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Sonar equations for planetary exploration

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Sonar equations for planetary exploration

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ABSTRACT

The set of formulations commonly known as 'the sonar equations' have for many decades been used to quantify the performance of sonar systems in terms of their ability to detect and localize objects submerged in seawater. The efficacy of the sonar equations, with individual terms evaluated in decibels, is well established in Earth's oceans. The sonar equations have been used in the past for missions to other planets and moons in our solar system, for which they are shown to be less suitable. Whilst it would be preferable to undertake high-fidelity acoustical calculations to support planning, execution and interpretation of acoustic data from planetary probes, to avoid possible errors for planned missions to such extraterrestrial bodies in future, doing so requires awareness of the pitfalls pointed out in this paper. There is a need to re-examine the assumptions, practices and calibrations that work well for Earth, to ensure that the sonar equations can be accurately applied in combination with the decibel to extraterrestrial scenarios. Examples are given for icy oceans such as exist on Europa and Ganymede, Titan's hydrocarbon lakes, and for the gaseous atmospheres of (e.g.) Jupiter and Venus.

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I. INTRODUCTION

Recent years have given rise to a growing realization of the role acoustics can play in future planetary exploration. Acoustic descriptions of distant nebulae reveal the density fluctuations that will form stars; oscillatory waves in the matter of stars have been used to reveal the presence of distant planets; acoustic sensors on distant planets and moons could record ice cracking, cryo-volcanoes, dust devils and lightning; and the way the resulting sounds propagate to the sensor can reveal the hidden material and chemical properties of the matter through which they pass. In addition to using natural sources of sound, active sources can be used to measure sound speed and acoustic absorption, data that can be used to infer the material and chemical properties of the gases, liquids and solids through which the sound passes. Active sources can also be used for anemometry and range finding.

In many of these applications, the amplitude and detectability of the received echo is key to a useful deployment when planning missions, for example in assessing absorption from measured signal loss, calculating the surface roughness from the reflected signal when a range-finding acoustic pulse propagates down through the atmosphere of Titan to the ground and back to the sensor, or sounding the depth of Titan's lakes. The ambient sound is key to both passive and active sonar, providing signal for one in terms of the natural processes that produce, absorb and scatter it and the noise for the other. Although the terminology of source level and noise level, propagation, transmission and absorption loss etc., for in both active and passive sonar been subsumed by approaches that use the sonar equation, they are also quantities that can be manipulated in a more high-fidelity approach [1, Ch9]. There is a strong argument to use such a high-fidelity approach, and the authors of the present paper support this approach. Both the high-fidelity and sonar equation approaches can be applied without introducing the errors and ambiguities discussed in the present paper, but examples of such unambiguous application are rare. In 1954, Horton lamented [2] “Restrictions which few overstep when dealing directly with such quantities are repeatedly disregarded when dealing with the logarithms of their ratios. The consequent errors, which are inevitable, are not committed solely by novices; nor are they trivial.” Horton was referring to the failure to mention or correct for differences in impedance between electrical circuits, but the statement applies equally well to any field of study, including acoustics, for which quantities not strictly proportional to power are reported as levels in decibels. It is especially relevant to planetary exploration because of the wide range of conditions encountered on extraterrestrial bodies.

While the validity of our arguments apply equally well to any approach for which results are reported in decibels, whether based on a high-fidelity solution to the wave equation or a power ratio (sonar equation) approach, given that past (and planned future) sonar missions have been formulated in terms of the sonar equations, we find it convenient to organise this paper around the familiar sonar equation terms, outlining errors and ambiguities that might have been introduced by conversion from linear to logarithmic quantities, and which should be borne in mind if the planetary probe research community continues to report its results in decibels.

Levels in decibels have been used for decades for sonar quantities measured on Earth, and although there have been some early but persistent ambiguities in definition, these have not revealed any systematic errors that might have caused significant hindering of operations because of the relatively small difference in impedance within Earth's oceans, lakes, atmosphere and sediments. However, the moment they are used for extraterrestrial environments, the familiar standard practice raises questions: What is the appropriate reference pressure for use in an alien atmosphere? If a signal propagates through Jupiter's atmosphere from pressures so great that it forms metallic hydrogen, to near-vacuum conditions at the top, how do we compare the source and received levels in terms of references?

The source of ambiguity goes beyond the simple and oft-cited issue of uncertainty in the reference value for a level in decibels. The signal to noise ratio (SNR) can be expressed in the form of a product of power ratios, leading to the so-called radar equation [3]. It follows that the logarithm of the SNR can be expressed as the sum of level differences, with each level difference equal to the logarithm of one of the power ratios, as is conventional with the sonar equation [4]. More specifically the level L of a quantity Q is defined as [5]

$$L = \log_r \frac{Q}{Q_0}, \tag{1}$$

where Q_0 is a specified reference value of the quantity Q and r is a specified base. By convention, L is expressed in decibels (dB), such that $r = e^2$ and Eq. (1) can be written [6]

$$L = 10\log_{10} \frac{Q}{Q_0}\ \mathrm{dB}. \tag{2}$$

When the decibel was first introduced as an alternative name for the transmission unit, it was understood that it would be used only in situations when Q/Q_0 was a strict power ratio [7]. If the value of Q_0 was provided, the value of the power Q followed immediately from Eq. (2). By the 1950s, the decibel has lost this original simplicity in meaning [2,8], and its ambiguity in underwater acoustics increased further during the 1980s as shown by Ref. [9]. Specific concerns include:

1) The value of Q_0 is not always stated explicitly, relying instead on convention to relay this crucial piece of information. Different conventions have arisen in different sub-fields of acoustics (especially airborne vs underwater acoustics) and in different branches of science (sonar vs radar), leading to the risk of misinterpretation if it is not stated which convention is being followed, a ubiquitous example being the use of different reference sound pressures in

gases and liquids [10]. Further, the reference value of propagation loss (1 m^2) (see Ref. 11) is frequently omitted.

2) Even when stated explicitly, the value of Q_0 is often incomplete, relying on convention to relay the missing information. Examples are the omission of "/Hz" in "dB re 1 $\mu Pa^2/Hz$" for spectral density level and of "m^2" in "dB re 1 μPa^2 m^2" for source level.

3) The nature of the physical quantity Q is rarely stated explicitly. Instead it is left to the reader to infer this information from the value of Q_0, such as stating a value of noise level in units of "dB re 1 $\mu Pa^2/Hz$" without specifying whether Q is the spectral density of the mean-square sound pressure (MSP) or of the equivalent plane wave intensity (EPWI, equal to MSP divided by the characteristic impedance), or characterising the "source level" of a surface ship in "dB re 1 μPa @ 1 m" without specifying whether the property in question is a conventional (monopole) source level or dipole source level [12], or a radiated noise level [13], all of which have identical reference values.

4) In the 1950s and 60s it was considered incorrect to use the decibel as a unit of a logarithmic ratio of any quantity Q not strictly proportional to power [14, 15, 16]. Today, however, it can no longer be assumed that Q/Q_0 is a ratio of powers [2, 9]. More specifically:

    a. since the 1950s, the decibel has been used to convey peak to peak [17], peak to valley [18], peak-equivalent [19], or zero to peak [20, 21] values of field quantities, even though the squares of such quantities are not proportional to power;

    b. since the 1980s, the decibel has been used in underwater acoustics to convey ratios of root-mean-square (rms) sound pressure without regard for the corresponding impedance ratio. It has been argued by Kuperman and co-workers [22, 23, 24], most recently in 2011 [Ref. 22], that the mean-square sound pressure must always be divided by the medium impedance before converting to decibels, but an extensive search published in 2005 [9] revealed no examples between 1981 and 2005 of an

> author having corrected for the impedance in this way, even by those who argue it is incorrect not to do so [22, 23, 24]. The *de facto* practice of not correcting for the impedance (henceforth referred to as the 'MSP convention') is so widespread that it is now required for compliance with international standard terminology [11].

Because of these ambiguities, some have called for an end to the use of the decibel [25, 26]. By contrast, Boute [27] makes a case for dropping all restrictions so that it may be applied to any ratio of like quantities, such as ratios of time, electrical resistance, temperature or frequency, while Chapman [28] and Chapman and Ellis [29] argue for moderation by calling for greater care in the continued use of the decibel.

Both the radar and sonar equations are used for quantifying the performance of sensor systems used for planetary exploration [30, 31, 32, 33, 34]. In the present paper we limit our scope to acoustics and therefore focus on the sonar equation. In underwater acoustics the use of the decibel is nearly universal, reporting levels with a reference sound intensity [4, 22] of 1 $\mu\mathrm{Pa}^2/\rho_0 c_0$, where $\rho_0 c_0$ is the impedance of seawater, assumed to be the same at all locations along the propagation path. On planets other than Earth, the characteristic acoustic impedance of the propagation medium is in general not equal to that of seawater on Earth, whether because the medium is a gas, a liquid other than water, or water subject to extremes of temperature or pressure. The decibel has been (and is being) used for planetary exploration, leading to ambiguity and confusion. Our Earth-centric conventions therefore need revisiting when applied to extraterrestrial acoustics. We consider each term in the sonar equation, evaluating potential for confusion by comparing a widely used textbook [4] with a recently developed international standard [11].

While we use the sonar equation to illustrate our point, primarily because doing so provides a convenient structure, our main point is that any calculation, no matter how carefully and precisely made, is rendered ambiguous if presented as a level, level difference or loss in decibels, unless the same care and attention is afforded to the definition of the level (or loss) as was taken in the original precise calculations. This paper explores this point by examining the behaviour of the terms in the 'sonar equations' as they are taken to other worlds.

In Section II the sonar equations of Urick [4] and the International Organization for Standardization (ISO) [11] are described, and the individual terms in each compared with the corresponding terms in the other. In Section III, the sonar equation terms are put into context by considering specific effects of extreme conditions such as high or low temperature or pressure. Conclusions are drawn in Section IV.

II. THE SONAR EQUATIONS

Active sonar uses the principle of echolocation. In other words, a pulse of sound (including infrasound or ultrasound) is transmitted by the sonar system, reflected from an object of interest (the sonar “target”), and the resulting echoes are sensed by the sonar receiver [14, 15]. The time delay between transmission and reception indicates the distance to the sonar “target” (i.e., the object of interest), while phase differences between receiver elements provide bearing information. Differences between the echo and the emitted pulses can be interpreted to infer properties of the target, such as its density and structure, which can provide characteristic ringing or resonances [35].

Unlike active sonar, passive sonar equipment does not transmit sound, listening instead for sounds radiated by the target or for perturbations in ambient sound caused by the target’s presence. Target bearing is estimated from the phase difference between receiver elements, in the same way as described above for active sonar. The target distance needs to be estimated by combining bearing estimates from different receivers, or from the rate of change of bearing on a single receiver.

In a variant of active sonar, if the source, receiver and reflector (if present) are well characterized, the received signal can be interpreted to identify parameter values associated with the medium. This has been discussed to provide ultrasonic anemometers for Mars [36; 37; 38] and devices to measure the sound speed on Titan [39], Venus [40; 41] and Jupiter, Saturn, Uranus and Neptune [42], such measurements having use in validating proposed chemical compositions for atmospheres, although the mountings of these have the potential to cause mis-readings if acoustical differences generated by transposing these structures to other worlds are not taken into account [43; 44].

The sonar equation takes a different form for passive and active sonar. Both forms are considered below, starting with the (simpler) passive sonar equation. In its most general form, the sonar equation relates the signal excess (symbol ΔL_{SE}) to the signal to noise ratio (abbreviation SNR, symbol R), via the equation [11]

$$\Delta L_{\mathrm{SE}} = 10\log_{10}\frac{R}{R_{\mathrm{T}}}\ \mathrm{dB}\,, \tag{3}$$

where R_{T} is the value of R required to accomplish a specified task (often the detection of an object) with a specified degree of confidence, characterized in terms of the probability of detection (often 0.5) and a specified probability of false alarm (typically between 10^{-12} and 10^{-4}). In other words, R_{T} is the SNR threshold above which the task is accomplished and ΔL_{SE} is the amount by which R exceeds that threshold, typically expressed in decibels.

The first sonar equations we are aware of are those of Horton [14], whose book was first published in 1957. Horton's "direct-listening equation" (in modern parlance, the passive sonar equation) for the signal to noise ratio R (Horton refers to $10\log_{10}R$ dB as the "signal differential", denoting it $\Delta L_{\mathrm{s/n}}$), with minor changes in notation to facilitate comparison with the notation of this paper (which follows Refs. 4 and 11), from p314 of Ref. 14, is

$$10\log_{10}R\ \mathrm{dB} = L_{\mathrm{sl}} - N_{\mathrm{pl}} - L_{\mathrm{nl}} + N_{\mathrm{di}}\,,$$

(4)

where L_{sl}, N_{pl}, L_{nl}, and N_{di} are referred to by Horton as the ‘index level of the signal’, ‘propagation loss’, ‘equivalent plane wave level of the interfering noise’ and ‘effective directivity index of the hydrophone system’, respectively. These were expressed as levels or level differences of EPWI. For example, L_{nl} is $10\log_{10}(J_{\mathrm{N}}/I_0)$ dB, where J_{N} is the noise EPWI and I_0 is a constant reference intensity

$$I_0 = 10\ \mathrm{kW/m^2},$$

(5)

equal to the unit of intensity in the centimetre-gram-second (CGS) system of units, i.e., 1 W/cm^2.

Similarly, Horton’s “echo-ranging equation” (now the active sonar equation) (p342)

$$10\log_{10} R\ \mathrm{dB} = L_{\mathrm{sl}} - N_{\mathrm{pl}} + N_{\mathrm{ts}} - N_{\mathrm{pl}} - L_{\mathrm{nl}} + N_{\mathrm{di}}\,,$$

(6)

where N_{ts} is the target strength and N_{pl} appears twice because the sound travels from sonar to target and back, the premise being that the return path experiences the same propagation loss as the forward path.

In 1967, Urick published the first edition of his widely used ‘Principles of Underwater Sound’ [15], including sonar equations for passive and active sonar corresponding to Horton’s listening and echo-ranging equations, respectively. In essence Urick’s sonar equations are the same as those of Horton, but they differ in one important detail: while Horton emphasized the need for a constant and well defined reference intensity, Urick (p13-14) introduced instead a value that depended on one’s choice of impedance, defined as the intensity of a plane wave whose root-mean-square sound pressure is equal to the reference sound pressure p_0 = 1 dyn/cm^2, i.e.,

$$I_0 = p_0^2 / \rho_0 c_0,$$

(7)

equal to approximately 6500 pW/m^2 if ρ_0 c_0 is chosen to be the characteristic impedance of seawater [6]. In the third and final edition of Urick's book [4], the reference pressure was updated to p_0 =1 μPa, corresponding to $I_0 = 6.5 \times 10^{-7}$ pW/m^2, calculated using Eq. (7). According to Ref. 4 , Eq. (7) was the American National Standard value for the reference intensity, but this claim is not borne out by the standard cited by Urick. Reference 45 (entry 9.040 Standard Sea Water Conditions) provides standard values of pressure (1 atm = 0.101325 MPa), temperature (15 deg C) and sound speed (1500 m/s), from which the salinity (31.60 parts per thousand), density (1023.38 kg/m^3) and impedance (1.53507 MPa s/m) are deduced. However, ASA Z24.1-1951 was superseded in 1960 by Ref. 46, leaving the value of the impedance ρ_0 c_0 unspecified. (A reference value of I_0 = 1 pW/m^2 has been the American national standard since 1960 and the international standard since 1994, obviating the need to standardize the value of ρ_0 c_0). Urick's equations have remained in use ever since, despite this ambiguity, appearing in his third (1983) edition and repeated by Ref. 22, unchanged except for the reference values of 1 dyn/cm^2 and 1 yd being replaced in modern texts by 1 μPa and 1 m, respectively, but with the same ambiguity in reference intensity.

In 2010, Ainslie published 'Principles of Sonar Performance Modeling', [12] with new sonar equations that removed this ambiguity by defining levels as ratios of MSP instead of EPWI. In 2012, ISO started the development of International Standard ISO 18405 Underwater Acoustics – Terminology, the purpose of which was to establish international standard definitions of quantities used in underwater acoustics. The ISO Working Group charged with the development of ISO 18405 published its second draft in April 2016 [11], including passive and active sonar equations, also based on MSP ratios. The planned publication date for the final International Standard is December 2016. The remainder of Sec. II compares the sonar equations of Urick [4] with those of ISO/DIS 18405.2 [11], henceforth abbreviated as "ISO 18405".

The passive and active sonar equations are introduced in Sec. II.A and in Sec. II.B, respectively, followed by an in-depth review of the passive equation terms, as applied to planetary exploration in

Sec. II.C. Section II.D considers those terms of the active sonar equation most influenced by extreme propagation conditions.

A. Passive sonar equation

The 'passive sonar equation' is the modern name given to Horton's 'direct-listening' equation. It relates the signal excess (the amount by which the SNR exceeds the threshold required to accomplish a specified task), to properties of the source of sound and of the sonar being used to detect the sound.

1. Urick (passive sonar)

A widely used form of the passive sonar equation is described by Ref. 4 (pp 22, 388), giving the signal excess (SE) in terms of the source level (SL), "transmission loss" (TL), noise level (NL), directivity index (DI) and detection threshold (DT)

$$\mathrm{SE} = \mathrm{SL} - \mathrm{TL} - \mathrm{NL} + \mathrm{DI} - \mathrm{DT} \,. \tag{8}$$

The term "transmission loss" is placed in inverted commas because what is meant is a quantity referred in the rest of this paper as propagation loss (abbreviated PL), while the term transmission loss is reserved hereafter to mean the difference between levels of like quantities at two different places [11]. Further, Urick uses DI as an approximation for the array gain (AG), and with these two changes, Eq. (8) becomes

$$\mathrm{SE} = \mathrm{SL} - \mathrm{PL} - \mathrm{NL} + \mathrm{AG} - \mathrm{DT} \,. \tag{9}$$

In both cases, our purpose in making the change is to facilitate comparison with the ISO sonar equation, below.

2. *ISO (passive sonar)*

An alternative to Urick's sonar equation is ISO 18405. According to this (draft) International Standard, the passive sonar equation is

$$\Delta L_{\mathrm{SE}} = L_{\mathrm{S}} - N_{\mathrm{PL}} - L_{\mathrm{N}} + \Delta L_{\mathrm{PG}} - \Delta L_{\mathrm{DT}}. \tag{10}$$

While Eq. (10) has the same form as Urick's sonar equation, the similarity is deceptive, as there are differences in the definitions of individual terms. The differences are analyzed in Sec. II.C. All terms in Eq. (10) are either levels (L_{S}, L_{N}), level differences (ΔL_{SE}, ΔL_{PG}, ΔL_{DT}) [6] or sensitivity levels (N_{PL}) [11]. All are conventionally expressed in decibels.

Equations (9) and (10) are also applicable to active sonar if the transmitted beam is received at the receiver after specular reflection from a surface such as the seabed. In this situation, the propagation loss term applies to the two-way path, and the resulting equation is referred to below as the "echo sounder equation".

The sonar equation is sometimes presented or described as an engineering approximation, but there is no approximation involved in the derivation of Eq. (10), each term of which is defined rigorously by ISO 18405. Any approximations incurred from the application from the sonar equation result not from the equation itself, but by further simplifications or assumptions made by its user.

3. *Examples (passive sonar)*

We are aware of three extraterrestrial acoustics papers that make use of the passive sonar equation. These are those of Arvelo and Lorenz [34], who use the echo sounder equation to investigate the performance of a sonar designed to measure the depth of *Ligeia Mare*, one of Titan's hydrocarbon lakes [47], Banfield et al. [33], who use the passive sonar equation to investigate the performance of a Martian anemometer, and Lee et al. [31], who assess the detectability of ice cracks on Europa with a view to using these to probe the upper 100 km of Europa's structure.

In addition, Leese et al. [48] refer to earlier work by Garry [49], who appears to use the echo sounder equation to investigate the height from which echoes from Titan's surface might be detected. We do not have access to Ref. 49.

B. Active sonar equation

The 'active sonar equation' is the modern name given to Horton's 'echo-ranging' equation. For high-power sonar in Earth's oceans, the performance of active sonar is often limited by self-noise in the form of reverberation [22]. For planetary missions we can expect less powerful transmitters to be available, and early exploration systems are more likely to be limited by ambient noise, electrical self-noise or even thermal noise. We therefore omit reverberation from our discussion of the active sonar equation.

1. Urick (active sonar)

Urick's active sonar equation (Ref. 4, p21, 388) is

$$\mathrm{SE} = \mathrm{SL} - \mathrm{PL} + \mathrm{TS} - \mathrm{PL} - \mathrm{NL} + \mathrm{AG} - \mathrm{DT} \tag{11}$$

2. ISO (active sonar)

The corresponding equation from Ref. 11 is

$$\Delta L_{\mathrm{SE}} = L_{\mathrm{S}} - N_{\mathrm{PL,Tx}} + N_{\mathrm{TS,eq}} - N_{\mathrm{PL,Rx}} - L_{\mathrm{N}} + \Delta L_{\mathrm{PG}} - \Delta L_{\mathrm{DT}}, \tag{12}$$

where the equivalent target strength ($N_{\mathrm{TS,eq}}$) is closely related to TS and the terms $N_{\mathrm{PL,Tx}}$ and $N_{\mathrm{PL,Rx}}$ replace the two PL terms in Eq. (11).

3. Examples (active sonar)

In 1969, Little [30] proposed a method for probing Earth's lower atmosphere using sonar, in order to measure parameters such as humidity, temperature and wind velocity profiles and 3D inhomogeneity, pointing out that fluctuations in the acoustic refractive index exceed those for their radio counterpart by a factor of 1000. For this purpose he employs a linear form of the active sonar equation based on the radar equation, in which the terms are multiplied instead of adding their logarithms as is customary for sonar. The same approach is adopted by Svedhem et al. [32], who examine the feasibility of using sonar to measure the properties of Titan's atmosphere such as precipitation rate.

C. Passive sonar equation: term by term comparison

The tolerances associated with many day-to-day measurements in acoustics (say ±3 dB) would seem extremely large to some branches of measurement physics, but are considered acceptable for a great deal of acoustical measurements, and it is therefore pertinent to ask 'how accurate do I need to be in practice?'. However, this is a different question to 'how accurate does a standard need to be?', since the latter must in principle apply for the most precise foreseeable measurement, including calibration. The practical implications of the currently tolerated inaccuracy are discussed in Sec. III. While some of the systematic errors caused by transposing familiar practices to extraterrestrial environments might seem small compared with uncertainties (both random and systematic) that can result from measurement error, for a *definition* such an ambiguity is both unnecessary and undesirable. It likely leads to unnecessary calibration errors: if, even on Earth, there is no consensus on whether to use the impedances specific for fresh/salt water if a sonar system is calibrated in one and used in the other, then we can never achieve the better than 0.5 dB calibration accuracy that Horton argued for in 1959.

We now consider the implications of the above considerations for the passive sonar equation. This is achieved by comparing each in term in Eq. (9) with its corresponding term in Eq. (10).

1. Source level

The ‘source level’ is a measure of the power radiated by a sound source – more precisely a measure of its far-field radiant intensity (power per unit solid angle) [12].

a. Source level (Urick)

Ref. 4 (henceforth referred to as “Urick 1983”) introduces projector source level (SL) as the term in the active sonar equation that characterizes the sonar transmitter. It is defined on p71 as “the intensity of the radiated sound in decibels relative to the intensity of a plane wave of rms pressure 1 μPa, referred to a point [at a reference distance, r_0, of] 1 yd from the acoustic center of the projector in the direction of the target”. We interpret this definition, in equation form, as

$$\mathrm{SL} \equiv 10\log_{10}\frac{I_{\mathrm{s},f}(r)\,r^2}{I_0 r_0^2 B_0^{-1}}\,\mathrm{dB}, \tag{13}$$

where the subscript f denotes a spectral density (here and throughout) and $I_\mathrm{s}(r)$ is the equivalent free-field intensity (the magnitude of the sound intensity that would exist in the free field if the source motion were unchanged [12, p576]) in the acoustic far field at distance r, i.e.,

$$I_\mathrm{s}(r) = \frac{p_\mathrm{s}^2(r)}{\rho_\mathrm{s} c_\mathrm{s}}, \tag{14}$$

where p_s is the rms free-field sound pressure in the far field of the source, again for identical source motion. The reference values for Eq. (13) are given by r_0 = 1 yd (converted here to r_0 = 1 m) and Eq. (7) for I_0 with p_0 = 1 μPa.

b. Source level (ISO)

Reference 11 (i.e., ISO 18405) defines source level as

$$L_\mathrm{S} \equiv 10\log_{10}\frac{p_\mathrm{s}^2(r)\,r^2}{p_0^2 r_0^2}\,\mathrm{dB}.$$

(15)

The difference between the Urick and ISO definitions of source level depends on the characteristic impedance at the source and on the receiver bandwidth, relative to the reference impedance and reference bandwidth, respectively

$$L_\mathrm{S} = \mathrm{SL} + 10\log_{10}\frac{\rho_\mathrm{s}c_\mathrm{s}}{\rho_0 c_0}\frac{B}{B_0}\,\mathrm{dB}.$$

(16)

c. Examples of source level

A recurring problem in the characterisation of sound sources in the context of the sonar equation is that their properties are reported in decibels, often without a clear description of the physical quantity being expressed as a level, leaving the reader to infer from the context what is intended.

Arvelo and Lorenz [34] calculate the source level required for an echo sounder in Titan's *Ligeia Mare* to detect an echo from the bottom of the ethane lake if the sonar is floating at the surface. They report a requirement of at least "150 dB re 1 μPa^2/Hz@1 m", where the "@1 m" is interpreted to mean scaled to a reference distance r_0 in the sense of Eq. (13), with r_0 = 1 m. In other words the source spectral density level is 150 dB re 1 μPa^2 m^2/Hz. As pointed out by Ainslie [6], when using the EPWI convention in a medium other than seawater on Earth, there is a need to specify the impedance used to determine the reference intensity. In other words, to use the stated information we at least need to know the value of I_o in Eq. (13), and possibly also the value of $\rho_\mathrm{s}\,c_\mathrm{s}$ in Eq. (14). Depending on the assumed impedance, the impedance correction term of Eq. (16) would have a value between -3.6 dB (source in methane; reference of seawater) and +2.5 dB (source in ethane; reference of methane), making for a total uncertainty of about 6 dB. There will always exist circumstances for which the characteristic acoustic impedance (whether a standard reference value, the value at source or the value at the receiver) must be specified. For example if a distributed source of known power (e.g. lightning on Mars, Venus or Titan) is modelled as launching pressure waves, whose forms are at the observer are numerically calculated and then summed to predict the received sound pressure field, this calculation hinges on knowing the correct value of impedance at the source. Furthermore, if the strength of that received field is then expressed as a level in decibels, both a standard reference impedance and the actual or assumed impedance at the receiver would need to be specified [50].

Towner et al. 2006 [51] describe the sonar transmitter on the Huygens landing probe as "resulting in a transmitted acoustic power of about 104 dB (with respect to 20μPa ...)". While such a phrase might be clear in light of widely adopted conventions for reference values in the context of terrestrial atmospheric acoustics, the present authors (who are used to the conventions of underwater acoustics) are unsure of its meaning. Judging from the reference value, it could refer to a sound pressure level of 104 dB re $(20\ \mu\text{Pa})^2$, corresponding to a mean-square sound pressure of 10.0 Pa^2 at some (unspecified) distance. It could also refer to a source level of 104 dB re $(20\ \mu\text{Pa})^2\ \text{m}^2$, implying a source factor [11] of 10.0 $\text{Pa}^2\ \text{m}^2$ for the MSP convention, and between 10 $\text{Pa}^2\ \text{m}^2$ and 31 $\text{Pa}^2\ \text{m}^2$ for the EPWI convention, depending on whether the impedance of air on Earth or nitrogen on Titan is chosen to determine the reference intensity, or some intermediate value.

2. Propagation loss

Propagation loss is the inverse of the transfer function from source to receiver. More specifically, it is the difference between the source level and the signal level received at the sonar. The term 'transmission loss' is sometimes used as a synonym [9], but we prefer 'propagation loss' to avoid confusion with the alternative meaning of transmission loss as the difference between two like quantities such as sound intensity level (SIL) [52].

a. Propagation loss (Urick)

Urick 1983 [4] defines propagation loss (PL) (p99) as $10\log_{10}(I_\mathrm{s}/J_\mathrm{S})$ dB, where I_s is “the intensity at the reference point located [1 m] from the “acoustic center” of the source ($10\log_{10} I_\mathrm{s}$ dB is the source level of the source)” and J_S is the “[equivalent plane wave] intensity at a distant point”. We interpret this definition, in equation form, as

$$\mathrm{PL} \equiv \mathrm{SL} - 10\log_{10}\frac{J_\mathrm{S}}{I_0}\,\mathrm{dB}, \tag{17}$$

where J_S is the EPWI of the signal at the sonar receiver.

b. Propagation loss (ISO)

ISO 18405 defines propagation loss as

$$N_\mathrm{PL} \equiv L_\mathrm{S} - 10\log_{10}\frac{p_\mathrm{S}^2}{p_0^2}\,\mathrm{dB}. \tag{18}$$

The Urick and ISO definitions of propagation loss are therefore related via the equation

$$N_\mathrm{PL} = \mathrm{PL} + 10\log_{10}(\rho_\mathrm{s}c_\mathrm{s}/\rho_\mathrm{r}c_\mathrm{r})\ \mathrm{dB}. \tag{19}$$

c. Examples of propagation loss

Propagation loss results are presented by Collins et al. [53] for Jupiter’s atmosphere, by Lee et al. [31] and Heaney and Campbell [54] for Europa’s icy ocean, and by Arvelo and Lorenz [34] for *Ligeia Mare*, on Titan. These three different scenarios lead unsurprisingly to very different propagation conditions, making the results intrinsically difficult to compare. Comparison is further (unnecessarily) complicated for a more mundane reason, namely that what is plotted is a different physical quantity in each case. Specifically, Arvelo and Lorenz use Urick’s definition of propagation loss, whereas Heaney and Campbell adopt that of ISO 18405, while Lee et al. [31] define propagation loss in terms of ratios of (mean-square) sound particle velocities instead of sound pressures. While there is nothing wrong with any one of these three definitions (in each case the choice of definition followed is clear), the proliferation of different definitions can lead to confusion. By contrast, Collins et al. [53] present graphs of propagation loss but do not state which definition is being used for this quantity. Possibilities include the MSP and EPWI conventions, and a third possibility involving ratios of the mean-square sound pressure divided by the density [55].

The term propagation loss, defined as the difference between source level and sound pressure level, is referred to by Urick as “transmission loss” and this practice is widely followed [22, 34, 53]. However, the term “transmission loss” has an alternative meaning as the difference between SIL at specified locations [56] (often either side of a barrier or boundary), and the ISO standard [11] reserves the term transmission loss for this second meaning. One example of the use of “transmission loss” with this ISO standard meaning, in the context of an echo sounder in Titan’s *Ligeia Mare* [34], is the decrease in SIL across the boundary between the solid transducer head, made of aluminium, and the liquid ethane in the lake.

Finally, we point out a third use of “transmission loss”, in the context of a Martian sonic anemometer [33], as a synonym of absorption loss, which is the contribution from absorption to propagation loss.

3. Noise level

The noise level is the level of the unwanted sound or non-acoustic noise that interferes with the sonar signal.

a. Ambient noise (Urick)

Urick 1983 [4] considers ambient noise (the ocean noise that would be present if the sonar and target signal were not) and self-noise (the noise due to the presence and operation of the sonar). Specifically, Urick (p202) defines 'ambient noise level' (NL) as "the intensity, in decibels, of the ambient background measured with a nondirectional hydrophone and referred to the intensity of a plane wave having an rms pressure of 1 μPa". We interpret Urick's definition of noise level, in equation form, as the level of the EPWI spectral density $J_{\mathrm{amb,N},f}$

$$\mathrm{NL} \equiv 10 \log_{10} \frac{J_{\mathrm{amb,N},f}}{I_0 B_0^{-1}} \,\mathrm{dB}, \tag{20}$$

where $B_0 = 1$ Hz.

In the event that self-noise is not negligible, the term $J_{\mathrm{amb,N},f}$ in Eq. (20) is replaced by $J_{\mathrm{amb,N},f} + J_{\mathrm{self,N},f}$, where $\mathrm{J}_{\mathrm{self,N},}$ is defined as $V_{\mathrm{self}}^2/(M^2 \rho_{\mathrm{r}}\, c_{\mathrm{r}})$, whereas M is the receiver sensitivity (receiver voltage per unit incident sound pressure) and V_{self} is the receiver voltage in the absence of signal and ambient noise. The subscript f denotes the spectral density.

b. Sonar noise level (ISO)

ISO 18405 [11] defines 'sonar noise level' as

$$L_{\mathrm{N}} \equiv 10 \log_{10} \frac{p_{\mathrm{N}}^2}{p_0^2} \,\mathrm{dB} \tag{21}$$

It follows that the ISO and Urick definitions are related via

$$L_{\mathrm{NL}} = \mathrm{NL} + 10 \log_{10} \frac{\rho_{\mathrm{r}} c_{\mathrm{r}}}{\rho_0 c_0} \frac{B}{B_0} \,\mathrm{dB}. \tag{22}$$

The need to correct for the impedance ratio was pointed out by Ainslie and Leighton [57]. For simplicity, the bandwidth term was excluded there by arbitrarily equating B to B_0. If NL is interpreted as a band-averaged spectral density level, no approximation is involved in the derivation of Eq. (22).

c. Examples of noise level

If the MSP convention is followed, the noise level of "40 dB re 1 μPa^2/Hz" quoted by Arvelo and Lorenz [34] for wind-generated noise in *Ligeia Mare* means the MSP is 10^4 μPa^2/Hz, precisely. In fact Arvelo and Lorenz [34] follow Urick's EPWI convention, for which either the reference impedance or reference intensity needs to be stated in order for the information to be interpreted unambiguously as an EPWI value. Possible values of reference intensity on Titan are between 6500 and 14900 aW/(m^2 Hz) [6, 57]. For interpretation in terms of a mean-square sound pressure, the characteristic impedance of the medium would also be needed.

The information used by Arvelo and Lorenz to arrive at their stated value of noise level originates from Figure 5 of Ref. 58, which plots the spectrum of the noise from a methanefall on Titan, and used the same sound power per bubble of Titan as it would have on Earth (revising earlier calculations by the same authors that used an estimate for the sound power on Titan as being roughly 10 times greater [59, 60]). These data were taken from examination of a waterfall on Earth, and so are illustrative only, since the Earth waterfall could have been more or less powerful. Following consultation with the originator of these graphs (personal communication, Dr P. R. White, January 2016), we can confirm that they were calculated using the MSP convention, without an impedance ratio.

The example of noise level in Titan's lakes teaches us that confusion can result when information from one paper making use of (say) the MSP convention is applied to another in which the EPWI convention is applied. The information is prone to misinterpretation unless a) the choice of MSP vs EPWI convention is clearly stated and b) the choice of reference impedance and assumed medium impedance is stated when making use of the EPWI convention.

Lee et al. [31] define noise level in Europa's ocean in terms of the spectral density of the mean square sound particle velocity. Their definition does not include an impedance ratio, making their approach comparable with the MSP convention but applied instead to particle velocity.

It is usually the case that as the frequency increases, acoustic sensors tend to be more prone to thermal noise, for which the same ambiguity applies when reported in decibels (see Sec. III.D). For high frequency uncorrelated noise generally, the dimensions of the receiving transducer is typically not small compared with the acoustic wavelength, in which case the usual concepts of receiver sensitivity need to be refined by averaging the sound pressure (or mean-square sound pressure) over the transducer's active surface.

4. Signal to noise ratio and processing gain

Sonar processing is designed to enhance performance, either by increasing the signal to noise ratio (R) or by decreasing the threshold required for detection (R_T). An increase in R (processing gain) can be achieved by combining signals from different hydrophones (spatial processing, known as beamforming, the resulting gain being called 'array gain') or by combining signals at different times (temporal processing, i.e. time-domain filtering such as a Fourier transform – the resulting gain is called 'filter gain').

a. Array gain (Urick)

In his sonar equation, Urick 1983 [4] approximates the array gain (AG) by the receiver directivity index (DI). As explained above, comparison with ISO is facilitated by replacing DI with AG. Urick 1983 defines AG (p34) as

$$\mathrm{AG} = 10\log_{10} G_\mathrm{A}\ \mathrm{dB}, \tag{23}$$

where

$$G_\mathrm{A} = \frac{R'_\mathrm{bf}}{R'_\mathrm{hp}} \tag{24}$$

and R' is the ratio of signal power to noise power spectral density, a quantity with dimensions of bandwidth. We interpret it as

$$R' \equiv BR\,, \tag{25}$$

where B is the receiver bandwidth and R is the ratio of signal power to noise power. The subscripts 'hp' and 'bf' indicate hydrophone and beamformer output, respectively. It follows from Eq. (25) that

$$G_\mathrm{A} = \frac{R_\mathrm{bf}}{R_\mathrm{hp}}. \tag{26}$$

b. Sonar processing gain (ISO)

ISO 18405 considers temporal and spatial processing combined and refers to the combined gain as 'sonar processing gain'. Specifically, the ISO 18405 definition of 'processing gain' is

$$\Delta L_\mathrm{PG} = 10\log_{10} G_\mathrm{P}\ \mathrm{dB} \tag{27}$$

$$G_\mathrm{P} = \frac{R_\mathrm{out}}{R_\mathrm{hp}}, \tag{28}$$

where the subscript 'out' indicates output of all processing, where the detection decision is made. The difference between processing gain and array gain can be written

$$\Delta L_{\mathrm{PG}} = \mathrm{AG} + 10\log_{10} G_{\mathrm{F}}\ \ \mathrm{dB}, \tag{29}$$

where G_{F} is the filter gain, defined as

$$G_{\mathrm{F}} \equiv \frac{G_{\mathrm{P}}}{G_{\mathrm{A}}}. \tag{30}$$

c. Examples of processing gain

In assessing the likely performance of a depth sounder in Titan's hydrocarbon seas, Arvelo and Lorenz [34] present results for DI, a useful proxy for AG. In line with the worst-case philosophy of that paper, this approximation will tend to underestimate the true AG because the transducer is facing down, away from the main noise source.

The processing gain term incorporates any change to the signal to noise ratio resulting from conversion of the sound to an electrical or (digital) electronic form, whether the change results from signal processing (e.g., beamforming or spectral filtering) or as an intended or unintended consequence of the hardware. For example a transducer whose active surface is large compared with the acoustic wavelength will have directional properties that will increase the strength of coherent signals arriving from the direction perpendicular to the transducer face, relative to that of thermal noise, or other uncorrelated high frequency noise.

5. Detection threshold

a. Detection threshold (Urick)

For a narrow-band source, Urick 1983 [4] (p378) defines DT as “the ratio, in decibel units, of the signal power (or mean-squared voltage) in the receiver bandwidth to the noise power (or mean-squared voltage), in a 1-Hz band, measured at the receiver terminals, required for detection at some preassigned level of correctness of the detection decisions." For a broadband source we interpret this definition, in equation form, as

$$\mathrm{DT} = 10\log_{10} R_{\mathrm{bf,T}}\ \mathrm{dB}, \tag{31}$$

where the subscript 'T' indicates the threshold required to achieve a specified detection probability and false alarm probability.

b. Detection threshold (ISO)

The ISO 18405 definition of 'detection threshold' is

$$\Delta L_{\mathrm{DT}} = 10\log_{10} R_{\mathrm{out,T}}\ \mathrm{dB}, \tag{32}$$

from which it follows that

$$\Delta L_{\mathrm{DT}} = \mathrm{DT} + 10\log_{10} G_{\mathrm{F}}\quad \mathrm{dB}. \tag{33}$$

c. Examples of detection threshold

The only planetary acoustics paper known to the authors to calculate detection threshold is Ref. 34. For the simple receiver considered, the filter gain is expected to be small or negligible, so the $10\log_{10}G_{\mathrm{F}}$ dB difference between Urick and ISO detection thresholds is of no consequence for this example.

6. Summary table

In summary we highlight three main differences between Urick's sonar equations [4] and those of ISO [11] (Table 1):

- various impedance ratios that are omitted from the ISO equations are implicit in Urick's equations, resulting in differences in the terms source level, propagation loss, and noise level;

- the ISO terms are band levels in the receiver frequency band, whereas Urick uses (band-averaged) spectral densities (affects noise level and source level);
- in the ISO equations, filter gain is included in the processing gain term, whereas Urick includes this effect in the detection threshold.

Table 1. ISO 18405 passive sonar equation terms and their relationship with corresponding terms from Urick 1983 **[4]**.

term	symbol	relation to Urick's corresponding sonar equation term	explanatory notes
source level	L_S	$L_S = \mathrm{SL} + 10\log_{10}\frac{\rho_s c_s}{\rho_0 c_0}\frac{B}{B_0}\,\mathrm{dB}$	$\rho_s c_s$ = impedance at source position $\rho_0 c_0$ = reference impedance (see text)
propagation loss	N_{PL}	$N_{PL} = \mathrm{PL} + 10\log_{10}\frac{\rho_s c_s}{\rho_r c_r}\,\mathrm{dB}$	$\rho_r c_r$ = impedance at receiver position
sonar noise level	L_N	$L_N = \mathrm{NL} + 10\log_{10}\frac{\rho_r c_r}{\rho_0 c_0}\frac{B}{B_0}\,\mathrm{dB}$	B = receiver bandwidth (assumed to exceed signal bandwidth) B_0 = 1 Hz
processing gain	ΔL_{PG}	$\Delta L_{PG} = \mathrm{AG} + 10\log_{10} G_F\ \mathrm{dB}$	G_F = filter gain (gain from all processing after the beamformer)
detection threshold	ΔL_{DT}	$\Delta L_{DT} = \mathrm{DT} + 10\log_{10} G_F\ \ \mathrm{dB}$	
signal excess	ΔL_{SE}	$\Delta L_{SE} = \mathrm{SE}$	

D. Active sonar equation: target strength and two-way propagation loss

The active sonar equation can be derived from the passive sonar equation by replacing the one-way transfer function, represented by the propagation loss term N_{PL}. If $N_{\mathrm{PL,Tx}}$ ($N_{\mathrm{PL,Rx}}$) is the one-way propagation loss to (from) the target, the one-way transfer function is replaced without approximation by the two-way transfer function, represented by the combination $N_{\mathrm{PL,Tx}}+N_{\mathrm{PL,Rx}}-N_{\mathrm{TS,eq}}$, where $N_{\mathrm{TS,eq}}$ is often approximated by TS. The issues of target strength and two-way propagation loss are discussed below.

1. Target strength and equivalent target strength

a. Target strength (Urick)

Urick 1983 [4] (p291) defines TS as "10 times the logarithm to the base 10 of the ratio of the intensity of the sound returned by the target, at a distance [r_0] from its "acoustic center" in some direction, to the incident intensity from a distant source". We interpret this definition, in equation form, for an incident plane wave of intensity I_{inc}, and back-scattered intensity at far-field distance r, $I_{\mathrm{sc}}(r)$, as

$$\mathrm{TS} \equiv 10 \log_{10} \frac{I_{\mathrm{sc}}(r)\, r^2}{I_{\mathrm{inc}} r_0^2} \,\mathrm{dB}. \tag{34}$$

b. Equivalent target strength (ISO)

ISO 18405 defines the target strength of an object for the same idealized conditions (incident plane wave and far-field free-field scattered wave) as Urick, albeit with one important difference, that the ISO standard defines target strength as a bistatic quantity, depending on both incident and scattered angles. However, this term is not used in the ISO sonar equation because of the requirement for it to be applicable in realistic situations (in which the idealized conditions are not met). Instead, the concept of an equivalent target strength ($N_{\mathrm{TS,eq}}$), applicable to realistic conditions such as scattering from a target in a shallow water waveguide, is introduced. This quantity is defined as

$$N_{\mathrm{TS,eq}} \equiv L_{\mathrm{TE}} + N_{\mathrm{PL,Tx}} + N_{\mathrm{PL,Rx}} - L_{\mathrm{S}}, \tag{35}$$

where L_{TE} is the target echo level, defined as the mean-square sound pressure level of the target echo.

For modelling work, $N_{\mathrm{TS,eq}}$ is often approximated by TS because $N_{\mathrm{TS,eq}}$ is more difficult to calculate, although this practice generates the new problem of knowing what values of incident and scattered angles to choose ($N_{\mathrm{TS,eq}}$ is independent of both). On the other hand, the measurement of TS is problematic because of the requirement for far-field condition and an incident plane-wave. The two quantities are equal if the target's differential scattering cross section is independent of the direction of both incident and scattered waves [12, pp607-610].

2. Propagation loss revisited (reciprocity)

It is often assumed that, for monostatic sonar, the sum of the two propagation loss terms in Eq. (12) or (35) can be replaced by two times one of them. This is an approximation that holds for narrow band sonar in a medium with zero mean flow and uniform impedance. For broadband sonar there is a difference between $N_{\mathrm{PL,Tx}}$ and $N_{\mathrm{PL,Rx}}$ that arises from differential absorption (the return path has a lower centre frequency than the outgoing path and therefore suffers less attenuation through absorption) [12]. The impedance ratio matters, and the precise way in which it matters depends on which sonar equation is used [12, p493]. The following discussion focuses on the impedance ratio because of the large contrasts in density and sound speed that can be encountered in planetary atmospheres.

The reciprocity principle applies to a situation with zero mean flow. In the presence of wind or strong currents, a modified form of the principle known as the "flow reversal theorem" is applicable [61]. The effect of vertical wind shear is known to be important for atmospheric acoustics [62].

Collins et al. 1995 [53] have developed methods for computing propagation loss with horizontal shear, leading to caustics due to horizontal refraction in the Jovian atmosphere.

a. Urick

Urick's active sonar equation, in the form quoted by Jensen et al. [22], replacing DI with AG for the same reason as previously, is

$$\mathrm{SE} = \mathrm{SL} - \mathrm{PL}_{\mathrm{Tx}} + \mathrm{TS} - \mathrm{PL}_{\mathrm{Rx}} - \mathrm{NL} + \mathrm{AG} - \mathrm{DT}\,, \tag{36}$$

where $\mathrm{PL}_{\mathrm{Tx}}$ is the propagation loss from sonar transmitter to target, $\mathrm{PL}_{\mathrm{Rx}}$ is the equivalent quantity for the return path from target to sonar receiver, and TS is the target strength.

It is widely assumed [22 (p714)] that $\mathrm{PL}_{\mathrm{Tx}} + \mathrm{PL}_{\mathrm{Rx}}$ in Eq. (36) may be replaced for monostatic sonar by $2\mathrm{PL}_{\mathrm{Tx}}$ (or $2\mathrm{PL}_{\mathrm{Rx}}$). However, in general $\mathrm{PL}_{\mathrm{Tx}}$ and $\mathrm{PL}_{\mathrm{Rx}}$ are not equal, even for monostatic sonar. If the speed of sound at the target position (c_{tgt}) differs from that at the sonar (c_{snr}), the two propagation loss terms are related by [12 (p493), 14 (p120)]

$$\mathrm{PL}_{\mathrm{Rx}} = \mathrm{PL}_{\mathrm{Tx}} + 10\log_{10}\frac{c_{\mathrm{snr}}^{2}}{c_{\mathrm{tgt}}^{2}}\,\mathrm{dB}\,, \tag{37}$$

where the subscripts 'snr' and 'tgt' indicate more generally a property of the sonar and target, respectively. Therefore the narrow-band active sonar equation can be written

$$\mathrm{SE} = \mathrm{SL} - 2\mathrm{PL}_{\mathrm{Tx}} + \mathrm{TS} - \mathrm{NL} + \mathrm{AG} - \mathrm{DT} + 10\log_{10}\frac{c_{\mathrm{tgt}}^{2}}{c_{\mathrm{snr}}^{2}}\,\mathrm{dB}\,. \tag{38}$$

b. ISO

The equation corresponding to Eq. (37) for N_{PL}, as defined by ISO 18405 is [63]

$$N_{\mathrm{PL,Rx}} = N_{\mathrm{PL,Tx}} + 10\log_{10}\frac{\rho_{\mathrm{tgt}}^{2}}{\rho_{\mathrm{snr}}^{2}}\,\mathrm{dB} \tag{39}$$

With this in mind, for a monostatic sonar, Eq. (12) can be written [12, 63]

$$\Delta L_{\mathrm{SE}} = L_{\mathrm{TE}} - L_{\mathrm{N}} + \Delta L_{\mathrm{PG}} - \Delta L_{\mathrm{DT}}, \tag{40}$$

where L_{TE} is the target echo level

$$L_{\mathrm{TE}} = L_{\mathrm{S}} + N_{\mathrm{TS,eq}} - 2N_{\mathrm{PL,Tx}} + 10\log_{10}\frac{\rho_{\mathrm{snr}}^{2}}{\rho_{\mathrm{tgt}}^{2}}\ \mathrm{dB}, \tag{41}$$

correcting a sign error on p493 of Ref. 12.

3. Signal level and noise level

Whether for active or passive sonar, the sonar equation can always be written in the form

$$\Delta L_{\mathrm{SE}} = L_{\mathrm{signal}} - L_{\mathrm{noise}} - \Delta L_{\mathrm{DT}}, \tag{42}$$

by writing $10\log_{10} R$ dB as $L_{\mathrm{signal}} - L_{\mathrm{noise}}$, where L_{signal} and L_{noise} noise are defined as signal and noise levels after processing. The same considerations described previously for noise level apply also to signal level. For example, Svedhem et al. [32] plot the echo level (signal level for an active sonar) in “dB,ref.20uPa”. The interpretation of this quantity depends on whether the MSP or EPWI convention is being used and, in the case of the EPWI convention, the choice of reference impedance.

III. FEATURES OF PLANETARY ACOUSTICS

The expense per bit of data for planetary exploration is very high, and so every effort must be made to foresee problems in definitions and calibrations that compromise either the design and effective use of equipment, or the end-users' ability to interpret the resulting measurements. In the study of planetary acoustics one inevitably encounters extreme conditions relative to those to which we are accustomed on Earth, resulting in the following issues:

- the small size of moons and some small planets result in curvature related effects that are usually negligible on Earth;
- the gaseous atmospheres in which acoustic sensors might operate have chemical, acoustical and thermodynamical properties very different to those encountered in Earth's atmosphere;
- the conditions on Saturn's moon Titan (low temperature combined with large reserves of light hydrocarbons) result in the formation of liquid hydrocarbon lakes;
- the limiting omnipresent thermal noise depends on the chemical and thermodynamical properties of oceans, lakes and atmospheres in a predictable way;
- high amplitudes, required when high absorption reduces the signal-to-noise ratio to unacceptable levels, create the need to consider non-linear effects.

These issues are addressed in turn below.

A. Small planets with high curvature: Europa and the icy moons

Water is a key ingredient for life, and as such its occurrence in vast quantities at the range of Jupiter and beyond (e.g. in Saturn's rings or on exoplanets) is of considerable interest, whether as a resource for human ventures, or as a possible harbour for extraterrestrial life. However, with solar radiation fluxes so weak at such distances within the Solar System, a power source is required to liquefy the ice, possibly occurring naturally through geothermal and tidal processes, etc. A surprising number of moons and dwarf planets are now thought to contain seas and oceans of liquid water (see Figure 1), some of them vast in extent compared to Earth's oceans [64]. Also noteworthy are the high curvature (illustrated by the section through Europa) and the high pressure, exceeding 1000 MPa on Ganymede and Titan. Candidate bodies include a subset of the moons of several distant planets: Jupiter's moons Europa, Ganymede and Callisto [65]; Saturn's moons Titan and Rhea; Uranus's moons Titania and Oberon; Neptune's moon Triton; the dwarf planet Ceres; and the minor planet Pluto [66].

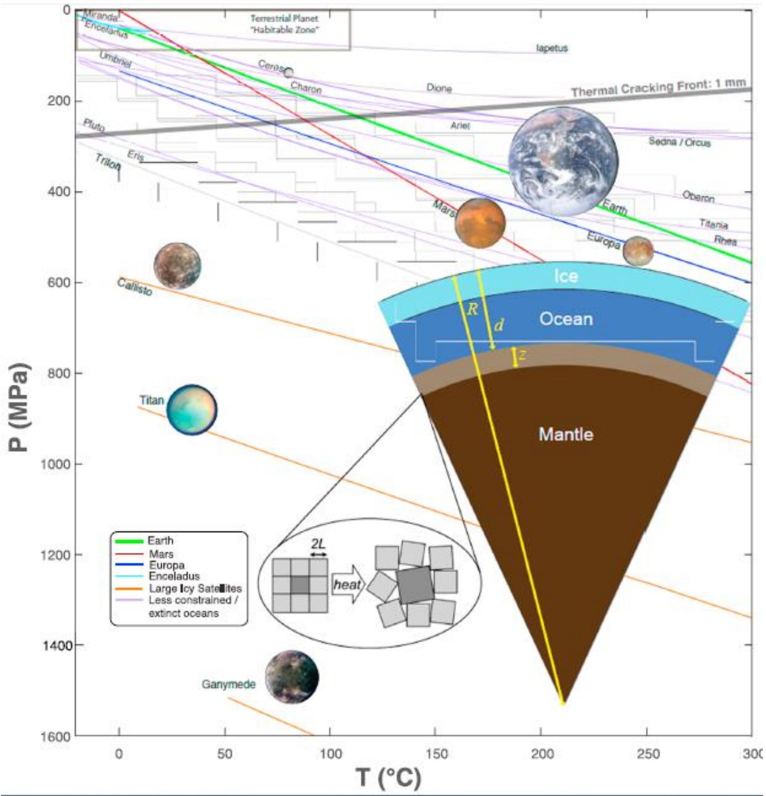


Figure 1. Temperature vs pressure profiles for worlds on which either liquid oceans are known to exist or the conditions for liquid water are thought to exist. Reproduced from Ref. [**64**].

Although the distance from the Sun causes the surface to freeze, beneath the ice, the combined effects of radiation, geothermal action, and the passage through massive planetary gravitational fields, is thought to create sufficient energy to maintain liquid water oceans beneath the frozen surface [67]. The evidence of rich chemistry on Europa [68; 69], and the knowledge that Earth supports some deep-ocean life that is not reliant on solar radiation, has stimulated planning for missions to these bodies. Given that acoustics provides by far the most useful radiation for sensing at distance in the ocean, it would be inconceivable not to equip such missions with sonar. Effective long range sonar requires propagation modelling e.g. to determine the acoustic path and path length to calculate the propagation loss. However, despite the apparent similarity to Earth's Arctic Ocean, the application of the familiar techniques developed for that environment would lead to errors in planning and interpreting sonar missions on Europa. Accounting for the effect of the curvature of small worlds when calculating the relative positions and geometries of sources, receivers and propagation paths is one requirement. Another, perhaps of greater importance is the correct calculation of hydrostatic pressure (P_h, a crucial parameter in ocean acoustics through its effect on the sound speed). This cannot be taken as equalling the product of density (ρ), acceleration due to gravity (g) and depth (h) on small worlds, partly because spherical, not Cartesian, coordinates must be used in integrating $\nabla P_h = \rho g$ (see Ref. 70), and partly because g itself is a function of depth, the depth of the ocean taking up a significant proportion of Europa's radius (Figure 1). The longer the propagation range in comparison with the planet radius, the more likely these effects would affect mission planning.

1. Europa

Sonar modelling has been done for both the ice and the ocean on Europa [31; 65, 70, 71; 72, 73]. In terms of the definitions discussed in the present paper, when calculating the terms in Table 1, various models can be used to calculate the ocean sound speeds at the base of the ice pack and bottom of the water column [73], but typical values might be 1500 and 1770 m/s, respectively. Even if variations in density are neglected [67], and 'flat world' calculations are assumed to be valid (though they are not), the change in sound speed with depth on its own will influence mission planning significantly if, say, the plan is to place a receiver at the base of the icecap in order to detect signals from a source sitting on the seabed [65].

The small size of Europa makes for interesting physics arising from high curvature, but the corresponding low pressure results in a relatively small impedance contrast. The seabed is an extreme environment for a manmade device but possibly is the location of geothermal or seismic sources of sound. In calculating the propagation loss, the $10\log_{10}(\rho_s c_s / \rho_r c_r)$ term creates a 0.7 dB ambiguity.

Whether on Europa or anywhere else, the potential for ambiguity arises the moment any physical quantity is expressed as a level in decibels, however sophisticated the calculations giving rise to that quantity. For example, Lee et al. [31] describe and execute a high fidelity procedure for predicting the sound particle velocity field as a function of time in Europa's water ocean after an ice cracking event. In the following, Eq. (n) and Fig. m from Ref. [31] are abbreviated as "LE-n" and "LF-m". Lee et al. [31] could have presented their results directly in terms of the magnitude and (if needed) the phase of this field, but chose instead to plot the quantity "horizontal velocity level" (and a corresponding quantity for the vertical component) vs time, related in an unspecified way to the horizontal component of the sound particle velocity, denoted $\dot{u}(r, z, t)$ (the caption of Fig. 13 refers to LE-C.18, but based on the evidence available to us we believe that what is plotted is the logarithm of $\dot{u}^2$, where the instantaneous quantity $\dot{u}(r, z, t)$ is given by LE-C.22). It is conventional when expressing a field quantity as a level in decibels to first convert it into a quantity proportional to power [74]. Because a squared field quantity is not itself proportional to power, an essential first step, before taking the logarithm, is to carry out a mean-square or envelope operation on the field quantity $\dot{u}(r, z, t)$ [16]. In LF-13 to LF-16 of Ref. [31] we see no evidence of any averaging (the deep nulls in LF-15 in particular are consistent with the zero crossings associated with an acoustic frequency of approximately 1.9 Hz, with no averaging), nor can we find any mention in the text of either a mean-square or envelope operation. This convention, combined with our interpretation of no averaging in LF-13 to LF-16, if confirmed, would lead to an error in the signal-to-noise ratio if any one of LF-13 to LF-16 were interpreted as the signal level; for a sine wave signal this error would be 3 dB. A further consequence of the convention is that nulls in the level of the time-averaged (or envelope) quantity can normally intuitively and unambiguously be interpreted as the consequence of coherent destructive interference between multipaths. Therefore, the pattern of deep nulls between successive peaks in these figures might be misinterpreted by some readers as the result of multi-path interference, when in reality this pattern is the trivial manifestation of successive zero crossings expected of *any* time varying oscillatory function. These avoidable ambiguities in an otherwise exemplary paper are the consequence of its authors' use of the decibel in presenting their results.

2. Ganymede

The huge pressure in Ganymede's interior of up to 1500 MPa (15,000 bar) leads to a larger discrepancy associated with a larger impedance contrast [75, 76, 77]. For example a sound speed for 2500 m/s combined with a density of 1100 kg/m^3 resulting in an impedance 2750 kPa s/m, corresponding to a 2.6 dB correction, a discrepancy that Horton's approach would eliminate.

3. Comparison with brine lakes and seawater on Earth

We can put the calculations for Europa and Ganymede into perspective by comparing these with extremes of pressure and salinity in seawater found on Earth. The highest pressures encountered in seawater are those at the bottom of the Mariana Trench, where sound speed = 1670 m/s and density = 1080 kg/m^3 are thought to occur [78]. High impedance can also arise from extreme salinity conditions in brine lakes, which can have a density of up to 1200 kg/m^3 [79] and a sound speed up to 1600 m/s (see Ref. 75). In both cases the correction is of order 0.8 dB, comparable to the situation on Europa.

B. Planets with gaseous atmospheres

Large planets with a strong gravitational attraction typically have dense gaseous [80], occasionally supercritical atmospheres [81, 82]. Sound propagates well in dense atmospheres because of its relatively low compressibility compared with the rarer atmospheres of smaller planets, making sound a useful alternative to electromagnetic waves for sensing planets like Jupiter

[53, 83] or Venus [84, 85, 86, 87 , 88]. Issues arise related to dense or rare atmospheres, large density changes (reciprocity), high mean flow (wind) and the choice of reference sound pressure and reference sound intensity.

1. Fluid loading

Several devices designed for planetary probes use components that vibrate in a known manner with known characteristics (such as the active acoustic transducers on anemometers [33] or sound speed measuring systems [88]). Other devices might have vibrations that we wish to damp out (such as in structural members of proposed instruments [89], dirigibles, ocean or land vehicles [90]). For such devices it is important that we know their vibrational characteristics, which are determined in large part by the stiffness, inertia and damping associated with the member. These latter two can, in particular, be strongly influenced by the density or compressibility fluid that surrounds the device, and if devices are designed, calibrated, tested and validated on Earth, then appropriate compensation needs to be made for the extraterrestrial environment. Leighton [44] illustrated simple trends in terms of the inertia. If the vibrating body is surrounded by an alien atmosphere that is more dense than the atmosphere on Earth in which it was calibrated, then the inertia associated with moving this fluid (from its "added mass") will tend to be greater than when the device was tested on Earth, reducing its resonance frequencies (as when a device tested in Earth's atmosphere is deployed at ground level on Venus or Titan). Conversely, if the transposition is instead to a rarer atmosphere such as on Mars, the inertia associated with fluid loading there will tend to be reduced, increasing the resonance frequencies of the device. Such effects would need to be taken into account if the changes of resonance frequencies of vibrating surfaces on Mars are to be interpreted in terms of an accumulation of mass upon that surface as a measure of some natural deposition process on Mars [91, 92]. Approaching any planet with an atmosphere, a probe would pass through regions where the density is less than that found at ground level on Earth, and on some (particularly giants like Jupiter, Saturn, Uranus and Neptune) could pass eventually into atmospheres far more dense than those of Earth, and it is difficult to estimate reliably how far into the atmosphere of such a giant a future acoustic probe would penetrate [44].

However, the extent to which the difference in the density of the fluid (between Earth and the deployment site) affects the inertia associated with the sensor, depends on the geometry of the structure in which the vibrating component is housed. If the fluid is allowed to move freely in all directions, the effect is far less than if the fluid motion is constrained [44] (for example in a tube [93] or between plates [94]) because such constraint increases the proportional contribution that the fluid makes to the inertia of the whole vibrator. Constraint within a rectangular casing [95] or pipe [96] can affect both resonant frequency and damping.

The effect of this on the sonar equations comes about when mounting is used to affect the directivity index, or changes the frequency-dependent voltage/motion/pressure transfer function of an emitter or sensor, so changing the source level or receiver sensitivity , and hence the sonar figure of merit [12].

2. Reciprocity

Use of sonar in any gaseous atmosphere is likely to encounter large differences in density, depending on the relative height of the source and receiver. In such situations the $10\log_{10}\rho_{\mathrm{tgt}}^2/\rho_{\mathrm{snr}}^2$ term (see Eq. (39)) will result in important corrections if the reciprocity principle is invoked to interchange the positions of source and receiver, whether for passive or monostatic active sonar.

3. Reference values for levels in decibels: EPWI and MSP conventions

When expressing a physical quantity as a level in decibels, the physical quantity is first divided by a reference value (see Table 2 and Introduction) of that quantity before taking a logarithm. In order to retrieve the original value of the physical quantity from its level, this operation needs to be reversed, which is only possible if the reference value is known. If the reference value is not reported the original value of the physical quantity is lost.

Table 2. International standard reference values of sound pressure, sound intensity, sound power and source factor in liquids, and where applicable in gases [10] (conventional values are included in brackets where these depart from the International Standard [11]).

medium	**sound pressure**	**sound intensity**	**sound power**	**radiant intensity**	**source factor**
gas	20 μPa	1 pW/m^2	1 pW	-	-
liquid	1 μPa	1 pW/m^2 (1 μPa^2/$\rho_0 c_0$)	1 pW (1 μPa^2 m^2/$\rho_0 c_0$)	(6.5×10^{-7} pW/sr)	1 μPa^2 m^2

The standard reference pressure for sound in water and other liquids (1 μPa) is different from that in gases (20 μPa) [10]. The standard reference intensity is $I_0 = 1\ \mathrm{pW/m^2}$, taking the same value for all gases and liquids, making its use uncontroversial for planetary exploration. However, the standard reference intensity is rarely (if ever) followed in underwater acoustics. Instead, sonar modellers use a reference intensity of $p_0^2/\rho_0 c_0 \approx 6.5\times10^{-7}$ pW/m^2, [6] based on the intensity of a plane wave in seawater whose rms sound pressure is 1 μPa (see Ref. 4). When using this convention, levels are then reported as the "level in dB re 1 μPa", giving the impression that a SPL is being reported, when it is actually the level of the equivalent plane wave intensity (EPWIL). In seawater, the difference between SPL (in dB re 1 μPa^2) and EPWIL (in dB re 6.5×10^{-7} pW/m^2) is very small, and for this reason the distinction between them is rarely made.

For sound in both gases and liquids, the standard reference sound power is 1 pW. However, the concept of sound power is rarely used in sonar modelling, being replaced by the radiant intensity, the integral of which over solid angle gives the source power. While values of source level in water are usually stated in units of "dB re 1 μPa @ 1 m" or similar [34], what is meant by this is the level of the source factor, in decibels relative to 1 μPa^2 m^2 (see Refs. 11, 12). This reference value of source factor corresponds to a radiant intensity of 6.5×10^{-7} picowatt per steradian (pW/sr). The differences between standard and convention, and between standards for gases and those for liquids, are bound to lead to confusion and misunderstandings when quantities are reported as levels in decibels, unless both the reference value and the convention being followed is stated explicitly each time, and even then any numerical comparison between a value of sound pressure level in a gas and one in a liquid is complicated by the lack of a common reference sound pressure. In principle this could be resolved by agreeing on a common reference sound pressure for gases and liquids, but such harmonization seems unlikely in the near future because the current values are firmly entrenches in standards [10, 11, 97, 98] and in practice [56].

The solution to this seemingly unsurmountable problem is surprisingly simple: all that is required is to follow Horton's 60-year old advice [14] to express the sonar equation terms in terms of EPWI ratios, and with the same standard reference intensity, regardless of circumstances. The EPWIL, L_J,

$$L_J \equiv 10\log_{10} J/I_0 \text{ dB} \tag{43}$$

is related to sound pressure level (SPL), L_p, via

$$L_J = L_p + 10\log_{10}\frac{p_0^2}{\rho c I_0}\text{ dB.} \tag{44}$$

The value of this correction is listed in Table 3 for situations representative of Earth's ocean and atmosphere, Titan's lakes and atmosphere, and Ganymede's ocean. Also included in the table (final column) is the EPWIL corresponding to an rms sound pressure of one pascal, ranging for the examples given from 54 dB re 1 pW/m^2 (Ganymede's ocean at 1000 MPa) to 100 dB re 1 pW/m^2 (Jupiter's atmosphere at 0.1 MPa).

Table 3. Corrections to convert from SPL to EPWIL, for a reference sound intensity of $I_0 = 1\ \mathrm{pW/m^2}$, for selected example conditions on Earth, Titan, Ganymede, Venus and Jupiter. The right-most column contains the value of EPWIL corresponding to an rms sound pressure of 1 Pa.

conditions	ρc / (kPa s/m)	p_0 / μPa	$10 \log_{10} \dfrac{p_0^2}{\rho c I_0}$	$L_J(p_{\mathrm{rms}}=$ 1 Pa) / dB
seawater @ 10 °C (Earth) [12]	1530	1	-61.8	58.2
air @ 10 °C (Earth)	0.4205	20	-0.2	93.8
hydrocarbon lake (Titan) [60]	1 285	1	-61.1	58.9
N_2-rich atmosphere @ 0.15 MPa (Titan) [60]	1.180	20	-4.7	89.3
water @ 1000 MPa (Ganymede) [75, 77]	3549	1	-65.5	54.5
CO_2-rich atmosphere @ 9.2 MPa (Venus)	26.65	20	-18.2	75.7
H_2-rich atmosphere @ 0.1 MPa (Jupiter)	0.095	20	6.2	100.2

4. *The gas giants Saturn and Jupiter*

The outer planets of the Solar System include the two gas giants (Saturn and Jupiter) consisting primarily of hydrogen and helium. The outer layer of molecular hydrogen contains clouds of crystalline ammonia, ammonia sulphide and water. There is no sharp boundary between the gaseous and liquid hydrogen layers in this so-called ‘inner atmosphere’, which is 21,000 km thick; it surrounds a peculiar zone that takes up most of the volume of the planet: a 40,000 km-thick layer of liquid hydrogen that has, under the extreme pressures, become electrically conducting and so is termed ‘metallic’. A rocky core, possibly molten, probably characterizes the centre of the gas giant.

This is a fascinating environment for acoustics. The fortuitous collision of Comet Shoemaker-Levy 9 into Jupiter allowed some authors to consider the propagation of pressure waves in the atmosphere [53, 83; 99], facilitated by data from the Voyager mission [100]. Leighton [44] considered the fluid-structure interactions on man-made probes introduced into Jupiter's atmosphere. He calculated conditions for two locations of possible interest for future probes to Jupiter: (i) the 1 bar altitude, at an equatorial radius of 71,500 km from Jupiter's centre, where P_h = 100 kPa (1 bar), ρ = 0.1 kg m^{-3}, and T ~ 165 K; and (ii) the estimated 'maximum operational penetration depth' of some future very robust probe, which he estimated by extrapolating from current terrestrial seismic sensors could withstand a maximum static pressure of P_h ~ 900 MPa, which he calculated to occur 69,600 km from Jupiter's centre, where T ~ 2000 K and ρ ~ 50 kg m^{-3}. An acoustic transmitter, dropped from the dirigible at the 1 bar altitude, would fall about 1,900 km before reaching this limit of operation. Ref. 44 compared the fluid loading on a range of structures at these two altitudes, and considered how the change in the density around them would affect their natural and resonance frequencies, concluding that the natural frequencies of some components, notably pipes, as the structure descended would be almost halved. Pipes were acoustically interesting for other reasons: Jiang et al. [43] considered an acoustical device that consisted of a pipe, with a sound source at one end and a receiver at the other, which was proposed for use on Venus and later the Jovian planets. The speed of sound pulses in the atmosphere, as measured by the propagation time in this pipe, could be used to infer atmospheric properties. However because space is limited on probes, this pipe was coiled into a spiral. Whilst this device worked well on Earth, Jiang et al. [43] showed that the dense atmosphere on Venus would couple to the material of the pipe walls and allow the acoustic pulse to 'short-cut' between arms of the spiral, artificially reducing the propagation time.

Fluid loading and coupling are just two of the acoustically-relevant fluid-structure interactions, and these calculations assume that the properties of the structure itself remain unaffected by the extreme change in conditions as it descends. The sound speed profile in the gas giants tends to favour the formation of an acoustic waveguide, with an axis close to Earth's atmospheric pressure [101] (see Figure 2).

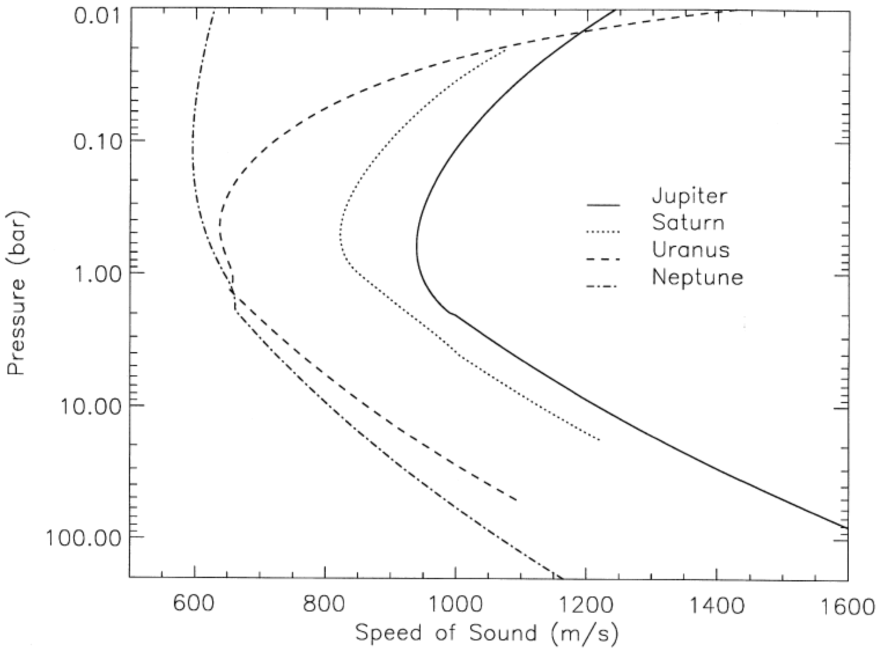


Figure 2. Sound speed vs pressure profiles for the gaseous atmospheres of Jupiter, Saturn, Uranus and Neptune. The significance of 100 kPa (1 bar) is that it correspondence approximately to atmospheric pressure on Earth. Reproduced from Ref. [**101**].

In a *tour de force*, Collins et al. [53] demonstrated the important effect of wind on sound propagation in the Jovian atmosphere, with wind speed up to 150 m/s at equatorial latitude, compared with a sound speed of 800 m/s at the channel axis (calculated from Lindal et al. [100] temperature profile assuming ideal diatomic gas – Lorenz [101] predicts a higher value, taking into account an expected increase in the specific heat ratio of hydrogen with decreasing temperature) caused by a temperature minimum (100 K), with the resulting horizontal wind shear resulting in caustics and focusing at predictable locations. Allison [102] documented a variety of waves observed propagating in Jupiter's atmosphere at speeds between 40 m/s and 70 m/s. In addition to acoustic waves, also gravity waves [103] are affected by the strong horizontal wind shear.

5. The ice giants Uranus and Neptune

While no opportunity has yet arisen to study acoustic waves on either of the ice giants Uranus or Neptune, the sound channel of Uranus (Figure 2) seems well suited to long distance propagation. Equatorial waves have been observed on both ice giants [104, 105] and gravity waves have been observed in Neptune's atmosphere [105]

6. Venus

On Venus the atmospheric density at the surface of the planet is not dissimilar to that at the 'maximum operational penetration depth' position discussed for Jupiter (above). On Venus's floor the atmosphere is about 50 times more dense (~65 kg/m^3) than Earth's (~1.29 kg/m^3) and its speed of sound is also greater (~410 m/s on Venus cf. ~340 m/s on the Earth). The increased density and sound speed of the ground-level atmosphere of Venus give it a characteristic acoustic impedance of about 27 kPa s/m, which is 60 times larger than that found in Earth's atmosphere, of 0.44 kPa s/m. This factor 60 leads to an ambiguity of about 18 dB (i.e., $10\log_{10}60$ dB) in the interpretation of levels expressed using the traditional conventions of underwater acoustics and sonar [4, 15, 22], as exemplified by Eq. (19).

Parts of Venus's atmosphere consist of supercritical fluid CO_2 [81], meaning that it behaves neither as a gas nor as a liquid. This inevitably raises the issue of reference value in extraterrestrial acoustics, and illustrates the need to harmonise standards for liquids and gases.

C. Titan's hydrocarbon lakes

Prior to the successful landing of the Huygens probe on Titan on 14 January 2005, there was considerable speculation and prior calculation on the acoustics of Titan, both by those who had built and planned the Huygens mission, and by other enthusiasts. Titan is a remarkable acoustical world, its surface temperature of 92 K allowing it to retain its mainly nitrogen-based atmosphere with a surface pressure of around 150 kPa (1.5 bar), giving lower acoustical absorption that Earth's own atmosphere [106]. The possibility of sound travelling to long distance prompted the prediction, prior to Huygens' landing, of the sounds that Titan's 'waterfalls' (made of liquid ethane and methane) might make, and whether a lander with a microphone might detect and observers might recognize such sounds as emanating from a methanefall [60] or a splashdown. The same opportunities for long-distance sound propagation at audio frequencies promoted the predictions of the sounds man-made structures might produce, musical instruments and voices being chosen for outreach purposes [88], but with the knowledge that these principles for extraterrestrial fluid-structure interactions must be elucidated to design extraterrestrial dirigibles and submersibles [44, 107]. With a dense atmosphere that has low acoustic absorption, and mysterious lakes and (at least for a period) flowing liquid, the possibilities for acoustic exploration of Titan are great.

In 2001, Garry and Towner [108] stated that "The Huygens probe *en route* to Titan carries a 15 kHz non-beam forming sonar…that delivers a signal of ~80 dB (ref 20 μPa) in the laboratory. In the event of landing in a sufficiently deep body of liquid, the sensor works as a bathometer, inferring the 'sea' depth from the echo's delay". The present authors have as yet been unable to ascertain either the distance from the source at which the reported level was measured or the medium in which the measurement was made. A laboratory representation of the expected atmosphere on Titan is mentioned and might have been used for these measurements, but the present authors have not yet been able to access the associated publications [49; 109]. Although it was designed for depth-finding in Titan's lakes, this ~15 kHz active sonar also provided good echoes from the surface as the probe descended through the atmosphere [39; 48]. According to Leese et al., [48], Garry [49] had estimated a sound pressure level of "around 100 dB near Titan's surface", corresponding to "a first return at 100 m altitude", with no indication given in Leese's paper either of the reference value or of the assumed conditions.

It was only after Huygens' actual landing that the presence of hydrocarbon lakes was confirmed, a notable one being *Ligeia Mare*, a several-hundred-kilometre wide lake near Titan's north pole. In 2013, Arvelo and Lorenz [34] described a possible future Titan Mare Explorer (TiME) mission, which would splashdown a capsule to operate for three months. Among TiME's scientific goals is the determination of the depth of *Ligeia*, using an acoustic depth sounder.

Arvelo and Lorenz conducted a theoretical study of the likely performance of this depth sounder. For the noise level term they used a prediction from Ref. 58 that the "power spectral density for bubble entrainment noise" was expected to be about 10 dB higher on Titan than on Earth for the frequency of interest, from which Arvelo and Lorenz estimated the wind-driven noise level to be "NLo = 40 dB//1 μPa^2/Hz ".

Not one of the above-mentioned publications mentions, in association with the signal or noise level in decibels, either the reference value of sound intensity or the impedance used to calculate that reference intensity, which means that the reader is left to guess. Our purpose in making this point is not to criticize any of the authors but to point out the complacency of conventional practice in underwater acoustics, and the consequences of this complacency if transferred to planetary exploration. If Ref. 34 adheres to Urick's definition of noise level as stated in Eq. (20), for example, does this imply the impedance of seawater is being assumed for the reference intensity or some other (unspecified) nominal characteristic acoustic impedance of the nitrogen atmosphere or the liquid of *Ligeia*? In the latter case, depending on the chosen value for impedance, the reference intensity might be anything from 6.5×10^{-7} pW/m^2 (if the impedance of seawater is used to define the reference intensity) to 14.9×10^{-7} pW/m^2 (using the impedance of liquid methane on Titan's surface). Without a clear specification of the reference intensity, any statement about noise level on Titan incorporates an inherent factor of 2.4 uncertainty in the intended value of $J_{\mathrm{amb,N},f}$ in Eq. (20), corresponding to 3.8 dB uncertainty in the level. If such calculations are being undertaken, the issues highlighted in this paper need to be addressed during the planning of any future Titan mission [107].

D. Extraterrestrial thermal noise

Whether in a gas or a liquid, all sonars are limited in their performance by noise, whether this be from ambient noise, reverberation, electrical noise, etc., and on how and where it is used. Any medium that supports sound is also a source of thermal noise [110], which determines the lower bound for noise level for all sonar. In general its value depends on temperature, pressure and the chemical composition of the medium.

The properties of thermal noise in any medium are related to the same thermodynamical properties of the medium that determine its density and speed of sound. Once we know the chemical composition of a planet's ocean or atmosphere, we can study thermal noise in that ocean or atmosphere from a theoretical perspective, using properties of the appropriate chemical elements or compounds measured on Earth. Conversely, a measurement of thermal noise tells us something about the chemistry, such as information about the molecular mass and specific heat ratio of a gas.

1. Thermal noise in any fluid

It is known [110] that the EPWI spectral density at frequency f caused by thermal noise, in any gas or liquid, is

$$J_{\mathrm{N},f} = \mu f^2, \tag{45}$$

where μ is a constant, henceforth referred to as the 'thermal noise coefficient'. It turns out that this constant has dimensions of mass, and for an ideal gas is proportional to (and an order of magnitude larger than) the molecular mass. The thermal noise coefficient is equal to about 500 yg for Ar, O_2, C_2H_6 (one yoctogram (1 yg) is equal to 10^{-27} kg = 1 aW/(m^2 kHz^3)), and for liquids is of order 10 yg. It thermal noise coefficient can be written in terms of Boltzmann's constant ($k = 1.38065 \times 10^{-23}$ J/K) and absolute temperature T

$$\mu = 4\pi\frac{kT}{c^2}. \tag{46}$$

2. Thermal noise in an ideal gas

a. Characteristic acoustic impedance

Consider an ideal gas of pressure P and density ρ that obeys Boyle's law in the form

$$\rho = Pm/kT, \tag{47}$$

where m is the mean molecular mass. The speed of sound in a gas with polytropic index Γ is

$$c = \sqrt{\frac{\Gamma kT}{m}}, \tag{48}$$

where Γ is equal to unity for isothermal fluctuations and to the specific heat ratio, γ, for adiabatic ones. In the high frequency limit, the fluctuations are expected to be isothermal [111, p351]. Combining Eqs. (47) and (48), the characteristic impedance is

$$\rho c = P\sqrt{\frac{\Gamma m}{kT}}, \tag{49}$$

consistent with [32].

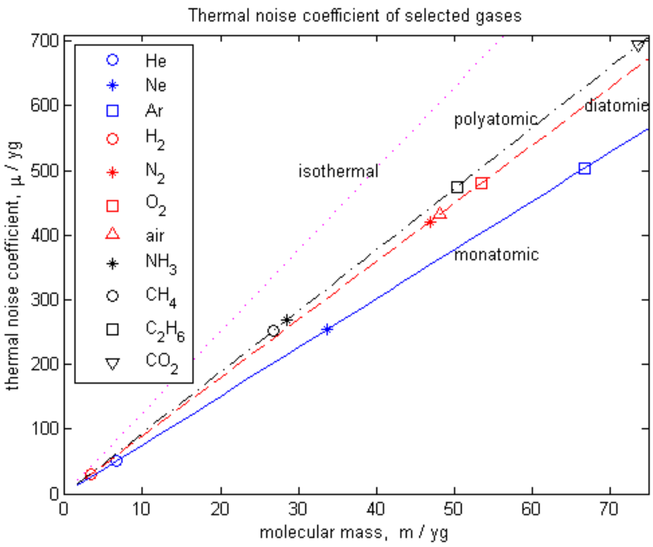
b. Thermal noise coefficient

Substituting Eq. (48) in Eq. (46) for μ gives

$$\mu = 4\pi \frac{m}{\Gamma}. \tag{50}$$

Equation (50) above, applicable to any ideal gas, is an extraordinarily simple result: at frequencies of interest, for which $\Gamma = \gamma$, the thermal noise coefficient depends only on the molecular mass and the specific heat ratio. The value of the thermal noise coefficient μ is then equal to about 30 yg for hydrogen and 430 yg for air, as illustrated by Figure 3a for gases with molecular mass up to 75 yg.

A consequence of this simple result is that the EPWI thermal noise in an ideal gas is independent of temperature and pressure. The MSP thermal noise is proportional to $P\ T^{-1/2}$ because of the additional impedance factor – see Eqs. (49) and (53).



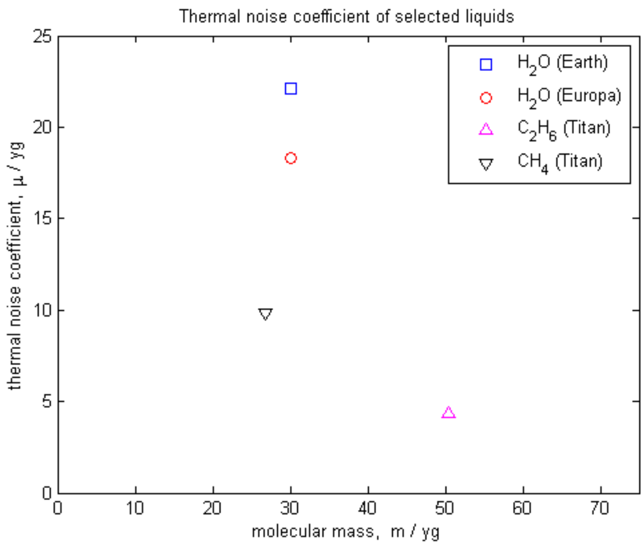


Figure 3. Thermal noise coefficient μ (constant of proportionality in Eq. **(45)**) vs molecular mass. Upper: for selected gases; the dotted line is for an isothermal gas ($\Gamma = 1$); other lines are for adiabatic gases with $\gamma = 4/3$ (polyatomic), 7/5 (diatomic) and 5/3 (monatomic). Lower: for selected liquids.

3. Thermal noise in liquids and non-ideal gases

The thermal noise coefficient in fluids other than ideal gases can be estimated using Eq. (46). This quantity depends directly on temperature and indirectly (through the speed of sound) on the pressure and chemical composition of the liquid or solid. Some examples for liquids from Table 4 are plotted in Figure 3b.

Table 4. Thermal noise coefficients of liquids.

Category	T/K	c / m s^{-1}	ρ / kg m^{-3}	ρc/ (kPa s/m)	$4\pi kT/c^2$ / yg
seawater on Earth	283	1490	1027	1530	22.1
water on Europa	270	1600	1000	1600	18.3
liquid ethane on Titan	95	1920	630	1210	4.5
liquid methane on Titan	95	1275	525	669	10.1

4. Use in the sonar equation(s)

For use in the sonar equation, Eq. (45) for $J_{\mathrm{N,f}}$ can be integrated over the receiver frequency band f_1 to f_2. The result can be expressed in terms of the arithmetic mean $f_{\mathrm{am}} = (f_1+f_2)/2$ and the geometric mean $f_{\mathrm{gm}} = (f_1\, f_2)^{1/2}$

$$\frac{p_{\mathrm{N}}^2}{\rho c} = \frac{\mu}{3} B\left(4f_{\mathrm{am}}^2 - f_{\mathrm{gm}}^2\right), \tag{51}$$

where $B = f_2 - f_1$. It follows from Eq. (21) that

$$L_{\mathrm{N}} = 10 \log_{10} \frac{\rho c \mu B\left(4f_{\mathrm{am}}^2 - f_{\mathrm{gm}}^2\right)/3}{p_0^2} \text{ dB}. \tag{52}$$

Converting to NL for Urick's sonar equation using Eq. (22) then gives

$$\mathrm{NL} = 10 \log_{10} \frac{\mu\left(4f_{\mathrm{am}}^2 - f_{\mathrm{gm}}^2\right)/3}{I_0 B_0^{-1}} \text{dB}. \tag{53}$$

E. Non-linear effects

Although by no means restricted to extraterrestrial acoustics, the issue of nonlinear propagation is mentioned because the high atmospheric absorption means that the detection of low frequency signals detected from a distance, and short-range high frequency signals emitted by planetary probes, might mean that these detected signals were high amplitude at source. Much of acoustics is based on models of low amplitude (linear) fluctuations, and such models generate errors if the material or convective nonlinearities become significant [112]). Nonlinear propagation might be expected in planetary acoustics in some circumstances, such as when sufficiently close to strong sources (volcanoes, meteorites, and meteors in thick atmospheres [113, 114]) or when such emissions in a higher density environment generate high sound particle velocities on reflection from the interface with a lower density environment (e.g., propagation of long-wavelength perturbations on Venus, crossing from its dense lower layer (reaching ~ 9 MPa (90 bar)) into the middle atmosphere (reaching < 0.1 MPa). Neglect of the presence of higher frequency energy that can result from such phenomena can lead to an underestimation of calculations of the absorption that occurs during propagation (and therefore an underestimate of propagation loss) and consequent underestimation of source level; or if the receiver is in the region where nonlinear propagation occurs and has insufficient bandwidth, it can fail to measure the higher frequency energy. This might be a consideration not only to the above examples of long distance propagation of low frequency sound or infrasound from natural sources, but also if (for example) man-made sources (e.g. in an anemometer) produce initially high amplitude signals to ensure a sufficient signal-to-noise ratio on reception.

IV. CONCLUSIONS

A term-by term comparison between the sonar equations originating from Urick's book [4] and those from a recently published international terminology standard [11] reveals strong superficial similarities but important differences in the definitions of the individual terms in these equations (see Table 1). The main differences involve ratios of impedance ratios (affecting source level, propagation loss, and noise level), bandwidth (source level and noise level), and filter gain (processing gain and detection threshold). Urick's sonar equation terms involve ratios of equivalent plane wave intensity (EPWI), with a reference intensity equal to $p_0^2/\rho_0 c_0$, where p_0 is the standard reference sound pressure (1 μPa), and $\rho_0 c_0$ is an unspecified reference impedance; this form of the sonar equation creates confusion because it conflicts with international standards and is of limited utility for extraterrestrial applications because of the ambiguity in reference intensity. The ISO sonar equation terms involve ratios of mean-square sound pressure (MSP), with a reference sound pressure of 1 μPa; this form of the sonar equation would be satisfactory for use in liquids, but the reference pressure conflicts with the international standard value for gases.

The solution to both problems is to adopt Horton's sonar equation expressed in terms of EPWI ratios, with the international standard reference sound intensity of 1 pW/m^2. Any confusion associated with uncertain reference pressure or failure to specify one's choice between MSP and EPWI conventions is eliminated by following Horton's convention.

When the value of a physical quantity is reported as a level in decibels, ambiguity results from the common practices such as a) use of a physical quantity that is not proportional to power, b) failure to specify the nature of the physical quantity, and c) partial or complete omission of the corresponding reference value. Perhaps the most common ambiguity is of type b), and in particular the failure to specify whether the EPWI or MSP convention is being followed. This ambiguity was introduced in the 1980s [see Ref. 9] and remains to this day, as illustrated by the examples provided in the present paper.

The atmospheres, lakes and oceans in which extraterrestrial acoustic sensors might operate have acoustical, chemical, and thermodynamical properties very different to typical conditions on Earth. The limiting omnipresent thermal noise depends on these properties in a predictable way. For example, the EPWI thermal noise coefficient is proportional to the ratio of molecular mass to specific heat ratio, independent of temperature T and pressure P. The corresponding MSP coefficient, on the other hand, is proportional to $P/T^{\frac{1}{2}}$.

Given that acoustics provides by far the most useful radiation for sensing at distance in liquid oceans, it would be inconceivable not to equip exploratory missions to Titan and other icy bodies with sonar. The ambiguities encountered on Earth are amplified by the exotic conditions found on moons and planets. Given the huge investment in resource to undertake such a mission, and the ~7 year transit time of a probe to the gas giants, it would be regrettable if avoidable errors in concepts were to prevent the successful acquisition or interpretation of mission data. The purpose of this paper is to alert its reader to possible errors and ambiguities in modelling the performance of acoustical systems intended for planetary exploration. Horton's sonar equations, with a single, already unified international standard reference intensity for gases and liquids provide an opportunity to start with a clean slate.

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FIGURE CAPTIONS

Figure 1. Temperature vs pressure profiles for worlds on which either liquid oceans are known to exist or the conditions for liquid water are thought to exist. Reproduced from Ref. [64].

Figure 2. Sound speed vs pressure profiles for the gaseous atmospheres of Jupiter, Saturn, Uranus and Neptune. The significance of 100 kPa (1 bar) is that it correspondence approximately to atmospheric pressure on Earth. Reproduced from Ref. [101].

Figure 3. Thermal noise coefficient μ (constant of proportionality in Eq. (45)) vs molecular mass. Upper: for selected gases; the dotted line is for an isothermal gas ($\Gamma = 1$); other lines are for adiabatic gases with $\gamma = 4/3$ (polyatomic), 7/5 (diatomic) and 5/3 (monatomic). Lower: for selected liquids.

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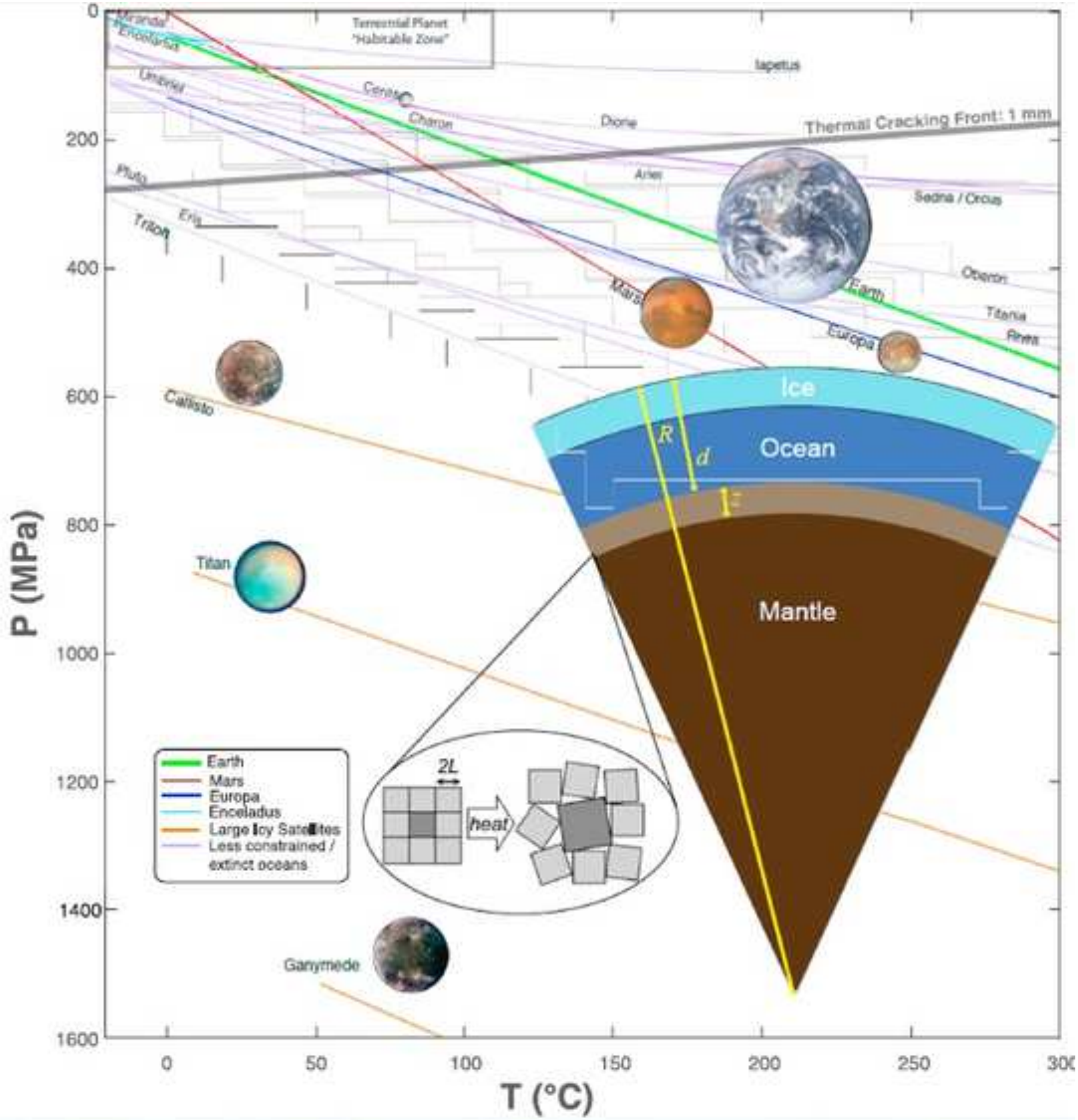
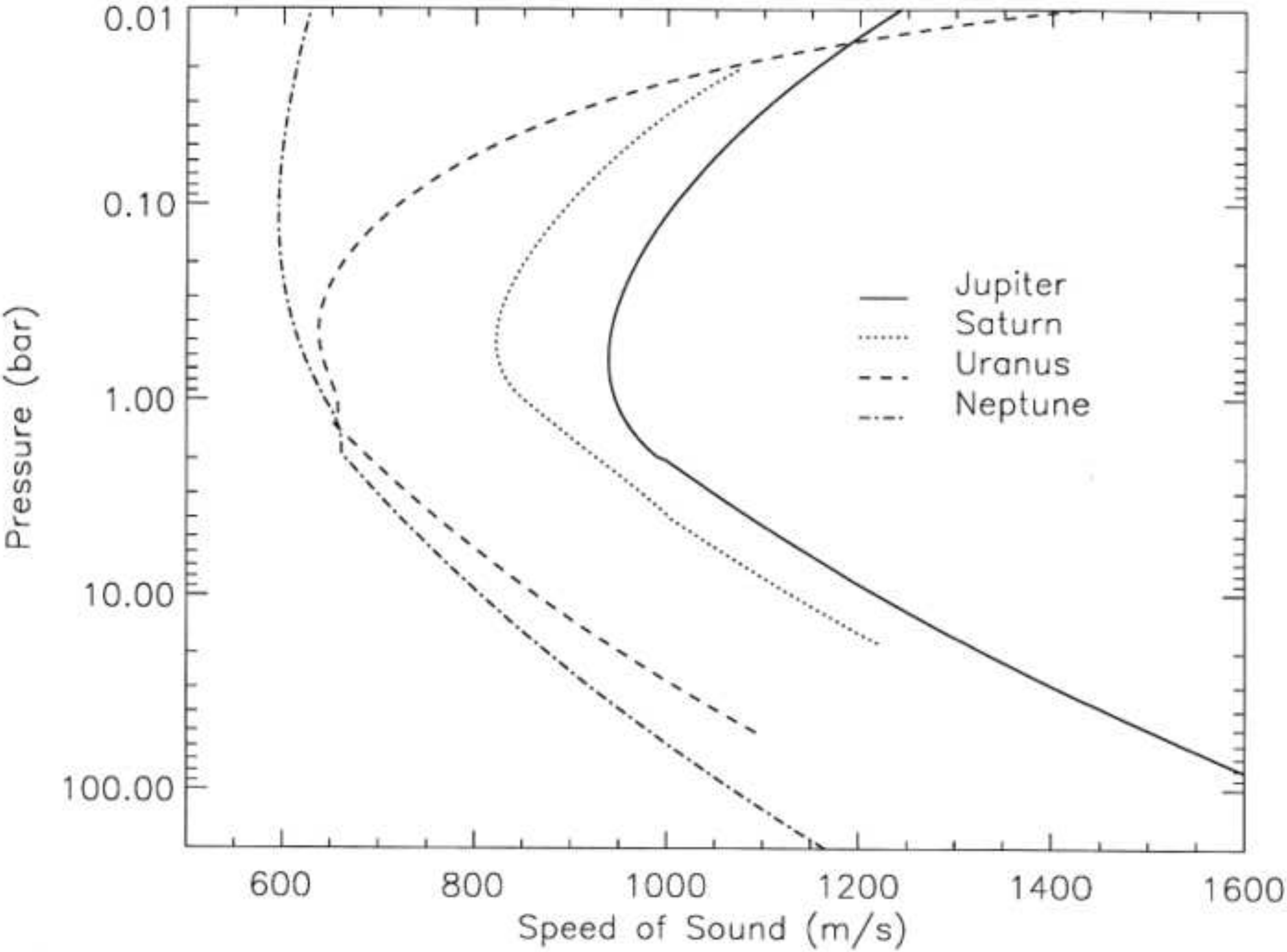
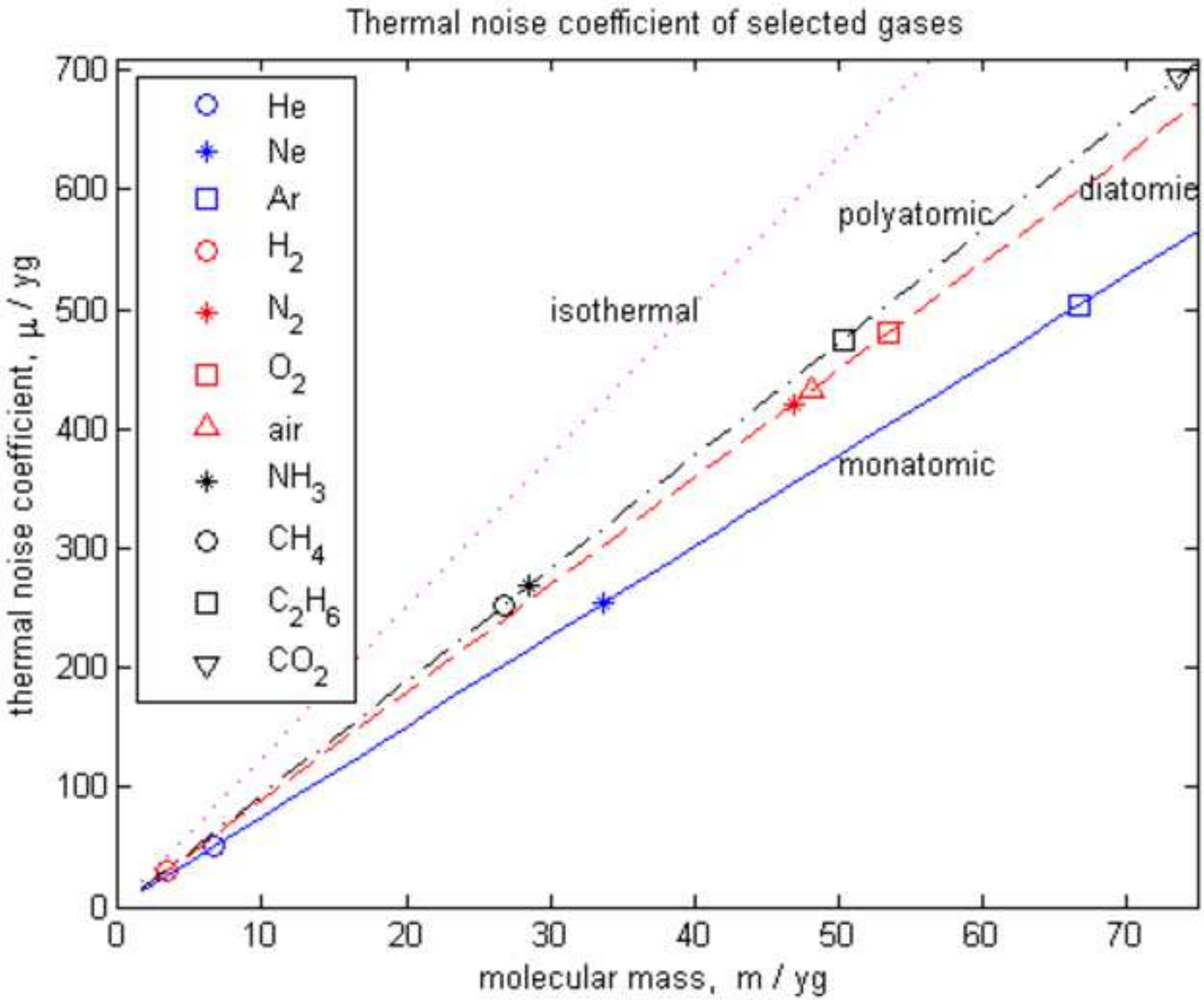
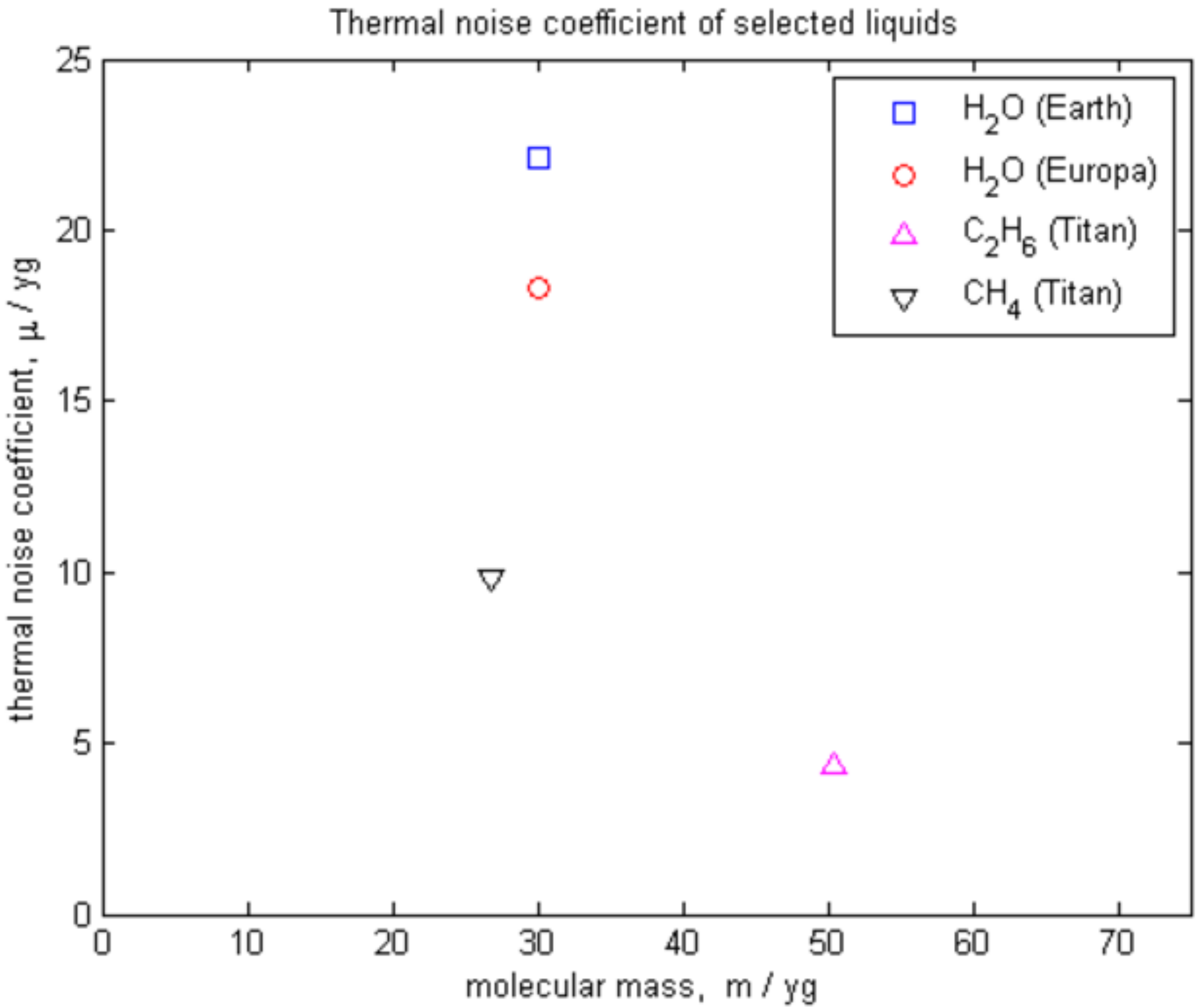



0.01
0.10
1.00
10.00
100.00
Pressure (bar)
600
800
1000
1200
1400
1600
Speed of Sound (m/s)
Jupiter
Saturn
Uranus
Neptune


Thermal noise coefficient of selected gases
thermal noise coefficient, μ / yg
molecular mass, m / yg
0
100
200
300
400
500
600
700
0
10
20
30
40
50
60
70
He
Ne
Ar
H2
N2
O2
air
NH3
CH4
C2H6
CO2
isothermal
polyatomic
diatomic
monatomic


Thermal noise coefficient of selected liquids
thermal noise coefficient, μ / yg
molecular mass, m / yg
0
5
10
15
20
25
0
10
20
30
40
50
60
70
H2O (Earth)
H2O (Europa)
C2H6 (Titan)
CH4 (Titan)