Correspondence

A Scalable Performance–Complexity Tradeoff for Constellation Randomization in Spatial Modulation

Christos Masouros, *Senior Member, IEEE*, and Lajos Hanzo, *Fellow, IEEE*

***Abstract*—It is widely recognized that traditional single radio frequency (RF)-chain-aided spatial modulation (SM) does not offer any transmit diversity gain. As a remedy, constellation randomization (CR), relying on transmit prescaling (TPS), has been shown to provide transmit diversity for single-RF-chain-aided SM. In this paper, we propose a low-complexity approach to SM with the aid of constellation randomization (SM-CR) that considerably improves the transmit diversity gain of SM at a reduced computational burden compared with conventional SM-CR. While conventional SM-CR performs a full search among a set of candidate TPS factors to achieve the maximum minimum Euclidean distance (MED) in the received SM constellation, here, we propose a thresholding approach, where, instead of the maximum MED, the TPS aims to satisfy a specific MED threshold. This technique offers a significant complexity reduction with respect to the full maximization of SM-CR, since the search for TPS is terminated once a TPS set is found that satisfies the MED threshold. Our analysis and results demonstrate that a scalable tradeoff can be achieved between transmit diversity and complexity by appropriately selecting the MED threshold, where a significant complexity reduction is attained, while achieving a beneficial transmit diversity gain for the single-RF SM.**

***Index Terms*—Constellation shaping, multiple-input single-output, spatial modulation (SM), transmit prescaling (TPS).**

I. Introduction

Spatial modulation (SM) has been shown to offer a low-complexity design alternative to spatial multiplexing, where only a subset (down to one) of radio frequency (RF) chains is required for transmission [1], [2]. Early work has focused on the design of receiver algorithms for minimizing the bit error ratio of SM at low complexity [1]–[5]. Matched filtering is shown to be a low-complexity technique for detecting the activated antenna index (AI) [1]–[3]. A maximum likelihood (ML) detector is introduced in [4] for reducing the complexity of classic spatial multiplexing ML detectors, whereas the complexity imposed can be further reduced by compressive sensing detection approaches [5]. In addition to receive processing, several transmit precoding (TPC) approaches have been proposed for receive antenna (RA)-aided SM, where the spatial information is mapped onto the RA index [6]–[8].

Relevant work has also proposed constellation shaping for SM [9]–[14]. Specifically, in [9], the transmit diversity of coded SM is analyzed for different *spatial constellations*, which represent the legitimate sets of activated transmit antennas (TAs). Furthermore, Yang *et al.* in [10] conceived a symbol constellation optimization technique for minimizing the bit error rate (BER). Indeed, spatial and symbol constellation shaping are discussed separately in the aforementioned reference. By contrast, the design of the received SM constellation that combines the choice of the TA as well as the transmit symbol constellation is the focus of this paper. A number of constellation shaping schemes [11]–[14] have also been proposed for the special case of SM, which is referred to as space shift keying, where the information is purely carried in the spatial domain, by the activated AI. However, the application of the above constellation shaping to the SM transmission, where the transmit waveform is modulated, is nontrivial.

Recent work has focused on shaping the receive SM constellation by means of symbol prescaling at the transmitter, aiming for maximizing the minimum Euclidean distance (MED) in the received SM constellation [15]–[17]. The constellation shaping approach in [15] and [16] aims at fitting the receive SM constellation to one of the existing optimal classic constellation formats in terms of minimum distance, such as, e.g., quadrature amplitude modulation (QAM). Due to the strict constellation fitting requirement imposed on both amplitude and phase, this prescaling relies on the inversion of the channel coefficients. In the case of ill-conditioned channels, this substantially reduces the received signal-to-noise ratio (SNR). This problem has been alleviated in [17], where a constellation shaping scheme based on phase-only scaling is proposed. Still, the constellation shaping used in the above schemes is limited in the sense that it only applies to multiple-input–single-output systems, where a single symbol is received for each transmission, and thus, the characterization and shaping of the receive SM constellation is simple.

Closely related to this work, a transmit prescaling (TPS) scheme was proposed for SM [19], where the received SM constellation is randomized by TPS for maximizing the MED between its points for a given channel. A number of randomly generated candidate sets of TPS factors are formed offline, known to both the transmitter and the receiver, and the transmitter then selects that particular set of TPS factors that yields the SM constellation having the maximum MED. Against this background, in this paper, we propose a low-complexity relaxation of the above optimization instead of an exhaustive search, where the first TPS factor set that is found to satisfy a predetermined threshold is selected, thus reducing the computational burden of the TPS operation. The proposed scheme is shown to provide a scalable tradeoff between the performance attained and the complexity imposed, by accordingly selecting the MED threshold.

This paper is organized as follows: In Section II, the basic system model is first introduced, and the proposed scheme is then discussed. The computational complexity of the proposed technique is analyzed in Section III, and its performance against the state of the art is evaluated in Section IV. Finally, in Section V, we draw the key conclusions of our study.

II. Spatial Modulation With Threshold Constellation Randomization (SM-TCR)

Consider a multiple-input multiple-output (MIMO) system, where the transmitter and the receiver are equipped with N_t and N_r antennas,

Manuscript received November 24, 2015; revised March 14, 2016; accepted May 17, 2016. This work was supported in part by the Royal Academy of Engineering, U.K. and in part by the Engineering and Physical Sciences Research Council through Project EP/M014150/1. The review of this paper was coordinated by Dr. Y. Xin.

C. Masouros is with the Department of Electrical and Electronic Engineering, University College London, London WC1E 6BT, U.K. (e-mail: chris.masouros@ieee.org).

L. Hanzo is with the School of Electronics and Computer Science, University of Southampton, Southampton SO16 7NS, U.K. (e-mail: lh@ecs.soton.ac.uk).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TVT.2016.2572760

respectively. For simplicity, unless stated otherwise, in this paper, we assume that the transmit power budget is limited to unity, i.e., $P = 1$. We focus on the single-RF-chain-aided SM approach, where the transmit vector is in the all-but-one zero form $\mathbf{s}_m^k = [0, \ldots, s_m, \ldots, 0]^T$, with $[.]^T$ denoting the transpose operator. Here, $s_m, m \in \{1, \ldots, M\}$ is a symbol taken from an M-order modulation alphabet that represents the transmitted waveform in the baseband domain conveying $\log_2(M)$ bits, whereas k represents the index of the activated TA (the index of the nonzero element in $\mathbf{s}_m^k$) conveying $\log_2(N_t)$ bits in the spatial domain. Clearly, since $\mathbf{s}$ is an all-zero vector apart from s_m^k, there is no interantenna interference.

For the per-antenna TPS approach, which is the focus of this paper, the signal fed to each TA is scaled by a complex-valued coefficient $\alpha_k, k \in \{1, \ldots, N_t\}$ for which we have $E\{|\alpha_k|\} = 1$, where $|x|$ denotes the amplitude of a complex number x, and $E\{.\}$ denotes the expectation operator. Defining the MIMO channel vector as $\mathbf{H}$ with elements $h_{m,n}$ representing the complex-valued channel coefficient between the nth TA and the mth RA, the received symbol vector can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{A}\mathbf{s}_m^k + \mathbf{w} \tag{1}$$

where $\mathbf{w} \sim \mathcal{CN}(0, \sigma^2\mathbf{I})$ is the additive white Gaussian noise component at the receiver, with $\mathcal{CN}(\mu, \sigma^2)$ denoting the circularly symmetric complex Gaussian distribution with a mean of μ and a variance of σ^2. Furthermore, $\mathbf{A} = \text{diag}(\mathbf{a})$ is the TPS matrix with $\mathbf{a} = [\alpha_1, \alpha_2, \ldots, \alpha_{N_t}]$, and $\text{diag}(\mathbf{g})$ represents the diagonal matrix with its diagonal elements taken from vector $\mathbf{g}$. Note that the diagonal structure of $\mathbf{A}$ guarantees having a transmit vector $\mathbf{x} = \mathbf{A}\mathbf{s}$ with a single nonzero element, so that the single-RF-chain aspect of SM is preserved.

At the receiver, a joint ML detection of both the TA index and the transmit symbol is obtained by the minimization, i.e.,

$$\begin{aligned}[\hat{s}_m, \hat{k}] &= \arg\min_i \|\mathbf{y} - \dot{\mathbf{y}}_i\| \\ &= \arg\min_{m,k} \|\mathbf{y} - \mathbf{H}\mathbf{A}\mathbf{s}_m^k\| \end{aligned} \tag{2}$$

where $\|\mathbf{x}\|$ denotes the norm of vector $\mathbf{x}$, and $\dot{\mathbf{y}}_i$ is the ith constellation point in the received SM constellation. By exploiting the specific structure of the transmit vector, this can be further simplified to

$$[\hat{s}_m, \hat{k}] = \arg\min_{m,k} \|\mathbf{y} - \mathbf{h}_k \alpha_m^k s_m\| \tag{3}$$

where $\mathbf{h}_k$ denotes the kth column of matrix $\mathbf{H}$. It is widely recognized that the performance of the detection as formulated above is dominated by the MED between the adjacent constellation points $\dot{\mathbf{y}}_i$ and $\dot{\mathbf{y}}_j$ in the receive SM constellation, i.e.,

$$d_{\min} = \min_{i,j} \|\dot{\mathbf{y}}_i - \dot{\mathbf{y}}_j\|^2, \ i \neq j. \tag{4}$$

Accordingly, to improve the likelihood of correct detection, constellation shaping TPS schemes conceived for SM aim at maximizing this MED. The optimum TPS matrix $\mathbf{A}^*$ can be found by solving the optimization problem of [20]

$$\begin{aligned} \mathbf{A}^* &= \arg\max_{\mathbf{A}} \min_{i,j} \|\dot{\mathbf{y}}_i - \dot{\mathbf{y}}_j\|^2, \ i \neq j \\ \text{s.t.c.} \quad & \text{trace}(\mathbf{A}^{*H}\mathbf{A}^*) \leq P \end{aligned} \tag{5}$$

and, additionally for single-RF-chain-aided SM, subject to $\mathbf{A}^*$ having a diagonal structure. In the above, $\mathbf{A}^H$ and $\text{trace}(\mathbf{A})$ represent the Hermitian transpose and trace of matrix $\mathbf{A}$, respectively. The above optimization, however, is an NP-hard problem, which makes finding the TPS factors prohibitively complex and motivates the conception of lower-complexity suboptimal techniques. Indeed, it has been shown that the TPS approach in [19], by selecting among a set of predetermined randomly generated TPS vectors instead of fully optimizing the TPS, offers a near-optimal performance with the lowest complexity among the TPS optimization approaches [20], [21].

TPS Vector Generation: Accordingly, with SM-TCR first, a number of D random candidate TPS vectors are generated, in the form of $\mathbf{a}_d$, where $d \in [1, D]$ denotes the index of the candidate set, and $\mathbf{a}_d$ is formed by the elements $\alpha_m^{k(d)} \sim \mathcal{CN}(0, 1)$. To ensure that the average transmit power remains unchanged, the scaling factors are normalized to unit power. These are made available to both the transmitter and the receiver before transmission. These assist in randomizing the received constellation, which is most useful in the critical scenarios where two points in the constellation of $\mathbf{H}\mathbf{s}_m^k, m \in [1, M], k \in [1, N_t]$ happen to be very close.

A. *Thresholded Selection of TPS*

For a given channel, based on knowledge of vectors $\mathbf{a}_d$, both the transmitter and the receiver can determine the received SM constellation for the dth TPS set by calculating the legitimate set of $[m, k]$ combinations in

$$\hat{\mathbf{y}} = \mathbf{H}\mathbf{A}_d\mathbf{s}_m^k \tag{6}$$

where $\mathbf{A}_d = \text{diag}(\mathbf{a}_d)$ is the diagonal matrix that corresponds to the candidate set $\mathbf{a}_d$. Then, for the given channel coefficients, the transmitter and the receiver can choose independently the scaling vector $\mathbf{a}_o$. Alternatively, if no channel state information is available at the transmitter (receiver), the receiver (transmitter) can inform the transmitter (receiver) concerning the optimum $\mathbf{a}_o$ by transmitting a number of $\lceil \log_2(D) \rceil$ bits. Contrary to the SM-CR in [19], where the maximum MED among all D possibilities is chosen, here, a threshold-based approach is introduced, where the search for TPS is terminated when a candidate TPS is found that satisfies a MED threshold. This optimization problem can be expressed as

$$\mathbf{A}_o = \begin{cases} \mathbf{A}_t, \text{if} \exists \mathbf{A}_t : & \min\limits_{\substack{\{m_i,k_j\} \neq \\ \{m_l,k_p\}}} \|\mathbf{H}\mathbf{A}_t\mathbf{s}_{m_1}^{k_1} - \mathbf{H}\mathbf{A}_t\mathbf{s}_{m_2}^{k_2}\|^2 \geq \\ & \quad \theta \min\limits_{\substack{\{m_i,k_j\} \neq \\ \{m_l,k_p\}}} \|\mathbf{H}\mathbf{s}_{m_1}^{k_1} - \mathbf{H}\mathbf{s}_{m_2}^{k_2}\|^2 \\ \arg\max_d \min\limits_{\substack{\{m_i,k_j\} \neq \\ \{m_l,k_p\}}} \|\mathbf{H}\mathbf{A}_d\mathbf{s}_{m_1}^{k_1} - \mathbf{H}\mathbf{A}_d\mathbf{s}_{m_2}^{k_2}\|^2, & \text{otherwise} \end{cases} \tag{7}$$

where θ represents the MED threshold with respect to the MED without TPS. Equivalently, for the case of single-RF-chain-based SM, this can be simplified to

$$\mathbf{A}_o = \begin{cases} \mathbf{A}_t, \text{if} \exists \mathbf{A}_t : & \min\limits_{\substack{\{m_i,k_j\} \neq \\ \{m_l,k_p\}}} \|\mathbf{h}_{k_1} a_{m_1}^{k_1} s_{m_1} - \mathbf{h}_{k_2} a_{m_2}^{k_2} s_{m_2}\|^2 \geq \\ & \quad \theta \min\limits_{\substack{\{m_i,k_j\} \neq \\ \{m_l,k_p\}}} \|\mathbf{h}_{k_1} s_{m_1} - \mathbf{h}_{k_2} s_{m_2}\|^2 \\ \arg\max_d \min\limits_{\substack{\{m_i,k_j\} \neq \\ \{m_l,k_p\}}} \|\mathbf{h}_{k_1} a_{m_1}^{k_1} s_{m_1} - \mathbf{h}_{k_2} a_{m_2}^{k_2} s_{m_2}\|^2, & \text{otherwise} \end{cases} \tag{8}$$

In other words, the search stops if a TPS set is found that satisfies the threshold; otherwise, the TPS that offers the maximum MED is returned, following a full search as in SM-CR. For completeness, we present the associated algorithm in Table I. It will be shown that this process offers significant computational benefits with respect to full SM-CR.

TABLE I
ALGORITHM SM-TCR

Input : $\mathbf{H}, M$, **Output** : $\mathbf{A}_o$

$\mathbf{e} = [\,]$
for $t = 1$ to D do
  $\mathbf{e}[t] := \min_{\{m_i,k_j\}\neq\{m_l,k_p\}} ||\mathbf{HA}_t\mathbf{s}_{m_1}^{k_1} - \mathbf{HA}_t\mathbf{s}_{m_2}^{k_2}||^2$
    % calculate the MED for the t-th TSP vector
  if $\mathbf{e}[t] \geq \theta \min_{\{m_i,k_j\}\neq\{m_l,k_p\}} ||\mathbf{Hs}_{m_1}^{k_1} - \mathbf{Hs}_{m_2}^{k_2}||^2$
  % check if the MED satisfies the MED threshold θ
    $\mathbf{A}_o := \mathbf{A}_t$
    % if so, select $\mathbf{A}_t$ and terminate the algorithm
    break
  end
end
$t_o = \arg\max_t \mathbf{e}$
  % if not, select the TPS vector with the max MED
$\mathbf{A}_o := \mathbf{A}_{t_o}$

TABLE II
COMPLEXITY FOR THE PROPOSED SM-TCR SCHEME

SM-TCR		Operations
Constellation Optimization		
$\mathbf{HA}_d\mathbf{s}_m^k, \forall m,k$	$\times t$	$(2N_r+1)N_tMt$
$\mathbf{f}_{m_1,m_2}^{k_1,k_2(d)} = ||\mathbf{HA}_d\mathbf{s}_{m_1}^{k_1} - \mathbf{HA}_d\mathbf{s}_{m_2}^{k_2}||$, $\forall m_1,m_2,k_1,k_2,m_1\neq m_2,k_1\neq k_2$	$\times t$	$2N_r\binom{N_tM}{2}t$
check: $\min\{\mathbf{f}_{m_1,m_2}^{k_1,k_2(d)}\} \geq \theta \min_{\{m_i,k_j\}\neq\{m_l,k_p\}} ||\mathbf{Hs}_{m_1}^{k_1} - \mathbf{Hs}_{m_2}^{k_2}||^2$	$\times t$	$\binom{N_tM}{2}t$
ML Detection		
$g_m^k = ||\mathbf{y} - \mathbf{HA}_o\mathbf{s}_m^k||^2, \forall m,k$	$\times B$	$2N_tMN_rB$
$\arg\min g_m^k$	$\times B$	N_tMB
Total: $(2Nr+1)\left[\binom{N_tM}{2} + N_tM\right]t + (2N_r+1)N_tMB$		

Based on (8), the transmitter sends $\mathbf{x} = \mathbf{A}_o\mathbf{s}_m^k$, and the receiver applies the ML detector according to

$$[\hat{s}_m, \hat{k}] = \arg\min_{m,k} \|\mathbf{y} - \mathbf{HA}_o\mathbf{s}_m^k\|. \tag{9}$$

It should be noted that, to dispense with the need for channel state information at the transmitter (CSIT), the receiver can select the best scaling factors using (8) and then feed the index of the scaling matrix $\mathbf{A}_o$ selected from the set of D candidates back to the transmitter, using $\lfloor\log_2(D)\rfloor$ bits. This constitutes major overhead savings for the proposed scheme with respect to the existing TPS schemes for SM that require full CSIT, while obtaining similar performance.

B. Transmit Diversity and Performance Trends

While the transmit diversity order of the single-RF SM is known to be one [9], the proposed TPS introduces an amplitude-phase diversity in the transmission, which is an explicit benefit of having D candidate sets of TPS factors to choose from. Accordingly, it was shown in [19] that the obtained transmit diversity order corresponds to the θ-dependent gain in the average MED associated with CR as

$$G(\theta) \hat{=} \frac{E\{\min_{m,k} \|\mathbf{HA}_o\mathbf{s}_{m_1}^{k_1} - \mathbf{HA}_o\mathbf{s}_{m_2}^{k_2}\|^2\}}{E\{\min_{m,k} \|\mathbf{Hs}_{m_1}^{k_1} - \mathbf{Hs}_{m_2}^{k_2}\|^2\}}. \tag{10}$$

In addition, SM systems with N_r uncorrelated RAs have been shown to experience a unity transmit diversity order and a receive diversity order of N_r. Accordingly, since the proposed scheme attains a θ-dependent transmit diversity order of $G(\theta)$, the total diversity order becomes $\delta = N_rG(\theta)$. The resulting probability of error P_e obeys the high-SNR trend of

$$P_e = \alpha\gamma^{-N_rG(\theta)} \tag{11}$$

where γ is the transmit SNR, and α is an arbitrary nonnegative coefficient. We verify the above theoretical performance trend against simulation in the following.

III. Computational Complexity

It is clear from the above discussion that the proposed SM-TCR leads to a computational complexity reduction with respect to conventional SM-CR, due to the early termination of the TPS search, after a calculation of $t \leq D$ out of D TPS sets. Here, we analyze this computational complexity reduction at the receiver. This analysis is complemented by the following results on the distribution of t. For reference, we have assumed a Long-Term Evolution Type-2 time-division duplex frame structure [18]. This has 10-ms duration that consists of ten subframes out of which five subframes, each containing 14 symbol time slots, are used for downlink transmission, yielding a frame size of $F = 70$ for the downlink, whereas the rest are used for both uplink and control information transmission. A slow-fading channel is assumed, where the channel remains constant for the duration of the frame.

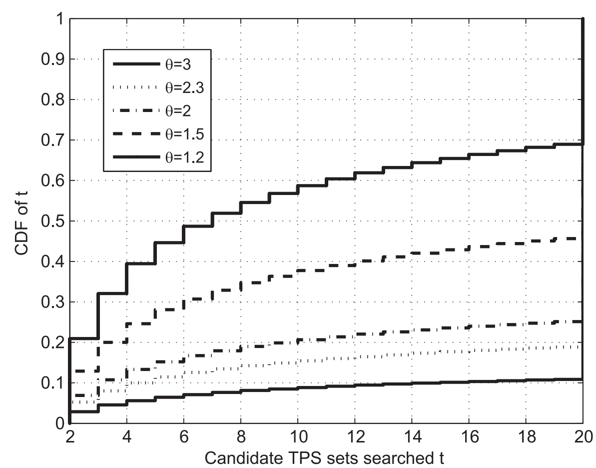

Fig. 1. Cumulative distribution function of the number of candidate TPS searched (t) for various values of θ, D = 20, 4-QAM.

Following the complexity analysis in [19], we quantify the number of operations required in each step of the SM-TCR search in Table II. From the table, we have a total SM-TCR receiver complexity of

$$C(t) = (2Nr+1)\left[\binom{N_tM}{2} + N_tM\right]t + (2Nr+1)N_tMB. \tag{12}$$

To complete this complexity discussion, in Fig. 1, we show the distribution of t as a function of the increasing threshold values of θ. It can be seen that low numbers of candidate TPS searched t are obtained with high probability, particularly in the cases of low MED thresholds θ. While large complexity savings can be observed in the figure, it is important to note that the complexity of SM-TCR is upper bounded by that of SM-CR, since $t \leq D$.

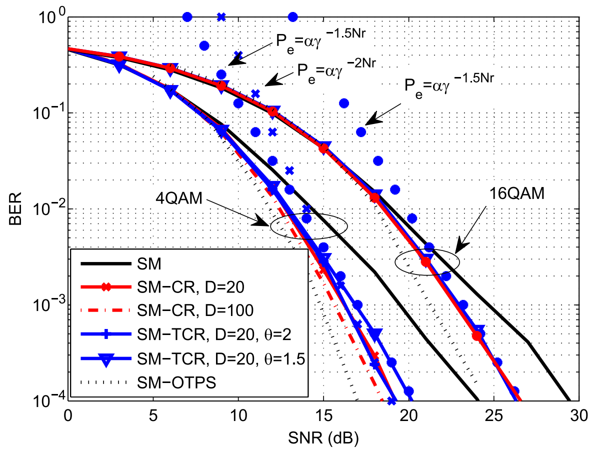

Fig. 2. BER versus SNR for a (4 × 2)-element MIMO with SM, SM-OTPS, SM-CR and SM-TCR, 4-QAM, and 16-QAM.

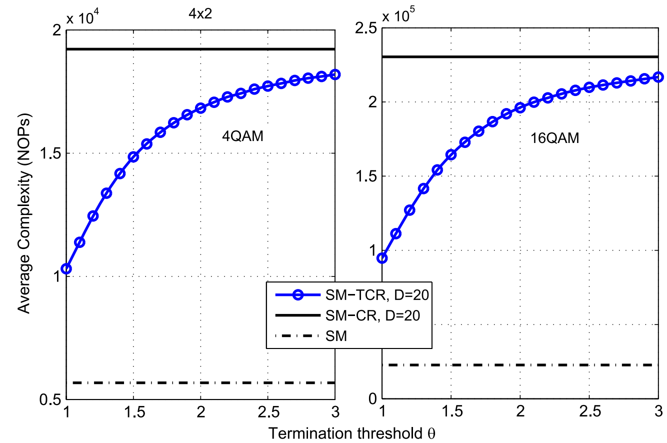

Fig. 3. Average complexity versus θ for a (4 × 2)-element MIMO with SM-TCR, 4-QAM, and 16-QAM.

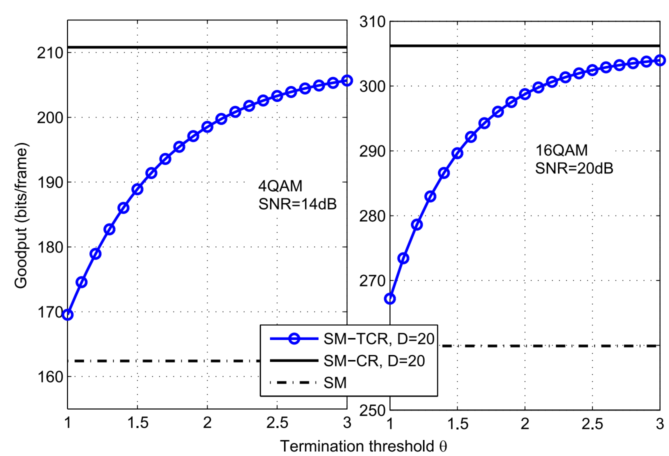

Fig. 4. Goodput versus θ for a (4 × 2)-element MIMO with SM-TCR, 4-QAM, and 16-QAM.

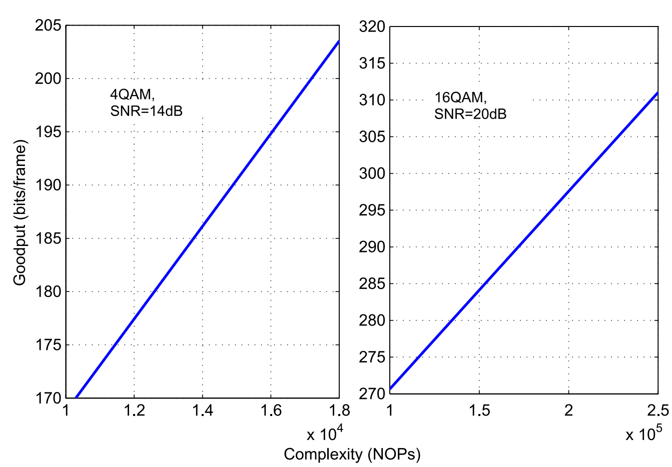

Fig. 5. Goodput versus average complexity for a (4 × 2)-element MIMO with SM-TCR, 4-QAM, and 16-QAM.

IV. Simulation Results

To evaluate the benefits of the proposed technique, this section presents numerical results based on Monte Carlo simulations of conventional SM without scaling (termed as SM in the figures), SM-CR, and the proposed SM-TCR. The channel's impulse response is assumed to be perfectly known at the transmitter. Without loss of generality, we assume that the transmit power is restricted to $P = 1$. MIMO systems with four TAs employing 4-QAM and 16-QAM modulation are explored, albeit it is plausible that the benefits of the proposed technique extend to larger-scale systems and higher-order modulation.

First, we characterize the attainable BER performance with an increasing transmit SNR for a (4 × 2)-element MIMO employing 4-QAM and 16-QAM, for various values of the MED threshold θ in Fig. 2. The performance of the highly complex TPS design in [21] based on convex optimization, and termed SM-OTPS in the figure, is also shown here for reference, where it can be seen that the proposed SM-TCR, with orders-of-magnitude less complexity than SM-OTPS, still performs within 1–2 dB from the optimization-based SM-OTPS. The theoretical trends of (11) are also shown, where it can be seen that they provide a close match for the high-SNR system behavior. It can be seen that the slope of the BER curves increases with increasing θ, which indicates an increase in transmit diversity order. Indeed, the BER of SM-TCR is identical to that of SM-CR for $\theta = 2$ in the case of 4-QAM and $\theta = 1.5$ in the case of 16-QAM. In both cases, significant complexity savings are obtained, as shown in the results that follow.

Fig. 3 shows the average computational complexity expressed in terms of numbers of operations (NOPs) for SM-TCR with increasing MED threshold values θ. The complexity of SM and SM-CR is also depicted for reference. It can be seen that as the MED threshold increases, the optimization becomes tighter, leading to complexity close to that of the full SM-CR. For reduced values of θ, however, significant complexity gains are obtained, where the NOPs for SM-TCR are down to less than 55% of those for SM-CR for 4-QAM and 40% for 16-QAM, respectively. A similar trend can be observed in Fig. 4, where the performance is shown for increasing θ, where performance is quantified in terms of goodput in bits per frame, i.e.,

$$T = \log_2(N_t M) \cdot F(1 - P_F) \tag{13}$$

with P_F denoting the frame error probability and $F = 70$ being the frame length used in these results, following [19]. The specific selection of the MED threshold θ in practice can be based on the desired tradeoff between the complexity in Fig. 3 and (12), the transmit diversity obtained, and the performance observed in Fig. 4 and (10) and (11). Finally, Fig. 5 shows the direct performance-versus-complexity tradeoff. A linear relation between goodput and complexity can be observed. More importantly, where previously either a low-complexity unit-diversity SM or a high-complexity high-diversity SM-CR alternative could be chosen, here, a scalable tradeoff is offered between these two extremes with the aid of SM-TCR, by selecting the MED thresholds θ accordingly.

V. Conclusion

A new low-complexity constellation shaping approach has been introduced for SM. While conventional CR offers a considerable transmit diversity gain at the cost of increased computational complexity compared with the conventional SM, the proposed scheme delivers a scalable tradeoff between the transmit diversity obtained and the complexity by appropriately selecting the MED threshold values. Complexity reduction of up to 60% over conventional CR was demonstrated, while still considerably improving the attainable performance of conventional SM.

References

[1] R. Mesleh, H. Haas, S. Sinanovic, C. W. Ahn, and S. Yun, "Spatial modulation," *IEEE Trans. Veh. Technol.*, vol. 57, no. 4, pp. 2228–2241, Jul. 2008.

[2] P. Yang *et al.*, "Single-carrier spatial modulation: A promising design for large-scale broadband antenna systems," *IEEE Commun. Surveys Tuts.*, to be published. AQ1

[3] M. Di Renzo and H. Haas, "Bit error probability of space modulation over Nakagami-m fading: Asymptotic analysis," *IEEE Commun. Lett.*, vol. 15, no. 10, pp. 1026–1028, Oct. 2011.

[4] M. Di Renzo, H. Haas, A. Ghrayeb, S. Sugiura, and L. Hanzo, "Spatial modulation for generalized MIMO: Challenges, opportunities, and implementation," *Proc. IEEE*, vol. 102, no. 1, pp. 56–103, Jan. 2014.

[5] A. Garcia and C. Masouros, "Low-complexity compressive sensing detection for spatial modulation in large-scale multiple access channels," *IEEE Trans. Commun.*, vol. 63, no. 7, pp. 2565–2579, Jul. 2015.

[6] R. Zhang, L. Yang, and L. Hanzo, "Generalised pre-coding aided spatial modulation," *IEEE Trans. Wireless Commun.*, vol. 12, no. 11, pp. 5434–5443, Nov. 2013.

[7] C. Masouros and L. Hanzo, "Dual layered downlink MIMO transmission for increased bandwidth efficiency," *IEEE Trans. Veh. Technol.*, vol. 65, no. 5, pp. 3139–3149, May 2016.

[8] C. Masouros and L. Hanzo, "Constructive interference as an information carrier by dual layered MIMO transmission," *IEEE Trans. Veh. Technol.*, to be published. AQ2

[9] M. Di Renzo and H. Haas, "On transmit diversity for spatial modulation MIMO: Impact of spatial constellation diagram and shaping filters at the transmitter," *IEEE Trans. Veh. Technol.*, vol. 62, no. 6, pp. 2507–2531, Jul. 2013.

[10] P. Yang, Y. Xiao, B. Zhang, S. Li, M. El-Hajjar, and L. Hanzo, "Star-QAM signaling constellations for spatial modulation," *IEEE Trans. Veh. Technol.*, vol. 63, no. 8, pp. 3741–3749, Oct. 2014.

[11] S. Sugiura, C. Xu, S. X. Ng, and L. Hanzo, "Reduced-complexity coherent versus non-coherent QAM-aided space–time shift keying," *IEEE Trans. Commun.*, vol. 59, no. 11, pp. 3090–3101, Nov. 2011.

[12] K. Ntontin, M. Di Renzo, A. Perez-Neira, and C. Verikoukis, "Adaptive generalized space shift keying," *EURASIP J. Wireless Commun. Netw.*, vol. 2013, pp. 1–15, Feb. 2013.

[13] S. Sugiura and L. Hanzo, "On the joint optimization of dispersion matrices and constellations for near-capacity irregular precoded space–time shift keying," *IEEE Trans. Wireless Commun.*, vol. 12, no. 1, pp. 380–387, Jan. 2013.

[14] M. Maleki, H. Bahrami, S. Beygi, M. Kafashan, and N. H. Tran, "Space modulation with CSI: Constellation design and performance evaluation," *IEEE Trans. Veh. Technol.*, vol. 62, no. 4, pp. 1623–1634, May 2013.

[15] X. Guan, Y. Cai, and W. Yang, "On the mutual information and precoding for spatial modulation with finite alphabet," *IEEE Wireless Commun. Lett.*, vol. 2, no. 4, pp. 383–386, Aug. 2013.

[16] J. M. Luna-Rivera, D. U. Campos-Delgado, and M. G. Gonzalez-Perez, "Constellation design for spatial modulation," *Procedia Technol.*, vol. 7, pp. 71–78, 2013.

[17] C. Masouros, "Improving the diversity of spatial modulation in MISO channels by phase alignment," *IEEE Commun. Lett.*, vol. 18, no. 5, pp. 729–732, May 2014.

[18] *Evolved Universal Terrestrial Radio Access (E-UTRA); LTE Physical Layer; General Description*, 3GPP TS 36.201, V11.1.0, Release 11, Mar. 2008.

[19] C. Masouros and L. Hanzo, "Constellation-randomization achieves transmit diversity for single-RF spatial modulation," *IEEE Trans. Veh. Technol.*, to be published. AQ3

[20] A. Garcia, C. Masouros, and L. Hanzo, "Pre-scaling optimization for space shift keying based on semidefinite relaxation," *IEEE Trans. Commun.*, vol. 63, no. 11, pp. 4231–4243, Nov. 2015.

[21] M.-C. Lee, W.-H. Chung, and T.-S. Lee, "Generalized precoder design formulation and iterative algorithm for spatial modulation in MIMO systems with CSIT," *IEEE Trans. Commun.*, vol. 63, no. 4, pp. 1230–1244, Apr. 2015.

AUTHOR QUERIES

AUTHOR PLEASE ANSWER ALL QUERIES

AQ1 = Please provide publication update in Ref. [2].
AQ2 = Please provide publication update in Ref. [8].
AQ3 = Please provide publication update in Ref. [19].

END OF ALL QUERIES

Correspondence

A Scalable Performance–Complexity Tradeoff for Constellation Randomization in Spatial Modulation

Christos Masouros, *Senior Member, IEEE*, and Lajos Hanzo, *Fellow, IEEE*

***Abstract*—It is widely recognized that traditional single radio frequency (RF)-chain-aided spatial modulation (SM) does not offer any transmit diversity gain. As a remedy, constellation randomization (CR), relying on transmit prescaling (TPS), has been shown to provide transmit diversity for single-RF-chain-aided SM. In this paper, we propose a low-complexity approach to SM with the aid of constellation randomization (SM-CR) that considerably improves the transmit diversity gain of SM at a reduced computational burden compared with conventional SM-CR. While conventional SM-CR performs a full search among a set of candidate TPS factors to achieve the maximum minimum Euclidean distance (MED) in the received SM constellation, here, we propose a thresholding approach, where, instead of the maximum MED, the TPS aims to satisfy a specific MED threshold. This technique offers a significant complexity reduction with respect to the full maximization of SM-CR, since the search for TPS is terminated once a TPS set is found that satisfies the MED threshold. Our analysis and results demonstrate that a scalable tradeoff can be achieved between transmit diversity and complexity by appropriately selecting the MED threshold, where a significant complexity reduction is attained, while achieving a beneficial transmit diversity gain for the single-RF SM.**

***Index Terms*—Constellation shaping, multiple-input single-output, spatial modulation (SM), transmit prescaling (TPS).**

I. Introduction

Spatial modulation (SM) has been shown to offer a low-complexity design alternative to spatial multiplexing, where only a subset (down to one) of radio frequency (RF) chains is required for transmission [1], [2]. Early work has focused on the design of receiver algorithms for minimizing the bit error ratio of SM at low complexity [1]–[5]. Matched filtering is shown to be a low-complexity technique for detecting the activated antenna index (AI) [1]–[3]. A maximum likelihood (ML) detector is introduced in [4] for reducing the complexity of classic spatial multiplexing ML detectors, whereas the complexity imposed can be further reduced by compressive sensing detection approaches [5]. In addition to receive processing, several transmit precoding (TPC) approaches have been proposed for receive antenna (RA)-aided SM, where the spatial information is mapped onto the RA index [6]–[8].

Relevant work has also proposed constellation shaping for SM [9]–[14]. Specifically, in [9], the transmit diversity of coded SM is analyzed for different *spatial constellations*, which represent the legitimate sets of activated transmit antennas (TAs). Furthermore, Yang *et al.* in [10] conceived a symbol constellation optimization technique for minimizing the bit error rate (BER). Indeed, spatial and symbol constellation shaping are discussed separately in the aforementioned reference. By contrast, the design of the received SM constellation that combines the choice of the TA as well as the transmit symbol constellation is the focus of this paper. A number of constellation shaping schemes [11]–[14] have also been proposed for the special case of SM, which is referred to as space shift keying, where the information is purely carried in the spatial domain, by the activated AI. However, the application of the above constellation shaping to the SM transmission, where the transmit waveform is modulated, is nontrivial.

Recent work has focused on shaping the receive SM constellation by means of symbol prescaling at the transmitter, aiming for maximizing the minimum Euclidean distance (MED) in the received SM constellation [15]–[17]. The constellation shaping approach in [15] and [16] aims at fitting the receive SM constellation to one of the existing optimal classic constellation formats in terms of minimum distance, such as, e.g., quadrature amplitude modulation (QAM). Due to the strict constellation fitting requirement imposed on both amplitude and phase, this prescaling relies on the inversion of the channel coefficients. In the case of ill-conditioned channels, this substantially reduces the received signal-to-noise ratio (SNR). This problem has been alleviated in [17], where a constellation shaping scheme based on phase-only scaling is proposed. Still, the constellation shaping used in the above schemes is limited in the sense that it only applies to multiple-input–single-output systems, where a single symbol is received for each transmission, and thus, the characterization and shaping of the receive SM constellation is simple.

Closely related to this work, a transmit prescaling (TPS) scheme was proposed for SM [19], where the received SM constellation is randomized by TPS for maximizing the MED between its points for a given channel. A number of randomly generated candidate sets of TPS factors are formed offline, known to both the transmitter and the receiver, and the transmitter then selects that particular set of TPS factors that yields the SM constellation having the maximum MED. Against this background, in this paper, we propose a low-complexity relaxation of the above optimization instead of an exhaustive search, where the first TPS factor set that is found to satisfy a predetermined threshold is selected, thus reducing the computational burden of the TPS operation. The proposed scheme is shown to provide a scalable tradeoff between the performance attained and the complexity imposed, by accordingly selecting the MED threshold.

This paper is organized as follows: In Section II, the basic system model is first introduced, and the proposed scheme is then discussed. The computational complexity of the proposed technique is analyzed in Section III, and its performance against the state of the art is evaluated in Section IV. Finally, in Section V, we draw the key conclusions of our study.

II. Spatial Modulation With Threshold Constellation Randomization (SM-TCR)

Consider a multiple-input multiple-output (MIMO) system, where the transmitter and the receiver are equipped with N_t and N_r antennas,

Manuscript received November 24, 2015; revised March 14, 2016; accepted May 17, 2016. This work was supported in part by the Royal Academy of Engineering, U.K. and in part by the Engineering and Physical Sciences Research Council through Project EP/M014150/1. The review of this paper was coordinated by Dr. Y. Xin.

C. Masouros is with the Department of Electrical and Electronic Engineering, University College London, London WC1E 6BT, U.K. (e-mail: chris.masouros@ieee.org).

L. Hanzo is with the School of Electronics and Computer Science, University of Southampton, Southampton SO16 7NS, U.K. (e-mail: lh@ecs.soton.ac.uk).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TVT.2016.2572760

respectively. For simplicity, unless stated otherwise, in this paper, we assume that the transmit power budget is limited to unity, i.e., $P = 1$. We focus on the single-RF-chain-aided SM approach, where the transmit vector is in the all-but-one zero form $\mathbf{s}_m^k = [0, \ldots, s_m, \ldots, 0]^T$, with $[.]^T$ denoting the transpose operator. Here, $s_m, m \in \{1, \ldots, M\}$ is a symbol taken from an M-order modulation alphabet that represents the transmitted waveform in the baseband domain conveying $\log_2(M)$ bits, whereas k represents the index of the activated TA (the index of the nonzero element in $\mathbf{s}_m^k$) conveying $\log_2(N_t)$ bits in the spatial domain. Clearly, since $\mathbf{s}$ is an all-zero vector apart from s_m^k, there is no interantenna interference.

For the per-antenna TPS approach, which is the focus of this paper, the signal fed to each TA is scaled by a complex-valued coefficient $\alpha_k, k \in \{1, \ldots, N_t\}$ for which we have $E\{|\alpha_k|\} = 1$, where $|x|$ denotes the amplitude of a complex number x, and $E\{.\}$ denotes the expectation operator. Defining the MIMO channel vector as $\mathbf{H}$ with elements $h_{m,n}$ representing the complex-valued channel coefficient between the nth TA and the mth RA, the received symbol vector can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{A}\mathbf{s}_m^k + \mathbf{w} \tag{1}$$

where $\mathbf{w} \sim \mathcal{CN}(0, \sigma^2\mathbf{I})$ is the additive white Gaussian noise component at the receiver, with $\mathcal{CN}(\mu, \sigma^2)$ denoting the circularly symmetric complex Gaussian distribution with a mean of μ and a variance of σ^2. Furthermore, $\mathbf{A} = \mathrm{diag}(\mathbf{a})$ is the TPS matrix with $\mathbf{a} = [\alpha_1, \alpha_2, \ldots, \alpha_{N_t}]$, and $\mathrm{diag}(\mathbf{g})$ represents the diagonal matrix with its diagonal elements taken from vector $\mathbf{g}$. Note that the diagonal structure of $\mathbf{A}$ guarantees having a transmit vector $\mathbf{x} = \mathbf{A}\mathbf{s}$ with a single nonzero element, so that the single-RF-chain aspect of SM is preserved.

At the receiver, a joint ML detection of both the TA index and the transmit symbol is obtained by the minimization, i.e.,

$$\begin{aligned}[\hat{s}_m, \hat{k}] &= \arg\min_i \|\mathbf{y} - \dot{\mathbf{y}}_i\| \\ &= \arg\min_{m,k} \|\mathbf{y} - \mathbf{H}\mathbf{A}\mathbf{s}_m^k\| \end{aligned} \tag{2}$$

where $\|\mathbf{x}\|$ denotes the norm of vector $\mathbf{x}$, and $\dot{\mathbf{y}}_i$ is the ith constellation point in the received SM constellation. By exploiting the specific structure of the transmit vector, this can be further simplified to

$$[\hat{s}_m, \hat{k}] = \arg\min_{m,k} \|\mathbf{y} - \mathbf{h}_k \alpha_m^k s_m\| \tag{3}$$

where $\mathbf{h}_k$ denotes the kth column of matrix $\mathbf{H}$. It is widely recognized that the performance of the detection as formulated above is dominated by the MED between the adjacent constellation points $\dot{\mathbf{y}}_i$ and $\dot{\mathbf{y}}_j$ in the receive SM constellation, i.e.,

$$d_{\min} = \min_{i,j} \|\dot{\mathbf{y}}_i - \dot{\mathbf{y}}_j\|^2, \ i \neq j. \tag{4}$$

Accordingly, to improve the likelihood of correct detection, constellation shaping TPS schemes conceived for SM aim at maximizing this MED. The optimum TPS matrix $\mathbf{A}^*$ can be found by solving the optimization problem of [20]

$$\begin{aligned} \mathbf{A}^* &= \arg\max_{\mathbf{A}} \min_{i,j} \|\dot{\mathbf{y}}_i - \dot{\mathbf{y}}_j\|^2, \ i \neq j \\ \text{s.t.c.} \quad & \mathrm{trace}(\mathbf{A}^{*H}\mathbf{A}^*) \leq P \end{aligned} \tag{5}$$

and, additionally for single-RF-chain-aided SM, subject to $\mathbf{A}^*$ having a diagonal structure. In the above, $\mathbf{A}^H$ and $\mathrm{trace}(\mathbf{A})$ represent the Hermitian transpose and trace of matrix $\mathbf{A}$, respectively. The above optimization, however, is an NP-hard problem, which makes finding the TPS factors prohibitively complex and motivates the conception of lower-complexity suboptimal techniques. Indeed, it has been shown that the TPS approach in [19], by selecting among a set of predetermined randomly generated TPS vectors instead of fully optimizing the TPS, offers a near-optimal performance with the lowest complexity among the TPS optimization approaches [20], [21].

TPS Vector Generation: Accordingly, with SM-TCR first, a number of D random candidate TPS vectors are generated, in the form of $\mathbf{a}_d$, where $d \in [1, D]$ denotes the index of the candidate set, and $\mathbf{a}_d$ is formed by the elements $\alpha_m^{k(d)} \sim \mathcal{CN}(0, 1)$. To ensure that the average transmit power remains unchanged, the scaling factors are normalized to unit power. These are made available to both the transmitter and the receiver before transmission. These assist in randomizing the received constellation, which is most useful in the critical scenarios where two points in the constellation of $\mathbf{H}\mathbf{s}_m^k, m \in [1, M], k \in [1, N_t]$ happen to be very close.

A. Thresholded Selection of TPS

For a given channel, based on knowledge of vectors $\mathbf{a}_d$, both the transmitter and the receiver can determine the received SM constellation for the dth TPS set by calculating the legitimate set of $[m, k]$ combinations in

$$\hat{\mathbf{y}} = \mathbf{H}\mathbf{A}_d\mathbf{s}_m^k \tag{6}$$

where $\mathbf{A}_d = \mathrm{diag}(\mathbf{a}_d)$ is the diagonal matrix that corresponds to the candidate set $\mathbf{a}_d$. Then, for the given channel coefficients, the transmitter and the receiver can choose independently the scaling vector $\mathbf{a}_o$. Alternatively, if no channel state information is available at the transmitter (receiver), the receiver (transmitter) can inform the transmitter (receiver) concerning the optimum $\mathbf{a}_o$ by transmitting a number of $\lceil \log_2(D) \rceil$ bits. Contrary to the SM-CR in [19], where the maximum MED among all D possibilities is chosen, here, a threshold-based approach is introduced, where the search for TPS is terminated when a candidate TPS is found that satisfies a MED threshold. This optimization problem can be expressed as

$$\mathbf{A}_o = \begin{cases} \mathbf{A}_t, \text{if} \exists \mathbf{A}_t : & \begin{aligned} &\min_{\substack{\{m_i,k_j\} \neq \\ \{m_l,k_p\}}} \|\mathbf{H}\mathbf{A}_t\mathbf{s}_{m_1}^{k_1} - \mathbf{H}\mathbf{A}_t\mathbf{s}_{m_2}^{k_2}\|^2 \geq \\ &\theta \min_{\substack{\{m_i,k_j\} \neq \\ \{m_l,k_p\}}} \|\mathbf{H}\mathbf{s}_{m_1}^{k_1} - \mathbf{H}\mathbf{s}_{m_2}^{k_2}\|^2 \end{aligned} \\ \arg\max_d \min\limits_{\substack{\{m_i,k_j\} \neq \\ \{m_l,k_p\}}} \|\mathbf{H}\mathbf{A}_d\mathbf{s}_{m_1}^{k_1} - \mathbf{H}\mathbf{A}_d\mathbf{s}_{m_2}^{k_2}\|^2, & \text{otherwise} \end{cases} \tag{7}$$

where θ represents the MED threshold with respect to the MED without TPS. Equivalently, for the case of single-RF-chain-based SM, this can be simplified to

$$\mathbf{A}_o = \begin{cases} \mathbf{A}_t, \text{if} \exists \mathbf{A}_t : & \begin{aligned} &\min_{\substack{\{m_i,k_j\} \neq \\ \{m_l,k_p\}}} \|\mathbf{h}_{k_1} a_{m_1}^{k_1} s_{m_1} - \mathbf{h}_{k_2} a_{m_2}^{k_2} s_{m_2}\|^2 \geq \\ &\theta \min_{\substack{\{m_i,k_j\} \neq \\ \{m_l,k_p\}}} \|\mathbf{h}_{k_1} s_{m_1} - \mathbf{h}_{k_2} s_{m_2}\|^2 \end{aligned} \\ \arg\max_d \min\limits_{\substack{\{m_i,k_j\} \neq \\ \{m_l,k_p\}}} \|\mathbf{h}_{k_1} a_{m_1}^{k_1} s_{m_1} - \mathbf{h}_{k_2} a_{m_2}^{k_2} s_{m_2}\|^2, & \text{otherwise} \end{cases} \tag{8}$$

In other words, the search stops if a TPS set is found that satisfies the threshold; otherwise, the TPS that offers the maximum MED is returned, following a full search as in SM-CR. For completeness, we present the associated algorithm in Table I. It will be shown that this process offers significant computational benefits with respect to full SM-CR.

TABLE I
ALGORITHM SM-TCR

Input : $\mathbf{H}, M$, **Output** : $\mathbf{A}_o$

$\mathbf{e} = [\,]$
for $t = 1$ to D do
  $\mathbf{e}[t] := \min_{\{m_i,k_j\}\neq\{m_l,k_p\}} ||\mathbf{HA}_t\mathbf{s}_{m_1}^{k_1} - \mathbf{HA}_t\mathbf{s}_{m_2}^{k_2}||^2$
    % calculate the MED for the t-th TSP vector
  if $\mathbf{e}[t] \geq \theta \min_{\{m_i,k_j\}\neq\{m_l,k_p\}} ||\mathbf{Hs}_{m_1}^{k_1} - \mathbf{Hs}_{m_2}^{k_2}||^2$
  % check if the MED satisfies the MED threshold θ
    $\mathbf{A}_o := \mathbf{A}_t$
    % if so, select $\mathbf{A}_t$ and terminate the algorithm
    break
  end
end
$t_o = \arg\max_t \mathbf{e}$
  % if not, select the TPS vector with the max MED
$\mathbf{A}_o := \mathbf{A}_{t_o}$

TABLE II
COMPLEXITY FOR THE PROPOSED SM-TCR SCHEME

SM-TCR		Operations
Constellation Optimization		
$\mathbf{HA}_d\mathbf{s}_m^k, \forall m,k$	$\times t$	$(2N_r+1)N_tMt$
$\mathbf{f}_{m_1,m_2}^{k_1,k_2(d)} = ||\mathbf{HA}_d\mathbf{s}_{m_1}^{k_1} - \mathbf{HA}_d\mathbf{s}_{m_2}^{k_2}||$, $\forall m_1,m_2,k_1,k_2, m_1 \neq m_2, k_1 \neq k_2$	$\times t$	$2N_r\binom{N_tM}{2}t$
check: $\min\{\mathbf{f}_{m_1,m_2}^{k_1,k_2(d)}\} \geq \theta \min_{\{m_i,k_j\}\neq\{m_l,k_p\}} ||\mathbf{Hs}_{m_1}^{k_1} - \mathbf{Hs}_{m_2}^{k_2}||^2$	$\times t$	$\binom{N_tM}{2}t$
ML Detection		
$g_m^k = ||\mathbf{y} - \mathbf{HA}_o\mathbf{s}_m^k||^2, \forall m,k$	$\times B$	$2N_tMN_rB$
$\arg\min g_m^k$	$\times B$	N_tMB
Total: $(2Nr+1)\left[\binom{N_tM}{2} + N_tM\right]t + (2N_r+1)N_tMB$		

Based on (8), the transmitter sends $\mathbf{x} = \mathbf{A}_o\mathbf{s}_m^k$, and the receiver applies the ML detector according to

$$[\hat{s}_m, \hat{k}] = \arg\min_{m,k} \|\mathbf{y} - \mathbf{HA}_o\mathbf{s}_m^k\|. \quad (9)$$

It should be noted that, to dispense with the need for channel state information at the transmitter (CSIT), the receiver can select the best scaling factors using (8) and then feed the index of the scaling matrix $\mathbf{A}_o$ selected from the set of D candidates back to the transmitter, using $\lfloor \log_2(D) \rfloor$ bits. This constitutes major overhead savings for the proposed scheme with respect to the existing TPS schemes for SM that require full CSIT, while obtaining similar performance.

B. Transmit Diversity and Performance Trends

While the transmit diversity order of the single-RF SM is known to be one [9], the proposed TPS introduces an amplitude-phase diversity in the transmission, which is an explicit benefit of having D candidate sets of TPS factors to choose from. Accordingly, it was shown in [19] that the obtained transmit diversity order corresponds to the θ-dependent gain in the average MED associated with CR as

$$G(\theta) \hat{=} \frac{E\{\min_{m,k} \|\mathbf{HA}_o\mathbf{s}_{m_1}^{k_1} - \mathbf{HA}_o\mathbf{s}_{m_2}^{k_2}\|^2\}}{E\{\min_{m,k} \|\mathbf{Hs}_{m_1}^{k_1} - \mathbf{Hs}_{m_2}^{k_2}\|^2\}}. \quad (10)$$

In addition, SM systems with N_r uncorrelated RAs have been shown to experience a unity transmit diversity order and a receive diversity order of N_r. Accordingly, since the proposed scheme attains a θ-dependent transmit diversity order of $G(\theta)$, the total diversity order becomes $\delta = N_rG(\theta)$. The resulting probability of error P_e obeys the high-SNR trend of

$$P_e = \alpha\gamma^{-N_rG(\theta)} \quad (11)$$

where γ is the transmit SNR, and α is an arbitrary nonnegative coefficient. We verify the above theoretical performance trend against simulation in the following.

III. Computational Complexity

It is clear from the above discussion that the proposed SM-TCR leads to a computational complexity reduction with respect to conventional SM-CR, due to the early termination of the TPS search, after a calculation of $t \leq D$ out of D TPS sets. Here, we analyze this computational complexity reduction at the receiver. This analysis is complemented by the following results on the distribution of t. For reference, we have assumed a Long-Term Evolution Type-2 time-division duplex frame structure [18]. This has 10-ms duration that consists of ten subframes out of which five subframes, each containing 14 symbol time slots, are used for downlink transmission, yielding a frame size of $F = 70$ for the downlink, whereas the rest are used for both uplink and control information transmission. A slow-fading channel is assumed, where the channel remains constant for the duration of the frame.

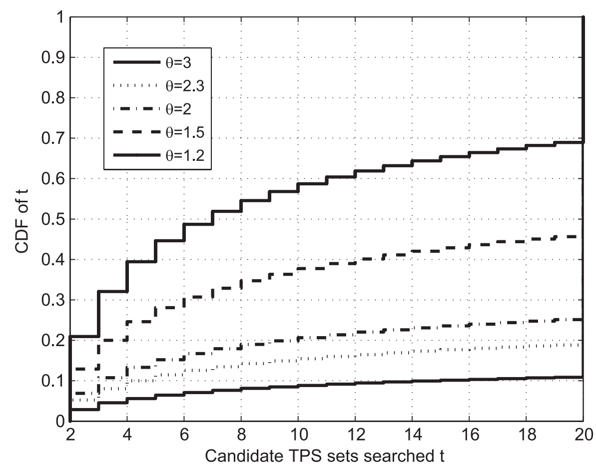

Fig. 1. Cumulative distribution function of the number of candidate TPS searched (t) for various values of θ, D = 20, 4-QAM.

Following the complexity analysis in [19], we quantify the number of operations required in each step of the SM-TCR search in Table II. From the table, we have a total SM-TCR receiver complexity of

$$C(t) = (2Nr+1)\left[\binom{N_tM}{2} + N_tM\right]t + (2Nr+1)N_tMB. \quad (12)$$

To complete this complexity discussion, in Fig. 1, we show the distribution of t as a function of the increasing threshold values of θ. It can be seen that low numbers of candidate TPS searched t are obtained with high probability, particularly in the cases of low MED thresholds θ. While large complexity savings can be observed in the figure, it is important to note that the complexity of SM-TCR is upper bounded by that of SM-CR, since $t \leq D$.

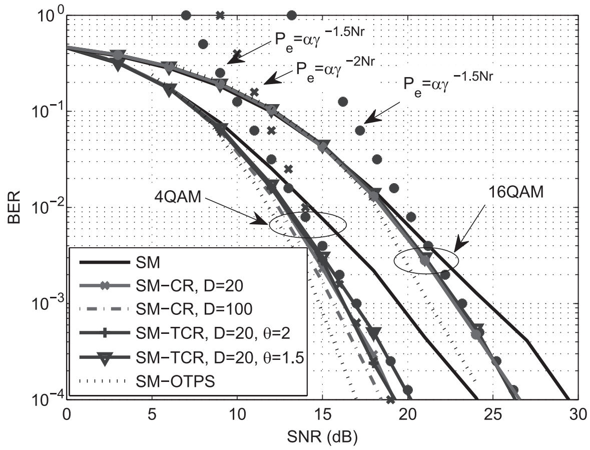

Fig. 2. BER versus SNR for a (4 × 2)-element MIMO with SM, SM-OTPS, SM-CR and SM-TCR, 4-QAM, and 16-QAM.

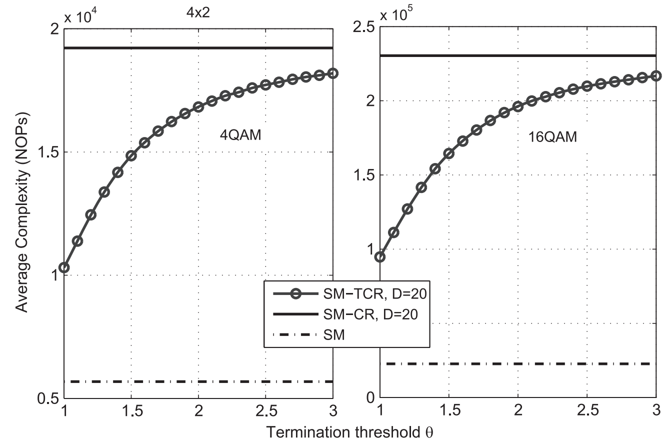

Fig. 3. Average complexity versus θ for a (4 × 2)-element MIMO with SM-TCR, 4-QAM, and 16-QAM.

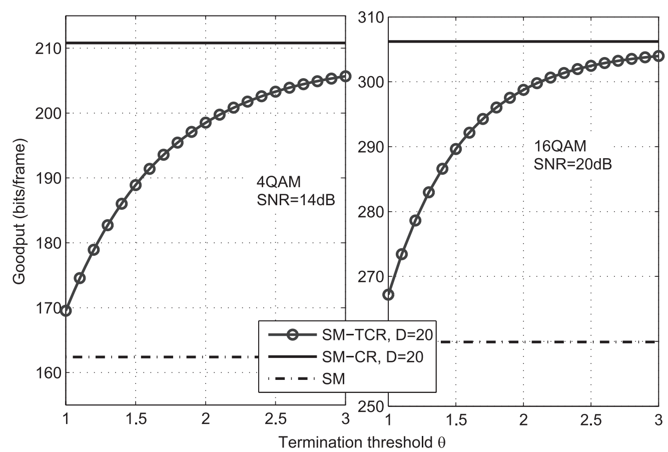

Fig. 4. Goodput versus θ for a (4 × 2)-element MIMO with SM-TCR, 4-QAM, and 16-QAM.

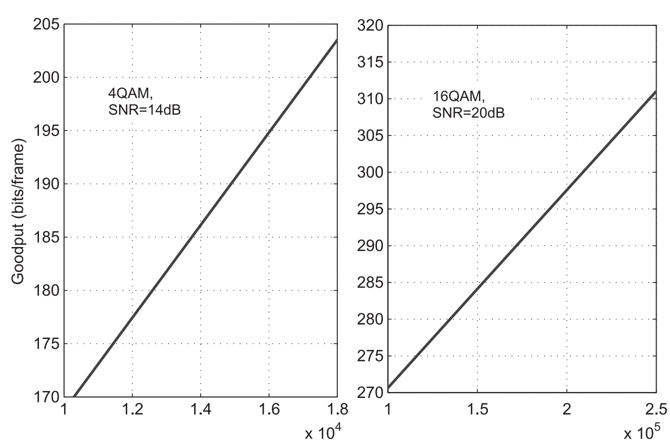

Fig. 5. Goodput versus average complexity for a (4 × 2)-element MIMO with SM-TCR, 4-QAM, and 16-QAM.

IV. Simulation Results

To evaluate the benefits of the proposed technique, this section presents numerical results based on Monte Carlo simulations of conventional SM without scaling (termed as SM in the figures), SM-CR, and the proposed SM-TCR. The channel's impulse response is assumed to be perfectly known at the transmitter. Without loss of generality, we assume that the transmit power is restricted to $P = 1$. MIMO systems with four TAs employing 4-QAM and 16-QAM modulation are explored, albeit it is plausible that the benefits of the proposed technique extend to larger-scale systems and higher-order modulation.

First, we characterize the attainable BER performance with an increasing transmit SNR for a (4 × 2)-element MIMO employing 4-QAM and 16-QAM, for various values of the MED threshold θ in Fig. 2. The performance of the highly complex TPS design in [21] based on convex optimization, and termed SM-OTPS in the figure, is also shown here for reference, where it can be seen that the proposed SM-TCR, with orders-of-magnitude less complexity than SM-OTPS, still performs within 1–2 dB from the optimization-based SM-OTPS. The theoretical trends of (11) are also shown, where it can be seen that they provide a close match for the high-SNR system behavior. It can be seen that the slope of the BER curves increases with increasing θ, which indicates an increase in transmit diversity order. Indeed, the BER of SM-TCR is identical to that of SM-CR for $\theta = 2$ in the case of 4-QAM and $\theta = 1.5$ in the case of 16-QAM. In both cases, significant complexity savings are obtained, as shown in the results that follow.

Fig. 3 shows the average computational complexity expressed in terms of numbers of operations (NOPs) for SM-TCR with increasing MED threshold values θ. The complexity of SM and SM-CR is also depicted for reference. It can be seen that as the MED threshold increases, the optimization becomes tighter, leading to complexity close to that of the full SM-CR. For reduced values of θ, however, significant complexity gains are obtained, where the NOPs for SM-TCR are down to less than 55% of those for SM-CR for 4-QAM and 40% for 16-QAM, respectively. A similar trend can be observed in Fig. 4, where the performance is shown for increasing θ, where performance is quantified in terms of goodput in bits per frame, i.e.,

$$T = \log_2(N_t M) \cdot F(1 - P_F) \tag{13}$$

with P_F denoting the frame error probability and $F = 70$ being the frame length used in these results, following [19]. The specific selection of the MED threshold θ in practice can be based on the desired tradeoff between the complexity in Fig. 3 and (12), the transmit diversity obtained, and the performance observed in Fig. 4 and (10) and (11). Finally, Fig. 5 shows the direct performance-versus-complexity tradeoff. A linear relation between goodput and complexity can be observed. More importantly, where previously either a low-complexity unit-diversity SM or a high-complexity high-diversity SM-CR alternative could be chosen, here, a scalable tradeoff is offered between these two extremes with the aid of SM-TCR, by selecting the MED thresholds θ accordingly.

V. CONCLUSION

A new low-complexity constellation shaping approach has been introduced for SM. While conventional CR offers a considerable transmit diversity gain at the cost of increased computational complexity compared with the conventional SM, the proposed scheme delivers a scalable tradeoff between the transmit diversity obtained and the complexity by appropriately selecting the MED threshold values. Complexity reduction of up to 60% over conventional CR was demonstrated, while still considerably improving the attainable performance of conventional SM.

REFERENCES

[1] R. Mesleh, H. Haas, S. Sinanovic, C. W. Ahn, and S. Yun, "Spatial modulation," *IEEE Trans. Veh. Technol.*, vol. 57, no. 4, pp. 2228–2241, Jul. 2008.

[2] P. Yang *et al.*, "Single-carrier spatial modulation: A promising design for large-scale broadband antenna systems," *IEEE Commun. Surveys Tuts.*, to be published. AQ1

[3] M. Di Renzo and H. Haas, "Bit error probability of space modulation over Nakagami-m fading: Asymptotic analysis," *IEEE Commun. Lett.*, vol. 15, no. 10, pp. 1026–1028, Oct. 2011.

[4] M. Di Renzo, H. Haas, A. Ghrayeb, S. Sugiura, and L. Hanzo, "Spatial modulation for generalized MIMO: Challenges, opportunities, and implementation," *Proc. IEEE*, vol. 102, no. 1, pp. 56–103, Jan. 2014.

[5] A. Garcia and C. Masouros, "Low-complexity compressive sensing detection for spatial modulation in large-scale multiple access channels," *IEEE Trans. Commun.*, vol. 63, no. 7, pp. 2565–2579, Jul. 2015.

[6] R. Zhang, L. Yang, and L. Hanzo, "Generalised pre-coding aided spatial modulation," *IEEE Trans. Wireless Commun.*, vol. 12, no. 11, pp. 5434–5443, Nov. 2013.

[7] C. Masouros and L. Hanzo, "Dual layered downlink MIMO transmission for increased bandwidth efficiency," *IEEE Trans. Veh. Technol.*, vol. 65, no. 5, pp. 3139–3149, May 2016.

[8] C. Masouros and L. Hanzo, "Constructive interference as an information carrier by dual layered MIMO transmission," *IEEE Trans. Veh. Technol.*, to be published. AQ2

[9] M. Di Renzo and H. Haas, "On transmit diversity for spatial modulation MIMO: Impact of spatial constellation diagram and shaping filters at the transmitter," *IEEE Trans. Veh. Technol.*, vol. 62, no. 6, pp. 2507–2531, Jul. 2013.

[10] P. Yang, Y. Xiao, B. Zhang, S. Li, M. El-Hajjar, and L. Hanzo, "Star-QAM signaling constellations for spatial modulation," *IEEE Trans. Veh. Technol.*, vol. 63, no. 8, pp. 3741–3749, Oct. 2014.

[11] S. Sugiura, C. Xu, S. X. Ng, and L. Hanzo, "Reduced-complexity coherent versus non-coherent QAM-aided space–time shift keying," *IEEE Trans. Commun.*, vol. 59, no. 11, pp. 3090–3101, Nov. 2011.

[12] K. Ntontin, M. Di Renzo, A. Perez-Neira, and C. Verikoukis, "Adaptive generalized space shift keying," *EURASIP J. Wireless Commun. Netw.*, vol. 2013, pp. 1–15, Feb. 2013.

[13] S. Sugiura and L. Hanzo, "On the joint optimization of dispersion matrices and constellations for near-capacity irregular precoded space–time shift keying," *IEEE Trans. Wireless Commun.*, vol. 12, no. 1, pp. 380–387, Jan. 2013.

[14] M. Maleki, H. Bahrami, S. Beygi, M. Kafashan, and N. H. Tran, "Space modulation with CSI: Constellation design and performance evaluation," *IEEE Trans. Veh. Technol.*, vol. 62, no. 4, pp. 1623–1634, May 2013.

[15] X. Guan, Y. Cai, and W. Yang, "On the mutual information and precoding for spatial modulation with finite alphabet," *IEEE Wireless Commun. Lett.*, vol. 2, no. 4, pp. 383–386, Aug. 2013.

[16] J. M. Luna-Rivera, D. U. Campos-Delgado, and M. G. Gonzalez-Perez, "Constellation design for spatial modulation," *Procedia Technol.*, vol. 7, pp. 71–78, 2013.

[17] C. Masouros, "Improving the diversity of spatial modulation in MISO channels by phase alignment," *IEEE Commun. Lett.*, vol. 18, no. 5, pp. 729–732, May 2014.

[18] *Evolved Universal Terrestrial Radio Access (E-UTRA); LTE Physical Layer; General Description*, 3GPP TS 36.201, V11.1.0, Release 11, Mar. 2008.

[19] C. Masouros and L. Hanzo, "Constellation-randomization achieves transmit diversity for single-RF spatial modulation," *IEEE Trans. Veh. Technol.*, to be published. AQ3

[20] A. Garcia, C. Masouros, and L. Hanzo, "Pre-scaling optimization for space shift keying based on semidefinite relaxation," *IEEE Trans. Commun.*, vol. 63, no. 11, pp. 4231–4243, Nov. 2015.

[21] M.-C. Lee, W.-H. Chung, and T.-S. Lee, "Generalized precoder design formulation and iterative algorithm for spatial modulation in MIMO systems with CSIT," *IEEE Trans. Commun.*, vol. 63, no. 4, pp. 1230–1244, Apr. 2015.

AUTHOR QUERIES

AUTHOR PLEASE ANSWER ALL QUERIES

AQ1 = Please provide publication update in Ref. [2].
AQ2 = Please provide publication update in Ref. [8].
AQ3 = Please provide publication update in Ref. [19].

END OF ALL QUERIES