EFFECTS OF SURFACE PROFILE ON A BOUNDARY-DRIVEN ACOUSTIC STREAMING FIELD

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Acoustic streaming fields in two-dimensional rectangular enclosures that have structured boundaries are simulated and the effects of surface profile amplitude on a boundary-driven acoustic streaming field are numerically investigated. The standing wave fields in the enclosures are generated by excitation of a boundary and a sine-wave shaped profile on a boundary parallel to the particle oscillations is considered. This surface profile is found to have a large influence on the magnitude of both outer and inner streaming velocities. In terms of streaming pattern, it is found that the number of inner streaming vortices is dependent on the wavelength of profile while this profile has a less significant effect on the outer vortex pattern.

1. Introduction

Acoustic streaming is a steady current in a fluid driven by the absorption of high amplitude acoustic oscillations. It can be generally regarded as any flow generated by the Reynolds stress arising from the presence of gradients in the time-averaged acoustic momentum flux in a fluid.

In acoustofluidic systems, acoustic streaming is usually associated with acoustic particle/cell manipulation, a technique in which ultrasonic standing waves are used to manipulate or pattern particles/cells to desired planes or positions. In these systems, particles will be subjected to both acoustic radiation force and acoustic streaming induced drag force. Acoustic streaming in ultrasonic particle manipulation devices is generally regarded as a ‘disturbance’ which disrupts the predictable particle movements driven by the primary acoustic radiation force. On the other hand, acoustic streaming has been found to be an excellent tool in many active applications, such as heat/mass transfer, fluid mixing, fluid pumping, particle/cell/droplet sorting and many others.[1] Therefore, while it is undeniably a problematic phenomenon in some circumstances, it can be an extremely useful tool if used correctly.[1] In order to optimise the use of this phenomenon, the first vital step is to understand the underlying mechanisms of diverse acoustic streaming patterns in various acoustofluidic systems, which is essential in the control of the acoustic streaming field and can provide effective guidance for microfluidic device designs for a variety of applications.

It has previously been demonstrated that boundary-driven streaming in standing wave fields can be effectively solved from the limiting velocity method provided that the curvature of the rigid boundary is large compared to the thickness of the viscous boundary layer, δ_v.[2, 3] In this work, we investigate the acoustic streaming field in systems that have structured boundaries to explore the effects of surface profile on the boundary-driven acoustic streaming field. While the surfaces in systems do not satisfy the condition stated above, the acoustic streaming field can be solved using an alternative method based on a Reynolds stress body force.[4, 5]

2. Numerical method

The finite element package COMSOL 4.3a[6] was used to implement the numerical simulations, which can be split into two steps.

Firstly, the first-order acoustic fields were simulated using the 'pressure acoustics' physics module, which solves the harmonic, linearized acoustic problem which takes the form:

$$\nabla^2 p + \frac{\omega^2}{c^2} p = 0, \tag{1}$$

where ω is the angular frequency, c is the sound speed, and p is the complex pressure defined at position r using the relation,

$$p_t(r,t) = \mathrm{Re}[p(r)e^{i\omega t}]. \tag{2}$$

In the second step, the 'creeping flow' physics module was used to simulate the second-order acoustic streaming fields as a response to the Reynolds stresses which can be calculated from the first step. This approximates the fluid as incompressible, and neglects inertial terms (Stokes flow) as the Reynolds numbers are much smaller than one in the devices presented in this paper. The governing equations for the streaming velocity field, u_2, and associated pressure field, p_2, are

$$\mu\nabla^2 u_2 = \nabla p_2 + F, \tag{3}$$

$$\nabla \cdot u_2 = 0, \tag{4}$$

where μ is the kinematic viscosity, and the driving term F is the Reynolds stress force which can be derived from the first order acoustic velocity field.[5]

3. Models, results and discussion

Figure 1(a) shows the model: a two-dimensional rectangular chamber with dimensions $l = 7.4\times10^{-3}$ m and $w = 4.24\times10^{-5}$ m is considered. It satisfies the condition for the generation of classical Rayleigh streaming[7] and Schlichting streaming[8], $l >> w >> \delta_v$. Figure 1(b) shows the magnified view of A on the top wall of the fluid chamber, where a sine wave shaped boundary is considered (this profile is also present in Figure 1(a), however due to its fine scale is not apparent). It is determined by two parameters, h_0 and λ_s, which are respectively the amplitude and wavelength of this sine wave. The standing wave field in this chamber is generated by the vibration of its left wall, which is driven at $f = 1$ MHz thus a half-wavelength resonance in the x direction of fluid chamber is established (for the fluid properties of $c = 1481.4$ m/s, $\rho = 999.62$ kg/m^3). Only half of the chamber is modelled for the numerical efficiency so in both steps the bottom wall of the fluid chamber is considered as a symmetric boundary. In this work, all the results presented are for $\lambda_s = 3.3\,\mu$m unless otherwise stated.

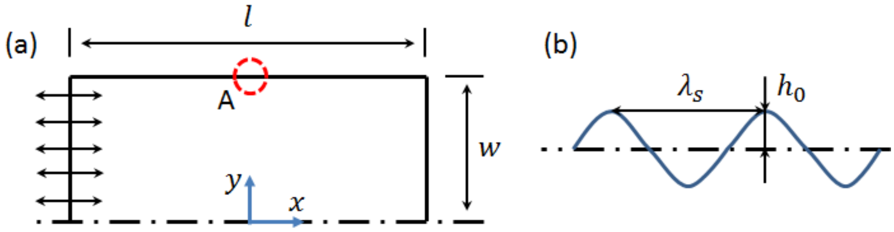

Figure 1. Illustration of the model: (a) excitation, coordinates and dimensions of the model; (b) showing a magnified view of A in (a), where λ_s and h_0 represent respectively its wavelength and amplitude.

In the first step the linear wave equation, Eq. (1) is solved in the frequency domain. Then, the Reynolds stress force can be calculated and with it the driving force for the second step, creeping flow, Eq. (3), solved by the stationary solution. A series of surface profiles with diverse h_0 ranging from 1×10^{-11} m to 2×10^{-5} m were studied. The modelled results in two cases respectively with $h_0 = 1\times10^{-10}$ m and $h_0 = 5.3\times10^{-7}$ m are shown and compared in Figure 2. The modelled acoustic pressure field, acoustic streaming field and a magnification of acoustic streaming field near the viscous boundary are presented. It can be seen from Figure 2 (a) and (c) that half wavelength resonances are formed in the x direction of the fluid chamber for both cases and the pressure magnitudes of them are similar, which means that the amplitude of the surface profile has little effect on the first-order acoustic pressure field. However, a huge difference is found for these cases on both magnitudes of acoustic streaming velocities and streaming patterns. Firstly, as is well known, Rayleigh-Schlichting streaming in devices with flat boundaries have some evident features, such as four vortices within each half wavelength in opposing directions (this number will decrease to two when the channel is sufficient narrow[9]). It is found that in cases where the amplitude of profile is small, e.g. Figure 2 (b), the modelled streaming field is the classical Rayleigh-Schlichting streaming. When the amplitude reaches to a certain value (around 10 nm), the number of inner vortices (Schlichting streaming) is found to be dependent on the wavelength λ_s of this profile such that there are two inner streaming vortices within each wavelength, Figure 2 (f). The mechanism underlying this might be attributed to the periodic structure of the acoustic velocity field near the structured boundaries, which thus creates a corresponding periodic Reynolds stress. However, there is less impact on the pattern of outer streaming (Rayleigh streaming), Figure 2 (d).

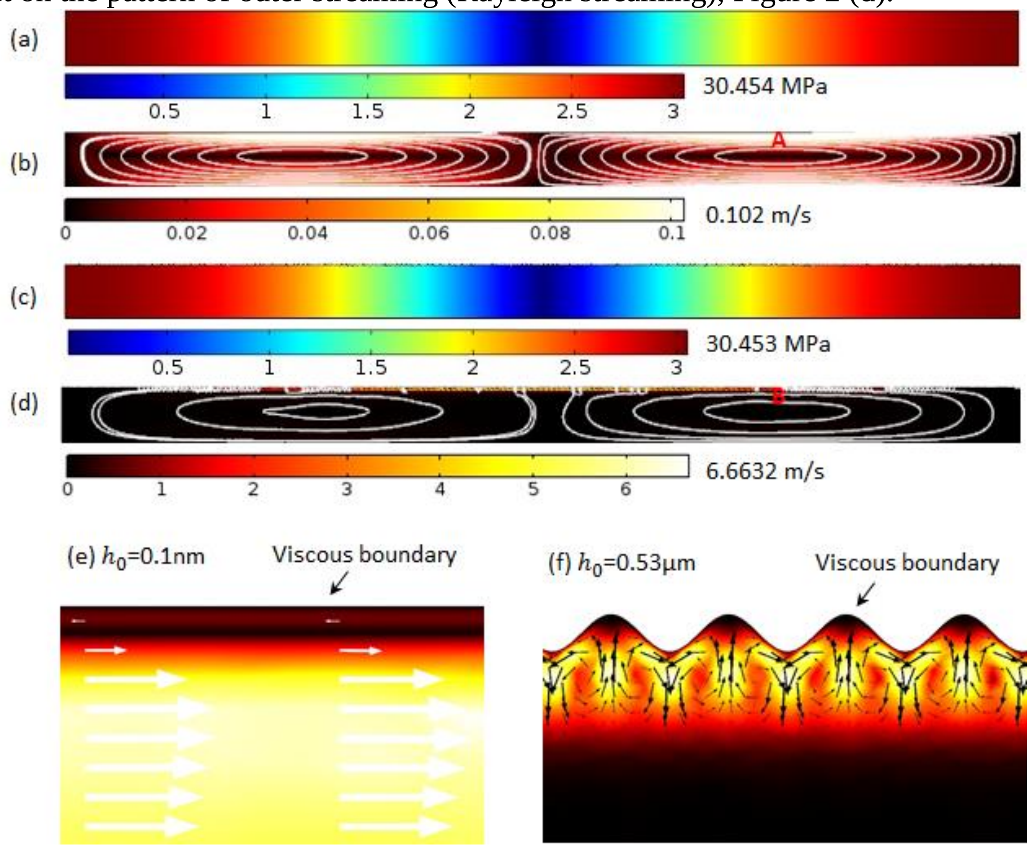

Figure 2. Modelled acoustic pressure and acoustic streaming fields: (a) acoustic pressure field in an enclosure with $h_0 = 1\times10^{-10}$ m; (b) acoustic streaming field in an enclosure with $h_0 = 1\times10^{-10}$ m; (c) acoustic pressure field in an enclosure with $h_0 = 5.3\times10^{-7}$ m; (d) acoustic streaming field in an enclosure with $h_0 = 5.3\times10^{-7}$ m; (e) magnification of A in (b) with arrow plot of streaming velocity field; (f) magnification of B in (d) with arrow plot of streaming velocity field, where A and B are local areas close to the boundary.

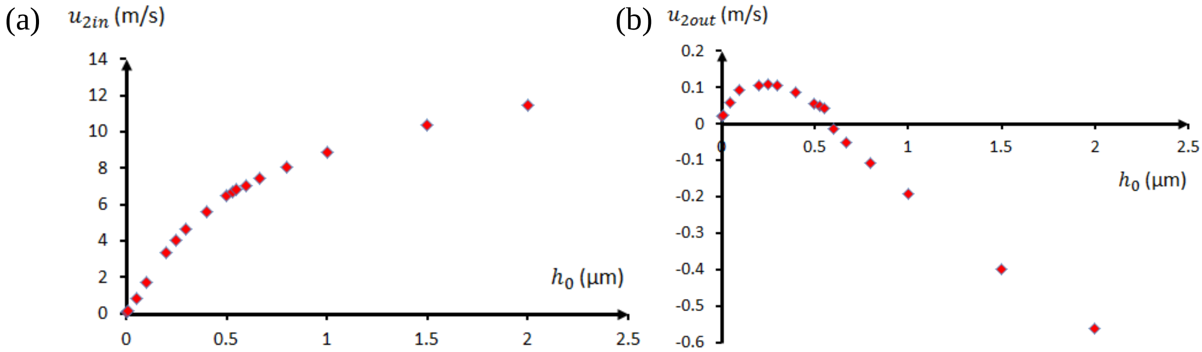

Figure 3. The relationship between maximum streaming velocity and the amplitude of the surface profile, where the red diamonds are the simulated streaming velocities, and u_{2in} shown in (a) and u_{2out} shown in (b) are respectively the maximum inner streaming velocity and maximum outer streaming velocity.

The spatial amplitude of the surface profile has a large influence on the magnitude of streaming velocities. Figure 3 (a) & (b) plot respectively the relationship between the maximum inner streaming velocity and the outer streaming velocity with the amplitude of the profile. It can be seen that a global growth trend is obtained for the maximum inner streaming velocity with the increase of h_0. The maximum inner streaming velocity grows less quickly after the amplitude of roughness exceeds δ_v, which is approximately 0.53 μm at water with a driving frequency of 1 MHz. More interestingly, with the increase of h_0, the maximum outer streaming velocities firstly increase rapidly to its peak when h_0 is approximately half of δ_v, 0.25 μm and then decreases to the maximum streaming velocity in an enclosure with flat boundaries when h_0 reaches close to δ_v. With the further increase of amplitude of roughness, the maximum outer streaming velocity will further decrease and then reverses in direction.

The mechanism underlying these changes is still to be analysed. But it might be attributed to the dramatic increase of Reynolds stress force in the y direction, F_y, from the structured surfaces within the viscous boundary layer, which might turns the dominant driving force for the Rayleigh-Schlichting streaming from F_x (Reynolds stress force in the x direction) in systems with flat boundaries to F_y in systems with this kind of structured surfaces.

4. Conclusions

The effects of surface profile on a boundary-driven acoustic streaming field have been numerically investigated for the case of a sine wave shaped profile on the boundary that is parallel to the direction of acoustic oscillations in rectangular enclosures. It was found that this kind of surface has huge influences not only on the magnitude of streaming velocities, but also on the streaming patterns.

The dramatic increase of inner vortices and magnitude of streaming velocity could significantly enhance mass transfer in acoustofluidic devices, which has huge potential in applications where acoustic streaming has a positive effect, such as microfluidic mixing, fluid-pumping and battery systems that are diffusion limited.

An important next step is to obtain an experimental verification of these numerical simulations.

5. Acknowledgement

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REFERENCES

1. Wiklund, M., R. Green, and M. Ohlin, *Acoustofluidics 14: Applications of acoustic streaming in microfluidic devices.* Lab on a Chip, 2012. **12**(14): p. 2438-2451.
2. Lei, J., P. Glynne-Jones, and M. Hill, *Acoustic streaming in the transducer plane in ultrasonic particle manipulation devices.* Lab Chip, 2013. **13**(11): p. 2133-43.
3. Lei, J., M. Hill, and P. Glynne-Jones, *Numerical simulation of 3D boundary-driven acoustic streaming in microfluidic devices.* Lab Chip, 2013. **14**(3): p. 532-41.
4. Bernassau, A.L., et al., *Controlling acoustic streaming in an ultrasonic heptagonal tweezers with application to cell manipulation.* Ultrasonics, 2014. **54**(1): p. 268-74.
5. Lighthill, J., *Acoustic Streaming.* Journal of Sound and Vibration, 1978. **61**(3): p. 391-418.
6. *COMSOL Multiphysics 4.3a*. 2013; Available from: http://www.comsol.com/.
7. Rayleigh, L., *On the circulation of air observed in Kundt's tube, and on some allied acoustical problems.* Phil. Trans., 1883. **175**: p. 1-21.
8. Schlichting, H., *Berechnung ebener periodischer Grenzschichtstromungen (Calculation of plane periodic boundary layer streaming).* Physikalische Zeitschrift, 1932. **33**(8): p. 327-335.
9. Hamilton, M.F., Y.A. Ilinskii, and E.A. Zabolotskaya, *Acoustic streaming generated by standing waves in two-dimensional channels of arbitrary width.* Journal of the Acoustical Society of America, 2003. **113**(1): p. 153-160.