Abstract—It is widely recognized that traditional single radio frequency (RF)-chain-aided spatial modulation (SM) does not offer any transmit diversity gain. As a remedy, constellation randomization (CR), relying on transmit prescaling (TPS), has been shown to provide transmit diversity for single-RF-chain-aided SM. In this paper, we propose a low-complexity approach to SM with the aid of constellation randomization (SM-CR) that considerably improves the transmit diversity gain of SM at a reduced computational burden compared with conventional SM-CR. While conventional SM-CR performs a full search among a set of candidate TPS factors to achieve the maximum minimum Euclidean distance (MED) in the received SM constellation, here, we propose a thresholding approach, where instead of the maximum MED, the TPS aims to satisfy a specific MED threshold. This technique offers a significant complexity reduction with respect to the full maximization of SM-CR, since the search for TPS is terminated once a TPS set is found that satisfies the MED threshold. Our analysis and results demonstrate that a scalable tradeoff can be achieved between transmit diversity and complexity by appropriately selecting the MED threshold, where a significant complexity reduction is attained, while achieving a beneficial transmit diversity gain for the single-RF SM.

Index Terms—Constellation shaping, multiple-input-single-output, spatial modulation (SM), transmit prescaling (TPS).

I. INTRODUCTION

Spatial modulation (SM) has been shown to offer a low-complexity design alternative to spatial multiplexing, where only a subset (down to one) of radio frequency (RF) chains is required for transmission [1], [2]. Early work has focused on the design of receiver algorithms [3] for minimizing the bit error ratio of SM at low complexity [1]–[5]. Matched filtering is shown to be a low-complexity technique for detecting the activated antenna index (AI) [1]–[3]. A maximum likelihood (ML) detector is introduced in [4] for reducing the complexity of classic spatial multiplexing ML detectors, whereas the complexity imposed can be further reduced by compressive sensing detection approaches [5]. In addition to receive processing, several transmit precoding (TPC) approaches have been proposed for receive antenna (RA)-aided SM, where the spatial information is mapped onto the RA index [6]–[8]. Relevant work has also proposed constellation shaping for SM [9]–[14]. Specifically, in [9], the transmit diversity of coded SM is analyzed for different spatial constellations, which represent the legitimate sets of activated transmit antennas (TAs). Furthermore, Yang et al. in [10] conceived a symbol constellation optimization technique for minimizing the bit error rate (BER). Indeed, spatial and 46 symbol constellation shaping are discussed separately in the aforementioned reference. By contrast, the design of the received SM constellation that combines the choice of the TA as well as the transmit symbol constellation is the focus of this paper. A number of constellation shaping schemes [11]–[14] have also been proposed for the special case of SM, which is referred to as space shift keying, whereas the information is purely carried in the spatial domain, by the activated AI. However, the application of the above constellation shaping to the SM transmission, where the transmit waveform is modulated, is nontrivial.

Recent work has focused on shaping the receive SM constellation by means of symbol prescaling at the transmitter, aiming for maximizing the minimum Euclidean distance (MED) in the received SM constellation. The constellation shaping approach in [15] and [16] aims at fitting the receive SM constellation to one of the existing optimal classic constellation formats in terms of minimum distance, such as, e.g., quadrature amplitude modulation (QAM). Due to the strict 60 constellation fitting requirement imposed on both amplitude and phase, 63 this prescaling relies on the inversion of the channel coefficients. In the 64 case of ill-conditioned channels, this substantially reduces the received 65 signal-to-noise ratio (SNR). This problem has been alleviated in [17], 66 where a constellation shaping scheme based on phase-only scaling is 67 proposed. Still, the constellation shaping used in the above schemes is 68 limited in the sense that it only applies to multiple-input–single-output 69 systems, where a single symbol is received for each transmission, and 70 thus, the characterization and shaping of the receive SM constellation 71 is simple.

Closely related to this work, a transmit prescaling (TPS) scheme was proposed for SM [19], where the received SM constellation is 74 randomized by TPS for maximizing the MED between its points for a 75 given channel. A number of randomly generated candidate sets of TPS factors are formed offline, known to both the transmitter and the receiver, and the transmitter then selects that particular set of TPS factors that yields the SM constellation having the maximum MED. Against this background, in this paper, we propose a low-complexity 80 relaxation of the above optimization instead of an exhaustive search, where the 81 first TPS factor set that is found to satisfy a predetermined threshold is 82 selected, thus reducing the computational burden of the TPS operation. The proposed scheme is shown to provide a scalable tradeoff between 84 the performance attained and the complexity imposed, by accordingly 85 selecting the MED threshold.

This paper is organized as follows: In Section II, the basic system 87 model is first introduced, and the proposed scheme is then discussed. 88 The computational complexity of the proposed technique is analyzed in Section III, and its performance against the state of the art is evaluated in Section IV. Finally, in Section V, we draw the key 91 conclusions of our study.

II. SPATIAL MODULATION WITH THRESHOLD CONSTELLATION RANDOMIZATION (SM-TCR)

Consider a multiple-input multiple-output (MIMO) system, where 95 the transmitter and the receiver are equipped with $N_t$ and $N_r$ antennas, 96...
optimization, however, is an NP-hard problem, which makes finding

\[ \mathbf{y} = \mathbf{H} \mathbf{A} \mathbf{s}_{m}^k + \mathbf{w} \]  

(1)

where \( \mathbf{w} \sim \mathcal{CN}(0, \sigma^2) \) is the additive white Gaussian noise com-

171 ponent at the receiver, with \( \mathcal{CN}(\mu, \sigma^2) \) denoting the circularly

178 symmetric complex Gaussian distribution with a mean of \( \mu \) and a

179 variance of \( \sigma^2 \). Furthermore, \( \mathbf{A} = \text{diag}(\mathbf{a}) \) is the TPS matrix with

180 \( \mathbf{a} = [a_1, a_2, \ldots, a_N] \), and \( \text{diag} \) represents the diagonal matrix

181 with its diagonal elements taken from vector \( \mathbf{a} \). Note that the diagonal

1822 structure of \( \mathbf{A} \) guarantees having a transmit vector \( \mathbf{x} = \mathbf{A} \mathbf{s} \) with a

183 single nonzero element, so that the single-RF-chain structure of SM is

184 preserved.

185 At the receiver, a joint ML detection of both the TA index and the

186 transmit symbol is obtained by the minimization, i.e.,

\[ [\hat{m}, \hat{k}] = \arg\min_{m,k} \|\mathbf{y} - \mathbf{H} \mathbf{A} \mathbf{s}_{m}^k\| \]  

(2)

187 where \( \|\mathbf{x}\| \) denotes the norm of vector \( \mathbf{x} \), and \( \mathbf{y} \) is the \( i \)th constellation

188 point in the received SM constellation. By exploiting the specific

189 structure of the transmit vector, this can be further simplified to

\[ [\hat{m}, \hat{k}] = \arg\min_{m,k} \|\mathbf{y} - \mathbf{h}_k \alpha_m \mathbf{s}_m\| \]  

(3)

190 where \( \mathbf{h}_k \) denotes the \( k \)th column of matrix \( \mathbf{H} \). It is widely recognized

191 that the performance of the detection as formulated above is dominated

192 by the MED between the adjacent constellation points \( \hat{\mathbf{y}} \) and \( \hat{\mathbf{y}}_i \) in the

193 receiver constellation, i.e.,

\[ d_{\min} = \min_{i,j} \|\hat{\mathbf{y}}_i - \hat{\mathbf{y}}_j\|^2, \quad i \neq j. \]  

(4)

194 Accordingly, to improve the likelihood of correct detection, con-

195 stellation shaping TPS schemes conceived for SM aim at maximizing

196 this MED. The optimum TPS matrix \( \mathbf{A}^* \) can be found by solving the

197 optimization problem of [20]

\[ \mathbf{A}^* = \arg\max_{\mathbf{A}} \min_{i,j} \|\hat{\mathbf{y}}_i - \hat{\mathbf{y}}_j\|^2, \quad i \neq j \quad \text{s.t.} \quad \text{trace}(\mathbf{A}^*^H \mathbf{A}^*) \leq P \]  

(5)

198 and, additionally for single-RF-chain-aided SM, subject to \( \mathbf{A}^* \) having

199 a diagonal structure. In the above, \( \mathbf{A}^H \) and trace(\( \mathbf{A} \)) represent the

200 Hermitian transpose and trace of matrix \( \mathbf{A} \), respectively. The above

201 optimization, however, is an NP-hard problem, which makes finding

the TPS factors prohibitively complex and motivates the conception

142 of lower-complexity suboptimal techniques. Indeed, it has been shown

143 that the TPS approach in [19], by selecting among a set of predetermined

144 mined randomly generated TPS vectors instead of fully optimizing the

145 TPS, offers a near-optimal performance with the lowest complexity

146 among the TPS optimization approaches [20], [21].

147

TPS Vector Generation: Accordingly, with SM-TCR first, a number of

148 \( D \) random candidate TPS vectors are generated, in the form of \( \mathbf{a}_d \),

149 where \( d \in [1, D] \) denotes the index of the candidate set, and \( \mathbf{a}_d \) is formed

150 by the elements \( \alpha_m \sim \mathcal{CN}(0, 1) \). To ensure that the average 151

152 transmit power remains unchanged, the scaling factors are normalized 153

154 to unit power. These are made available to both the transmitter and the 155

156 receiver before transmission. These assist in randomizing the received 157

158 constellation, which is most useful in the critical scenarios where two 159

160 155 points in the constellation of \( \mathbf{H}_m \mathbf{s}_m \), \( m \in [1, M], k \in [1, N] \) happen to 156

161 be very close.

162

A. Thresholded Selection of TPS

163

For a given channel, based on knowledge of vectors \( \mathbf{a}_d \), both the 159

164 transmitter and the receiver can determine the received SM constel-

165 lation for the \( d \)th TPS set by calculating the legitimate set of \( [m, k] \) 161

166 combinations in

\[ \hat{y} = \mathbf{H} \mathbf{A} \mathbf{s}_n^k \]  

(6)

where \( \mathbf{A}_d = \text{diag}(\mathbf{a}_d) \) is the diagonal matrix that corresponds to 163

167 the candidate set \( \mathbf{a}_d \). Then, for the given channel coefficients, the 164

168 transmitter and the receiver can choose independently the scaling 165

169 vector \( \mathbf{a}_d \). Alternatively, if no channel state information is available 166

170 at the transmitter (receiver), the receiver (transmitter) can inform the 171

172 transmitter (receiver) concerning the optimum \( \mathbf{a}_d \) by transmitting a 168

173 number of \( \lceil \log_2(D) \rceil \) bits. Contrary to the SM-CR in [19], where the 169

174 maximum MED among all \( D \) possibilities is chosen, here, a threshold-

175 based approach is introduced, where the search for TPS is terminated 171

176 when a candidate TPS is found that satisfies a MED threshold. This 172

177 optimization problem can be expressed as

\[ \mathbf{A}_t, \mathbf{A}_r \in \begin{cases} \min_{m,k} \|\mathbf{H}_m \mathbf{s}_m^k - \mathbf{H}_m \mathbf{s}_m^k\| \geq \theta \quad &\text{if } \exists \mathbf{A}_t \colon \min_{m,k} \|\mathbf{H}_m \mathbf{s}_m^k - \mathbf{H}_m \mathbf{s}_m^k\| \geq \theta \\ \arg\max_{m,k} \min_{m,k} \|\mathbf{H}_m \mathbf{s}_m^k - \mathbf{H}_m \mathbf{s}_m^k\|, \text{otherwise} \end{cases} \]  

(7)

where \( \theta \) represents the MED threshold with respect to the MED 174

178 without TPS. Equivalently, for the case of single-RF-chain-based SM, 175

179 this can be simplified to

\[ \mathbf{A}_t, \mathbf{A}_r \in \begin{cases} \min_{m,k} \|\mathbf{H}_m \mathbf{s}_m^k - \mathbf{H}_m \mathbf{s}_m^k\| \geq \theta \quad &\text{if } \exists \mathbf{A}_t \colon \min_{m,k} \|\mathbf{H}_m \mathbf{s}_m^k - \mathbf{H}_m \mathbf{s}_m^k\| \geq \theta \\ \arg\max_{m,k} \min_{m,k} \|\mathbf{H}_m \mathbf{s}_m^k - \mathbf{H}_m \mathbf{s}_m^k\|, \text{otherwise} \end{cases} \]  

(8)

In other words, the search stops if a TPS set is found that satisfies 177

178 the threshold; otherwise, the TPS that offers the maximum MED is 178

179 returned, following a full search as in SM-CR. For completeness, 179

180 we present the associated algorithm in Table I. It will be shown that 180

181 this process offers significant computational benefits with respect to 181

182 full SM-CR.
TABLE I
ALGORITHM SM-TCR

<table>
<thead>
<tr>
<th>Input: $H, M$</th>
<th>Output: $A_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e = []$</td>
<td>$e$</td>
</tr>
<tr>
<td>for $t = 1$ to $D$ do</td>
<td>$e(t) := \min_{(m,k) \neq (m',k') \in \mathcal{T}} |HA_{m,k} - HA_{m',k'}|^2$</td>
</tr>
<tr>
<td>$\leq e(t) \geq \theta$</td>
<td>$\min_{(m,k) \neq (m',k')} |HA_{m,k} - HA_{m',k'}|^2$</td>
</tr>
<tr>
<td>% calculate the MED for the $t$-th TPS vector</td>
<td>% check if the MED satisfies the MED threshold $\theta$</td>
</tr>
<tr>
<td>$A_o := A_t$</td>
<td>$A_o := A_t$</td>
</tr>
<tr>
<td>% if so, select $A_t$ and terminate the algorithm</td>
<td>% if so, select $A_t$ and terminate the algorithm</td>
</tr>
<tr>
<td>break</td>
<td>break</td>
</tr>
<tr>
<td>$t_o = \arg\max_e e$</td>
<td>$t_o = \arg\max_e e$</td>
</tr>
<tr>
<td>if not, select the TPS vector with the max MED</td>
<td>if not, select the TPS vector with the max MED</td>
</tr>
</tbody>
</table>

TABLE II
COMPLEXITY FOR THE PROPOSED SM-TCR SCHEME

<table>
<thead>
<tr>
<th>SM-TCR Optimization</th>
<th>Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constellation $\mathcal{C}$</td>
<td>$2N_t(M_t + 2N_r + 1)N_t M_t$</td>
</tr>
<tr>
<td>$\mathcal{C}_m, \forall m, k$</td>
<td>$2N_t(M_t + 2N_r + 1)N_t M_t$</td>
</tr>
<tr>
<td>check: $\min_{(m,k) \neq (m',k')} |HA_{m,k} - HA_{m',k'}|^2$</td>
<td>$2N_t(M_t + 2N_r + 1)N_t M_t$</td>
</tr>
<tr>
<td>$\arg\min g^r_m$</td>
<td>$2N_t M_t N_t B$</td>
</tr>
<tr>
<td>Total: $2N_t + 1$</td>
<td>$(N_t M_t + N_t M_t + t + 2N_r + 1)N_t M_t$</td>
</tr>
</tbody>
</table>

Fig. 1. Cumulative distribution function of the number of candidate TPS searched ($\theta$) for various values of $\theta$, $D = 20$, 4-QAM.
IV. SIMULATION RESULTS

To evaluate the benefits of the proposed technique, this section presents numerical results based on Monte Carlo simulations of conventional SM without scaling (termed as SM in the figures), SM-CR, and the proposed SM-TCR. The channel’s impulse response is assumed to be perfectly known at the transmitter. Without loss of generality, we assume that the transmit power is restricted to $P = 1$. MIMO systems with four TAs employing 4-QAM and 16-QAM modulation are explored, albeit it is plausible that the benefits of the proposed technique extend to larger-scale systems and higher-order modulation.

Fig. 2. BER versus SNR for a $(4 \times 2)$-element MIMO with SM, SM-OTPS, SM-CR and SM-TCR, 4-QAM, and 16-QAM.

First, we characterize the attainable BER performance with an increasing transmit SNR for a $(4 \times 2)$-element MIMO employing 4-QAM and 16-QAM, for various values of the MED threshold $\theta$ in Fig. 2. The performance of the highly complex TPS design in [21] based on convex optimization, and termed SM-OTPS in the figure, is also shown here for reference, where it can be seen that the proposed SM-TCR, with orders-of-magnitude less complexity than SM-OTPS, still performs within 1–2 dB from the optimization-based SM-OTPS. The theoretical trends of (11) are also shown, where it can be seen that they provide a close match for the high-SNR system behavior. It can be seen that the slope of the BER curves increases with increasing $\theta$, which indicates an increase in transmit diversity order. Indeed, the BER of SM-TCR is identical to that of SM-CR for $\theta = 2$ in the case of 4-QAM and $\theta = 1.5$ in the case of 16-QAM. In both cases, significant complexity savings are obtained, as shown in the results that follow.

Fig. 3 shows the average computational complexity expressed in terms of numbers of operations (NOPs) for SM-TCR with increasing MED threshold values $\theta$. The complexity of SM and SM-CR is also depicted for reference. It can be seen that as the MED threshold increases, the optimization becomes tighter, leading to complexity close to that of the full SM-CR. For reduced values of $\theta$, however, significant complexity gains are obtained, where the NOPs for SM-TCR are down to less than 55% of those for SM-CR for 4-QAM and 40% for 16-QAM, respectively. A similar trend can be observed in Fig. 4, where the performance is shown for increasing $\theta$, where performance is quantified in terms of goodput in bits per frame, i.e.,

$$T = \log_2 (N_t M) \cdot F (1 - P_F)$$  \hspace{1cm} (13)\

with $P_F$ denoting the frame error probability and $F = 70$ being the frame length used in these results, following [19]. The specific selection of the MED threshold $\theta$ in practice can be based on the desired tradeoff between the complexity in Fig. 3 and (12), the transmit diversity obtained, and the performance observed in Fig. 4 and (10) and (11). Finally, Fig. 5 shows the direct performance-versus-complexity tradeoff. A linear relation between goodput and complexity can be 275
observed. More importantly, where previously either a low-complexity unit-diversity SM or a high-complexity high-diversity SM-CR alter-
native could be chosen, here, a scalable tradeoff is offered between these two extremes with the aid of SM-TCR, by selecting the MED thresholds \( \theta \) accordingly.

V. Conclusion

A new low-complexity constellation shaping approach has been introduced for SM. While conventional CR offers a considerable transmit diversity gain at the cost of increased computational complexity compared with the conventional SM, the proposed scheme delivers a scalable tradeoff between the transmit diversity obtained and the complexity by appropriately selecting the MED thresholds accordingly. Complexity reduction of up to 60% over conventional CR was demonstrated, while still considerably improving the attainable performance of conventional SM.

REFERENCES


AUTHOR QUERIES

AUTHOR PLEASE ANSWER ALL QUERIES

AQ1 = Please provide publication update in Ref. [2].
AQ2 = Please provide publication update in Ref. [8].
AQ3 = Please provide publication update in Ref. [19].

END OF ALL QUERIES
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I. INTRODUCTION

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Consider a multiple-input multiple-output (MIMO) system, where the transmitter and the receiver are equipped with $N_t$ and $N_r$ antennas, respectively.
where optimization, however, is an NP-hard problem, which makes finding a solution intractable. In [20], it is demonstrated that the TPS approach in [19], by selecting among a set of predetermined randomly generated TPS vectors instead of fully optimizing the TPS, offers a near-optimal performance with the lowest complexity among the TPS optimization approaches [20], [21].

\[ \mathbf{y} = \mathbf{H} \mathbf{A} \mathbf{s}_{\mathbf{m}} + \mathbf{w} \]  

For a given channel, based on knowledge of vectors \( \mathbf{a}_{d} \), both the transmitter and the receiver can determine the received SM constellation for the \( d \)th TPS by calculating the legitimate set of \( \{ m, k \} \) combinations in

\[ \hat{\mathbf{y}} = \mathbf{H} \mathbf{A} \mathbf{s}_{\mathbf{m}}^k \]  

where \( \mathbf{A}_{d} = \text{diag}(\mathbf{a}_{d}) \) is the diagonal matrix that corresponds to the candidate set \( \mathbf{a}_{d} \). Then, for the given channel coefficients, the transmitter (receiver) can inform the receiver (transmitter) concerning the optimum \( \mathbf{a}_{d} \) by transmitting a number of \( \log_2(D) \) bits. Contrary to the SM-CR in [19], where the maximum MODD among all D possibilities is chosen, here, a threshold-based approach is introduced, where the search for TPS is terminated when a TPS set is found that satisfies the MED threshold with respect to the MED constellation, which is most useful in the critical scenarios where two points in the constellation of \( \mathbf{H} \mathbf{s}_{\mathbf{m}} \) happen to be very close.

A. Thresholded Selection of TPS

For a given channel, based on knowledge of vectors \( \mathbf{a}_{d} \), both the 159 transmitter and the receiver can determine the received SM constellation for the \( d \)th TPS by calculating the legitimate set of \( \{ m, k \} \) combinations in

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\[ \mathbf{A}_{d} = \begin{cases} \min_{\{ m, k \} \neq \{ m_{i}, k_{i} \}} \| \mathbf{H} \mathbf{A} \mathbf{s}_{\mathbf{m}} \|^2 & \text{if } \mathbf{A}_{d} \neq \emptyset \\ \emptyset & \text{otherwise} \end{cases} \]  

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where \( \mathbf{A}_{d} = \text{diag}(\mathbf{a}_{d}) \) is the diagonal matrix that corresponds to the candidate set \( \mathbf{a}_{d} \). Then, for the given channel coefficients, the transmitter and the receiver can choose independently the scaling factor \( \mathbf{a}_{d} \). Alternatively, if no channel state information is available at the transmitter (receiver), the receiver (transmitter) can inform the transmitter (receiver) concerning the optimum \( \mathbf{a}_{d} \) by transmitting a number of \( \log_2(D) \) bits. Contrary to the SM-CR in [19], where the maximum MODD among all D possibilities is chosen, here, a threshold-based approach is introduced, where the search for TPS is terminated when a TPS candidate is found that satisfies a MSED threshold. This MSED optimization problem can be expressed as

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TABLE I
ALGORITHM SM-TCR

<table>
<thead>
<tr>
<th>Input: $H, M$</th>
<th>Output: $A_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e = 1$</td>
<td>$e$</td>
</tr>
<tr>
<td>for $t = 1$ to $D$ do</td>
<td>$e[t] := \min_{(m_1, k_1), (m_2, k_2)}</td>
</tr>
<tr>
<td>if $e[t] \geq \theta$</td>
<td>$A_o := A_t$</td>
</tr>
<tr>
<td>% calculate the MED for the $t$-th TPS vector</td>
<td>if so, select $A_t$ and terminate the algorithm</td>
</tr>
<tr>
<td>check if the MED satisfies the MED threshold $\theta$</td>
<td>break</td>
</tr>
<tr>
<td>$A_o := A_t$</td>
<td>$t_o = \arg\max_t e$</td>
</tr>
<tr>
<td>% if not, select the TPS vector with the max MED</td>
<td>$A_o := A_t$</td>
</tr>
</tbody>
</table>

183 Based on (8), the transmitter sends $x = A_o s_{m_t}$, and the receiver 184 applies the ML detector according to

$$[s_{m_t}, k] = \arg \min_{m,k} ||y - HA_s^{k}||.$$  \hspace{1cm} (9)

185 It should be noted that, to dispense with the need for channel state 186 information at the transmitter (CSIT), the receiver can select the best 187 scaling factors using (8) and then feed the index of the scaling matrix 188 $A_o$ selected from the set of $D$ candidates back to the transmitter, 189 using $[\log_2(D)]$ bits. This constitutes major overhead savings for the 190 proposed scheme with respect to the existing TPS schemes for SM that 191 require full CSIT, while obtaining similar performance.

192 B. Transmit Diversity and Performance Trends
193 While the transmit diversity order of the single-RF SM is known to 194 be one [9], the proposed TPS introduces an amplitude-phase diversity 195 in the transmission, which is an explicit benefit of having $D$ candidate 196 sets of TPS factors to choose from. Accordingly, it was shown in 197 [19] that the obtained transmit diversity order corresponds to the 198 $\delta$-dependent gain in the average MED associated with CR as

$$G(\delta) = \frac{E[\min_{m,k} ||HA_s^{m} - HA_s^{k}||^2]}{E[\min_{m,k} ||HA_s^{m} - HA_s^{k}||^2]}.$$  \hspace{1cm} (10)

199 In addition, SM systems with $N_r$ uncorrelated RAs have been 200 shown to experience a unity transmit diversity order and a receive 201 diversity order of $N_r$. Accordingly, since the proposed scheme attains 202 a $\delta$-dependent transmit diversity order of $G(\delta)$, the total diversity order 203 becomes $\delta = N_r G(\delta)$. The resulting probability of error $P_e$ obeys the 204 high-SNR trend of

$$P_e = \alpha \gamma^{-N_r G(\delta)}.$$  \hspace{1cm} (11)

205 where $\gamma$ is the transmit SNR, and $\alpha$ is an arbitrary nonnegative 206 coefficient. We verify the above theoretical performance trend against 207 simulation in the following.

III. COMPUTATIONAL COMPLEXITY
209 It is clear from the above discussion that the proposed SM-TCR 210 leads to a computational complexity reduction with respect to conven-211 tional SM-CR, due to the early termination of the TPS search, 212 after a calculation of $t \leq D$ out of $D$ TPS sets. Here, we analyze this 213 computational complexity reduction at the receiver. This analysis is

complemented by the following results on the distribution of $t$. For ref-214 erence, we have assumed a Long-Term Evolution Type-2 time-division 215 duplex frame structure [18]. This has 10-ms duration that consists of 216 ten subframes out of which five subframes, each containing 14 symbol 217 time slots, are used for downlink transmission, yielding a frame size of 218 $F = 70$ for the downlink, whereas the rest are used for both uplink and 219 control information transmission. A slow-fading channel is assumed, 220 where the channel remains constant for the duration of the frame.

Following the complexity analysis in [19], we quantify the number of operations required in each step of the SM-TCR search in Table II. 223 From the table, we have a total SM-TCR receiver complexity of

$$C(t) = (2N_r + 1)(\frac{N_r M}{2}) + N_r M t + (2N_r + 1)N_r M B.$$  \hspace{1cm} (12)

To complete this complexity discussion, in Fig. 1, we show the dis-225 tribution of $t$ as a function of the increasing threshold values of $\theta$. It can 226 be seen that low numbers of candidate TPS searched $t$ are obtained with 227 high probability, particularly in the cases of low MED thresholds $\theta$. 228 While large complexity savings can be observed in the figure, it is 229 important to note that the complexity of SM-TCR is upper bounded by 230 that of SM-CR, since $t \leq D$. 231

Fig. 1. Cumulative distribution function of the number of candidate TPS searched ($t$) for various values of $\theta$, $D = 20$, 4-QAM.
To evaluate the benefits of the proposed technique, this section presents numerical results based on Monte Carlo simulations of conventional SM without scaling (termed as SM in the figures), SM-CR, and the proposed SM-TCR. The channel’s impulse response is assumed to be perfectly known at the transmitter. Without loss of generality, we assume that the transmit power is restricted to $P = 1$. MIMO systems with four TAs employing 4-QAM and 16-QAM modulation are explored, albeit it is plausible that the benefits of the proposed technique extend to larger-scale systems and higher-order modulation.

First, we characterize the attainable BER performance with an increasing transmit SNR for a $(4 \times 2)$-element MIMO employing 4-QAM and 16-QAM, for various values of the MED threshold $\theta$ in Fig. 2. The performance of the highly complex TPS design in [21] based on convex optimization, and termed SM-OTPS in the figure, is also shown here for reference, where it can be seen that the proposed SM-TCR, with orders-of-magnitude less complexity than SM-OTPS, still performs within 1–2 dB from the optimization-based SM-OTPS. The theoretical trends of (11) are also shown, where it can be seen that they provide a close match for the high-SNR system behavior. It can be seen that the slope of the BER curves increases with increasing $\theta$, which indicates an increase in transmit diversity order. Indeed, the BER of SM-TCR is identical to that of SM-CR for $\theta = 2$ in the case of 4-QAM and $\theta = 1.5$ in the case of 16-QAM. In both cases, significant complexity savings are obtained, as shown in the results that follow.

Fig. 3 shows the average computational complexity expressed in terms of numbers of operations (NOPs) for SM-TCR with increasing MED threshold values $\theta$. The complexity of SM and SM-CR is also depicted for reference. It can be seen that as the MED threshold increases, the optimization becomes tighter, leading to complexity close to that of the full SM-CR. For reduced values of $\theta$, however, significant complexity gains are obtained, where the NOPs for SM-TCR are down to less than 55% of those for SM-CR for 4-QAM and 40% for 16-QAM, respectively. A similar trend can be observed in Fig. 4, where the performance is shown for increasing $\theta$, where performance is quantified in terms of goodput in bits per frame, i.e.,

$$T = \log_2(N_t M) \cdot F(1 - P_F)$$

with $P_F$ denoting the frame error probability and $F = 70$ being the frame length used in these results, following [19]. The specific selection of the MED threshold $\theta$ in practice can be based on the desired tradeoff between the complexity in Fig. 3 and (12), the transmit diversity obtained, and the performance observed in Fig. 4 and (10) and (11). Finally, Fig. 5 shows the direct performance-versus-complexity tradeoff. A linear relation between goodput and complexity can be seen.
observed. More importantly, where previously either a low-complexity
unit-diversity SM or a high-complexity high-diversity SM-CR alter-
native could be chosen, here, a scalable tradeoff is offered between
these two extremes with the aid of SM-TCR, by selecting the MED
thresholds $\theta$ accordingly.

281 V. Conclusion

282 A new low-complexity constellation shaping approach has been
introduced for SM. While conventional CR offers a considerable
transmit diversity gain at the cost of increased computational com-
plexity compared with the conventional SM, the proposed scheme
delivers a scalable tradeoff between the transmit diversity obtained
and the complexity by appropriately selecting the MED threshold
values. Complexity reduction of up to 60% over conventional CR
was demonstrated, while still considerably improving the attainable
performance of conventional SM.

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AQ1 = Please provide publication update in Ref. [2].
AQ2 = Please provide publication update in Ref. [8].
AQ3 = Please provide publication update in Ref. [19].

END OF ALL QUERIES