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Political sustainability of pension system in endogenous growth model

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Abstract

Cooley and Soares (1999) show that a pay-as-you-go pension system can be an outcome of a political equilibrium and rather relatively large size of it can be introduced. This note assumes endogenous growth model rather than the exogenous one, which establishes a link between savings rate and economic growth. Since the introduction of the pay-as-you-go system lowers the savings rate, it has an adverse effect on growth rate and hence on future pensions. When this additional incentive is taken into account, then the level of pay-as-you-go system chosen in political equilibrium is much lower than in exogenous growth model of Cooley and Soares. Still their qualitative conclusion holds, at least some level of pay-as-you-go system will be introduced in equilibrium.

Keywords: Pension system, endogenous growth, political equilibrium, reform.

JEL Classification: H55, O4, D72

1 Motivation

This paper investigates political sustainability of pension systems. It is shown that if voters understand that pay-as-you-go pension system discourages savings then the motive for introduction of large pay-as-you-go system is significantly diminished.

Old-age pension systems experience many serious problems in most of the developed countries in the world. Among many things on the reform agenda is the choice between pay-as-you-go and a funded system. Macroeconomists compared these two schemes and the classical answer is that when the rate of return on capital is greater than the rate of growth of population, then the funded system is superior.[1]

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[1] For standard exposition see Blanchard and Fisher (1989)

This does not take into account political sustainablity of a pension system. Why do we observe pay-as-you-go systems in so many countries? Is there any political 'tension' that leads societies to choose this system, despite its inferior economic performance? (For a recent survey of related litarature see Mulligan and Sala-i-Martin (2004)).

To explain why pay-as-you-go system is so dominant, Cooley and Soares (1999) construct an overlapping generation model where agents take economic decisions as well as vote for some level of pay-as-you-go system. The resulting political equilibrium provides an explanation why this system is introduced and sustained. It turns out that if the pension system is decided in voting in which only current generations participate, the resulting game is akin to Prisioners' Dilemma. Fully funded pension system is Pareto efficient, but each generation has incentive to vote in favour of a pay-as-you-go system.

This note examines how voting decisions may affect economic growth. It is well known that in pay-as-you-go system incentives for savings are lower. Lower savings would have an adverse effect on economic growth, *as long as endogenous growth model is assumed*. Voters have to bear in mind that increasing pay-as-you-go debt will decrease the economic growth in the future. Recognizing this link, they will vote for smaller pay-as-you-go system in the first place. This link is absent in models which assume constant exogenous rate of growth.

There is a vast literature discussing dynamic and inter-generational effects of various tax systems. But whenever computational models are constructed to quantitatively assess these effects, essentially only exogenous growth models are used as a tool of analysis (for instance see Bassetto (1999) and the literature mentioned there).

Next section presents the model of Cooley and Soares with exogenous growth. It is followed by an alternative model with endogenous growth. Finally, the implications for the size of the pension system are discussed in the last section.

2 Exogenous growth model

Cooley and Soares consider four-generation overlapping generation model with exogenous technological progress. The population grows at the constant rate n per period. Therefore, if μ_i is a share of population of age i, we have $\mu_{i+1} = (1+n)\,\mu_i$, and $\mu_1 + \mu_2 + \mu_3 + \mu_4 = 1$.

Lifetime utility of an agent born in period κ is given by

$$U_\kappa = \sum_{i=1}^{4} \beta^{i-1} \frac{\left(c_{i,\kappa+i-1}^{\sigma} l_{i,\kappa+i-1}^{1-\sigma}\right)^{1-\rho}}{1-\rho},$$

where $c_{i,t}$ is consumption of agent of age i in period t, and $l_{i,t}$ stands for leisure. Parameter ρ is the coefficient of risk aversion and σ is coefficient of Cobb-Douglas function on consumption.

Agents have their income $y_{i,t}$ from labor or from pension system. They can spend it on consumption or asset accumulation $a_{i,t}$. Therefore the budget constraint can be written as

$$a_{i+1,t+1} = (1 + r_t)\, a_{i,t} + y_{i,t} - c_{i,t} \tag{1}$$

where r_t is rate of return of these assets. By assumption agents are born with no assets, and clearly this preference structure imply that they leave no bequests, $a_{1,t} = a_{5,t} = 0$. It is assumed that agents can work for three initial periods of their lives, but they have to retire at age of $i = 4$. While young, agents supply $h_{i,t} = 1 - l_{i,t}$ hours of labor and in this way they earn $w_t h_{i,t} \epsilon_i$, where w_t is real wage and ϵ_i is age-specific exogenous efficiency index. After retirement agents do not work any more, instead they may get pension benefit b_t, if there is established pay-as-you-go pension system. The crucial parameter describing the magnitude of this system is the replacement ratio θ_t, defined as a ratio of pension benefit to average wage in the economy in this period, $b_t = \theta_t w_t \sum_{i=1}^{3} \mu_i h_{i,t} \epsilon_i$. These assumptions imply that agent's income is

$$y_{i,t} = \begin{cases} (1 - \tau_t)\, w_t h_{i,t} \epsilon_i & \text{for } i = 1, 2, 3 \\ b_t & \text{for } i = 4 \end{cases} \tag{2}$$

where τ_t is a pension tax on labor income.

The production technology is described by constant-returns-to-scale production function,

$$Y_t = \Psi K_t^{1-\alpha} \left[L_t \left(1 + \gamma\right)^t \right]^\alpha$$

where $\Psi > 0$, $\alpha \in (0, 1)$ and γ is the exogenous rate of labor-augmenting technological progress. This assumption is important, because in the next section we will analyze the same economy but with endogenous technological progress generated by diffferent production function. Variables K_t and L_t are total capital and efficiency labor input, respectively. It is assumed that capital depreciates with constant rate of δ.

Economic decisions are modeled in a standard way. Households want to find streams of consumption, labor supply and asset accumulation to maximize their discounted utility in current period, given current and future prices as well as replacement ratios θ_t. Firms, also taking prices as given, choose labor and capital demand in order to maximize their profit.

2.1 Political decisions

Cooley and Soares show that it is possible to introduce and sustain pay-as-you-go system in nonaltruistic world. The driving force is assumed trigger expectation formation,[2] in which everybody expects that only one default on pay-as-you-go debt destroys pension system's reputation forever.

The government here is very simple, it only obeys voters who choose pay-as-you-go replacement ratio θ_t. Then it fixes pension tax rate τ to balance the budget. We restrict our attention to constant sequences of replacement ratios.

[2] See Cooley and Soares (1999) for details. We provide here just an overview.

Let θ^R be the replacement ratio prescribed by law. Since each agent is infinitesimally small, there is no strategic considerations in taking a political decision, we simply assume sincere voting. Votes are aggregated according to a majority rule.

The political outcome comes in two steps:

1. *Why is pay-as-you-go system sustainable once introduced?* Assume that pension system already exists, $\theta^R > 0$, and we want to check whether it is politically sustainable. Suppose the rule is θ^R and voting outcome is θ, then we let $V_i\left(a, A, \theta, \theta^R\right)$ be i's value function once economic variables were optimized out (which depends on current asset holdings of an individual a and distribution of assets among generations A). Agents vote sincerely to $\max_\theta V_i\left(a, A, \theta, \theta^R\right)$. Assumed trigger expectation formation implies that agents essentially vote either in favor of existing θ^R or against it. Let $\Theta_{ag}\left(A, \theta^R\right) \in \left\{0, \theta^R\right\}$ be an aggregate outcome of the majority voting. We will call θ^R a *sustainable* rule if it is a fixed point of this aggregate outcome operator $\theta^R = \Theta_{ag}\left(A, \theta^R\right)$ in each period along the equilibrium path. Let the set of all sustainable θ^R be denoted as $\Omega(A)$.

   Why a positive level of pay-as-you-go may be sustainable? This is because the median voter, who decides whether to destroy or to sustain the system, is in her middle age. Such a person treats her contributions as sunk and the perspective of retirement is relatively close to her. The benefit from lower pension taxes today has to be compared to a vanishing benefit of having a pension while old. Comparison of these two effects results in voting in favor of θ^R.

2. *Why is pay-as-you-go system implementable?* Once we identified the set of sustainable equilibria for current state of the economy we can consider a case when initially there is no pay-as-you-go system. Let $\bar{V}_i\left(a, A, \theta^R\right) \equiv V_i\left(a, A, \theta^R, \theta^R\right)$ be i's utility resulting from introduction of a brand new pay-as-you-go system θ^R. The voting problem then asks to solve $\max_{\theta^R \in \Omega(A)} \bar{V}_i\left(a, A, \theta^R\right)$. If preferences are single-peaked then the median voter theorem can be used. The resulting political equilibrium is therefore described by maximizing utility of median voter.

2.2 Equilibrium

Definition 1 *An* equilibrium *is a sequence of quantities*

$$\left(K_t, L_t, h_{1,t}, h_{2,t}, h_{3,t}, a_{2,t+1}, a_{3,t+1}, a_{4,t+1}, c_{1,t}, c_{2,t}, c_{3,t}, c_{4,t}\right)_{t=1}^{\infty},$$

prices and taxes $(w_t, r_t, \tau_t)_{t=1}^{\infty}$, *replacement rate* θ^R, *such that, given initial conditions* $a_{1,1}, a_{2,1}, a_{3,1}$, *the following is satisfied:*

1. Households, given prices, expectational mechanism and rule θ^R maximize their utility over consumption, asset accumulation and labor supply such that equations (1) and (2) are satisfied,

2. Firms solve their maximization problem

$$\max_{K_t, L_t} \Psi K_t^{1-\alpha} \left[L_t (1+\gamma)^t \right]^{\alpha} - w_t L_t - (r_t + \delta) K_t,$$

3. Markets clear. In particular $K_t = \sum_{i=2}^{4} \mu_i a_{i,t}$ and $L_t = \sum_{i=1}^{3} \mu_i h_{i,t} \epsilon_i$.

4. If $\theta^R > 0$, households solve their voting problem $\max_{\theta \in \{0, \theta^R\}} V_i \left(a, A, \theta, \theta^R \right)$. If initially $\theta^R = 0$ then their problem is $\max_{\theta^R \in \Omega(A)} \bar{V}_i \left(a, A, \theta^R \right)$

5. Replacement ratio is the political outcome. If $\theta^R > 0$ we have $\theta^R = \Theta_{ag} \left(A, \theta^R \right)$. If initially $\theta^R = 0$ then problem is $\max_{\theta^R \in \Omega(A)} \bar{V}_\kappa \left(a, A, \theta^R \right)$

6. The government chooses tax to finance the pension system $\tau_t = \theta_t \mu_4$.

2.3 Solution

Before we start solving this model it will be necessary to transform the above variables into different ones, to assure stationarity of the solution. The reason is that the economy just described in steady state grows whenever γ is positive. Next, we will find the solution path for this rescaled economy. We assume that after the reform, a new replacement ratio $\theta_t = \theta$ will be constant and therefore $\tau = \theta \mu_4$. First, we have to characterize equilibrium path of the economy as a function of θ. Once we do that, we can investigate θ's impact on the utility of any individual, which is the relevant criterion to determine agents' political decision.

In order to solve this model we need to rescale some variables to assure stationarity of the equilibrium path. Let the tilde describe a variable divided by $(1+\gamma)^{t-1}$. Let also $\hat{K}_t = \frac{K_t}{L_t (1+\gamma)^{t-1}}$ and $\tilde{\beta} = \beta (1+\gamma)^{\sigma(1-\rho)}$.

Now households born in $\kappa = -2, -1, 0, 1$, and therefore living in $t = 1$, want to maximize their remaining lifetime utility

$$U_\kappa = \sum_{i=2-\kappa}^{4} \tilde{\beta}^{\kappa+i-2} \frac{\left[\tilde{c}_{i,\kappa+i-1}^{\sigma} \left(1 - h_{i,\kappa+i-1}\right)^{1-\sigma} \right]^{1-\rho}}{1-\rho}$$

subject to rescaled constraints (1) and (2). Similarly firms' profit has to be expressed in terms of rescalled variables,

$$L_t (1+\gamma)^{t-1} \left[\Psi (1+\gamma)^{\alpha} \hat{K}_t^{1-\alpha} - \tilde{w}_t - (r_t + \delta) \hat{K}_t \right]$$

Before the reform, agents accumulated $\tilde{a}_{2,1}, \tilde{a}_{3,1}, \tilde{a}_{4,1}$. Fix a new size of the pay-as-you-go system θ. The initial condition and new θ determine the path of

13 economic endogenous variables through the following 13 equations for each period $t = 1, 2, ...$

(1) Firm's first order condition

$$(1-\alpha)\,\Psi(1+\gamma)^{\alpha}\hat{K}_t^{-\alpha} - (r_t+\delta) = 0$$

(2) Firm's condition for zero profit

$$\Psi(1+\gamma)^{\alpha}\hat{K}_t^{1-\alpha} - \tilde{w}_t - (r_t+\delta)\,\hat{K}_t = 0$$

(3) Market clearing for $\hat{K}_t$,

(4)-(6) Household's first order condition for labor supply (intratemporal) for $i = 1, 2, 3$,

(7)-(9) Household's first order condition for level of asset holdings (intertemporal Euler equation) for $i = 1, 2, 3$,

(10)-(13) Definitions of consumption for $i = 1, 2, 3, 4$.

All agents are rational and can easily compute the path of the endogenous economic variables, should the new pay-as-you-go system be introduced. In particular, they can find $a(\theta^*)$ and $A(\theta^*)$. Therefore they can evaluate the effect of this reform. Each representative member of each generation living in period $t = 1$ compares utilities $\bar{V}_i\,(a(\theta^*), A(\theta^*), \theta^*)$ for all θ^* and makes voting decision accordingly.

The same parameters for calibration as in Cooley and Soares (1999) were chosen.[3] Figure 1 shows the utilities for various levels of new pension system θ^*. Note that for each generation utility as a function of replacement ratio is single-peaked and that the older an individual is, the more in favor of pay-as-you-go system she is. The median voter's utility level is maximized by $\theta^*_{\max} = 0.8325$. Therefore, the prediction is that societies will introduce positive level of pay-as-you-go system.[4]

3 Endogenous growth model

Cooley and Soares build the model with exogenous technology improvement. But by allowing endogenous growth we introduce an additional private disincentive for implementing pay-as-you-go, which should be revealed in voting. The chain of reasoning is as follows: pay-as-you-go decreases savings, savings decrease growth rate, lower growth means that when working generations eventually retire in the future, there will be a smaller pie to divide.

3.1 Model

Now we assume $\gamma = 0$, so there is no technological progress from outside the model. To get endogenous growth let us suppose that economy-wide capital

[3] $n = 0.1782$, $\rho = 1.5$, $\beta = 0.8109$, $\sigma = 0.235$, $\alpha = 0.6$, $\delta = 0.6823$, $\gamma = 0.2008$, $\epsilon_1 = 0.9043$, $\epsilon_2 = 1.11828$ and $\epsilon_3 = 1.1873$.

[4] Thus we have tackled the question of implementability. The other question is whether $\theta^*_{\max}$ is sustainable. We do not show this here.

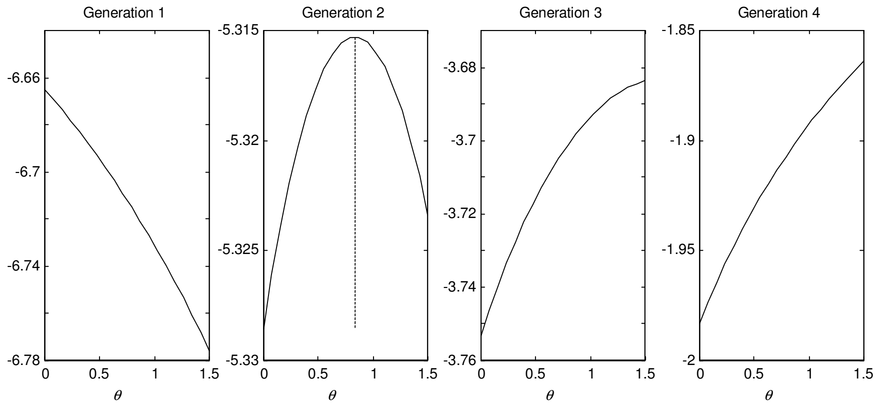

Figure 1: Utilities in exogenous growth model

per capita delivers positive externalities in the production. In particular let Ψ be no longer a constant, but rather let $\Psi \equiv \Phi \hat{Z}_t^{\alpha}$,where $\hat{Z}_t$ is a level of capital per capita in the economy taken by all firms as a constant. If we let Z_t be the economy-wide level of capital and let N_t denote the level of employment both treated by the firms as given, then $\hat{Z}_t = \frac{Z_t}{N_t}$ and production function is

$$Y_t = \Phi K_t^{1-\alpha} \left(L_t \hat{Z}_t \right)^{\alpha}$$

In this simple way we get essentially an AK model. Apart from this difference, the model is exactly the same as before. The definition of an equilibrium is also the same, although there is one additional condition: in equilibrium, firm's decision variable has to be equal to respective average variable in the economy, $K_t = Z_t$ and $L_t = N_t$.

Again, the first step is to rescale the model in order to get stationarity of the steady state.

3.2 Rescalation and solution

Let upperbar denote variables divided by $\hat{Z}_{t-1}$. It is interesting to note that variables with upperbars are essentially the same as variables previously with tildes (strictly speaking "proportional to..."). As previously, let hat denote variables in per capita terms $\hat{K}_t = \frac{K_t}{L_t}$ and $\hat{Z}_t = \frac{Z_t}{N_t}$, and finally let lower case letters denote grow rate of per capita capital $k_t = \frac{\hat{K}_t}{\hat{Z}_{t-1}}$ and $z_t = \frac{\hat{Z}_t}{\hat{Z}_{t-1}}$.

In this formulation household wants to maximize its lifetime utility[5]

$$U_\kappa = \sum_{i=2-\kappa}^{4} \beta^{\kappa+i-2} \left(\prod_{j=\kappa}^{\kappa+i-2} z_j \right)^{\sigma(1-\rho)} \frac{\left[\bar{c}_{i,\kappa+i-1}^{\sigma} \left(1 - h_{i,\kappa+i-1}\right)^{1-\sigma} \right]^{1-\rho}}{1-\rho}$$

subject to rescaled constraints (1) and (2). Firms' rescaled profit is

$$L_t \hat{Z}_{t-1} \left[\Phi k_t^{1-\alpha} z_t^a - \bar{w}_t - (r_t + \delta) k_t \right]$$

Now let us characterize the economy equilibrium by discussing parallel equations to those in exogenous model (1)-(13). We will denote these new ones by primes, (1')-(13'). For given initial conditions $\bar{a}_{2,1}, \bar{a}_{3,1}, \bar{a}_{4,1}$ the path of the economy is described by

(1') Firm's first order condition

$$\Phi(1-\alpha) - (r_t + \delta) = 0$$

(2') Firm's condition for zero profit

$$\Phi z_t - \bar{w}_t - (r_t + \delta) z_t = 0$$

Remaining equations (3')-(13') are almost the same as (3)-(13), the only change is that tilde is replaced by upperbar, and $\hat{K}_t$ and $(1+\gamma)$ are replaced by z_t. This reflects the fact mentioned earlier that from the point of view of the consumer nothing changed. Only equations describing production side of the economy (1') and (2') are different than their counterparts (1) and (2). In particular (1') is diametrically different. It indicates that rate of return on capital is independent of amount of capital in the economy, which is what keeps economy endogenously growing forever.

3.3 Results

Making two models comparable requires additional attention. We want such values of calibration that both models with zero replacement rate have the same steady state growth rate. By choosing $\Phi = 4.1655$ and $\hat{Z}_0 = 1$ in endogenous model, we obtain growth rate of capital in steady state $z = 1.2008$, the same as assumed exogenous growth rate in the Cooley and Soares' model.[6] Respective rates of return on capital and levels of efficiency labor supply are the same.

Now, we are in the position to run an experiment that will compare the behavior of Cooley and Soares' model and endogenous growth model presented in this note. Results are shown on Figure (2). We can see that the replacement ratio $\theta^*_{\max}$ which maximizes utility of the second generation is significantly different. For these parameter values, the above exogenous growth model generates $\theta^*_{\max} = 0.8325$, whereas in endogenous growth model it is 0.416. That is, taking

[5] Define $\prod_{j=t}^{t-1} z_j = 1$

[6] Let $\Psi = 4.1655$ in exogenous growth model too.

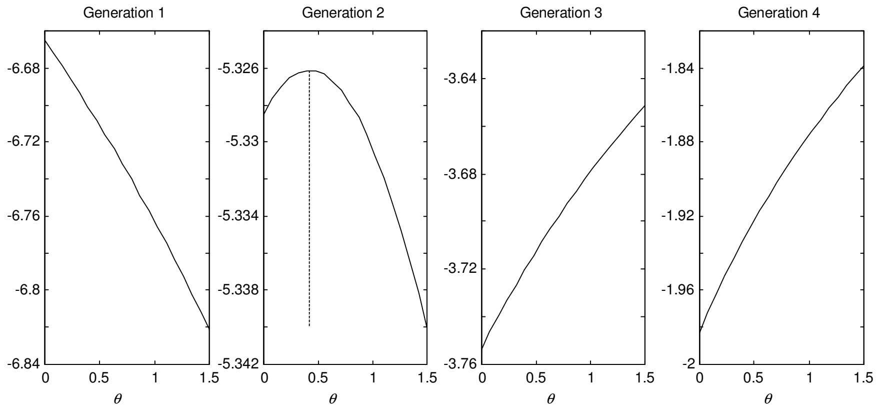

Figure 2: Utilities in endogenous growth model

effect of voting on economic growth into account can have very sizable impact on our understanding of what is a politically sustainable scale of pay-as-you-go system. The reason is the adverse effect of the large pay-as-you-go system on savings. Lower savings in turn reduce growth rate which affects future utility. In steady state, the economy grows endogenously at the rate of 0.2008 for no pay-as-you-go, 0.1489 if political outcome is $\theta^* = 0.416$ and only 0.1009 if political outcome is $\theta^* = 0.8325$. This effect is absent in Cooley and Soares' model because growth is constant by assumption.

4 Conclusion

A long standing postulate of many reformers is to replace pay-as-you-go pension system with a funded one. But the analysis of voters' incentives suggests that there is a fundamental pressure towards pay-as-you-go system. Simply current voters do not take future generations' well-being into account. The society votes in favor of pay-as-you-go system because its introduction boosts the utility of the majority, especially older members of the society. However, if it turns out that the larger saving rate increases growth rate like in endogenous growth model, then incentives of current generations to introduce pay-as-you-go system are significantly tempered.

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