



TURBULENCE-CASCADE INTERACTION NOISE USING AN ADVANCED DIGITAL FILTER METHOD

Fernando Gea-Aguilera, and James Gill

University of Southampton, Faculty of Engineering and the Environment, SO16 7QF Southampton, UK

Xin Zhang

The Hong Kong University of Science and Technology, Department of Mechanical and Aerospace Engineering, Clear Water Bay, Kowloon, Hong Kong SAR, China

email: aexzhang@ust.hk

Thomas Nodé-Langlois

Airbus Operations S.A.S., Acoustics Department, Numerical Methods, 31060 Toulouse, France

Fan wakes interacting with outlet guide vanes is a major source of noise in modern turbofan engines. In order to study this source of noise, the current work presents two-dimensional simulations of turbulence-cascade interaction noise using a computational aeroacoustic methodology. An advanced digital filter method is used for the generation of isotropic synthetic turbulence in a linearised Euler equation solver. A parameter study is presented to assess the influence of airfoil thickness, mean flow Mach number, stagger angle and gap-to-chord ratio on noise. Results are validated against predictions from an analytical method for two-dimensional flat plate cascades. Fan wake modelling is also addressed by extending the advanced digital filter method to account for spatial variations of the turbulence intensity.

1. Introduction

Airworthiness authorities are becoming increasingly strict in terms of noise pollution. Stringent targets have been set in Europe for noise reduction by 2050, which intend to reduce perceived aircraft noise by 65% against a baseline from the year 2000 [1]. In order to keep a sustainable growth of aviation, a reduction of aircraft noise emissions is necessary in the coming years. Engine fan broadband noise is one of the main contributors to noise pollution in the current generation of commercial aircraft. Particularly, the interaction of fan wakes with Outlet Guide Vanes (OGVs) has been found to be the dominant source of broadband noise in experiments of a 18-inch fan rig [2].

The objective of the current work is to improve understanding of the turbulence-cascade interaction noise through a number of two-dimensional Computational AeroAcoustic (CAA) simulations. Early works in this area focused on the interaction of harmonic gusts with airfoil cascades. For instance, Namba and Schulten [3] proposed a three-dimensional benchmark case to study a simplified cascade geometry. Fan wakes were modelled by a discrete sum of harmonic gusts. Another benchmark case was proposed by Envia [4] on a quasi two-dimensional cascade configuration with a realistic OGV geometry. Three harmonic gusts were introduced into the computational domain and convected downstream by the mean flow.

More realistic three-dimensional configurations of turbulence-cascade interaction noise were examined by Polacsek *et al.* [5]. Fan wakes were generated by a stochastic method based on a summation of Fourier modes, whose amplitude was modulated by an isotropic turbulence spectrum. The

study was limited to a discrete number of frequencies and only velocity fluctuations perpendicular to the airfoil chord were considered.

Wohlbrandt *et al.* [6] adapted the Random Particle-Mesh (RPM) method [7] for the prediction of two-dimensional turbulence-cascade interaction noise using a CAA solver. The method was applied to reproduce the fan wake with Gaussian spectra and different assumptions, such as space-varying turbulence intensity and length scale. However, synthetic turbulence was only introduced in a small region of the CAA domain, so that only a few OGVs interacted with the fan wakes, and spurious noise reflections were found at the edge the CAA domain.

In this work, a Linearized Euler Equation (LEE) solver is used along with an advanced digital filter method to generate synthetic turbulence for the fan wake modelling [8]. This synthetic turbulence method combines the advantages of RPM methods [7, 9], for the mathematical background, and synthetic eddy methods [10, 11], for the numerical implementation. Unlike Fourier mode methods, the resulting fluctuating velocity field is not periodic and has a continuous turbulence spectrum. The isotropic von Kármán spectrum, which was reported to closely match the spectral content of fan wakes in experiments [2], is preferred herein to extend the validity of the sound power spectra to higher frequencies than in previous CAA studies. The fluctuating velocity field is introduced into the CAA domain upstream of the OGVs all along the circumferential direction. The fluctuating velocity field is convected downstream by the mean flow, thus interacting with all vanes in the CAA domain.

CAA simulations of cascades are validated with Cheong *et al.*'s analytical model [12], which was adapted by Jenkins [13] to be used with both CFD and experimental input data from fan wakes. The analytical model predicts the sound power spectra from the interaction of isotropic turbulence with an unloaded two-dimensional flat plate cascade using the LEEs. Recently, this analytical method was used by Kim *et al.* [14] to validate a synthetic turbulence inflow based on the RPM method.

The current work is organized as follows. The CAA method is described in Section 2. A parameter study is presented in Section 3 to investigate geometry effects on sound power spectra due to cascades interacting with isotropic synthetic turbulence. Finally, Section 4 extends the advanced digital filter method to account for fan wakes with spatial variations in turbulence intensity.

2. CAA method

The characteristics of the CAA solver used in this work can be found in a previous study of leading edge noise from isolated airfoils [8]. Numerical simulations presented herein were run with mean flow density $\rho_0 = 1.2 \text{ Kg/m}^3$ and speed of sound $c_0 = 340 \text{ m/s}$. The reference value for the sound PoWer Level (PWL) calculations is $1 \times 10^{-12} \text{ W}$. All computations used the IRIDIS 4 high performance computing facility at the University of Southampton.

2.1 Computational setup

A schematic of the two-dimensional cascade configuration that is investigated in this work is shown in Fig. 1(a), where $M_x = U_x/c_0$ and $M_y = U_y/c_0$ are the mean flow Mach numbers in the axial and circumferential directions, α is the stagger angle, s is the inter-vane gap and c is the airfoil chord. This cascade configuration is similar to that considered in Cheong *et al.*'s analytical model [12] for flat plate cascades in isotropic turbulence. In this work, airfoils are symmetric and unloaded, with $c = 0.15 \text{ m}$. The mean flow is aligned in the chordwise direction.

Numerical sound power spectra upstream (\mathcal{P}^+) and downstream (\mathcal{P}^-) of the airfoil cascade are compared to analytical predictions using Cheong *et al.*'s model [12]. Numerical noise predictions are obtained from an array of monitor points at $|x_u| = 3c$ upstream and at $|x_d| = 4c$ downstream of the airfoil cascade. Potential cut-off modes in the acoustic pressure field are expected to be damped before reaching the monitor points, so that cut-off modes do not contribute to the sound power spectra. Assuming that the mean flow is uniform where the monitor points are located, numerical sound power spectra [14] can be evaluated by using

$$\mathcal{P}^{\pm}(\omega) = \frac{Bs}{\rho_0} \text{Re} \left\{ \sum_{l=-\infty}^{\infty} |\hat{p}_l(\omega)|^2 \frac{\omega}{|\omega + U_x k_{x,l}^{\pm} + U_y k_{y,l}|^2} \left[-k_{x,l}^{\pm*} + \frac{U_x}{c_0^2} (\omega + U_x k_{x,l}^{\pm} + U_y k_{y,l}) \right] \right\}, \quad (1)$$

where B is the number of vanes, $\omega = 2\pi f$ is the angular frequency, $\hat{p}_l(\omega)$ is the amplitude of the fluctuating pressure from a spatio-temporal Fourier transform, the acoustic wavenumber in the circumferential direction is $k_{y,l} = 2\pi l/(Bs)$ with l being an integer that represents the order of the acoustic mode in the circumferential direction, and the acoustic wavenumber in the axial direction is

$$k_{x,l}^{\pm} = \frac{M_x(\omega/c_0 + M_y k_{y,l}) \pm \sqrt{(\omega/c_0 + M_y k_{y,l})^2 - (1 - M_x^2)k_{y,l}^2}}{1 - M_x^2}. \quad (2)$$

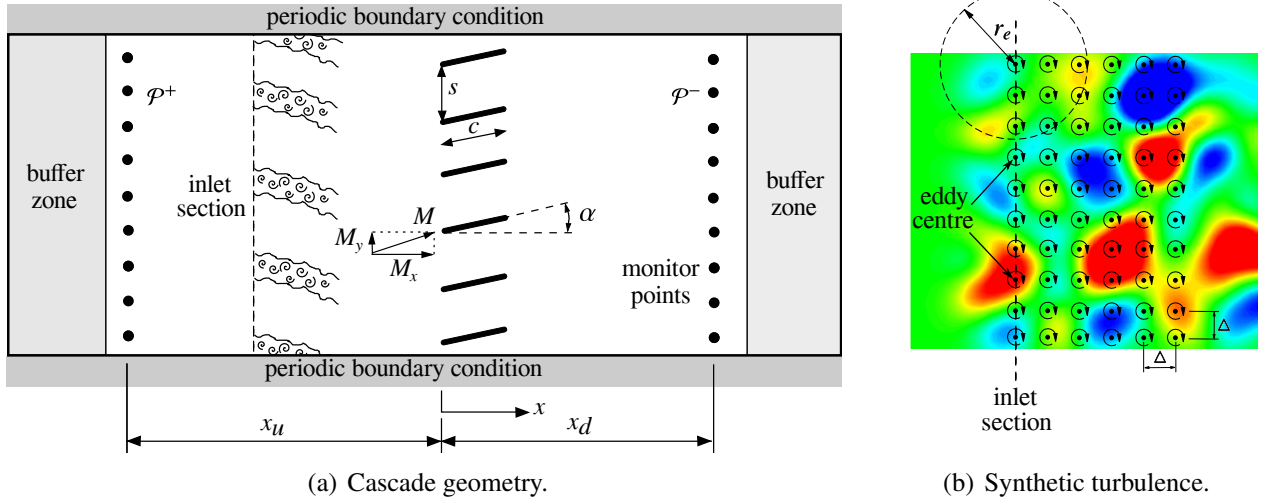


Figure 1: Two-dimensional airfoil cascade in the CAA domain.

2.2 Synthetic turbulence modelling

In this work, the digital filter method proposed by Gea-Aguilera *et al.* [8] is used to generate two-dimensional isotropic synthetic turbulence. The resulting turbulent flow is obtained from a superposition of Gaussian eddies. Each Gaussian eddy introduces a divergence-free fluctuating velocity field that vanishes far from its centre, therefore acting as a spatial filter. Figure 1(b) shows numerical implementation of the synthetic turbulence method in the CAA solver. The fluctuating velocity field introduced by each eddy in the inlet section is

$$u_x(\mathbf{x}) = -\epsilon \Delta \sum_{i=1}^{N_e} \frac{\sqrt{2\pi u_{rms,i}^2}}{\Lambda_i^2} (y - y_e) \exp\left(-\frac{\pi r^2}{2\Lambda_i^2}\right), \quad (3)$$

$$u_y(\mathbf{x}) = +\epsilon \Delta \sum_{i=1}^{N_e} \frac{\sqrt{2\pi u_{rms,i}^2}}{\Lambda_i^2} (x - x_e) \exp\left(-\frac{\pi r^2}{2\Lambda_i^2}\right), \quad (4)$$

where $u_{rms,i}$ and Λ_i control the amplitude and length scale of the Gaussian eddies, respectively, r is the distance between a point in the flow field (x, y) and the eddy centre (x_e, y_e) , Δ is the separation between eddy centres, and ϵ randomly takes the values ± 1 . The eddies are injected into the CAA domain through an inlet section, without requiring an additional boundary condition, and are convected downstream by the mean flow for their interaction with the vanes. The inlet section extends along the whole circumferential length, and periodic boundary conditions are used on the upper and

lower edges of the CAA domain. The correct turbulence statistics are recovered provided that the eddy spacing and radius satisfy $\Delta \leq \min \{\Lambda_i\} / 2$ and $r_e \geq 3 \max \{\Lambda_i\} / 2$, respectively.

The Gaussian superposition methodology is applied to generate two-dimensional isotropic turbulence with the spectral content of the von Kármán energy spectrum,

$$E^{2D}(k) = \frac{110u_{rms}^2\Lambda}{27\pi} \frac{\left(\frac{k}{k_e}\right)^4}{\left[1 + \left(\frac{k}{k_e}\right)^2\right]^{17/6}}, \quad (5)$$

where $k_e = [\sqrt{\pi}\Gamma(5/6)] / [\Lambda\Gamma(1/3)]$ with $\Gamma()$ as the Gamma function. The turbulence intensity and integral length scale are set to $u_{rms}/U = 0.017$ and $\Lambda = 0.008$ m, respectively. Table 1 shows the parameters necessary to recover the von Kármán energy spectrum from the Gaussian superposition at different mean flow Mach numbers.

Table 1: Parameters for Gaussian superposition using the advanced digital filter method.

| Λ_i [m] | $u_{rms,i}^2$ [m ² /s ²] ($M = 0.3$) | $u_{rms,i}^2$ [m ² /s ²] ($M = 0.5$) | $u_{rms,i}^2$ [m ² /s ²] ($M = 0.6$) |
|------------------------|--|--|--|
| 2.524×10^{-2} | 5.194×10^{-2} | 1.405×10^{-1} | 2.077×10^{-1} |
| 1.401×10^{-2} | 2.152×10^{-1} | 5.772×10^{-1} | 8.608×10^{-1} |
| 7.285×10^{-3} | 3.012×10^{-1} | 8.845×10^{-1} | 1.205 |
| 3.023×10^{-3} | 4.667×10^{-1} | 1.261 | 1.867 |
| 2.238×10^{-3} | 8.929×10^{-3} | 2.981×10^{-2} | 3.566×10^{-2} |

3. Parameter study of isotropic turbulence-cascade interaction noise

CAA simulations of a 4-vane cascade of NACA 0001 airfoils with $s/c = 0.75$ are performed to validate the numerical methodology for the prediction of upstream and downstream PWL spectra. The mean flow is uniform, with a freestream Mach number of $M = 0.6$, so that the turbulent structures are not distorted by mean flow gradients in the leading edge region of the airfoils. This assumption has been shown to be suitable for leading edge noise predictions of thin airfoils [15], and is consistent with Cheong *et al.*'s analytical model [12]. The CAA mesh extends approximately 5 chordlengths upstream and downstream of the airfoil cascade and contains 1,400,000 grid points. The grid was designed to propagate the smallest vortical waves with at least 8 points-per-wavelength. For the numerical calculation of \mathcal{P}^\pm , a total number of 3.18×10^4 fluctuating pressure samples are collected every 566 non-dimensional time steps $dt c_0/L_{ref} = 4 \times 10^{-6}$, with $L_{ref} = 1$ m.

Figure 2 shows a comparison between sound power spectra of the NACA 0001 airfoil cascade, at $\alpha = 0^\circ$ and $\alpha = 30^\circ$, and their corresponding flat plate prediction. Numerical noise predictions of the NACA 0001 airfoil cascade are able to reproduce the correct shape of peaks and valleys in the spectra due to the interference of several cut-on modes propagating upstream and downstream of the cascade. Overall, numerical PWL spectra show an agreement within 2 – 3dB of the analytical prediction for the majority of frequencies, which validates the numerical approach.

Additionally, the sound power spectrum from an isolated flat plate was calculated and scaled by the number of vanes, $B = 4$, as shown in Fig. 2. Cheong *et al.* [12] reported that sound power spectra have a low sensitivity to the vane count at low frequencies. In contrast, sound power spectra scale with the number of vanes at high frequencies, in which case cascade effects on noise are negligible. Similar results were also shown by Blandeau *et al.* [16]. The comparison of analytical PWL spectra in Fig. 2 between the cascade and isolated flat plate predictions shows a good agreement, especially at high frequencies.

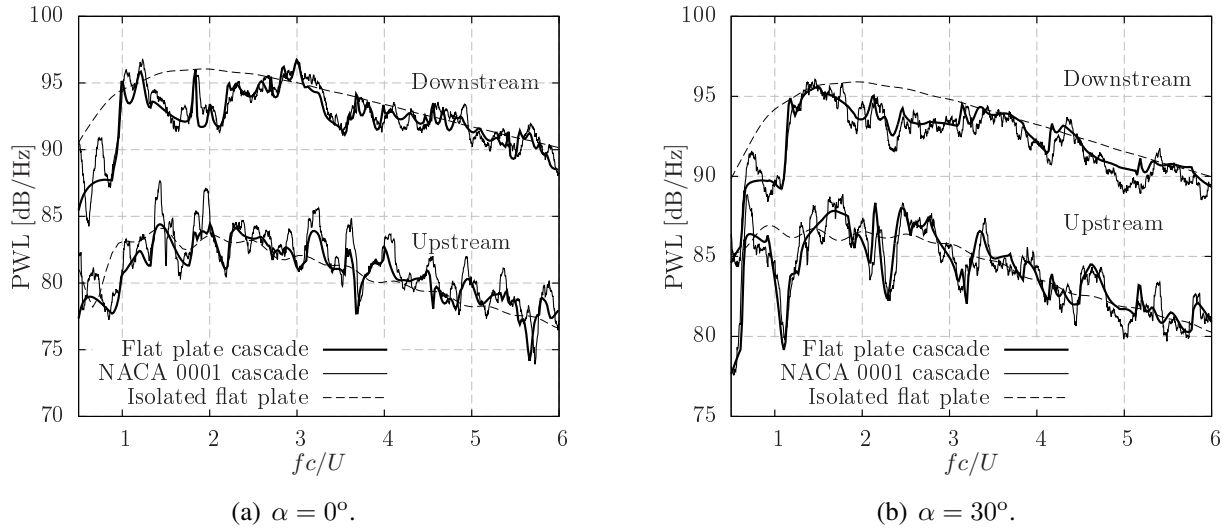


Figure 2: Upstream and downstream PWL spectra of NACA 0001 airfoil cascades in a $M = 0.6$ flow with $B = 4$ and $s/c = 0.75$.

3.1 Airfoil thickness and freestream Mach number effects on noise

In this section, turbulence-cascade interaction noise is investigated using a 4-vane cascade of NACA 0012 airfoils with $s/c = 0.75$ and $\alpha = 0^\circ$. An inviscid steady mean flow at $M = 0.3$ and $M = 0.5$ is used in the CAA simulations to study freestream Mach number effects on noise, while keeping a subsonic mean flow around the airfoil cascade. Instantaneous contours of non-dimensional fluctuating vorticity and pressure are shown in Fig. 3 for isotropic turbulence interacting with a 4-vane cascade of NACA 0012 airfoils. Fluctuating pressure contours show a number of upstream- and downstream-going cut-on modes. These are radiated from the leading edge of the airfoils, following a dipole source pattern.

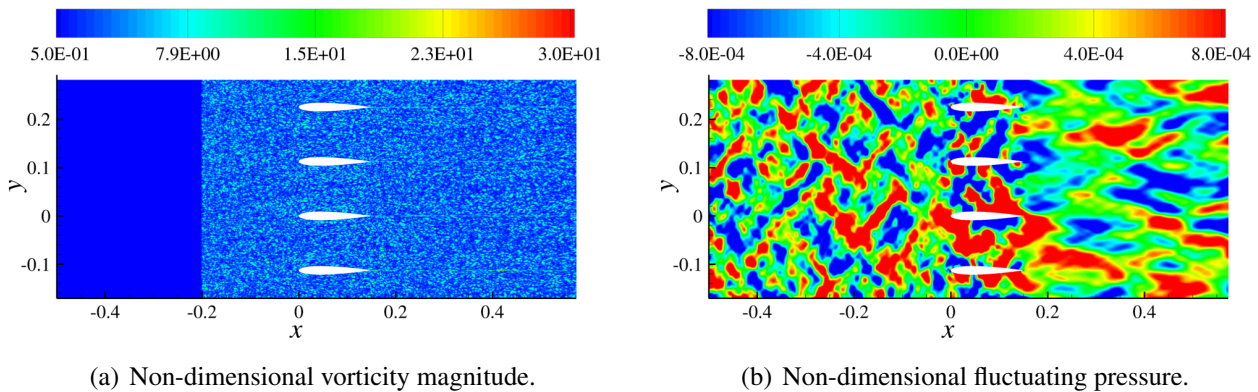


Figure 3: Instantaneous plots of a 4-vane cascade with NACA 0012 airfoils in a $M = 0.5$ flow.

Figure 4 shows numerical PWL spectra of a 4-vane cascade with NACA 0012 airfoils and the corresponding analytical prediction for a flat plate cascade. A significant noise reduction due to airfoil thickness is found at high frequencies, especially for downstream observers. These findings are consistent with previous studies on isolated airfoils [8, 15]. Additionally, $\Delta\text{PWL} = \text{PWL}|_{\text{flat plate}} - \text{PWL}|_{\text{NACA 0012}}$ is larger in the lower Mach number simulation for most reduced frequencies, fc/U . This suggests that ΔPWL as a function of fc/U may not overlap in cascades with thick airfoils, as is the case for isolated airfoil configurations [17].

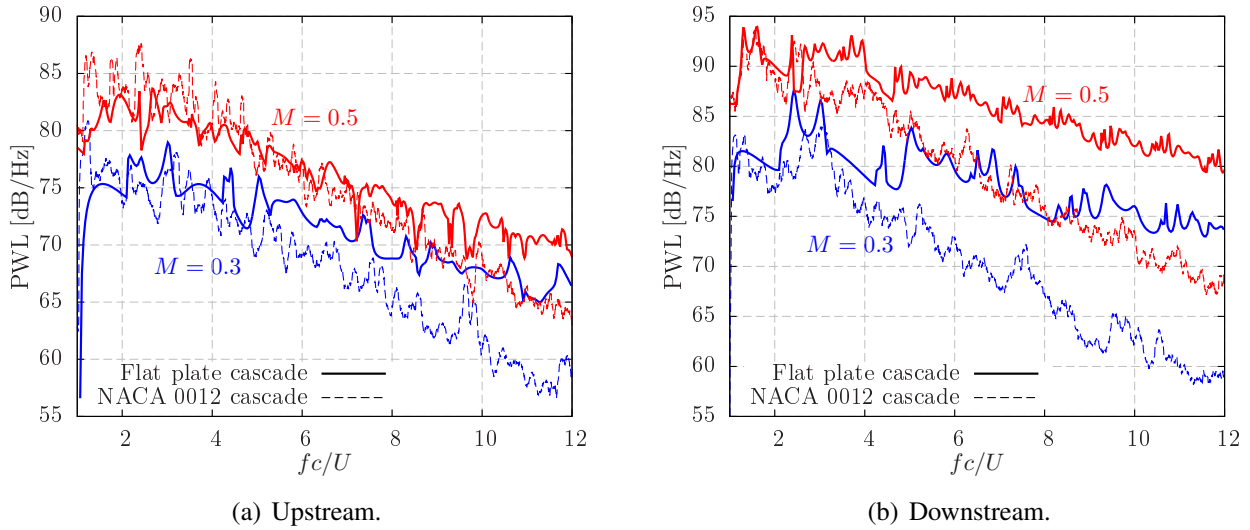


Figure 4: PWL spectra of NACA 0012 airfoil cascades in freestream flows at $M = 0.3$ and $M = 0.5$.

3.2 Gap-to-chord ratio and stagger angle effects on noise

The s/c ratio and α are varied independently in the NACA 0012 airfoil cascade that was used in Section 3.1. Thus, the baseline cascade configuration corresponds to $B = 4$, $s/c = 0.75$, $\alpha = 0^\circ$ and $M = 0.3$. Figure 5(a) shows downstream PWL spectra for a variation of 30° in the stagger angle with respect to the baseline cascade configuration. This change in stagger angle leads to a modification on the spectral peaks due to a different distribution of the cut-on circumferential modes that contribute to the radiated downstream PWL spectrum. However, similar noise levels are found for NACA 0012 airfoil cascades at $\alpha = 0^\circ$ and $\alpha = 30^\circ$. This result shows that variations in stagger angle of cascades with thick airfoils may have little effect on noise levels, as reported by Cheong *et al.* [12] for flat plates.

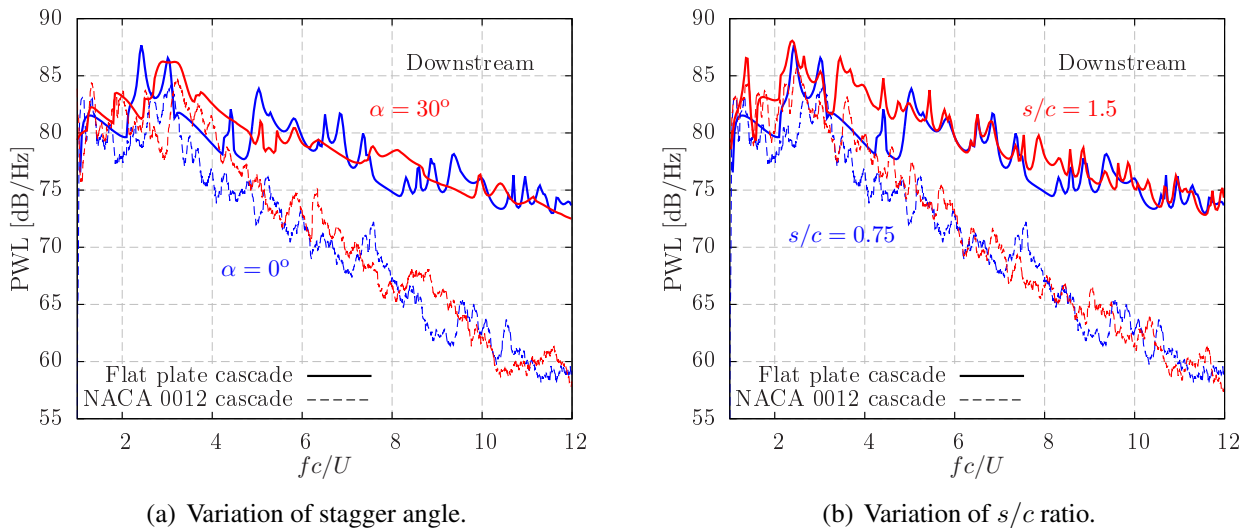


Figure 5: PWL spectra of NACA 0012 airfoil cascades for varying α and s/c .

The effect of the s/c ratio on cascade noise is assessed and results are shown in Fig. 5(b), which gives downstream PWL spectra for an increased inter-vane gap of $s/c = 1.5$ with respect to the baseline cascade configuration. The increase in the s/c ratio leads to a slight increase in noise levels at low frequencies and similar noise levels at high frequencies. It should be noted that as the s/c ratio increases, cascade effects become less important, and the PWL spectrum of the cascade with NACA

0012 airfoils is expected to become closer to that of the isolated airfoil response, scaled by the number of vanes.

4. Fan wake modelling for turbulence-cascade interaction noise

This section presents a simple strategy to generate a train of two-dimensional fan wakes with a Gaussian shape, based on the cyclostationary analysis of Jurdic *et al.* [18]. Turbulent fan wakes are assumed to be locally isotropic, with a constant integral length scale but including spatial variations in the turbulence intensity. This can be achieved by multiplying the fluctuating velocity components of each eddy in Eqs. 3 and 4 by the amplitude scaling factor

$$A_w(y_e, t) = \sum_{i=-\infty}^{+\infty} \left\{ \exp \left[-\frac{\ln 2}{h_w^2} (y_e - \Omega R(t + iT_w))^2 \right] \right\}^{1/2}, \quad (6)$$

where h_w is the semi-wake width, Ω is the angular velocity of the fan wakes, $R = Bs/(2\pi)$ is the radius of the wrapped cascade, and $T_w = Bs/(n_w \Omega R)$ is the period of the fan wakes, with n_w the integer number of wakes within the CAA domain. The present fan wake modelling is based on Dieste's work [9], in which the suitability of different implementation strategies was investigated to study noise from an isolated flat plate interacting with fan wakes.

Figure 6 shows instantaneous non-dimensional contours of fluctuating vorticity magnitude in fan wakes and acoustic pressure. The parameters for the fan wake are $n_w = 4$, $\Omega = 3640$ rad/s and $h_w = \Lambda/0.42$, where the semi-wake width value was chosen according to Jurdic *et al.*'s work [18]. The present methodology allows a more realistic representation of the fan wake-OGV interaction than the isotropic turbulence assumption. It should be noted that the root-mean-square fluctuating velocity of the fan wakes at a fixed point upstream of the cascade is proportional to $A_{w,rms}$. Consequently, the resulting sound power spectra are similar in shape to those of the simulations using isotropic turbulence (see Section 3), but with a scaled amplitude.

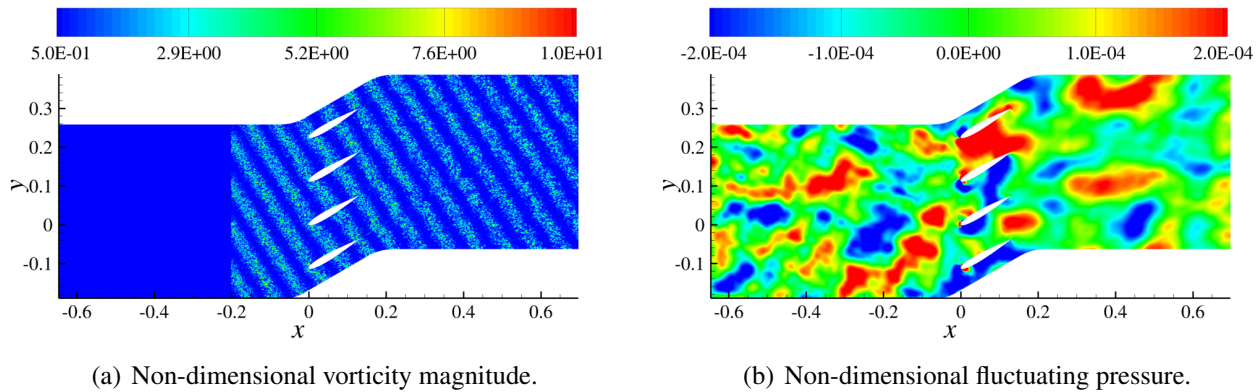


Figure 6: Instantaneous plots of a 4-vane cascade with NACA 0012 airfoils in a $M = 0.3$ flow.

5. Conclusions

This paper has presented a CAA methodology to investigate two-dimensional turbulence-cascade interaction noise using isotropic synthetic turbulence. A number of cascade configurations has been studied to assess potential geometry effects on noise. Cascades with thick airfoils have been found to reduce noise levels at high frequencies, especially downstream of the cascade. Increasing mean flow Mach number has been reported to decrease $\Delta\text{PWL} = \text{PWL}|_{\text{flat plate}} - \text{PWL}|_{\text{NACA 0012}}$ at a fixed reduced frequency. PWL spectra presented similar noise levels with a 30° variation in the stagger angle and a significant increase in the s/c ratio, although the shape of the peaks and valleys was significantly altered.

Acknowledgements

The research funding for this project was partially provided by the University of Southampton and Innovate UK (Technology Strategy Board), as part of the wHole AiRcraft Multidisciplinary nOise design sYstem (HARMONY) programme.

REFERENCES

1. European Commission, Report of the High Level Group on Aviation, Flightpath 2050 Europe's Vision for Aviation, (2011).
2. Ganz, U. W., Joppa, P. D., Patten, T. J., Scharpf, D. F., NASA Contractor Report CR-1998-208704, Boeing 18-Inch Fan Rig Broadband Noise Test, (1998).
3. Namba, M., Schulten, J. B. H. M., NASA Conference Publication CP-2000-209790, Third Computational Aeroacoustics (CAA) Workshop on Benchmark Problems - Category 4 - Fan Stator with Harmonic Excitation by Rotor Wake, (2000).
4. Envia, E., NASA Conference Publication CP-2004-212954, Fourth Computational Aeroacoustics (CAA) Workshop on Benchmark Problems - Category 3 - Problem 2 - Cascade-Gust Interaction, (2004).
5. Polacsek, C., Clair, V., Le Garrec, T., Reboul, G., Jacob, M. C., Numerical Predictions of Turbulence/Cascade-Interaction Noise Using Computational Aeroacoustics with a Stochastic Model, *AIAA Journal*, **53** (12), 3551-3566, (2015).
6. Wohlbrandt, A. M., Guérin, S., Ewert, R., Extension of the Random Particle Mesh Method to Periodic Turbulent Flows for Fan Broadband Noise Prediction, *21st AIAA/CEAS Aeroacoustics Conference*, Dallas, Texas, No. AIAA 2015-2383, (2015).
7. Ewert, R., Broadband Slat Noise Prediction based on CAA and Stochastic Sound Sources from a Fast Random Particle-Mesh (RPM) Method, *Journal of Computers and Fluids*, **37** (4), 369-387, (2008).
8. Gea-Aguilera, F., Zhang, X., Chen, X., Gill, J., Nodé-Langlois, T., Synthetic Turbulence Methods for Leading Edge Noise Predictions, *21st AIAA/CEAS Aeroacoustics Conference*, Dallas, Texas, No. AIAA 2015-2670, (2015).
9. Dieste, M., *Random-Vortex-Particle Methods Applied to Broadband Fan Interaction Noise*, Thesis for the degree of Doctor of Philosophy, ISVR, University of Southampton, (2011).
10. Jarrin, N., Benhamadouche, S., Laurence, D., Prosser, R., A Synthetic-Eddy-Method for generating Inflow Conditions for Large-Eddy Simulations, *International Journal of Heat and Fluid Flow*, **27** (4), 585-593, (2006).
11. Sescu, A., Hixon, R., Towards Low-Noise Synthetic Turbulent Inflow Conditions for Aeroacoustic Calculations, *International Journal for Numerical Methods in Fluids*, **73** (12), 1001-1010, (2013).
12. Cheong, C., Joseph, P., Lee, S., High Frequency Formulation for the Acoustic Power Spectrum due to Cascade-Turbulence Interaction, *Journal of the Acoustical Society of America*, **119** (1), 108-122, (2006).
13. Jenkins, G., *Models for the Prediction of Rear-Arc and Forward-Arc Fan Broadband Noise in Turbofan Engines*, Thesis for the degree of Doctor of Philosophy, ISVR, University of Southampton, (2013).
14. Kim, D., Heo, S., Cheong, C., Time-domain Inflow Boundary Condition for Turbulence-airfoil Interaction Noise Prediction using Synthetic Turbulence Modeling, *Journal of Sound and Vibration*, **340**, 138-151, (2015).
15. Gill, J., Zhang, X., Joseph, P., Symmetric Airfoil Geometry Effects on Leading Edge Noise, *Journal of the Acoustical Society of America*, **134** (4), 2669-2680, (2013).
16. Blandeau, V. P., Joseph, P. F., Jenkins, G., Powles, C. J., Comparison of Sound Power Radiation from Isolated Airfoils and Cascades in a Turbulent Flow, *Journal of the Acoustical Society of America*, **129** (6), 3521-3530, (2011).
17. Paruchuri, C., Gill, J., Narayanan, S., Joseph, P., Vanderwel, C., Zhang, X., Ganapathisubramani, B., Aerofoil Geometry Effects on Turbulence Interaction Noise, *21st AIAA/CEAS Aeroacoustics Conference*, Dallas, Texas, No. AIAA 2015-2830, (2015).
18. Jurdic, V., Joseph, P., Antoni, J., Investigation of Rotor Wake Turbulence Through Cyclostationary Spectral Analysis, *AIAA Journal*, **47** (9), 2022-2030, (2009).