \pm 1\%$ for the Ti:Sapph tuning range used. The Ga:La:S refractive index could be more accurately determined by prism coupling into a smooth waveguide and measuring the values at each of the Ti:Sapph wavelengths used.

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>Ga:La:S Refractive Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>632.8</td>
<td>2.528</td>
</tr>
<tr>
<td>825</td>
<td>2.474</td>
</tr>
<tr>
<td>870</td>
<td>2.467</td>
</tr>
<tr>
<td>910</td>
<td>2.461</td>
</tr>
</tbody>
</table>

Table 5.1 - Ga:La:S dispersion data interpolated from reference 2.
The next stage is to prism couple directly into the patterned waveguide with the conical Ga:La:S prism and the Ti:Sapph laser. It is very useful to calculate the expected coupling angles, $\alpha$, for the invisible Ti:Sapph light so that waveguide modes can be located more easily. The waveguide index and thickness are measured with the He:Ne laser and the conical prism. For the top left hexagonal patterned region on sample 29/10B the TE and TM coupling angles are $26.525^\circ \pm 0.005^\circ$ and $17.936^\circ \pm 0.005^\circ$ giving a waveguide effective index of $2.046 \pm 0.05\%$ and thickness of $253.2\text{nm} \pm 0.05\%$. Figure 5.1 shows the predicted Ti:Sapph coupling angles for the same region. Note that it is not possible to measure the patterned waveguide loss with the conical prism as there is no guided streak to observe. The loss can be measured by ray coupling with the rutile prism and is considered in section 5.6.

**Figure 5.1** – Predicted coupling angle vs. wavelength for the Ga:La:S conical prism and sample 29/10B in the patterned regions.
5.3 Observation of Band Gaps

Bandgaps can be found by rotating the manual turntable and watching the shape of the dark $m$-line on the screen. When the range of directions coupled into the waveguide straddles a bandgap then a gap is seen in the $m$-line. Bandgaps occur on the wavevector diagram where the average index circles intersect and two different plane waves couple together. Figure 5.2(a) shows a TM-TE bandgap observed in a triple grating structure (29/10B). The image shows the bright spot reflected from the base of the conical prism, which is viewed on a screen with a vidicon camera. The dark $m$-lines can be seen more easily by viewing the picture at a steep angle so that the vertical axis is compressed. The screen is marked with a coarse 5mm grid and a fine 1mm grid and

![Image](120203a.bmp)
![Image](120203b.bmp)

(a) (b)

- $0.02$
- $0.025$
- $0.03$
- $0.035$
- $0.04$
- $0.045$
- $0.05$

- $1.059$
- $1.056$

$k_y \Lambda / \pi$

![Image](120204a.bmp)
![Image](120204b.bmp)

(c) (d) (e) (f)

- $0.16$
- $0.165$
- $0.17$
- $0.175$
- $0.18$
- $0.185$

$k_y \Lambda / \pi$

![Image](120205a.bmp)
![Image](120205b.bmp)

- $1.044$
- $1.04$

$k_x \Lambda / \pi$

**Figure 5.2** - (a) TM-TE bandgap as viewed on screen with vidicon camera. (b) The image in (a) is processed to accentuate the dark $m$-lines. (c) The dark $m$-lines are transformed onto a wavevector diagram. The dotted line corresponds to equation (5.4) which is fit to the dark $m$-lines to determine the coupling strength. (d) TM-TM bandgap as viewed on screen with vidicon camera. (e) The image in (d) is processed to accentuate the dark $m$-lines. (f) The dark $m$-lines are transformed onto a wavevector diagram. The dotted line corresponds to equation (5.4) which is fit to the dark $m$-lines to determine the coupling strength. (Sample 29/10B, $\lambda = 876.4$nm.)
is placed approximately 1m away from the prism. Figure 5.2(b) shows the image after processing to extract the dark \( m \)-lines. The two dark vertical lines are dark \( m \)-lines which correspond to light that has been coupled into the zero order TM waveguide mode and so is missing from the bright spot. The gap between the lines is a photonic bandgap. In this region the waveguide mode is suppressed and so light is forbidden from coupling into the waveguide and is reflected from the coupling spot at the prism base.

In order to transfer this data to a wavevector diagram the horizontal and vertical axes can be calibrated. The horizontal scale is directly related to the electronic turntable angle, \( \alpha \). This is converted to the effective refractive index of the waveguide, \( n_{\text{eff}} \), by equation (4.1) and into normalised frequency, \( \nu_{\text{eff}} = k_{\varphi} n_{\text{eff}} \Lambda \). The vertical scale can be calibrated directly to the manual turntable angle, which gives the propagation angle in the plane of the waveguide, \( \phi \). By using these conversions, each point of the image can be transformed to a point on the wavevector diagram using the simple relations:

\[
\begin{align*}
k_x \Lambda / \pi &= \nu_{\text{eff}} \cos \phi / \pi \\
k_y \Lambda / \pi &= \nu_{\text{eff}} \sin \phi / \pi
\end{align*}
\]  

(5.3)

Figure 5.2(c) shows the dark \( m \)-lines as they appear in wavevector space. A simplified two-wave bandgap function can be fit to the dark lines to determine the coupling constant. The coupling equation is given simply by\(^3\)

\[
u u = \kappa^2 / 2
\]  

(5.4)

where the \( u \) and \( v \) axes are defined as shown in Figure 5.3. The band gap is due to coupling between a zero order TM partial wave and a first order TE partial wave. The relevant average index circles are shown in Figure 5.3. The wavevectors \( k_{\text{TE}} \) and \( k_{\text{TM}} \) point from the centres of the circles to the centre of the bandgap, which occurs at the intersection of the two circles. The \( u \) (\( v \)) axis is along the \( k_{\text{TE}} \) (\( k_{\text{TM}} \)) direction with the origin centred at the intersection. The width of the gap is determined by the value of the coupling constant, \( \kappa \). The asymptotes to the band gap function are straight lines perpendicular to each of the \( u \) and \( v \) axes. These are only good approximations to the average index circles close to the \( u \)-\( v \) origin. The dotted line on Figure 5.2(c) shows the band gap function superimposed on the transformed band gap image. The dark \( m \)-
Images of band gaps for TM-TE, TM-TM, TE-TE and TE-TM coupling are shown in figures 5.2 and 5.4 and the normalised coupling constants, $\kappa_N$, are shown in Table 5.2.

**Figure 5.3** - Geometry for a TM-TE band gap function. The band gap occurs at the intersection of the TM and TE average index circles shown. The u (v) axis is along the $k_{TE}$ ($k_{TM}$) direction with the origin centred at the intersection. The asymptotes to the band gap function are straight lines perpendicular to each of the u and v axes.

Lines seen in the image do not extend up to the edges of the gap because of the finite angular beam divergence of the input beam.

<table>
<thead>
<tr>
<th>Mode Types</th>
<th>Measured Coupling Constant, $\kappa_N$</th>
<th>Coupling Constant from Reference 3, $\kappa_N$</th>
<th>Figure No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE-TM</td>
<td>0.00057±0.00007</td>
<td>0.0100</td>
<td>Figure 5.4(d)</td>
</tr>
<tr>
<td>TE-TE</td>
<td>0.00021±0.00007</td>
<td>&lt; 0.0015</td>
<td>Figure 5.4(a)</td>
</tr>
<tr>
<td>TM-TE</td>
<td>0.00071±0.00007</td>
<td>0.0105</td>
<td>Figure 5.2(a)</td>
</tr>
<tr>
<td>TM-TM</td>
<td>0.00035±0.00007</td>
<td>0.0075</td>
<td>Figure 5.2(d)</td>
</tr>
</tbody>
</table>

**Table 5.2** - Normalised coupling constants

Images of band gaps for TM-TE, TM-TM, TE-TE and TE-TM coupling are shown in figures 5.2 and 5.4 and the normalised coupling constants, $\kappa_N$, are shown in Table 5.2,
where

$$\kappa_N = \frac{\kappa}{K} \left( \frac{\pi}{\Lambda} \right) = \frac{\kappa}{2} \quad (5.5)$$

For comparison the coupling constants previously obtained for a doubly periodic square lattice$^3$ are also shown in the table. Note that the coupling obtained in the triple grating structure here is very weak. This is because difficulties are encountered in the photoresist processing stages of the fabrication, which means that only very shallow gratings can be produced in the photoresist. The problems stem from poor exposure contrast due to the polarisation drifting in the fibre interferometer and ‘washing out’ of gratings by slight angular misalignment in the multiple exposure method. The poor

Figure 5.4 - (a) TE-TE bandgap as viewed on screen with vidicon camera. (b) The image in (a) is processed to accentuate the dark $m$-lines. (c) The dark $m$-lines are transformed onto a wavevector diagram. The dotted line corresponds to equation (5.4) which is fit to the dark $m$-lines to determine the coupling strength. (d) TE-TM bandgap as viewed on screen with vidicon camera. (e) The image in (d) is processed to accentuate the dark $m$-lines. (f) The dark $m$-lines are transformed onto a wavevector diagram. The dotted line corresponds to equation (5.4) which is fit to the dark $m$-lines to determine the coupling strength. (Sample 29/10B, $\lambda = 895.7$nm.)
contrast in the exposure means that only very shallow photoresist gratings are produced. To etch the waveguide the photoresist layer has to be very thin so that the over exposed grating lines are fully developed, this introduces additional problems due to the poor adhesion of very thin (<80nm) photoresist films. The observation of weak coupling does demonstrate the sensitivity of the conical prism coupling technique for measuring fine detail on the wavevector diagram.

5.4 Grating Periods and Angles

The simplest method for measuring the grating periods and angles is to observe the diffracted spots when the waveguide is illuminated at normal incidence. Unfortunately, in the samples produced here, the gratings are so weak that the diffracted spots are not visible. We can however use the fact that the gratings are weak to develop a prism coupling method for determining the grating periods and angles. Because the gratings are weak the intersections of the average index circles can be accurately located and by modelling their angular positions with wavelength it is possible to extract the grating periods and angles.

![Figure 5.5 – Geometry of average index circle intersections.](image)

Bandgaps occur where the average index circles of modes intersect. Figure 5.5 shows a schematic wavevector diagram. The circle centred on the origin corresponds to the
mode launched into the waveguide by the prism coupler and has a normalised frequency $\nu_1$. The upper circle is centred exactly $2\pi/\Lambda$ above the origin and corresponds to a first order reflection from a grating with grating vector parallel to the $k_y$ axis. The normalised frequency of the upper mode is $\nu_2$. The dashed circles correspond to first order reflections from the other two gratings, which are not involved here. The position of intersection A is given by the relationship

$$
\left(\frac{\nu_2}{\pi}\right)^2 = \left(\frac{\nu_1}{\pi}\right)^2 + 4\left(1 - \frac{\nu_1}{\pi}\cos\phi\right) \quad (5.6)
$$

The normalised frequencies, $\nu_1$ and $\nu_2$, take the values $\nu_{TE}$ or $\nu_{TM}$ corresponding to TE and TM waveguide modes. If $\nu_1$ and $\nu_2$ are equal, corresponding to the case of TE-TE or TM-TM coupling, then (5.6) reduces to

$$
\frac{\nu_1}{\pi}\cos\phi = 1 \quad (5.7)
$$

The TE and TM normalised frequencies are plotted in Figure 2.3 and it can be seen that over the required tuning range they obey the approximate relation

$$
\nu_{TE} = \nu_{TM} + \delta \quad (5.8)
$$

where $\delta = 0.32$. The solution for TE-TM coupling can be found by substituting (5.8) into (5.6) and solving for $\nu_{TE}$ to give

$$
\nu_{TE} = \frac{\pi((\delta / \pi)^2 - 4)}{2\delta / \pi - 4\cos\phi} \quad (5.9)
$$
Note that the experimental in plane angle, $\phi$, is measured with reference to an arbitrary zero point, so it is necessary to subtract an angle $\phi_0$ so that $\phi = 0$ corresponds to the direction of the grating vector. Recalling that $\nu_{TE} = k n_{av} \Lambda$, it is possible to determine $\Lambda$ and $\phi_0$ by measuring the gap positions as the wavelength varies and then doing a least squares fit of the data to equations (5.7) or (5.9). Figure 5.6 shows the experimental points plotted with the theoretical fits, the numbers refer to the particular band gap positions labelled in Figure 5.7. The curve fitting yields values for the grating period (error < 0.1%) and the angle corresponding to the direction of the grating vector (error < 0.5%) and these are listed in Table 5.3. The angle corresponding to the triple point on the $k_x$ axis can be located as this occurs when the two-wave band gaps of the same type intersect i.e. at the intersections of lines 1 and 2, or, 3 and 4 on Figure 5.6. The intersections give a value for the in plane angle, $\phi$, at the triple point of 135.57°. If we assume that the triple point is 180° from the x-axis then the grating vectors, based on average values from the table, are at 151.17 ± 0.36° and 208.78 ± 0.19°. In an ideal structure these angles would be 150° and 210°, the errors are due to angular misalignment in the fabrication process. This angular misalignment has serious

**Figure 5.6** – Experimental band gap positions as a function of normalised frequency, $\nu$. The curves are fit to the data using equations (5.6) to (5.9). The numbers refer to the crossings marked on Figure 5.7. The triple point occurs at the intersection of curves of the same type such as 1 and 2.
consequences for the quality and strength of the gratings produced. In a perfect structure the average index circles representing higher order interactions between the gratings will all perfectly overlap and reinforce each other, whereas in the misaligned structure the gratings tend to wash each other out and produce extra spurious lines. This may also explain the variation in measured grating period as each pair of gratings will interact to produce another grating with a slightly different period. There will also be a contribution from the interferometer beams as shown in Figure 3.3. The average grating period is 281 ± 1.3nm which is very close to the 282nm design period.

<table>
<thead>
<tr>
<th>Intersection No.</th>
<th>Coupling Type</th>
<th>Grating Period, Λ (nm)</th>
<th>Grating Vector Angle, φ₀ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TE-TE</td>
<td>280.89</td>
<td>106.96</td>
</tr>
<tr>
<td>2</td>
<td>TE-TE</td>
<td>279.51</td>
<td>164.17</td>
</tr>
<tr>
<td>3</td>
<td>TE-TM</td>
<td>280.25</td>
<td>164.33</td>
</tr>
<tr>
<td>4</td>
<td>TE-TM</td>
<td>281.44</td>
<td>106.85</td>
</tr>
<tr>
<td>5</td>
<td>TM-TE</td>
<td>282.72</td>
<td>164.66</td>
</tr>
<tr>
<td>6</td>
<td>TM-TE</td>
<td>280.45</td>
<td>107.05</td>
</tr>
<tr>
<td>7</td>
<td>TM-TM</td>
<td>282.50</td>
<td>164.25</td>
</tr>
<tr>
<td>8</td>
<td>TM-TM</td>
<td>283.29</td>
<td>106.13</td>
</tr>
</tbody>
</table>

*Table 5.3* - Grating periods and angles derived from fits to experimental data.
Figure 5.7 – Expanded view of wavevector diagram (Figure 5.18(a)) close to a high symmetry point based on average index circles with intersections numbered. Experimentally the distance to the origin and the angular position of each intersection are measured. As the normalised frequency is increased the circle radii increase as illustrated in Figure 2.15 and the intersections change position. $\lambda = 861.0\text{nm}$, $\nu_{TE} = 3.716$, $\nu_{TM} = 3.393$. 
5.5 Triple Points

Triple points occur close to where three average index circles on the wavevector diagram intersect. Around these points small shapes form which are expected to exhibit novel ray propagation phenomena. Figure 5.8(a) shows the approximate wavevector diagram based on average index circles at a wavelength of 871.2nm. The grating period used is 281.4nm and the grating vectors are 90°, 151.2° and 208.8° from the x-axis. Note how the triple points at ~120° and 180° are quite different. Figure 5.8(b) shows an enlarged view of the triple point at 180° corresponding to region A on Figure 5.8(a). The regions marked A, B and C are investigated by prism coupling. Figure 5.9 shows the same region modelled by the coupled wave approximation in section 2.5.2. The partial waves used in this approximation cannot be changed to account for the angular misalignment but a good fit can be made by shifting the triple point to correspond with the triple point position in Figure 5.8(b). The coupling constant values for the calculation are taken from Table 5.2.

Figure 5.10(a) shows the dark $m$-line image corresponding to region A on Figure 5.8(b) and Figure 5.10(b) shows the dark lines after image processing. Both gaps are due to TM-TE coupling. Figure 5.10(c) shows the dark $m$-lines transformed onto a wavevector diagram by using (5.3). The dotted lines show the average index circles and the band gap positions are a good match to the intersections of the dotted lines. Figure 5.10(c) shows the same region of the diagram calculated by the coupled wave approximation. The parts of the diagram corresponding to the dark $m$-lines are emphasised. The size of the band gaps is approximately correct although the positions are slightly wrong. This is because the average index circles are effectively approximated as straight lines in the coupled wave method and the accuracy of the intersections decreases away from the triple point.
Figure 5.8 - (a) Wavevector diagram based on average index circles. $\lambda = 871.2\text{nm}$, $\nu_{\text{TE}} = 3.666$, $\nu_{\text{TM}} = 3.344$. Solid lines represent TE modes and dashed lines represent TM modes. (b) Enlarged view of region A in (a). The regions marked A, B and C are investigated by prism coupling.
Figure 5.11 shows diagrams corresponding to region B in Figure 5.8(b). The upper gap is TM-TE and the lower gap is TM-TM. The position of the lower gap in (c) matches the intersection of the dotted lines but the upper gap is some way out. The gap sizes in (c) do not seem to match very well with the gap sizes in (d). There are some extra lines crossing the lower TM-TM gap which are due to spurious higher order modes created by the misaligned gratings.

Figure 5.12 shows diagrams corresponding to region C in Figure 5.8(b). The upper gap is TM-TM and the lower gap is TM-TE. The position of the lower gap in (c) matches the intersection of the dotted lines but the upper gap is some way out. The gap sizes in (c) match the gap sizes in (d) fairly well. There are some extra lines crossing the lower TM-TM gap which are due to spurious higher order modes created by the misaligned gratings.

Figure 5.9 - Wavevector diagram based on coupled wave method corresponding to region A on Figure 5.8(a). $\lambda = 871.2\text{nm}$, $\nu_{\text{TE}} = 3.666$, $\nu_{\text{TM}} = 3.344$. 
Figure 5.10 – Wavevector diagram detail corresponding to region A on Figure 5.8(b) showing TM-TE band gaps. \( \lambda = 871.2 \text{nm} \), \( \nu_{\text{TE}} = 3.666 \), \( \nu_{\text{TM}} = 3.344 \). (a) Bright spot with dark \( m \)-lines as viewed on a screen with a coarse 5mm grid. (b) Dark \( m \)-lines extracted from (a). (c) Dark \( m \)-lines transformed onto the wavevector diagram. The dotted lines correspond to the average index circles. (d) Wavevector diagram detail from coupled wave method. The parts of the diagram corresponding to the dark \( m \)-lines are emphasised.
Figure 5.11 - Wavevector diagram detail corresponding to region B on Figure 5.8(b). The upper band gap is TM-TE and the lower band gap is TM-TM. $\lambda = 871.2$nm, $\nu_{\text{TE}} = 3.666$, $\nu_{\text{TM}} = 3.344$. (a) Bright spot with dark $m$-lines as viewed on a screen with a coarse 5mm grid. (b) Dark $m$-lines extracted from (a). (c) Dark $m$-lines transformed onto the wavevector diagram. The dotted lines correspond to the average index circles. (d) Wavevector diagram detail from coupled wave method. The parts of the diagram corresponding to the dark $m$-lines are emphasised.
Figure 5.12 - Wavevector diagram detail corresponding to region C on Figure 5.8(b). The upper band gap is TM-TM and the lower band gap is TM-TE. $\lambda = 871.2\text{nm}$, $\nu_{TE} = 3.666$, $\nu_{TM} = 3.344$. (a) Bright spot with dark $m$-lines as viewed on a screen with a coarse 5mm grid. (b) Dark $m$-lines extracted from (a). (c) Dark $m$-lines transformed onto the wavevector diagram. The dotted lines correspond to the average index circles. (d) Wavevector diagram detail from coupled wave method. The parts of the diagram corresponding to the dark $m$-lines are emphasised.
As the normalised frequency is increased the triangular shapes at A, B and C in Figure 5.13(b) will gradually shrink and disappear as the gaps at their vertices come together. Figure 5.13 shows the wavevector diagram as the frequency is increased slightly, the wavelength is now 866.3nm. Figure 5.14 shows that the triangular shapes should disappear and large gaps are expected at these points.

Figure 5.15 shows diagrams corresponding to point A on Figure 5.13(b). The band gap is caused by TE-TM-TM coupling. The upper branch of the gap in (a) is quite clear but the lower branch has a faint tail that extends into the gap this is another artefact of the grating imperfection. If the tail is ignored then the gap in (c) seems to be much larger than the gap in (d). This could be caused by the finite angular bandwidth of the beam, which cannot cover the wavevector diagram as it curves away at the band gap.

Figure 5.16 shows diagrams corresponding to point B on Figure 5.13(b). The band gap is caused by TE-TE-TM coupling. The features are very similar to those in Figure 5.15. (c) shows that the position of the gap does not correspond well with the average index circles.

Figure 5.17 shows diagrams corresponding to point C on Figure 5.13(b). The band gap is caused by TE-TE-TM coupling. Both branches of the gap in (a) are clear but there are faint tails in the gap.
Figure 5.13 - (a) Wavevector diagram based on average index circles. $\lambda = 866.3\text{nm}$, $\nu_{\text{TE}} = 3.690$, $\nu_{\text{TM}} = 3.368$. Solid lines represent TE modes and dashed lines represent TM modes. (b) Enlarged view of region A in (a). The regions marked A, B and C are investigated by prism coupling.
Figure 5.14 - Wavevector diagram based on coupled wave method corresponding to region A on Figure 5.13(a). $\lambda = 866.3\text{nm}$, $\nu_{\text{TE}} = 3.690$, $\nu_{\text{TM}} = 3.368$. 
Figure 5.15 - Wavevector diagram detail corresponding to region A on Figure 5.13(b). The band gap is a TE-TM-TM triple point. $\lambda = 866.3\text{nm}$, $\nu_{\text{TE}} = 3.690$, $\nu_{\text{TM}} = 3.368$. (a) Bright spot with dark $m$-lines as viewed on a screen with a coarse 5mm grid. (b) Dark $m$-lines extracted from (a). (c) Dark $m$-lines transformed onto the wavevector diagram. The dotted lines correspond to average index circles. (d) Wavevector diagram detail from coupled wave method. The parts of the diagram corresponding to the dark $m$-lines are emphasised.
Figure 5.16 - Wavevector diagram detail corresponding to region B on Figure 5.13(b). The band gap is a TE-TE-TM triple point. $\lambda = 866.3\text{nm}$, $\nu_{\text{TE}} = 3.690$, $\nu_{\text{TM}} = 3.368$. (a) Bright spot with dark $m$-lines as viewed on a screen with a coarse 5mm grid. (b) Dark $m$-lines extracted from (a). (c) Dark $m$-lines transformed onto the wavevector diagram. The dotted lines correspond to average index circles. (d) Wavevector diagram detail from coupled wave method. The parts of the diagram corresponding to the dark $m$-lines are emphasised.
Figure 5.17 - Wavevector diagram detail corresponding to region C on Figure 5.13(b). The band gap is a TE-TE-TM triple point. $\lambda = 866.3\text{nm}$, $\nu_{\text{TE}} = 3.690$, $\nu_{\text{TM}} = 3.368$. (a) Bright spot with dark $m$-lines as viewed on a screen with a coarse 5mm grid. (b) Dark $m$-lines extracted from (a). (c) Dark $m$-lines transformed onto the wavevector diagram. The dotted lines correspond to average index circles. (d) Wavevector diagram detail from coupled wave method. The parts of the diagram corresponding to the dark $m$-lines are emphasised.
Figure 5.18 - (a) Wavevector diagram based on average index circles. $\lambda = 861.0\text{nm}$, $\nu_{\text{TE}} = 3.716$, $\nu_{\text{TM}} = 3.393$. Solid lines represent TE modes and dashed lines represent TM modes. (b) Enlarged view of region A in (a). The regions marked A, B and C are investigated by prism coupling.
Figure 5.18 shows the wavevector diagram as the frequency is increased further, the wavelength is now 861.0nm. Note that the triangular shapes at A, B and C in Figure 5.19 have reappeared and are facing in opposite directions to those in Figure 5.9.

Figure 5.20 shows diagrams corresponding to region A in Figure 5.17(b). The bandgaps are TM-TE. The size and position of the upper gap in (c) matches well with (d). The lower gap is obscured by grating artefacts.

Figure 5.21 shows diagrams corresponding to region B in Figure 5.17(b). The upper bandgap is TE-TE and the lower bandgap is TE-TM. The size of the upper gap in (c) matches well with (d). The lower gap is obscured by grating artefacts.

Figure 5.22 shows diagrams corresponding to region C in Figure 5.17(b). The upper bandgap is TE-TM, the lowest bandgap is TE-TE and there is an extra spurious gap created by the grating imperfection. The sizes of the upper gap and lowest gap in (c) match well with (d).

Figure 5.19 - Wavevector diagram based on coupled wave method corresponding to region A on Figure 5.18(a). $\lambda = 861.0\text{nm}$, $\nu_{\text{TE}} = 3.716$, $\nu_{\text{TM}} = 3.393$. 
Figure 5.20 - Wavevector diagram detail corresponding to region A on Figure 5.18(b). The band gaps are TM-TE. $\lambda = 861.0\text{nm}$, $\nu_{TE} = 3.716$, $\nu_{TM} = 3.393$. (a) Bright spot with dark $m$-lines as viewed on a screen with a coarse 5mm grid. (b) Dark $m$-lines extracted from (a). (c) Dark $m$-lines transformed onto the wavevector diagram. The dotted lines correspond to average index circles. (d) Wavevector diagram detail from coupled wave method. The parts of the diagram corresponding to the dark $m$-lines are emphasised.
Figure 5.21 - Wavevector diagram detail corresponding to region B on Figure 5.18(b). The upper band gap is TE-TE and the lower is TE-TM. $\lambda = 861.0\text{nm}$, $\nu_{\text{TE}} = 3.716$, $\nu_{\text{TM}} = 3.393$. (a) Bright spot with dark $m$-lines as viewed on a screen with a coarse 5mm grid. (b) Dark $m$-lines extracted from (a). (c) Dark $m$-lines transformed onto the wavevector diagram. The dotted lines correspond to average index circles. (d) Wavevector diagram detail from coupled wave method. The parts of the diagram corresponding to the dark $m$-lines are emphasised.
Figure 5.22 - Wavevector diagram detail corresponding to region C on Figure 5.18(b). The upper band gap is TE-TM and the lower is TE-TE. $\lambda = 861.0\text{nm}$, $\nu_{\text{TE}} = 3.716$, $\nu_{\text{TM}} = 3.393$. (a) Bright spot with dark $m$-lines as viewed on a screen with a coarse 5mm grid. (b) Dark $m$-lines extracted from (a). (c) Dark $m$-lines transformed onto the wavevector diagram. The dotted lines correspond to average index circles. (d) Wavevector diagram detail from coupled wave method. The parts of the diagram corresponding to the dark m-lines are emphasised.
5.6 Ray Coupling

The dark $m$-line images seen in the previous section can be checked by coupling a divergent beam into the triple grating structures. The beam is coupled into a smooth region of the waveguide with a rutile half prism. The beam travels through a short transition region into the triple grating structure. The divergent beam can be thought of as a bundle of rays with each ray travelling in a slightly different direction. Each of the rays excites a single point on the wavevector diagram and so the divergent beam will excite a region of the wavevector diagram. In the smooth waveguide the wavevector direction and the group velocity direction are the same, this is because the group velocity direction is normal to the wavevector dispersion surface, which is a sphere for a homogeneous material. For a photonic crystal, small features with high curvature appear on the wavevector diagram close to the triple points as calculated in section 2.5.2. These features create interesting ray propagation phenomena.

The experimental set up is shown schematically in Figure 5.23. A broad divergent beam of light is formed by using a x10 microscope objective and a 40mm focal length lens. The beam divergence at the waveguide is approximately 100mrad. The waveguide is viewed through a $\times 5$ microscope objective mounted with a sliding microscope tube on an infrared vidicon camera. Figure 5.24 (a) shows a plan view image of a divergent TM polarised beam coupled into a photonic crystal region. The area of the image is 2.09 x 2.09 mm$^2$. The divergent beam from the Ti:Sapph laser travels from right to left in the image. The dark vertical strip on the right of the image

![Figure 5.23](image)

- Schematic layout for launching a divergent beam into a waveguide.
is a region of low loss smooth waveguide. There are some bright spots visible in this region that are caused by scratches created by the rutile prism, which is off the image to the right. The light coupled in at the prism base traverses approximately 1mm of smooth waveguide and then goes into the photonic crystal region where it becomes easily visible due to the strong scattering of the hexagonal pattern.

The image in Figure 5.24(a) was captured at a wavelength of 871.2nm and the coupling direction corresponds to the region shown on Figure 5.10. Figure 5.24(c) shows the detail of the wavevector diagram calculated by the coupled wave method (Figure 5.10(d)) and scaled so that the band gap positions correspond to the intersections on the average index circle diagram (Figure 5.10(c)). The single headed arrows represent the group velocity directions of the rays excited in the smooth region of the waveguide. These rays couple, through a transition region at the interface, into the photonic crystal region where it becomes easily visible due to the strong scattering of the hexagonal pattern.

**Figure 5.24** – Ray propagation at 871.2nm. (a) Plan view image of a divergent TM polarised beam propagating, from right to left, into a photonic crystal region where it is focussed by the curvature of the wavevector diagram. (b) Schematic of (a). The double headed arrows represent the ray directions in the photonic crystal (grey shaded region) of the modes excited at the interface (thick lines). (c) Wavevector diagram based on average index and coupled wave approximations. The single headed arrows represent the group velocity directions of the rays excited in the smooth region of the waveguide. These rays couple, through a transition region at the interface, into the photonic crystal modes marked with thick lines. The double headed arrows indicate the group velocity directions of the rays in the photonic crystal which are normal to the wavevector dispersion surface. The central part of the beam is focussed by the curved edges of the central triangular feature. The band gaps prevent small sections of the beam from entering the photonic crystal. The outer parts of the beam go straight through into the photonic crystal.
the photonic crystal modes marked with thick lines. The double headed arrows indicate the group velocity directions of the rays in the photonic crystal which are normal to the wavevector dispersion surface. The central part of the beam is focussed by the curved edges of the central triangular feature. The TM-TE band gaps prevent small sections of the beam from entering the photonic crystal. The outer parts of the beam go straight through into the photonic crystal. Figure 5.24(b) shows a schematic diagram of the ray propagation in the photonic crystal region (shaded grey). The different rays in the beam hit the interface at different positions so that the modes marked by the thick lines in Figure 5.24(c) map onto the positions at the interface marked in Figure 5.24(b). The double headed arrows indicate the group velocity directions of the corresponding rays in the photonic crystal.

Figure 5.25 shows ray propagation at a wavelength of 866.3nm. The normalised frequency has increased and the TM-TE gaps have come closer together. The TM-TE band gaps block a large central portion of the beam. A small central triangular feature has been added to the wavevector diagram (Figure 5.25(c)) to correspond with the
tightly focused beam on Figure 5.25(a). The bright feature is not centrally placed in
the gap, this is due to a wavevector tail (caused by the grating imperfection) extending
into the gap as shown in Figure 5.15. The bright focussed beam does not extend far
into the waveguide, this is because the large curvature on the wavevector diagram has
a very small group velocity associated with it. The explanation for this reduced group
velocity is that the rays go through many multiple reflections as they travel through the
grating. These low group velocity rays will therefore experience a much larger
absorption coefficient and decay more quickly. This multiple reflection argument also
explains why the refracted parts of the beam appear brighter as they will be more prone
to scattering by microscopic imperfections in the waveguide. The waveguide loss, \( \alpha \), is
measured by fitting the horizontal intensity profile in the pictures to equation (5.2).
The loss in the patterned regions varies from approximately 26 to 46 dB/cm. This large
loss would seriously affect any attempt to make a high-\( Q \) micro-resonator structure.
This loss can partly be attributed to the non-ideal sinusoidal grating profile, which acts
as a blazed grating and causes radiative scattering into the air. A rectangular profile
would produce much less radiative scattering and therefore a lower loss.
As the normalised frequency is increased further the central triangular shape on the
wavevector diagram disappears completely and then reappears facing the other
direction. Figure 5.26 shows the ray propagation and wavevector diagram at a
wavelength of 861nm. The central part of the beam in Figure 5.26(a) travels straight
on at the interface. Small sections of the beam are blocked by the TM-TE band gaps.
Parts of the beam are refracted by the outer edges of the band gaps on the wavevector
diagram (Figure 5.26(c)).
Figure 5.26 - Ray propagation at 861nm. (a) Plan view image of a divergent TM polarised beam propagating, from right to left, into a photonic crystal region. The central part of the beam travels straight on at the interface. The outer parts of the beam are refracted by the outer edges of the band gaps on the wavevector diagram. (b) Schematic of (a). The double headed arrows represent the ray directions in the photonic crystal (grey shaded region) of the modes excited at the interface (thick lines). (c) Wavevector diagram based on average index and coupled wave approximations. The straight side of the central triangle causes the central part of the beam to go straight into the photonic crystal. The curved band gap edges cause partial focusing of the outer parts of the beam.

References

2. D. Brady, Private Communication.
6. CONCLUSIONS

6.1 Discussion

6.2 Future Applications
6. Conclusions

This chapter summarises the work presented in the thesis and draws conclusions from the modelling and experiments undertaken. The relevance of this work within the context of past and current work on photonic crystals is discussed. Finally, applications of this work are proposed.

6.1 Discussion

In this thesis we have considered the properties of photonic crystal waveguides with a view to getting an intuitive feel for the behaviour of individual Bloch modes. We concentrated particularly on modes close to photonic band gap edges, which exhibit novel phenomena. This contrasts with most other work in the field, which has concentrated on suppressing the Bloch modes with large photonic band gaps and then putting isolated defect modes within the gaps to use as resonators. Analytical models have been developed for the modes in one and two-dimensional photonic crystal waveguides. We have also interpreted the results from a numerical real space method to more accurately predict the behaviour of a 2-D hexagonal photonic crystal waveguide with a large index contrast. Weak two-dimensional photonic crystal waveguides have been fabricated and they have been assessed with a novel conical prism coupling method. There is good agreement between the analytical models and the experimental results.

The modelling of one-dimensional photonic crystals has shown the existence of two different types of fully guided Bloch modes with zero group velocity in the waveguide plane. These standing wave modes can be viewed as stationary resonances with a high effective $Q$-factor and hence are suitable as micro-resonators for enhancing dipole-field coupling. The electromagnetic field microstructures of these modes show that the field intensity is concentrated in either the high index regions of the waveguide or the air gap regions of the waveguide. The theory also predicts the existence of stationary modes consisting of a mixture of TE and TM polarisations, which are completely novel. A very similar structure has been analysed by Fan et al\(^1\) and they predict similar field microstructures and band diagrams although they do not show any TE-TM mode mixing. They also model a high-$Q$ defect structure and propose its use as a microlaser.
resonator. Such a microlaser may produce thresholdless and low noise behaviour, but the power output will be severely limited. The stationary resonances proposed in this thesis are much more like distributed feedback laser structures and the associated gain volumes can be significantly larger\(^2\). The fast mode, which has its intensity concentrated in the air gaps, might be used as the basis of a gas sensor or even a gas laser.

The work on two-dimensional photonic crystals has concentrated on the hexagonal structure. The modelling of two-dimensional photonic crystal waveguides has shown the existence of symmetry points, which exhibit stationary modes in a similar way to the one-dimensional structures. In addition the ability of photonic crystals to suppress ranges of modes in multimode waveguides has also been shown. More importantly we have found that the actual grating pitch required in a waveguide is larger than for the in-plane case, which lessens the technological difficulty of making practical photonic crystal waveguides. In contrast, the theoretical work of other groups has mainly concentrated on in-plane theories and in-plane propagation.

Weak hexagonal photonic crystal waveguides have been fabricated. The fabrication method was far from ideal, the fibre interferometer introduced many problems and the triple sequential exposure method seems to “wash out“ the gratings due to the superposition of slightly different gratings. A three beam interferometer built from bulk optics with stable mounts and vibration isolation would probably have performed better and may not have required active stabilisation. A very elegant solution for producing the hexagonal intensity pattern has been demonstrated by Berger et al\(^3\). The pattern is created by illuminating a photolithographic mask containing three separate diffraction gratings so that three diffracted beams interfere at the sample.

A novel prism coupling method has been used to measure the wavevector dispersion around the symmetry points. The prism consisted of a cone shape so that the sample and prism could be rotated with out changing the coupling. A spacer layer was used so that the prism didn’t come into contact with the fragile photonic crystal region. The fine detail around the symmetry points was found to agree well with the analytical models developed in the earlier parts of the thesis. Ray coupling in to the photonic crystal waveguides revealed the extent of the wavevector band gaps and the group
velocity behaviour at the edges of the gaps. Even though the gratings were very weak they still exhibited very similar behaviour to the cubic gratings investigated by Zengerle.

6.2 Future Applications

An application where it is essential to control wavevector magnitude and group velocity direction is wavelength conversion in non linear materials. One of the most important applications of nonlinear frequency conversion is optical parametric oscillation (OPO). In this process a strong pump beam at frequency $\omega_p$ propagating through a crystal is down-converted to signal and idler beams at frequencies $\omega_s$ and $\omega_i$. The interacting beams must conserve energy such that $\omega_p = \omega_s + \omega_i$. For the energy exchange to be efficient the beams must also be phase matched such that $k_p = k_s + k_i$, where $k$ are the wavevectors at the different frequencies in the nonlinear crystal. In traditional nonlinear crystals this phase matching is achieved by using the natural birefringence of the material and either temperature tuning or angle tuning the crystal. To achieve phase matching over a large frequency range, nonlinear crystals with a high birefringence are required. However, a major problem is that the birefringence is a result of anisotropy and it is very difficult to grow large single crystals of highly anisotropic materials. Nonlinear crystals also have limited transparency ranges, for instance it is very difficult to operate a lithium niobate OPO above 4$\mu$m due to the high intrinsic losses.

An alternative to traditional nonlinear crystals is to start with an isotropic material with good transparency and a high nonlinear coefficient and engineer the birefringence by introducing artificial microstructure. Ideal materials for this application are GaAs and AlGaAs, which have very high second order susceptibilities ($\chi^2$(GaAs) $\approx$ 240pm/V in the near infrared), are widely transparent in the infrared and are routinely grown as high quality single crystals in the microelectronics industry. Recently much attention has been given to quasi-phase matching (QPM), where phase matching is achieved by periodically reversing the polarisation of the nonlinear coefficient, this produces a grating that makes up the phase mismatch and couples the different wavelengths
together. These materials can be constructed by stacking plates and then thermally bonding them together under high compression. For instance, to frequency double a 10.6\(\mu\)m carbon dioxide laser requires GaAs plates 106\(\mu\)m thick.

Alternatively birefringent phase matching can be used, where the birefringence is created by either a waveguide\(^9\) or a fine laminar structure\(^10\). Typical waveguide birefringence is shown in Figure 2.2. For a given waveguide thickness there are a variety of refractive indices available in the different mode orders and polarisations so by careful design it is possible to achieve phase matching. The problem with this technique is that the spatial overlap of different mode orders limits the conversion efficiency, and tuning of the output wavelength is difficult. The birefringence of a laminar structure is shown in figures 2.5(a) and 2.5(b). This behaviour is the same as a traditional uniaxial birefringent crystal. The refractive index of the TM polarised beam changes with angle so that phase matching can be achieved by rotating the crystal. It is also possible to angle tune the output wavelength by rotating the crystal. The problem with this method is that generally the group velocities of the interacting waves are not collinear so that as the beams travel through the crystal they will go in slightly different directions and eventually the spatial overlap will be lost. This is commonly known as group velocity walkoff. One way to prevent group velocity walkoff is to use non-critical phase matching. This would require the use of beams travelling only in the \(\beta\) direction on Figure 2.5 so that the group velocities are collinear. Phase matching can still be achieved by varying the magnitude of the birefringence, which depends on the refractive index contrast of the layered structure.

In this thesis we have considered hexagonal photonic crystal waveguides and shown that they have very special wavevector diagrams. Figure 2.19 shows a typical wavevector diagram for a large index contrast hexagonal photonic crystal. The most notable feature of this diagram are the very flat sides of the dispersion surface. For example in the region of the X direction all the modes within an angle of around 20\(^\circ\) have group velocities which point in the X direction. This means that, within this region, group velocity walkoff is no longer a problem and long interaction lengths and high gains become feasible. Angular tuning is still possible because the refractive index is related to the distance to the origin and clearly this changes with angle and should
enable some degree of wavelength tuning. The pump acceptance angle will be very high and this will enable such an OPO to be pumped with a source with low beam quality. For the effects described here to work it is necessary to design the crystal so that similar wavevector diagrams occur at the different interacting wavelengths while also maintaining phase matching. This should be possible given the choice of parameters available, such as waveguide thickness, filling fraction and perhaps even grating profile. It may also be possible to tailor the unit cell so that the material dispersion is perfectly compensated, i.e. so that phase matching occurs simultaneously over a large bandwidth. This has recently been demonstrated with non collinear phase matching in a standard BBO crystal\textsuperscript{11} and has applications in remoting sensing\textsuperscript{12}. In summary, the design options afforded by photonic crystal waveguides should open up a whole new range of future possibilities for high efficiency nonlinear frequency conversion.

References


Appendix A: TRANSLATION MATRIX ELEMENTS

The matrix \( M_{21} \) relating the field in the second layer to the field in the first layer is:

\[
\begin{pmatrix}
  a_N^2 \\
  b_N^2
\end{pmatrix} = M_{21} \begin{pmatrix}
  a_N^1 \\
  b_N^1
\end{pmatrix} = \begin{pmatrix}
  A_{21} & B_{21} \\
  C_{21} & D_{21}
\end{pmatrix} \begin{pmatrix}
  a_N^1 \\
  b_N^1
\end{pmatrix}, \quad (A.1)
\]

where

\[
A_{21} = c_1c_2 - (\xi_1 p_1 \Lambda / \xi_2 p_2 \Lambda) s_1s_2, \\
B_{21} = s_1c_2 + c_1s_2 (\xi_1 p_1 \Lambda), \\
C_{21} = -\xi_1 p_1 A s_1c_2 - \xi_2 p_2 A c_1s_2, \\
D_{21} = c_1c_2 - (\xi_2 p_2 \Lambda / \xi_1 p_1 \Lambda) s_1s_2, \\
\det(M_{21}) = 1,
\]

where the terms \( s_j \) and \( c_j \) are shorthand for:

\[
c_j = \cos(p_j h_j / 2), \quad s_j = \sin(p_j h_j / 2). \quad (A.3)
\]

The matrix \( M_{12} \) relating the field in the first layer of the \((N+1)\)th period to the field in the second layer of the \(N\)th period is then

\[
\begin{pmatrix}
  a_{N+1}^1 \\
  b_{N+1}^1
\end{pmatrix} = M_{12} \begin{pmatrix}
  a_N^1 \\
  b_N^1
\end{pmatrix} = \begin{pmatrix}
  D_{21} & B_{21} \\
  C_{21} & A_{21}
\end{pmatrix} \begin{pmatrix}
  a_N^1 \\
  b_N^1
\end{pmatrix}, \quad (A.4)
\]

The analysis can either be based on the translation matrix \( M = M_{12}M_{21} \) (with a state vector representing the field in layers with index \( n_1 \)) or equivalently on the matrix \( M' = M_{21}M_{12} \) (state vector representing the field in layers with index \( n_2 \)). \( M \) is

\[
\begin{pmatrix}
  a_{N+1}^1 \\
  b_{N+1}^1
\end{pmatrix} = M \begin{pmatrix}
  a_N^1 \\
  b_N^1
\end{pmatrix} = \begin{pmatrix}
  A & B \\
  C & D
\end{pmatrix} \begin{pmatrix}
  a_N^1 \\
  b_N^1
\end{pmatrix}, \quad (A.5)
\]

where

\[
A = D = A_{21}D_{21} + B_{21}C_{21}, \\
B = 2D_{21}B_{21}, \quad C = 2A_{21}C_{21}. \quad (A.6)
\]

\( A \) can be re-arranged as

\[
A = \cos(p_1 h_1)\cos(p_2 h_2) - \frac{1}{2} \left( \frac{p_1 \xi_1}{p_2 \xi_2} + \frac{p_2 \xi_2}{p_1 \xi_1} \right) \sin(p_1 h_1)\sin(p_2 h_2) \quad (A.7)
\]
but $B$ and $C$ are most conveniently expressed as the product of two factors as above.

The elements of the alternative matrix $M'$ are:

$$A' = D' = A,$$
$$B' = 2A_{21}B_{21}, \quad C' = 2D_{21}C_{21}. \quad (A.8)$$