MATHEMATICAL SOURCE REFERENCES

Original source references for common mathematical ideas

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INTRODUCTORY REMARKS

This list of references originated from a notebook in which were jotted down interesting original references. Subsequently it seemed a useful project to extend and complete the list as far as time permitted, the task being an unending one.

Having reached a certain stage of completeness it is made available hoping it is found useful for those interested in mathematical origins. Generally speaking, the topics mentioned are those met in a degree course in mathematics. For each entry the list attempts to give an exact source reference with comments about priority. There are now available other historical reference sources for mathematics on the internet though with a different style of presentation*.

The work has been completed in its present form while Visitor to the Institute of Sound and Vibration Research, University of Southampton.

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* E.g. *Earliest known uses of some of the words in mathematics*
http://jeff560.tripod.com/a.html

A

AFFINE CONNEXION

Defined by Weyl in his thesis and used by him in General Relativity.

H. Weyl: Reine Infinitesimal Geometrie, Diss. Math. Ann. 78 1917

H. Weyl: Raum, Zeit, Materie, Berlin 1918 (Springer),
Engl. transl. from 4th edition: Space, Time, Matter 1922 (Methuen), rpt. NY (Dover)

ALGEBRA

The word derives from the Arabic "al-jabr" which occurred in the title of a book of the Arabic mathematician al-Khwarizmi (fl. 813-850) who taught at the University of Cordoba. His book was translated into Latin in 1145 by Robert of Chester from where the form "algebra" originated. The word had the meaning of restoration and this meaning survived in Spanish as "algebrista", a bone-setter (e.g. in Don Quixote, part 2, chap.15 where the Don goes to an algebrister after falling from his horse)

Musa ibn al-Khwarizmi: Hisab al-jabr wa'l-muqabalah, Cordoba, 9th century

Robert of Chester: Liber algebrae et muqabala, Segovia 1145

ARCHIMIDEAN SPIRAL

Archimedes: On Spirals, Engl. tr. in T.L. Heath: The Works of Archimedes

ARGAND DIAGRAM

Wallis was the first to attempt, though unsuccessfully, to find a geometric representation of complex numbers. The usual geometric representation was first published by the Norwegian surveyor Caspar Wessel who wrote a memoir on this subject in 1797. His memoir though, written in Danish, received little attention until a hundred years later when the Danish Academy published a French version. In 1806 in France the Abbe Bueé and Argand both put forward the now familiar geometrical representation. With Bueé, only vector addition was present but Argand proposed the multiplication rule also. Argand's pamphlet was published anonymously and only came to light in 1813 when there was correspondence on the subject in Gergonne's Annales and Argand drew attention to his work by saying that he had previously communicated it to Legendre. At the same time he put forward new ideas including an attempt at a geometrical proof of the fundamental theorem of algebra. Gauss, it has been said, also discovered the geometrical representation in 1797 but did not publish it. Several similar attempts were made shortly after Wessel.
(cf. Smith: History II, 263-267, Smith's Source Book, Tait: Sci. Papers II pp.446-7).

J. Wallis: De Algebra Tractatus ('Treatise on Algebra') Book II chap.6, London 1685 (Extract in Smith's Source Book I)

C. Wessel: 'Om Direktionens analytiske Betegning, et forsøg anvendt førnemmelig til plane og sphaeriske polygoners opløsning.' Nye Samling af det køngelige Danske Videnskabernas Selskabs Shrifter 5 (1797) 1799 469-518,
Fr. transl: H.G. Zeuthen: Essai sur la représentation analytique de la direction, Copenhagen & Paris (1879); partial Engl. tr.: 'On the analytical representation of direction; an attempt applied chiefly to the solution of plane and spherical polygons' (Smith's Source Book I, 55-66)

J.R. Argand: 'Essai sur une manière de représenter les quantités imaginaires dans les constructions géométriques ', Paris 1806, published anonymously, rpr. Annales Math. 4 1813 133-147, 2nd ed. (ed. Houel) Paris 1874 (Gauthier-Villars), Paris 1971 (Blanchard) includes papers from Gergonnes Annales: Engl.tr. A.S. Hardy:'Imaginary Quantities, their geometric Representation', New York 1881 (van Nostrand)

Abbe A.Q. Bueé: Mémoire sur les quantités imaginaires, Phil. Trans. 96 1806 23-88

ASCOLI-ARZELÀ THEOREM

G. Ascoli: Le curve limiti di una varieta data di curve,
Rend. Lincei 18 1883/84 521-586

C. Arzelà: Sulle serie di funzioni, Bologna,
Acad. Sci. Mem. 8 1899-1900 131-186; 701-747

AUTOCORRELATION FUNCTION

This function arose in G.I.Taylor's experimental work on turbulence it being called the 'correlation function'. This name was then used in theoretical investigations such as those of Khinchin. When many variables are involved it becomes necessary to distinguish between auto- and cross-correlation which was done by Cramér in 1940.

G.I. Taylor: Diffusion by continuous movements,
Proc. Lond. Math. Soc. 212 1920 196-212

A. Khinchin: Korrelationstheorie der stationären stochastischen Prozesse,
Math. Ann. 109 1934 604-615

H. Cramér: On the theory of stationary random processes,
Annals Math. 41 1940 215-230

AUTOREGRESSIVE SCHEME

This term was introduced by M.G. Kendall when talking of phenomena in economics, meteorology and geophysics. He says on p. 96 of the cited paper:

> *'It appears that all these phenomena are subject to disturbances*
> *of the "casual" or "stochastic" type which, once they have occurred,*

are integrated into the system and influence its future motion.'

The next page gives the definition:

$$u_t = f(u_{t-1}, u_{t-2}, \ldots u_{t-m}) + \varepsilon_t$$

with the remark '*such a scheme I call autoregressive*'.

M.G. Kendall: On the analysis of oscillatory time series,
J. Roy. Stat. Soc. 108 l945 93-141

B

BANACH SPACE

Independent definitions were given about the same time by Banach and Wiener with Banach having slight priority. At first the spaces were known as Wiener-Banach spaces but, since Wiener did not continue to publish on their theory, they became known as Banach spaces especially after the publication of Banach's book.

S. Banach: Sur les opérations dans les ensembles abstraites et leurs applications aux équations intégrales, thesis, Univ. Lwow 1920; Fund. Math. 3 1922 133-181

N. Wiener: Limit in terms of continuous functions,
Bull. Soc. Math. France 50 1922 119-134

S. Banach: Théorie des opérations linéaires,
Warsaw 1932 (Monographie Matematyczni); rpr. New York (Chelsea)

BANACH FIXED POINT THEOREM

This was first proved in Banach's thesis by contraction mapping although that term was not used.

S. Banach: Sur les opérations dans les ensembles abstraits et leurs applications aux équations intégrales, thesis, Univ. Lwow. 1920;
Fund. Math. 3 1922 133-181, (cf. p. 160, Theorem 6)

BAYES' THEOREM

Named after the Reverend Thomas Bayes (1701-61) but Bayes, Laplace and Gauss all share credit for this method.

T. Bayes: Essay towards solving a Problem in the Doctrine of Chances,

Posthumously published in Phil. Trans. 53 1763 370-418; 54 1764 296-325;
Ostwald's Klassiker no. 69

P.S. Laplace: Mémoire sur la probabilité des causes par les événements,
Mém. pres. div. savants. 6 1774 621-656; Œuvres Complètes VIII 325-366
Rpr. in Dale: History of inverse probability,
Engl. tr. S.M. Stigler, Stat. Sci. 1 1986 359-378

C. F. Gauss: Theoria motus corporum coelestium in sectionibus conicis'' solem ambientium, Hamburg l809 (Perthes & Besser) (Book II, sect. 3, art.176)
C.H. Davis (tr.): Theory of the Motion of Heavenly Bodies Moving about the Sun in Conic Sections, 1857; rpr. New York 1963 (Dover) pp.255-256

P.S. Laplace: Théorie analytique des Probabilités, Paris 1812 (chapter 23)

G.A. Barnard: Thomas Bayes' essay towards solving a problem in the doctrine of chances, Biometrika 45 1958 293-315 (Bayes' essay is reprinted on pp. 296-315)

A.I. Dale: Bayes or Laplace?: An examination of the origin and early applications of Bayes' theorem, Arch. Hist. Exact. Sci 27 1982 23-47

A.I. Dale: A History of Inverse Probability, New York-Berlin 1991 (Springer)

BELTRAMI'S REPRESENTATIONS OF HYPERBOLIC GEOMETRY

Beltrami was the first to use Riemannian space for hyperbolic geometry and he described the three representations usually ascribed to Klein and Poincare. See Stillwell for original versions, English translations and comments

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E. Beltrami: Saggio di interpretazione della geometria non-euclidea, Giornale di Mat. VI 1868 284-312; Opere I 374-405; original and English transl. "Essay on the interpretation of Noneuclidean Geometry" in Stillwell 1996

E. Beltrami: Teoria fondamentale degli spazii di curvatura costante, Annali di mat. pura appl, ser II (1868) 232-255; original and English transl. "Fundamental theory of spaces of constant curvature" in Stillwell 1996

Stillwell J: Sources of Hyperbolic Geometry,
London Math. Soc. & Amer. Math. Soc. 1996

BERNOULLI NUMBERS

Although Bernoulli only computed the first 5 numbers he convincingly demonstrated their usefulness. Euler extended Bernoulli's calculations.

Jakob Bernoulli: Ars Coniectandi, Basel 1713, (95-98, see p.97)
Engl. tr. in Smith's Source Book I 85-90)

L. Euler: Inst. Calc. Int., St Petersb.-Berlin 1768-70, Chapter 6, art 122 p.420, 'De summatione progressionum per series infinitas'; Opera Omnia (1) X 337-367

L. Euler: De summis serierum numeros Bernoullianos involventium,
Novi comm. acad. sci. Petrop. 1 1770 129-167; Opera Omnia (1) XI 91-130

BESSEL FUNCTION

In Bessel's work the functions of integral order n occurred coefficients of the Fourier series solution of the Kepler equation, a solution which had already been investigated by Lagrange. The later 1824 paper explicitly gave the expression for the nth order function for integral n. The series had previously been considered for special cases by the Bernoullis, Euler, and Fourier. (cf. Whittaker & Watson, Kline Gr-Guiness)

Jakob Bernoulli: Letter to Leibnitz, Oct 3rd 1703
(He used a series for Bessel function of order one third)

Daniel Bernoulli: Theorems on oscillations of bodies connected by a flexible thread and of a vertically suspended chain, Comm. Acad. Sci. Petrop. 6 1732-33 108-22; published 1738 (He used a series for a Bessel function of order zero)

L. Euler: De oscillationibus fili flexibilis quotcunque ponduslis onusti,
Comm. Acad. sci. Petersb. 8 1736 30-47; Opera ser. 2 X 35-49; ibid.(1764) 1766

J.L. Lagrange: Sur le problème de Kepler,
Mem. Acad. Berlin 25 1769; Œuvres III 113-138

J.B.J. Fourier: Sur la propagation de la chaleur, Paris 1807
Library MS, Ecole National des Ponts et Chaussées
In this Fourier used the series form of a Bessel function of order zero.
(See below Gr-Guiness 1969 and 1972, the memoir is reproduced in his 1972 book.)

F.W. Bessel: Analytische Auflösung der Kepler'sche Aufgabe,
Abh. Ber. Akad. Wiss. 1816-17 49-55; Werke I 17-20

F.W. Bessel: Untersuchungen des Theils der planetärischen störungen,
und welcher aus der Bewegung der Sonne entsteht,
Abh. Ber. Acad. Wiss. l824 1-52; Werke I 84-109

I.Gr-Guiness: Joseph Fourier and the Revolution in Mathematical Physics,
J. Inst. Maths. Appls. 5 1969 230-253

I.Gr-Guiness: Joseph Fourier 1768-1830, Camb. Mass. 1972 (MIT Press)

BESSEL'S INEQUALITY

Bessel stated the inequality just for partial sums of a Fourier series.

F.W. Bessel: Über die Bestimmung des Gesetzes einer periodischen Erscheinung,
Astr. Nachr. 6 l828 333-348; Abh. II 364 -

BETA FUNCTION

Euler defined this function and developed its theory relating it to the gamma function which he also defined. The usual notation is due to Binet.

L. Euler: De expressione integralium per factores,
Novi comm. acad. Petrop. 6 (1756/7) 1761 115-154; Opera Omnia (1) XVII 233-247

J.P.M. Binet: Mémoire sur les intégrales définies Eulériennes,
J. Ec. Poly. 16 1839 123-343

BETTI NUMBERS

Betti's paper was the first to discuss connectivity in n dimensional space.

Poincaré, following his ideas, first used the term 'Betti number'.

E. Betti: Sopra gli spazi di un numero qualunque di dimensionali,
Annali di Mat. pura. appl. 4 1871 140-158; Opere II 273-290

H. Poincaré: Analysis situs, J. Ec. Poly. 1 1895 1-121; Œuvres VI 193-288

BINOMIAL THEOREM

A knowledge of the theorem for powers up to eight was widespread in medieval Arabic and Chinese mathematics (see Pascal triangle). The first general proof for integer index by combinatorial methods was by Jakob Bernoulli published in 1713. The theorem for a fractional index was discovered by Newton in 1664-65 and described by him in letters to Oldenburg, the Secretary of the Royal Society. Also at about the same time Gregory discovered similar results. These discoveries were not immediately published. Newton's discovery was published in 1685 by Wallis in his book on algebra but Gregory's discoveries only appeared in print with the publication of his letters. (cf. Boyer, Kline and refs. below)

I. Newton: Letters of June 13, October 24, 1676 to Henry Oldenburg, Commercium Epistolicum 1712, transl. in Smith's Source Book I 224-231; Struik SB 284-291

J. Wallis:The Doctrine of Infinite Series, further prosecuted by Mr Newton, in Algebra, London 1685, chapter 91, transl. in Smith's Source Book I 219-223

J. Gregory: Enclosure to a letter to Collins dated Nov.23 1670,
cf. 131-133 in R.W. Turnbull: James Gregory

Jakob Bernoulli: Ars Conjectandi, Basel 1713

D.T. Whiteside: Newton's discovery of the general binomial theorem,
Math. Gazette. 45 961 175-180

M. Yadegari: The binomial theorem: a widespread concept in Medieval Islamic mathematics, Hist. Math. 7 1980 401-406

M. Pensivy: The binomial theorem, in Gr-Guiness (ed): Companion Enc.

BOLZANO-WEIERSTRASS THEOREM

Bolzano asserted the existence of a least upper bound to a bounded set. The usual formulation that every infinite set has a point of accumulation is due to Weierstrass. The name 'Bolzano-Weierstrass' appears to be due to Schwarz. (cf. Gr-Guiness 1970)

B.P.J.N. Bolzano: Rein analytischer Beweis des Lehrsatzes..., Prague l8l7 (See art 12)

K.T.W. Weierstrass: Lectures at Berlin (unpublished)

H.A. Schwarz: Zur integration der partielle differentialgleichung, J. Math. (Crelle) 74 1872 218-253 (cf. 221 footnote); Papers 2 175-210 (cf.187)

BOOLEAN ALGEBRA

Boole's two classics are referred to below. The second applies the theory to probability. Both are reproduced in his collected works.

G. Boole: The Mathematical Analysis of Logic, being an Essay on the Calculus of Deductive Reasoning. Cambridge 1847; rpr. Oxford 1948.

G. Boole: An Investigation of the Laws of Thought, on which are founded the Mathematical Theories of Logic and Probabilities, London 1854 (MacMillan); rpr. New York 1958 (Dover)

Boole: Collected Logical Works. Vol.1: Studies in Logic and Probability, La Salle, Illinois 1952 (Open Court)

Studies in Logic and Probability, ed. R.Rhees, London 1952

BOREL MEASURE

E. Borel: Leçons sur la théorie des fonctions, Paris 1898 (Gauthier-Villars)

BROWNIAN MOTION

Robert Brown was the Keeper of the Botany Department at the British Museum. The movement of pollen grains which he discovered attracted much attention when it was later realized that it gave visible proof of the existence of atomic motions. The name 'Brownian motion' appears to have come into use after Einstein's 1905 paper.

R. Brown: A brief description of Microscopical Observations made in the months of June, July and August 1827, on the particles contained in the Pollen of Plants and on the general existence of active molecules in organic and in inorganic bodies, Ann. Phys. 14 1828 294-313 (German); Phil. Mag. 4 1828 161-173

A. Einstein: Zur Theorie der Brownschen Bewegung, Ann. Phys. 19 1905 371-; Engl. tr. rpr. New York (Dover)

C

CALCULUS OF VARIATIONS

What is now understood as calculus of variations originated in Euler's book 'Methodus inveniendi...' of 1744. However the name 'calculus of variations' was only later given by Euler to the improved notation and technique subsequently introduced by Lagrange to treat these problems using a variation of the function itself. Lagrange referred to this method as 'the method of variations'. Subsequently the name 'calculus of variations' came to have a wider use.

L. Euler: Methodus inveniendi ...,
Lausanne-Geneva 1744 Opera Omnia(1) XXIV; Ostwald's Klassiker no.46

J.L. Lagrange: Essai d'une nouvelle méthode pour determiner les maxima et les minima des formules intégrales indéfinies,
Misc. Taurin. 1760/61 173-95; Œuvres I 332-362; 363-468

L. Euler: Elementa calculi variationum, [1764]
Novi Comm. Acad. Sci. Petrop. 10 1766 51-93

J.L. Lagrange: Sur la méthode des variations,
Misc. Taurin. 1766/69 IV; Œuvres II 37-66

L. Euler: De calculo variationum,
Inst. Calc. Int., Lausanne 1770, vol. III appendix; Opera Omnia(1) XIII 371-458

L. Euler: Methodus novo et facilis calculum variationi tractandi,
Nov. comm. Petrop. (1771) 1772

CAUCHY'S CONVERGENCE TESTS

These were given in Cauchy's 1821 Cours d'Analyse

A-L Cauchy: Des séries convergentes et divergentes; Règles sur la convergence des séries; Sommation de quelques séries convergentes. Cours d'analyse - Analyse algébrique, Paris 1821, sect.2, chap.6; Œuvres ser. 2 III 230-273 (Cauchy test: Theorem I; d'Alembert test: Theorem II; Cauchy condensation test: Theorem III)

CAUCHY'S GENERAL PRINCIPLE OF CONVERGENCE

A-L Cauchy: Sur la convergence des séries,
Exerc. Math. 1827 221-232; Œuvres ser.2 VII 267-279

CAUCHY'S INEQUALITY

Cauchy proved this inequality for any number of variables as one of many inequalities in an appendix to his Cours d'Analyse. Among them is the general inequality between arithmetic and geometric means. See also 'Schwarz's inequality'.

A-L. Cauchy: Sur les formules qui résultent de l'emploie du signe > ou <, et sur les moyennes entre plusieurs quantités, Theorem XVI, Cours d'Analyse, 1er Partie: Analyse algébrique 1821; Œuvres ser.2 III 373-377

CAUCHY'S INTEGRAL TEST

This was initially due to MacLaurin and then rediscovered by Cauchy.

C. MacLaurin: A Treatise on Fluxions, Edinburgh 1742 (See art 350 p.289)

A-L. Cauchy: Sur la convergence des séries,
Exerc. Math. 1827 221-232; Œuvres ser. 2 VII 267-279

CAUCHY'S INTEGRAL THEOREM

This theorem was foreshadowed in a letter from Gauss to Bessel of 1811 in which Gauss observed that the value of the complex integral of dx/x depends on the path of integration. Cauchy's earlier paper referenced below proved the theorem for a rectangle. As Smithies has since shown, the general proof of the theorem valid for any simple closed curve was only derived much later in the 1831 2nd Turin memoir For notes on various subsequent hypotheses see Watson 1914, Smithies 1998.

K.F. Gauss: 1811 letter to Bessel in Briefwechsel zwischen Gauss und Bessel, Berlin (1880) (cf. 156-157)

A-L Cauchy: Mémoire sur les intégrales définies, [1814];
Mém. pres. div. savans 1 1827 601-799; Œuvres ser.1 I 329-506

A-L. Cauchy: Mémoire sur les rapports qui existent entre le calcul des résidus et le calcul des limites, et sur les avantages qu'offrent ces nouveaux calculs dans la résolution des équations algébriques ou transcendantes.
Lithograph, Turin 1832/33 Presented to the Turin Academy 27 November 1831; Œuvres ser.2 XV 182-261; A paper of the same title was published in Bull. Sci. Math (Ferussac) 16 1831 116-119 Œuvres ser.2 II 169-172

Smithies: Cauchy and the creation of Complex Function Theory,
Cambridge 1998 (Univ. Press)

CAUCHY-KOVALEVSKI THEOREM

Cauchy had used the method of 'calcul des limites' (see next entry) to demonstrate the existence of convergent solutions to systems of analytic ordinary differential equations and in 1842 he extended it to partial differential equations. Kovalevski applied this method to systems of partial differential equations in her inaugural dissertation written under direction of Weierstrass who had contributed to the theory at the same time as Cauchy.

A-L Cauchy: Mémoire sur l'emploi du nouveau calcul des limites dans l'intégration d'un système d'équations différentielles, C.R. Acad. Sci. Paris 15 1842 14-25

A-L Cauchy: Mémoire sur l'emploi du calcul des limites dans l'intégration des équations aux dérivées partielles, ibid. 44-59, 85-101

A-L Cauchy: Mémoire sur les systèmes d'équations aux dérivées partielles d'ordres quelconques et sur leur réduction … des systèmes d'équations linéaires du premier ordre, ibid. 131-138

S.V. Kovalevski: Zur theorie der partiellen Differentialgleichungen,
Inaug. diss. Univ. Stockholm 1874; J. Math.(Crelle) 80 1874 1-32,
(Extract in Birkhoff's Source Book)

CAUCHY'S METHOD OF CALCUL DES LIMITES (MAJORANTS)

This was called by Cauchy the 'calcul des limites' since he used it to determine the limits within which a series converges.

A-L Cauchy: Extrait du mémoire présenté à l'Académie de Turin le 11 octobre 1831 Lithographed

A-L Cauchy: Formules pour le développement des fonctions en séries, Calcul des limites. Exercices d'analyse et de physique mathématique 2 1841 50-109 (see 65); Œuvres (2) XII 58-112. (French translation of the part of the 1831 Turin memoir dealing with 'Calcul des limites')

CAUCHY POLYGON

A-L. Cauchy: Mémoire sur l'intégration des équations différentielles,
Œuvres ser.2 XI 399-465 (Extract in Birkhoff's source book)

CAUCHY REMAINDER FOR TAYLOR'S THEOREM

A-L. Cauchy: Leçons de calcul différentiel, Paris 1829;
Œuvres ser.2 IV 9th and 10th lessons (Extract in Birkhoff's Source Book)

CAUCHY-RIEMANN CONDITIONS

These conditions first occurred in d'Alembert's analysis of two dimensional fluid motion, an analysis which was further developed by Euler in the third part of his fundamental memoir on fluid dynamics. Euler there credits to d'Alembert the technique of considering u+√-1 v as a function of x+ √-1 y where x, y are Cartesian coordinates and u, v the corresponding velocity components. The topic was later mentioned briefly by Euler in 1793 in connection with complex integration. Cauchy first used these equations in his 1814 memoir (which was only published in 1825) but as Smithies observed, he did not then fully realize their significance. Complete clarity came later at the time of Riemann's discussion of the condition in connection

with the founding of a theory of functions of a complex variable.
cf. Dugas 1950, Gratton-Guiness 1970, Smithies 1998.

J le R D'Alembert: Essai d'une nouvelle théorie de la résistance des fluides, Paris l752 (Durand)

L. Euler: Continuation des recherches sur la théorie du mouvement des fluides, Mém. Acad. Berlin 1755 p.316; Opera omnia ser. 2 XII 92-132

L. Euler: De integrationibus maxime memorabilibus ex calculo imaginorum oriundis, Nova Acta Acad. Sci Petrop. 7 (1789) 1793 99-133;1797; Opera Omnia ser.1 XIX 1-44; 168-186

A.L. Cauchy: Mémoire sur les intégrales définies [1814], Mém. pres. div. sav. 1825 601-799; Œuvres ser.1 I 329-506; (Analysed by Gr-Guiness and Smithies)

B. Riemann: Grundlage für eine allgemeine Theorie der Funktionen einer veränderlichen complexen Grösse. Inaug. diss. Göttingen 1851; Ges. math. Werke I

CAUCHY THEORY OF RESIDUES

Cauchy developed his theory in 1826 in the first few volumes of the then newly issued journal 'Exercices de mathematiques.' The first paper is referenced below. Further details are given by Smithies in his book on Cauchy.

A.L. Cauchy: Sur un nouveau genre de calcul analogue au calcul infinitésimal, [1826] Exerc. Math. 1 1826 11-24; Œuvres 2 6, 23-37

CAYLEY ALGEBRA

A. Cayley: On Jacobi's elliptic functions in reply to the Rev. B. Bronwin; and on Quaternions, Phil. Mag. 26 1845 208-211; Coll. Papers I p.127

CAYLEY-HAMILTON THEOREM

Cayley verified the result for 2nd and 3rd orders while Hamilton stated an equivalent result for quaternions. The proof for general matrices was given by Frobenius in 1878.

W.R. Hamilton: Lectures on Quaternions, Dublin 1853 (See pp. 566-569)

A. Cayley: A memoir on the theory of matrices, Phil. Trans. 148 1856 17-37; Coll. Papers II 475-498 (cf. 482-3)

W.R. Hamilton: On the existence of a symbolic and biquadratic equation, which is satisfied by the symbol of linear operation in quaternions, Proc. Roy. Irish Acad. 8 1864 190-1; Phil. Mag. 24 1862 127-128; Math.Papers III 350-2

G. Frobenius: Über lineare Substitutionen und bilineare Formen, J. Math.(Crelle) 84 1878 1-63

CAYLEY-KLEIN METRIC

A. Cayley: Sixth Memoir upon Quantics,
Phil. Trans. 44 1859 61-90; Coll. Works II 561-592

F. Klein: Über die sogenante nichteuklidische Geometrie,
Math. Ann. 4 1871 573-625: Ges. Math. Abh. I,254. revised as ibid, 6 1873 112-145

F. Klein: Autographierte Vorlesungen über nichteuklidische Geometrie,
Fr. Schilling (ed.) 1892, 2nd. ed. 1893;
reissued: Rosemann (ed.) 'Nichteuklidische Geometrie', Berlin 1928 (Springer)

CAYLEY-KLEIN PARAMETERS

A. Cayley: On the correspondence between homographies and rotations,
Math. Ann. 15 1879 238-240; Coll. Works X 153-154

F. Klein: Das Ikosaeder. Leipzig 1884
Engl. tr.: 'Lectures on the Icosahedron' rpr. New York 1956 (Dover)

CAYLEY'S THEOREM ON FINITE GROUPS

Cayley's theorem states that every finite group is isomorphic to a subgroup of a permutation group. It was proved in the paper cited below which also was the first to use the multiplication table for groups.

A. Cayley: On the theory of groups as depending on the symmetrical equation
$\theta^n = 1$, Phil. Mag. 7 1854 40-47; 408-409; Coll. Papers II 123-130; 131-132

CAYLEY TRANSFORM

A. Cayley: Sur quelques propriétés des déterminants gauches,
J. Math. (Crelle) 32 1846 119-123; Coll. Papers I 332-336

CENTRAL LIMIT THEOREM

This theorem was first proved for the binomial distribution by De Moivre and Laplace. Lyapunov introduced the use of the characteristic function. The standard form needing only finite first and second moments is due to Lindeberg and Lévy.
cf Hardy, Littlewood, Polya, Dudley p.260

A. de Moivre: Approximatio ad Summam Terminorum Binomii $(a+b)^n$ in Seriem Expansi, London 1733 Rpr. R.C. Archibald, Isis 8 1926 671-676

P.S. de Laplace: Théorie analytique des probabilités, Paris 1812

A.M. Lyapunov: Nouvelle forme du théorème sur la limite de probabilité,
Mém. Acad. St. Petersb. 12(5) 1901 1-24; Coll. Works I 157-176

J.W. Lindeberg: Eine neue Herleitung des Exponential gesetzes in der Wahrschein-lichkeitsrechnung, Math. Z, 15 1922 211-225

P. Lévy: Calcul des probabilités, Paris 1925

CENTRIFUGAL FORCE

C. Huyghens: De Vi Centrifuga [1659]; Œuvres (posthumous) 1703

CHAPMAN-KOLMOGOROV EQUATION

This is a generalization of the Fokker-Planck equation.

S. Chapman: On the Brownian displacements and thermal diffusion of grains suspended in a non-uniform liquid, Proc. Roy. Soc. 119A 1928 34-54

A.N. Kolmogorov: Über die analytischen Methoden in der Wahrscheinlich=keitsrechnung, Math. Ann. 104 1931 415-458

CHARACTERISTIC EXPONENTS

H. Poincaré: Les Méthodes nouvelles de la Mécanique céleste,
Paris 1892 (Gauthier Villars) I chap.4

CHI-SQUARED DISTRIBUTION

F.R. Helmert: Die Ausgleichungsrechnung nach der Methode der kleinsten Quadrat, Leipzig-Berlin 1907

COLLINEATION

A term introduced by Möbius.

A.F. Möbius: Der barycentrische Calcul, Leipzig l827 (See Vorrede S XII)

COMPACTNESS

This idea had its origins in the Heine-Borel theorem and in Dirichlet's Principle qv.
It was defined explicitly by Fréchet.

M. Fréchet: Généralisation d'un théorème de Weierstrass,
C.R. Acad. Sci. Paris 139 1904 848-849

COMPLEX NUMBER

The phrase 'complex number' first occurs in Gauss' work and he was also the person who introduced into common use i as the symbol for √-1. Being preoccupied with the 'true metaphysics of √-l ', Gauss in 1831 considered a complex number as a number couple. The same idea was independently proposed by Hamilton, his 1837 paper hinting at the idea of number couples developed in detail in his 1843 book on quaternions.

C.F. Gauss: Zusätze zu Seeber's Werke über die ernäheren quadratischen Formen, Gött. gel. Anz. 1831; Werke II

C.F. Gauss: Theoria residuorum biquadraticorum,
Comm.Gött.6 1823-27 27-56; 7 1828-31 89-148; Gött. gel. Anz. 1831; Werke II 95-

W.R. Hamilton: Theory of conjugate functions, or algebraic couples; with a preliminary and elementary essay on algebra as the science of pure time. Read 1833, 1835
Trans. Roy. Irish Acad. 17 1837 293-422; Math. Papers III, 3-100

C.F. Gauss: Letter to W. Bolyai 1837; Werke II, p.102

W.R. Hamilton: Lectures on Quaternions, Dublin 1843
(See the preface, in particular, para. 17)

E. Cartan: Nombres Complexes, Enc. sci. math. 1 329-468;
Œuvres Complètes pt II vol.1 107-246

CONFORMAL MAPPING

Gauss solved the problem of conformal representation of one surface on to another in 1822. He introduced the word conformal ('conform') only later.

K.F. Gauss: Allgemeine Auflösung der Aufgabe: Die Theile einer gegebener Fläche auf einer andern gegebenen Fläche so abzubildenn, dass die Abbildung dem Abgebildeten in den Kleinsten Theilen, ähnlich wird,
Entry for the prize problem of Roy. Soc. Sci. Copenhagen, 1822.
Published in:Astron. Abh. 3 1825 1-30; Phil. Mag. 4 1828 104-113; 206-215;
Werke IV 192-216 (Engl. tr. in Smith's SB)

K.F. Gauss: Untersuchungen über die Gegenstände der höheren Geodäsie,
Many investigations were printed in Gött. Abh. 1842-1847; Ostwald's Klassiker 177

CONICS

Conic sections were first used by Menaechmus who solved the Delian problem of duplication of the cube by the intersection of two parabolae. They are mentioned by other ancient Greek writers, notably Euclid and Archimedes. The principal contribution was by Apollonius of Perga who published a work on conics in 8 volumes. He was the first to recognize the three cases parabola, ellipse, hyperbola and these names are due to him. The Conics of Apollonius was for long only partially known in Europe, the translation of all seven extant books being completed in 1710 by Edmund Halley. The first treatment of conics by analytical geometry was by Wallis.

Apollonius of Perga: Conica 8 vols.

Halley E: Apollonii Pergaei conicorum libri octo et Sereni Antissensis de sectione

cylindri et coni, Oxford 1710

J. Wallis: Tractatus de sectionibus conicis, London 1655

T L Heath (tr. & ed.): Apollonius: Treatise on Conic Sections,
Cambridge 1896 (Univ. Press) rpr. 1961

CONTINUITY

A.L. Cauchy: Cours d'analyse - Analyse algébrique, Paris 1821,
Œuvres (2) III 43 - (Extract in Birkhoff's Source Book)

CONVERGENCE IN MEAN

E Fischer: Sur la convergence en moyenne,
C.R. Acad. Sci. Paris 1907, 1, 1022-1024

CORIOLIS FORCE

G.G. Coriolis Traité de la mécanique des corps solides, Paris 1844

CORRELATION

The idea of correlation has a long history being present in Gauss' work on least squares. The statistical theory of correlation as understood nowadays is principally due to Karl Pearson who developed the standard theory of correlation of scatter diagrams over many years. Initially he worked in association with Francis Galton who was interested to develop a mathematical theory for Darwinian evolution and inheritance. Pearson lectured on correlation in l893 and started publishing his main work soon after. The history is reviewed in his 1920 paper.

F. Galton: Types and their inheritance,
Brit. Assoc. Report, Aberdeen 1885, 1206-1214

F. Galton: Co-relations and their measurement, chiefly from anthropomorphic data
[1883] Proc. Roy. Soc. 45 1889 135-145

K. Pearson: Contributions to the mathematical theory of evolution,
Phil Trans. 185A pt I 1894 71-110

K. Pearson: On lines and planes of closest fit to systems of points in space,
Phil. Mag. 2 1901 559-572

K. Pearson: On the systematic fitting of curves to observations and measurements,
Biometrika 1 1902 265-; 2 1902 1-

K. Pearson: Notes on the history of correlation, Biometrika 8 1920 25-45

COVARIANT & CONTRAVARIANT

G. Ricci: Delle derivazioni covarianti e contravarianti e del loro uso nella analysi applicata, Padua 1888 (Univ. Studies)

CRAMÉR'S RULE

Cramér's rule was an early example of the use of determinants of orders two and three. This solution was previously known to MacLaurin.

C. MacLaurin: A Treatise on Algebra, London 1748

G. Cramér: Introduction … l'analyse des lignes courbes algébriques
Geneva l750 (Cramér & Philibert) (See pp. 656-659)

CRYSTALLOGRAPHIC GROUPS

These were discovered independently by Feodoroff, Schoenfliess and Barlow.
The subject was unified by Bieberbach.

E. Feodorov: Symmetrie der regelmässigen Systeme der Figuren, (originally in Russian), Crystallography & Minerology (St. Petersburg) 21 1885; 28 1891

A. Schoenfliess: Krystallsysteme und Krystallstruktur,
Leipzig 1891 (Teubner); 2nd ed. 1923

L. Bieberbach: Über die Bewegungsgruppen der Euklidischen Räume,
Math. Ann. 70 1911 297-336; 72 1912 400-412

CUBIC EQUATION

Girolamo Cardan (also known in Latin as Hieronymous Cardanus) was the first to publish the solution but Nicolo Tartalia of Brescia claimed priority saying that he had communicated the solution to Cardan under promise not to publish. Cardan himself said that the method was discovered by Scipio del Ferro of Bologna (also known as Ferraro) 30 years previously.

Girolamo Cardano: Ars Magna, Nürenberg 1545, 1570 (Bosch), See Chap. XI 'De cubo & rebus aequalibus numero', (Engl. tr. of 1545 edition is in Smith's Source Book I 203-206; a facsimile of two pages are in Smith's Hist. Math. II 462-463)

CURL

C. Maxwell: On the mathematical classification of physical quantities,
Proc. Lond. Math. Soc. 3 1871 33-43; Papers II 257-266 (see p.265)

D

D'ALEMBERT'S PRINCIPLE

Hermann and Euler had previously stated a similar idea. D'Alembert in his later publication 1749, applied the principle to study the precession of the equinoxes. D'Alembert's Principle is usually known in the form Lagrange took as the foundation of his dynamical equations which makes use of the principle of virtual work. D'Alembert had stated the Principle differently.
See Voss (Enz. Mat. Wiss.), Dugas 1950, Rosenberg 1968, Fraser 1985.

J. Hermann: Phoronomia, sive de viribus et motibus corporum solidarum et fluidorum, libri duo, Amsterdam 1716 (Wetstenios)

L. Euler: De minimis oscillationibus corporum tam rigidorum quam flexibilium. Methodus nova et facilis, Comm. Acad. Sci. Imp. Petrop. 7 1740 99-122

J le R d'Alembert: Traité de Dynamique 2nd augmented edition, Paris 1743 (David); rpr. Brussels 1967 (Culture et Civilization)
(See second part: 'General principle to find the motion of several bodies which act upon each other in an arbitrary fashion, and several applications of this principle.' In particular chap.1, p.72)

J le R d'Alembert: Recherches sur la précession des équinoxes, et sur la nutation de l'axe de la terre, dans le système Newtonien,
Paris 1749 (David); rpr. Brussels 1967 (Culture et Civilization)

J.L. Lagrange: Méchanique analitique, Paris 1788
(See Seconde Partie: La Dynamique, Première section)

R.M. Rosenberg: D'Alembert and others on d'Alembert's Principle
J. Engrg. Educ, (ASME) 58 1968 959-960

C. Fraser: D'Alembert's principle: the original formulation and application in Jean d'Alembert's Traité de Dynamique, Centaurus 28 1985 31-61, 145-159

DANIELL INTEGRAL

P.J. Daniell: A general form of integral,
Annals of Math. 19 1917/18 279-294

DE MOIVRE'S THEOREM

The formula

$$(\cos \varphi + i \sin \varphi)^n = \cos n\varphi + i \sin n\varphi$$

was not given explicitly by de Moivre in any of his published works although in several places he used its equivalent, e.g. in his Miscellanea analytica. The formula was stated explicitly for integral values of n by Euler in 1748 and for rational values of n by him in 1769. The name de Moivre's theorem' is possibly due to Lagrange who in 1806 gave a clear statement of the theorem and its consequences, ascribing it to de Moivre in his Miscellanea analytica:

' Il parait que Moivre est le premier qui ait trouvé cette belle formule'

Translated extracts of Euler's contributions are in Smith's Source Book II.

A. de Moivre: Miscellanea analytica, London 1730, p.1

L. Euler: De quantitatibus transcendentibus ex circulo ortis,
(On transcendental quantities derived from the circle),
Intro. Anal. Inf. 1748 I chap. 8 97-98; Opera Omnia ser.1 VIII 140-141

L. Euler: Recherches sur les racines imaginaires des équations,
Hist. Acad. Berlin 5 1749 222-288 (cf. p.265)

J.L. Lagrange: Leçons sur la calcul des fonctions..., 2nd ed. Paris 1806;
Œuvres X p.111,leçon 10

I. Schneider: Der Mathematiker Abraham de Moivre (1667-1754)
Arch. Hist. Exact Sci. 5 1968 177-317

DECIMALS

First systematically discussed by Simon Stevin (Stevenius)

S. Stevin: La Thiende (Flemish), Leyden 1585 (Plantijn); La Disme - la Pratique d'Arithmétique (French), Œuvres Math; English edition ed. R.Morton, 'The Art of Tenths or Decimall Arithmetik' l608, English Extracts are in Smith's Source Book I

DEGREE OF ARC

The subdivision of the circle into 360 degrees originates from the Babylonian division of the 12 signs of the zodiac into 30 (or 60) subdivisions. In consequence the Greeks divided the circle into 360 subdivisions, particularly with astronomical applications in mind. It is believed that this was first done by Hipparcus although the first recorded written use of it is by Hypsicles (fl.2BC). The usage was established by Ptolemy's Almagest where the Babylonian sexagesimal fractions are used to further divide the degrees into what are now called minutes and seconds of arc. Ptolemy used the small circle to denote degrees and also the signs ', " for minutes and seconds of arc. The use of degrees of arc was well established in medieval times (see Chaucer's Canterbury Tales). Subdivision into minutes and seconds of arc and the corresponding notation ', " was used in the period 1580-60 by many authors, e.g. Tycho Brahé and Kepler, (cf. Cajori: Hist. Math. Notns.)

Hypsikles: On Risings (See the extract in the Loeb selections, II 395)

DEL

The operator was first introduced by Hamilton in his 1853 lectures on quaternions. Formulae for the operator were first stated in quaternion form by Tait 1870. The corresponding formulae in Gibbs' form of vector analysis are due to Heaviside l883 who stated the usual vector formulae for grad, div and curl. In a later presentation, Heaviside showed the effects of scalar and vector products with the operator. (cf. Crowe 1967). The name "del" came later in 1901 in Wilson's Vector Analysis.

W.R. Hamilton: Lectures on Quaternions, Dublin 1853 (See section 620)

P.G. Tait: Green's and allied theorems,
Trans. Roy. Soc. Edin. 20 l870 169-184; Sci.Papers I 136-150

P.G. Tait: On some quaternion integrals,
Proc. Roy. Soc. Edin. 1870 318-320; 1872 784-788; Sci.Papers I 159-163

O. Heaviside: The electromagnetic wave surface,
The Electrician l883; Phil.Mag. 19 1885 397-419

E.B. Wilson: Vector Analysis, New York 1901 (cf. p.138)

DELTA FUNCTION

The delta function achieved mathematical notoriety after its use in Dirac's form of Quantum Mechanics. Its use is however implicit in many investigations in mathematical physics based on the superposition principle and its origins can be traced back to Cauchy, Poisson and Kirchhoff. The delta function itself was first used by Heaviside.

A.L. Cauchy: Mémoire sur la théorie de la propagation des ondes a la surface d'un fluide pesant d'une profondeur indéfinie, [1815]
Mém. pres. div. sav. 1(1) 1827 3-312; Œuvres ser.1 I 5-318

S.D. Poisson: Mémoire sur la théorie des ondes,
[1815] Mém. Acad. Sci. Paris 1 1816 71-186

G.R. Kirchhoff: Vorlesungen über Mathematische Optik,
Leipzig 1891 (Teubner) (See p.24)

O Heaviside: Electromagnetic Theory,
London 1912 (cf. vol.II, para.271, eqns 54, 55)

P.A.M. Dirac: The physical interpretation of the quantum mechanics,
Proc. Roy. Soc. 113A 1926-7 621-641

P.A.M. Dirac: The Principles of Quantum Mechanics, Oxford 1930 (Clarendon Press)

B. van de Pol: Heaviside's Operational Calculus,
London 1950 (Inst. Electr. Engrs.), Heaviside Centenary Volume

DETERMINANT

The concept of a determinant in its expanded form occurred in a 17th century manuscript by the Japanese mathematician Seki Kowa. The first European statement by Leibnitz occurred shortly afterward. Then came the independent contribution by Cramér for determinants of orders 2 and 3 which became well-known as 'Cramér's Rule'. The first general definition and systematic treatment of properties is due to Cauchy in 1812 who in his extensive memoir also introduced double suffix notation to characterize the double array. The use of vertical lines is due to Cayley 1841 who, in further papers (e.g. that quoted below for 1843) gave an essentially modern treatment. See: Muir 1906, Mikami 1913, Smith 1929, Kline 1972

Seki Kowa: Kai Fukudai no Ho (Solving with determinants) 1683

G. Leibnitz: Letter to l'Hôpital April 28 1693 and Math. Schriften ser.1 II 238-240 (unpublished manuscript) both reproduced in Muir's History and Smith's Source Book

G. Cramér: Introduction … l'analyse des lignes courbes algébriques,
Geneva 1750 (Cramér & Philibert)

A-L Cauchy: Mémoire sur les fonctions qui ne peuvent obtenir que deux valeurs égales et de signes contraires par suite des transpositions opérées entre les variables qu'elles renferment J. Ecole. Poly. 10 1812 29-112; Œuvres ser. 2 I 91-169

A. Cayley: On a theorem in the geometry of position,
Camb. Math. J. 2 1841 267-271; Math. Papers I 1-4. (See p.1)

A. Cayley: On the theory of determinants, Trans. Camb. Phil. Soc. 8 1842 1-16;

T. Muir: Theory of Determinants in the Historical Order of Development 2 vols.
London 2nd ed 1906 (MacMillan)

DIOPHANTINE EQUATIONS

Diophantus wrote on this subject about 250 A.D. The 1621 printing of the 'Arithmetica' of Diophantus, edited and published by Bachet, was the first to publish the original Greek text (with notes in Latin). Bachet's 1621 edition was made famous through Fermat's annotation.

Diophantus Alexandrini: Arithmetica, Greek and English versions in:
I. Thomas (ed.): Greek mathematical works II (Loeb edition)

Diophantus of Alexandria: Arithmetica Bachet (ed.) Toulouse 1621

DIRICHLET CONDITIONS

Dirichlet R G L: Sur la convergence des séries trigonométriques qui servent à représenter une fonction arbitraire entre des limites données,
J. Math. (Crelle) 4 1829 157-169; Werke I 117-132

Dirichlet R G L: Ueber die Darstellung ganz willkürlicher Functionen durch Sinus- und Cosinus reihen, Dove's Repert. der Physik, Berlin 1937; Oswald's Klassiker 116

I. Gratton-Guiness: The Development of the Foundations of Mathematical Analysis from Euler to Riemann. 1970 MIT Press (See chapter 5)

DIRICHLET'S PRINCIPLE

The name comes from Riemann's use of this principle to establish the existence of harmonic functions on Riemann surfaces. He said that the principle had been used by Dirichlet in his lectures on inverse square attraction to show the existence of a solution of Laplace's equation in 3 dimensions, a method also used by Gauss. The lectures are presumably those of 1856/7 at Göttingen which Riemann had attended and which were published posthumously in 1876. Actually Gauss had previously given considerable attention to the method and a similar idea had also been used by Green and Thomson in mathematical physics. Their statements did not however have the clarity of Dirichlet's.

G. Green: On the determination of the exterior and interior attractions of ellipsoids of variable density. Trans. Camb. Phil. Soc. 5 1835 395-430; Math. Papers 187-222

C.F. Gauss: Allgemeine Lehrsätze in Beziehung auf die im verkehrten Verhältnisse des Quadrats der Entfernung wirkenden Anziehungs- und Abstoss-Kräftes, 1840; J. Math.(Liouville) 7 1842 273-324; Werke V 194-242; Ostwald's Klassiker no.2

Thomson W: Note sur une équation aux différences partielles qui se présente dans plusieurs questions de physique mathématique, J. Math. (Liouville) 12 1847 493-96

Thomson W: Theorems with reference to the solution of certain partial differential equations, Camb. & Dublin Math. J. 3 1848

R. G. Lejeune-Dirichlet: Vorlesungen über die im umgekehrten Verhältnisse des Quadrats der Entfernung wirkenden Kräfte. Göttingen 1856-7; F. Grube (ed.) Leipzig 1876 (extract in Birkhoff's Source Book 84-87); Math. & Phys. Papers I 93-96

B. Riemann: Theorie der Abel'sche Funktionen, J. Math. (Crelle) 1857 54-110; Werke 88-144. (Section 3, pp.97,98 in Dover edition)

A.F. Monna: Dirichlet's Principle: A Mathematical Comedy of Errors and its Influence on the Development of Analysis, Utrecht 1975 (Oosthoek, Scheltema & Holkema)

DIVERGENCE

W.K. Clifford: Elements of Dynamics, 2 vols
London 1878 (MacMillan) (Divergence defined on p.209)

E

THE NUMBER E

The use of the letter e for 2.717.. is due to Euler. It first appeared in an unpublished manuscript of 1727-28. The first occurrence in print was in Euler's Mechanica of 1736. Euler afterwards reverted to the use of letter a, e.g. in his Analysi infinitorum of 1748 where the value is quoted to 23 decimal places. (See Smith's Source Book)

L. Euler: Meditatio in experimentua explosione tormentorum nuper instituta 1727/28 (Meditation upon experiments made recently and on the firing of canon);
Opera posthuma, Petrop.1862 II 800-804; Engl. tr. in Smith's Source Book

L. Euler: Specimen de constructione aequationum differentiatum Mechanica,
St Petersb. 1736; Œuvres ser.2 I & II (See I p.68; II p.251 etc.)
Extract tr. in Smith's Source Book

L. Euler: Intro. Anal. Inf., Lausanne 1748 sect. 122, 123 De quantitatum exponentialium ac logarithmorum explicatione

EDGEWORTH EXPANSION

As was pointed out in the 1954 textbook of Gnedenko & Kolmogorov, this form of asymptotic expansion was used by Chebyshev before Edgeworth.

P.L. Chebyshev: Sur le développement des fonctions à une seule variable,
Bull. Acad. Sci. St. Petersb. (3)1 1860 193-200
F.Y. EdgeworthThe asymmetrical probability curve, Phil. Trans. 41 1896 90-99

F.Y. Edgeworth: The law of error, Proc. Camb. Phil. Soc. 20 1905 36-65

EIGENVALUE

The German word "eigenwerte" was first used by Hilbert in his lectures on integral equations, deriving from "eigen" characteristic and "werte" value. It was translated as the hybrid word "eigenvalue" in the English version of Courant & Hilbert's book and became current in quantum mechanics.

D. Hilbert: Grundzüge einer allgemeinen Theorie der linearen Integralgleichungen
Nachr. Ges. Wiss. Gött. Math.-Phys. Kl. 1904 -1910. Leipzig 1912 (Teubner); New York 1952 (Chelsea)

R. Courant R. & D. Hilbert: Methoden der Mathematischen Physik,
Berlin 1924 (Springer); Engl. tr: New York 1953 (Interscience)

ENERGY

The term 'energy' was introduced by Thomas Young to denote the product of mass and velocity squared, a quantity previously introduced by Leibnitz and called 'vis viva' i.e. living force. The half-factor is due to Coriolis, 1844 In 1853 Rankine used the term 'potential energy' to distinguish between potential and actual energy, a distinction which was at that time generally recognized (Helmholtz 1847). Later Thomson and Tait used the term 'kinetic energy' instead of actual energy.

T. Young: Natural Philosophy, Royal Institution Lectures 1801 (Lecture 8)

G. Coriolis: Traité de la Méchanique ... Paris 1844

H von Helmholtz: Über die Erhaltung der Kraft, Berlin 1847;
Wiss. Abh. 1 12-75; Ostwalds Klassiker no.1

W.J.M. Rankine: On the general law of transformation of energy,
Proc. Glasgow Phil. Soc. 3 1853 p.276; Misc. Sci. Papers p.203

W. Thomson & P.G.Tait: Article in 'Good Words' (ed. Charles Dickens) 1862
(cf. Whittaker 1951 II p.214): Elements of Dynamics, 1868

ERGODIC HYPOTHESIS

The German term "ergoden" is derived from ancient Greek "ergon" work and "odos" path was translated into English as "ergodic". It was introduced by Boltzmann in his work on the kinetic theory of gases. His use of the term was different from that which it subsequently had owing to the influence of the article by the Ehrenfests in Enz. Math. Wiss. (see Brush in the English translation of Boltzmann's book). The assumption that any trajectory will eventually transverse all points on a surface of constant energy was made by Maxwell and was subsequently commented upon by Rayleigh.

L. Boltzmann: Studien über das Gleichgewicht der lebendigen Kraft zwischen bewegten materiellen Punkten, Sitz. Akad. Wien 58 1868 517-560

L. Boltzmann: Einige allgemeine Sätze über Wärmegleichgewicht,
Wien Ber 63(2) 1871 cf. p.679

C. Maxwell: On Boltzmann's Theorem on the average distribution of energy in a system of material points, Camb. Phil. Soc. Trans. 12 1879; Sci. Papers II 713-741

L. Boltzmann: Vorlesungen über Gas Theorie, 2 vols. Leipzig (Barth) pt I 1896, pt II 1898 rpr.1910-12. Engl. tr. S.G. Brush, 1964 (Calif. Univ. Press & Camb. Univ. Press) ('Ergoden' used in Pt.II, chap.3, section 32)

Lord Rayleigh: The law of partition of kinetic energy,
Phil. Mag. 49 1900 95-118; Sci. Papers IV 433-451

L. Boltzmann: Wissenschaftliche Abhandlungen, (ed. F. Hasenohr)
Leipzig 1909 (Barth) 3 vols.

ERGODIC THEOREM

In the mathematical literature the term 'ergodic theorem' is used, following Birkhoff, to imply the existence of a time average along any trajectory. In this sense, the theorem was first proved, under differing assumptions, by both von Neumann and Birkhoff. This result does not however imply ergodicity in the older physical sense unless the system has the property of metrical transitivity (qv.) which is roughly equivalent to ergodicity in the older sense.

J von Neumann: Proof of the quasi-ergodic hypothesis,
Proc. Nat. Acad. Sci. USA, 18 1932 70-82; Coll. Works II 261-273

G.D. Birkhoff: Proof of the ergodic theorem,
Proc. Nat. Acad. Sci. USA, 17 1932 656-666

ERLANGER PROGRAMME

Klein's lectures on this subject were given as his inaugural lectures on admission to the University of Erlangen

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F. Klein: Vergleichende Betrachtungen über neuere Geometrische Forschungen
Programm zum Eintritt in die philosophische Fakultät der Univ. zu Erlangen. 1872 (Deichert); Math. Ann. 43 1893 63-100; Ges. Math. Abh. I 460-497; Engl. trans. 'A Comparative Review of Recent Advances in Geometry' Bull. New York Math. Soc. vol.2 1892-93, 215-249

D.E. Rowe: Felix Klein's 'Erlanger Antrittsrede': a transcription with English translation and commentary, Hist. Math. 12 1985 123-41

EUCLID'S ALGORITHM

Euclid: Elements, Prop.2 of book 7 (which is on number theory)

EULER'S ANGLES

These were referred to by Jacobi in 1827 in his work on dynamics where the 1775 reference was quoted.

Euler: Problema algebraicum ob affetiones prorsus singulares memorabile,
Novi Comm. Petrop. 15 1771 75-106; Comm. Arith. Coll. I 427-443

L. Euler: Formulae generales pro translatione quacunque corporum rigidorum,
Nov. Comm. Petrop. 20 1775 189-207

C.G.J Jacobi: Euleri formulae de transformatione coordinatarum,
J. Math (Crelle) 2 1827 188-189; Math. Werke VII 3-5

A. Cayley: Report on the Progress of the Solution of Certain Special Problems of Dynamics. Rep. Brit. Ass. Adv. Sci. 1862 184-252; Math. Papers IV 513-593 (see 522-559)

EULER'S EQUATIONS FOR ROTATION OF A RIGID BODY

These equations were the subject of Euler's treatise of 1760 on motion of rigid bodies. They had been proposed in a previous memoir of 1758 (which, as Cayley remarked, appeared in more than one version). Euler appreciated the importance of his equations and said of them in his treatise that the sum total of the theory of the motion of rigid bodies is contained with greatest simplicity in these three equations:

> *'summa totius theoriae motus corporum rigidorum*
> *his tribus formulis satis simplicibus continebitur'*

cf. Dugas 1950

L. Euler: Du mouvement de rotation des corps solides autour d'un axis variable, Mem. de Berlin 1758 158-193; printed 1765

L. Euler: Theoria motus corporum solidorum seu rigidorum, Rostock 1760, 2nd. ed. 1790; Opera Omnia ser.2 III

A. Cayley: Reproduction of Euler's memoir of 1758 on the rotation of a solid body, Quart. Math. J. 7 1868 361-373; Coll. Works VI 135-

EULER'S FORMULA

Formulae for sin x and cos x in terms of exp ix and exp (−ix) and their consequences are described by Euler 1743 but, strangely enough, without actually quoting

$$\exp (ix) = \cos x + i \sin x$$

Exponential and trigonometrical functions with imaginary arguments were however very fully treated in Euler's Analysin Infinitorum of 1748. The equivalent formula:

$$ix = \ln (\cos x + i \sin x)$$

had been previously stated in words by Cotes.

R. Cotes: Harmonia Mensurarum, Cambridge 1722

L. Euler: De summis serierum reciprocarum ex potestibus numerorum naturalium ortarum, Misc. Berolin.7 1743 172-192; Opera omnia ser.1 XIV 138-155 (cf. Struik) 3

L. Euler: Introductio in Analysin infinitorum, St. Petersb. 1748 Chap. 8, 'De quantitatibus transcendentibus ex circulo ortis', Opera Omnia ser.1 VIII 148-

EULER-LAGRANGE EQUATION

The Euler-Lagrange formula of the calculus of variations is due to Euler as Lagrange himself acknowledged:

> *'cette équation est celle qu'Euler a trouvée le premier'*

The 'new method' in the title of Lagrange's paper refers to the technique of making a variation in the function. Euler's priority here is sometimes forgotten, e.g. Hilbert called the equation just 'Lagrange's equation.' (see his Ges. math. Abh.)

L. Euler: Methodus inveniendi lineas curvas maximimi minimive gaudentes, 1744: Opera Omnia ser.1 XXIV; Ostwald's Klassiker 46, chap.2, sect. 21)

J.L. Lagrange Essai d'une nouvelle méthode pour déterminer les maxima et les minima des formules intégrales indéfinies,
Misc. Taurin. 1760-1761 173-95; Œuvres I 332-362; 363-468

EULER-MACLAURIN SUMMATION FORMULA

This formula was discovered independently by Euler and MacLaurin.

L. Euler: Methodus generalis summandi progressiones, Comm. Acad. Soc. Sci. Petrop. 6 (1732/3) 1738 68-97; Opera Omnia ser.1 XIV 42-72 (p.43)

C. MacLaurin:Treatise on Fluxions, Edinburgh 1742 (See II art 829, p.673)

L. Euler: Inst. Calc. Diff., St Petersb.-Berlin 1755 sect. 121, 'Investigation summae serierum ex termino generali'; Opera Omnia X p. 320

EULER-MASCHERONI'S CONSTANT

L. Euler: Inst.Calc.Diff., Lausanne 1755 chap. 7, art 143. 'De summatione progressionum per series infinitas'; Opera Omnia ser.1 X 339

L. Euler: De summis serierum numeros Bernoullianos involventium
Novi Comm Acad. Sci. Petrop. 1770 I 129-167; Opera Omnia ser.1 XI 91-130 (p.ll5)
commentary ibid. XII p.423 et seq., esp. p.431

L. Mascheronio: Adnotationes ad Calculo integralem Euleri, Paris 1790
Reprinted as an appendix in Euler's Opera Omnia XII cf. p. 431

EULER'S METHOD FOR NUMERICAL SOLUTION OF DIFFERENTIAL EQUATIONS

L. Euler: De integratione aequationum differentialium per approximationem,
Inst. Calc. Integralis., St Petersb. 1768 vol.I sect.2, chap.6, scholium 1;
Opera Omnia ser.1 XI p.424

EULER'S NUMBERS

L. Euler: De usu calculi differentialis in formandis seriebus, Inst. Calc. Differentialis, St. Petersb. 1755, pt 2, chap.7, art 224; Opera Omnia ser.1 X 419

EULER-POINCARÉ FORMULA

The formula was mentioned in a letter of Euler dated 1750. It was stated and proved for three dimensional convex polyhedra in the two parts of his extensive memoir of 1752/3. Its modification for multiply-connected surfaces was analyzed in detail by Listing. Poincaré generalized it to curvilinear polyhedra on multidimensional manifolds.

L. Euler: Letter to Goldbach, Nov. 1750. Opera omnia ser.4 I 158, letter 863,
(Rpr. in N. Biggs: 'The development of topology', in Fauvel 1993)

L. Euler: Elementa doctrinae solidorum,
Novi Comm. Acad. Sci. Petrop. 4 (1752-53) 1758 109-140; Opera ser.1 XXVI 71-93

L. Euler: Demonstratio nonnullarum insignium proprietatum quibus solida hedris plan-is inclusa sunt proedit, ibid. 140-160; Opera ibid. 94-108

J.B. Listing: Der Census räumlicher Complexe oder Verallgemeinerung des Euler'schen Satzes von den Polyedern. Gött. Abh. 10 1861/62 97-180; Gött. Nachr. 1861 352-358

H. Poincaré: Sur la généralisation d'un théorem d'Euler relatif aux polyèdres,
C.R. Acad. Sci. Paris 117 1893 144-145

H. Poincaré: Analysis situs, J. Ec. Poly. 1 1895 1-121 art 16; Œuvres VI p.270

F

FERMAT'S LAST THEOREM

Fermat wrote numerous comments in the margin of his own copy, now lost, of the 1621 printing of the Arithmetica of Diophantus. His comments were reprinted in the

2nd 1670 edition of this work edited by his son. The 'Last Theorem' of 1637 is written in the margin alongside problem 8, book 2 on finding Pythagorean triples.

Bachet (ed.): Diophantus Alexandrini: Arithmetica Toulouse 1621 (Bachet)

S. de Fermat (ed.): Diophanti Alexandrini Arithmetica ... cum observationibus
P. Fermat. Toulouse 1670. Rpr. Œuvres de Fermat, Paris 1891
(The 'Last Theorem' is written in Latin in vol.I, p. 291)

FERMAT'S PRINCIPLE OF LEAST TIME

P. Fermat: Letter dated Aug 1657 to Cureau de la Chambre,
Œuvres II 354-359 (A detailed description)

FINSLER SPACE

This was suggested by Carathéodory's 1906 paper on calculus of variations and discussed in detail in Finsler's 1918 doctoral thesis written under his direction. The idea however goes back to Hamilton whose contribution has been described by Synge.

W.R. Hamilton: Calculus of principal relations, Math. Papers II 1836, p.332
(See also the notes by Synge in the introduction to vol. II)

C. Carathéodory: Über die starken Maxima und Minima bei einfachen Integralen,
Math. Ann. 62 1906 449-503; Ges. math. Schr. I 80-142

P. Finsler: Über Kürven und Flächen in allgemeinen Räumen,
Diss. Göttingen 1918; rpr. Basel 1951 (Birkhäuser)

J.M. Synge: A generalization of the Riemannian line element,
Trans. Amer. Math. Soc. 27 1925 61-67

J.M. Synge et al. (eds.): The Mathematical Papers of Sir William Rowan Hamilton,
Cambridge l931, vol.II

FOCUS

The geometry of the burning point of a parabolic mirror was known to Diocles, a little known 2nd century BC Greek writer and was well-known in Arabic optics and was described by Al-Hazen. The Latin word 'focus', meaning hearth or fireplace, was used in 1604 in relation to a conic by Kepler in his optical work and then used in 1609 in relation to the elliptic orbit of Mars. This usage was followed by Newton.

Diocles: On burning mirrors (Greek) English version from Arabic translation of lost original: by G.J. Toomer 'Diocles on burning mirrors' Springer 1976,
Extract in Fauvel & Gray 1987

Ibn al-Haythan (Al-Hazen 965-c.1040): On paraboloidal burning mirrors (Arabic),
13th century manuscript now in Florence (cf. Dict. Sci. Bib.)

J. Kepler: Ad Vitellionem Paralipomena... Pars Optica traditur, Frankfurt 1604;
Ges. Werke, München 1939 vol.II, chap.4, sect.4.

J. Kepler: Astronomia nova Aitiologitis.. , Prague 1609

I. Newton: Principia Mathematica, London 1687
(See e.g. book I, section III:'the motion of bodies in eccentric conic sections')

I. Newton: Opticks, London 1704,
('focus' is defined immediately after axiom 7 in book 1, part 1)

FOKKER-PLANCK EQUATION

A.D. Fokker: Sur les mouvements browniens dans le champ du rayonnement noir,
Diss. Leiden 1913; Archives Neérlandaises 4 1914, 379-401

M. Planck: Über einen Satz der statistischen Dynamik und seine Erweiterung in
der Quantentheorie, Sitzber. Preuss. Akad. Wiss. Berlin 1 1917 324-341

FOURIER INTEGRAL

This was used by Fourier to solve the problem of heat conduction for an infinite one-dimensional flow in an early work dated from 1811 (see details in Gratton-Guinness). His principal contributions were only published later in his 1822 treatise. In the meantime, Poisson and Cauchy had applied the Fourier integral to infinite wave motion.

A.L. Cauchy: Théorie de la propagation des ondes à la surface d'un fluide pesant d'une profondeur indéfinie, 1815 Prize Essay; Mém. prés. div. sav. 1 1827 3-312; Œuvres(1) I 4-318. (Extract in Birkhoff's Source Book)

S.D. Poisson: Mémoire sur la théorie des ondes, Mém. Acad. Sci. Paris 1 1816

J.B.J. Fourier: La théorie analytique de la chaleur, Paris 1822
(See Section IV, arts 415, 416)

Burkhardt: Trigonometrische Reihen und Integrale bis etwa 1850,
Enz. Mat. Wiss. II A 12

I Gratton-Guiness: Joseph Fourier and the Revolution in Mathematical Physics,
J. Inst. Maths. Appls. 5 1969 230-253

FOURIER SERIES

Fourier series first occurred in Daniel Bernoulli's solution of the vibrating string problem obtained using the principle of superposition of vibrations, there stated for the first time. His claim that the solution was general led to an extended controversy with Euler and D'Alembert over the possibility of representing an arbitrary function by such a series. Fourier's first paper of 1807, asserting that trigonometrical series could be used to represent an arbitrary function, was rejected for publication. Fourier's early work, mostly unpublished until later, is well documented and described by Grattan-Guiness. Fourier's classic 1822 work, apart from showing applications of such series, also demonstrated convergence in certain cases. W. Thomson (Lord Kelvin) who was an enthusiastic supporter of Fourier's work, was to a large extent responsible for introducing Fourier methods into engineering and physics. (cf Riemann 1854, Hobson 1907, Gratton-Guiness 1969, 1972)

D. Bernoulli: Réflexions et éclaircissements sur les nouvelles vibrations des chordes, Mém. Acad. Berlin 9 1755 147-172

D. Bernoulli: Sur le mélange de plusieurs espèces de vibrations simples isochrones, qui peuvent coexister dans un même système de corps, ibid. 173-195

J.B.J. Fourier: Sur la propagation de la chaleur, Paris 1807 Library MS, École National des Ponts et Chaussées; rpr. in Gratton-Guiness: Joseph Fourier 1768-1830

J.B.J. Fourier: La théorie analytique de la chaleur, Paris 1822

I. Gratton-Guiness: Joseph Fourier 1768-1830, 1972 (MIT Press)

FREDHOLM EQUATION

I. Fredholm: Sur une nouvelle méthode pour la résolution du problème de Dirichlet, Öfversigt af Köngliga Svenska Vetenskaps-Akademiens Förhandlingar Stockholms (Summary of Transactions of the Royal Swedish Academy of Sciences) 57 1900 39-46

I. Fredholm: Sur une classe d'équations fonctionnelles,
Acta Math 27 1903 365-390

FUNCTION NOTATION

The notation f(x) for a function of x is due to Herschel. He defined also powers $f^2(x)$ as f(f(x)) etc. resulting in the notation f^{-1} (x) for the inverse function cf. Cajori 1909

J.F.W. Herschel: Calculus of Finite Differences, Cambridge 1820

FUNDAMENTAL GROUP

Poincaré's definition of the fundamental group immediately follows the definition of the Betti numbers in the memoir below.

H. Poincaré: Analysis situs,
J. Ecole. Poly. 1 1895 1-121; Œuvres VI 193-288 (sect.12)

FUNDAMENTAL THEOREM OF ALGEBRA

This theorem was first proved rigorously by Gauss in his doctoral thesis of 1799. There were previous failed attempts at proof such as those of Euler and Legendre which were reviewed by Gauss. Two proofs were included in Gauss' Disquisitiones and he later deduced a further four proofs.

C.F. Gauss: Demonstratio nova theorematis. ...
Doctoral diss. Helmstedt 1799: Werke III 1-30

C.F. Gauss: Disquisitiones Arithmeticae, Leipzig 1801
See articles 135 (Werke I 104-) 262 (Werke I 292-); Engl. tr. in Smith's Source Book with notes and references to other proofs

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G

GALOIS THEORY

Galois communicated his ideas on groups in a letter written to his friend Auguste Chevalier on the evening before his fatal duel in 1832. His letter summarized his findings and referred to three memoirs. The first and the most celebrated of these memoirs is that referenced below. It was unknown until made public by Liouville in Paris Acad. Sci. July 1843. In 1846 he published it with Galois' other writings.

E. Galois: Letter to Auguste Chevalier, May 29 1832
Engl. tr. in Smith's Source Book I 278-285

E. Galois: Mémoire sur les conditions de résolubilité des équations par radicaux, (posthumous, J. Liouville ed.), J. Math.(Crelle) 1846

GAMMA FUNCTION

The function was defined and its properties investigated by Euler who, in his later 1771 paper used a form of factorial notation. The name and usual capital gamma notation are due to Legendre, cf. Cajori 1928.

L. Euler: Letter to Goldbach, Oct 13 1729, Opera Omnia ser. 4 I letter 715

L. Euler: De progressionibus transcendentibus, seu quarum temini generales algebraice dari nequeunt, Novi Comm. Acad. Sci. Petrop. 5 (1730/1) 1738 36-57; Opera Omnia ser.1 XIV 1-24

L. Euler: Evolutio formulae integralis $\int dx(-lx)^n$ integration a valore x=0 ad x=1 extensa, Novi comm. acad. sci. Petrop 16 (1771) 1772 91-139

A.M. Legendre: Exercices de calcul intégral, Paris 1814 cf. II p.4

A.M. Legendre: Traité des fonctions elliptiques, Paris 1826 (See II p.406)

GAUSSIAN CURVATURE

C.F. Gauss: Disquisitiones generales circa superficies curvas,
Göttingen 1823-2, 99-146; Werke IV (cf. arts 7 to 11)

GAUSSIAN QUADRATURE

Gauss used hypergeometric functions to derive his result. Jacobi gave the simpler method which is normally followed. (cf Chabert et al.)

C.C. Gauss: Methodus nova integralium valores per approximationem inveniendi, Comment. Gött. 3 1814/15 39-76; Werke III 163-196

Jacobi: Ueber Gauss' neue Methode die Werte der Integrale nährungsweis zu finden, J. Math. (Crelle) 1 1826 301-308,

GERSCHGORIN'S THEOREM

This theorem has a somewhat complex history which was reviewed by Bodewig.
It was first stated by Hadamard and then Gerschgorin rediscovered his result and demonstrated its usefulness in numerical analysis. The theorem has also been ascribed to Brauer but without justification.

J. Hadamard: Leçons sur le propagation des ondes,
Collège de France 1898-9, Paris l903 (See pp.707-8)

S. Gershgorin: Über die Abgrenzung der Eigenwerte einer Matrix,
Izv. Akad. Nauk SSSR 7 1931 749-754.

E. Bodewig: Matrix Calculus, 1956 (North-Holland)

GIBBS' PHENOMENON

The Gibbs phenomenon was noted in 1848 by Wilbraham. Later, Michelson and Stratton, using a wave analyzer which could analyze and synthesize Fourier components up to the 80 harmonic, found the phenomenon experimentally. Gibbs maintained that the phenomenon is mathematical. That it is, was finally clarified in the paper of Gronwall 1912. cf. Carslaw 1925, Hewitt & Hewitt 1979

H. Wilbraham: On a certain periodic function, Camb. & Dublin Math. J. 3 1848 198-201, (Extract in Birkhoff's Source Book)

A.A. Michelson & S.W. Stratton: A new harmonic analyzer,
Phil. Mag. 45 1898 85-91

J.W. Gibbs: Fourier series,
Nature 59 1898-99, pp.200, 606; Coll. Works II 258-260

T.H. Gronwall: Zur Gibbschen Erscheinung,
Ann. Math. 31 1930 232-240

H.S. Carslaw: A historical note on Gibb's phenomenon in Fourier series and integrals, Bull AMS 31 1925 420-424

E. Hewitt & R.E. Hewitt: The Gibbs-Wilbraham phenomenon: an episode in Fourier analysis, Arch. Hist. Exact Sci. 21 1979 129-160

GÖDEL'S THEOREM

K. Gödel: Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme, Monatsh. Math. Phys. 38 1931 173-198

GOLDBACH'S THEOREM

In a 1742 letter to Euler Goldbach made a conjecture on primes valid for any integer which can be written as the sum of two primes. Euler later replied saying he was certain any even integer could be so written but could not prove it.(cf. e.g. Wikipedia).

L. Euler: Letter to Goldbach, June 30 1742, Opera Omnia ser. 4 I

GRADIENT (GRAD)

B. Riemann-Weber (ed.): Die partielle Differentialgleichungen der mathematischen Physik, Brunswick 1900 (Gradient defined on p.213)

GRAEFFE'S ROOT-SQUARING METHOD

The method was discovered independently by Dandelin (1826), Lobachevski (1834) and Graeffe (1833, 1837)

G.P. Dandelin: Recherches sur la résolution des équations numériques, Mém. Acad. Sci. Bruxelles 3 1826 7-71, 153-159

C.H. Graeffe: Beweis eines Satzes aus der Theorie der numerischen Gleichungen, J. Math (Crelle) 10 1833 288-291

N.I. Lobachevski: Algebra ili vychislenie konechnykh, (Algebra or Calculus of Finites) Kazan 1834; Pol. Sobr. Soch. IV (see para. 257)

C.H. Graeffe: Auflösung der höheren numerischen Gleichungen, Zürich 1837

A.S. Householder: Dandelin, Lobachevski or Graeffe? Amer. Math. Monthly 66 1959 464-466

GRAM-CHARLIER SERIES

The contributions of Chebyshev and Bruns are usually overlooked.

J.P. Gram: Om Raekkendviklinger bestemte ved Hjaelp af de mindste Kvadraters Methode, Diss. Univ. Copenhagen 1879

P.L. Chebyshev: Sur deux théorèmes relatifs aux probabilités, Mem. Acad. Sci. St. Petersb. no.6 1887, supplement to vol.15; Acta Math. 14 1890-1891 305-315; Œuvres II 481-491 (cf. p.490)

H. Bruns: Ueber die Darstellung von Fehlergesetzen, Astr. Nachr. 143 1897 329-340

H. Bruns: Wahrscheinlichkeitsrechnung und Kollectivmasslehre, Leipzig-Berlin 1906 (Teubner)

C.V.L. Charlier: Ueber das Fehlergesetz, Arkiv för Math. 2 1905-1906

GRAM-SCHMIDT METHOD

Gram's contribution was overlooked until Schmidt's reference to it in 1907.

J.P. Gram: Om Raekkendviklinger bestemte ved Hjaelp af de mindste Kvadraters Methode, Diss. Univ. Copenhagen 1879

J.F. Gram: Ueber die Entwicklung reeller Functionen in Reihen mittelst der Methode der kleinsten Quadrate, J. Math. (Crelle) 94 1883 41-73

E. Schmidt: Entwicklungen willkürlicher Funktionen nach Systemem vorgeschriebener, Diss. Göttingen 1905, (published in next reference)

E. Schmidt: Zur Theorie der linearen und nichtlinearen Integralgleichungen,
Math. Ann. 65 1907 433-476

GREEN'S FUNCTION

This was used in Green's 1828 essay where it made its appearance in the special sense of the electric potential induced by a system of point charges; it was the direct generalization to electricity of the gravitational potential as used by Laplace.

G. Green: An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism, Nottingham 1828, reprinted later (see next entry)

GREEN'S THEOREM

After Gauss' statement of this theorem in 1840 it was found that Green had previously proved it in his forgotten 1828 essay which was subsequently reprinted in Crelle's Journal. Independently, Ostrogradski had proved an n-dimensional form of the divergence theorem in 1831 but his presentation is less clear than Green's and unrelated to any physical application. Osgood observed that the representation in 3 dimensions of volume integrals by surface integrals goes back to Lagrange in 1760.
The 2 dimensional version of the theorem, also called Green's theorem in analysis books, was used by Riemann in 1851 as a basis for treating functions of a complex variable (see Cauchy-Riemann).

J.L. Lagrange: Nouvelles recherches sur la nature et la propagation du son,
Misc. Taurin. II 1760-1761; Œuvres I 151-318 (see p.263)

G. Green: An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism, Nottingham 1828;
J. Math.(Crelle) 39 1850; 44 1852; 47 1854 (cf. sect.3)

M.V. Ostrogradski: Mémoire sur le calcul des variations des intégrales multiples,
Mém. Acad. Sci. St. Petersb. 1831(6) 3 pt 1, 36-58; J. Math.(Crelle) 15 1836 332-354

C.F. Gauss: Allgemeine Lehrsätze in Beziehung auf die im verkehrten Verhältnisse des Quadrats der Entfernung wirkenden Anziehungs= und Abstossungs= kräfte, Leipzig 1840; Ges. Werke V 224-

GREGORY-NEWTON INTERPOLATION FORMULA

J Gregory: Letter to Collins 1670

I Newton: Methodus differentialis, 1676; published 1711

H

HAHN-BANACH THEOREM

H. Hahn: Über lineare Gleichungssysteme in linearen Räumen,
J. Math.(Crelle) 157 1927 214-229

S. Banach: Sur les fonctionnels linéaires,
Studia Math. 1929 I 211-216; II 223-239

S. Banach: Théorie des Operations Linéaires, Warsaw 1932

H. Hochstadt: Eduard Helly, father of the Hahn-Banach Theorem,
The Math. Intell. 2(3) 1980 123-125

HAMILTON'S CANONICAL EQUATIONS

These first occurred in a special form in the investigations of Lagrange and Poisson on perturbations of planetary elements caused by disturbing forces. Hamilton's derivation used his principal function S (the action). The equations were first announced in his brief British Association report of 1934 and then discussed in a more detailed way in connection with Hamilton's Principle in his 1935 papers quoted below.
cf. Cayley 1857, 1862, Whittaker 1904.

J.L. Lagrange: Mécanique analytique, Paris 1787,
2nd part-dynamics, sect.V, para.II; Œuvres XI p.357

S.D. Poisson: Sur les inégalités séculaires des moyens mouvements des planètes,
J. Ec. Poly. 8 1809 1-56

W.R. Hamilton: On the application to dynamics of a general mathematical method previously applied to optics, Brit. Assn. Rep 1834 513-518; Math. Papers II 212-216

HAMILTON'S PRINCIPLE

Hamilton's first statement of this principle occurred in the second of his 1935 papers quoted below. At the end of the first paper he had defined the function

$$S = \int (T + U)\, dt$$

where T is the kinetic energy and U the negative of the potential V.
In the second paper he further developed the properties of S and named it the principal function, then remarking:

> *'..; and it is worth observing, that when S is expressed by this definite integral, the conditions for its variation vanishing (if the final and initial coordinates and the time be given) are precisely the differential equations of motion ... under the forms assigned by Lagrange'*

For Hamilton the importance of this observation is that it showed that his method for dynamics based on the use of S is entirely equivalent to direct use of Lagrange's equations. That this is so follows from a simple application of the calculus of variations and it is strange that Lagrange, who was himself so much concerned with the calculus of variations, should never have made the same observation. Ostrogradski however did and in Russia the principle is called the Ostrogradski-Hamilton Principle. Hamilton's ideas were taken up by Jacobi to whom the name 'Hamilton's Principle' is due.
(cf. Cayley 1857, 1862)

W.R. Hamilton: On a general method of dynamics; by which the study of the motion of all free systems of attracting or repelling points is reduced to the search for and differentiation of one central relation, or characteristic function, Phil. Trans. pt II 1835 247-308; Math. Papers II 101-161

W.R. Hamilton: Second essay on a general method in dynamics,
Phil. Trans. 1835 pt.I 95-144; Math. Papers II 162-211

C.G.J. Jacobi: Vorlesungen über Dynamik, Königsberg 1842/3;
Rpr. New York 1969 (Chelsea) (cf. supplement p.58 in Ges. Werke.VIII)

HAMILTON – JACOBI EQUATION

The equation is central to Hamilton's method in dynamics based on the use of the function S (cf. last entry). The differential equation for S is mentioned briefly in the first essay and developed in full in the second essay. The method was developed further by Jacobi in his Königsberg lectures on dynamics quoted above.

HARDY SPACE

G.H. Hardy: The mean value of the modulus of an analytic function,
Proc. Lond. Math. Soc. 14 1915 269-279

HAUSDORFF MEASURE, SPACE

F. Hausdorff: Grundzüge der Mengenlehre Leipzig 1914; 2nd. rev. ed. 'Mengenlehre' Berlin-Leipzig 1927; J.R. Aumann (tr.) 'Set Theory' New York 1957 (Chelsea)

HEINE-BOREL THEOREM

The underlying idea first occurred in Heine's proof that a continuous function in a closed interval is uniformly continuous. Borel then gave the first explicit statement and proof of the theorem. Young gave it the name 'Heine-Borel' and generalized it to any number of dimensions for a countable set of overlapping intervals.

E.H. Heine: Über trigonometrischer Reihen,
J. Math.(Crelle) 71 1870 353-365 (esp. 361)

E.H. Heine: Die Elemente der Funktionlehre,
J. Math.(Crelle) 74 1874 172-188

E. Borel: Sur quelques points de la théorie des fonctions,
Ann. Sci. Ec. norm. sup. 12 1895 9-55 (Note on p.51)

W.H. Young: Overlapping intervals,
Proc. Lond. Math. Soc. 35 1902 384-388 (cf. 387)

W.H. Young: The Theory of Sets of Points,
Cambridge 1906 (Univ. Press); 2nd. ed. 1972 New York (Chelsea)

HELMHOLTZ EQUATION

H. Helmholtz: Theorie de Luftschwingen in Röhren mit offenen Enden,
J. Math. (Crelle) 57 1860 1-72 ; Wiss. Abh. I 303- Extract in Birkhoff's Source Book

HENON MAP

M. Hénon: A two-dimensional mapping with a strange attractor,
Commun. Math. Phys. 50 1976 69-77

HERMITE POLYNOMIALS

These polynomials first occurred in Laplace's work on probability and, although they are barely recognizable as such, their principal properties were proved. They were subsequently fully discussed in an essentially modern presentation by Chebyshev in 1859. Hermite's paper was therefore misnamed since the development of a function in terms of these polynomials was not new. His contribution was that in his 1865 papers he defined and discussed the multidimensional polynomials.

P.S. Laplace: Théorie Analytique des Probabilités 1812; Livre 2ième
(polynomials on p.321, orthogonality on p.323)

P.L. Chebyshev: Sur le développement des fonctions à une seule variable,
Bull. Phil-Math. Imp. Acad. Sci. St. Petersb. 1 1859 193-200; Œuvres I 499-508
(rpr. New York Chelsea 1989)

C. Hermite: Sur un nouveau développement en série de fonctions,
C.R. Acad. Sci. Paris, 58 1864 93-100, 266-273, Œuvres II, Paris 1908 293-308.

C. Hermite: Sur quelques développements en série de fonctions de plusieurs variables, C.R. Acad. Sci. Paris, 60 1865 370-377; 432-440; 461-466; 512-518; Œuvres II, 319-346

HILBERT'S LIST OF UNSOLVED MATHEMATICAL PROBLEMS

D. Hilbert: Mathematische Probleme, Intern. Conf. Math. Paris 1900; Gött. Nachr 1900 253-297 Ges. math. Werke III 290-329; Engl. tr. Bull. AMS (2) 8 1901-02 437-479

HILBERT SPACE

Hilbert space evolved from Hilbert's articles on integral equations. In Hilbert's definition the elements of the space were sequences, the geometrical interpretation as an infinite dimensional Euclidean space being mainly due to Schmidt. The axiomatic treatment came much later from von Neumann and Stone when the development of Hilbert space theory owed much to its use in quantum mechanics.

D. Hilbert: Grundzüge einer allgemeinen Theorie der linearen Integralgleichungen, Gött. Nachr. 1904-1910
Published as a book: Leipzig 1912 (Teubner); rpr. New York 1952 (Chelsea)

E. Schmidt: Über die Auflosung linearer Gleichungen mit abzählbare unendliche vielen unbekannten, Rend. Cir. Mat. Palermo 25 1908 53-77

J. von Neumann: Allgemeine Eigenwertetheorie Hermitescher Funktionaloperatoren, Math. Ann. 102 1929 49-131; Coll. Works II 3-85

P.A.M. Dirac: The Principles of Quantum Mechanics, Oxford U.P. etc. 1930

J. von Neumann: Mathematische Grundlagen der Quantenmechanik, Berlin 1931 (Springer)

M.H. Stone: Linear Transformations in Hilbert Space, New York 1932 (AMS Colloquium Pub.)

HÖLDER CONDITION

O.L. Hölder: Beiträge zur Potentialtheorie, Diss. Univ. Tübingen 1882

HÖLDER'S INEQUALITY

Hölder proved it for finite sums acknowledging that it had been previously stated in this form by Rogers. The integral form is due to Riesz. (cf Dudley 1989)

L.J. Rogers: An extension of a certain theorem in inequalities, Mess. Math. 17 1888 154-150

O.L. Hölder: Ueber einen Mittelwertsatz, Nach. Ges. Wiss. Gött. 1889 38-47

F. Riesz: Untersuchungen über Systeme integrierbarer Funktionen, Math. Ann. 69 1910 449-497

HOMOLOGY

This was defined by Poincaré at the beginning of his second memoir on analysis situs.

H. Poincaré: Complément … l'analysis situs,
Rend. Circ. Mat. Palermo 13 1899 285-343; Œuvres VI 290-337

M. Bollinger: Geschichliche Entwicklung des Homologiebegriffs,
Arch. Hist. Exact Sci. 9 1971 94-166

HOPF MAP

E. Hopf: Über die Abbildungen der dreidimensionalen Sphäre auf die Kügelflache,
Math. Ann. 104 1931 639-665; rpr. in Selecta Heinz Hopf 1964 38-63

HORNER'S METHOD

It is commonly thought that Horner's method was described in his 1819 paper. That paper however describes another method which used a similar tabular layout of the calculation. Horner only described the familiar method, which he called synthetic division, in a later publication of 1830. This method had however been previously published as a supplement to the 1820 book of Holdred. Already in 1804 Ruffini had proposed what was essentially the same idea though with a different layout. Popularization of the method under Horner's name was principally due to De Morgan. Techniques having a similarity to it were known long before in China in the 13th century and by al Kashi in the 14th century. [Boyer, Mikami]

Ch'in chiu-shao: Su-shu Chiu-chang: (Nine Sections of Mathematics) 1247
P. Ruffini: Sopra la Determinazione delle Radici nelle Equazione Numeriche de Qualunque Grado. Modena 1804 (Societ… Italiana delle Scienze) [see pp. 22-25]

W.G. Horner: A new method of solving numerical equations of all orders, by continuous approximation, Phil. Trans. II 1819 308-335; rpr. Ladies Diary 1838 49-72; Extract in Smith's Source Book

T. Holdred: A new method of solving equations with ease and expedition; by which the true value of the unknown quantity is found without previous reduction.
London 1820 (Davies & Dickson)

W.G. Horner: Horae arithmeticae, Mathematical Repository (Leybourn ed.) 5 1830 21-75; posthumously reprinted in another version in 'The Mathematician' 1 1845 108-112

F. Cajori: Horner's method of approximation anticipated by Ruffini,
Bull. AMS 17 1911 409-414

A.T. Fuller: Horner versus Holdred: an episode in the history of root computation,
Hist. Math. 26 1998 29-51

HUYGHENS' PRINCIPLE

C. Huyghens: Traité de la lumière, [1678] Paris 1690

HYPERBOLIC FUNCTIONS

Thee were introduced by V Riccati the son of Count Riccati known for his equation. They were further developed by Lambert. cf. Cajori: Hist. math. notns.

V Riccati: Opusculorum ad Res Physicas et Mathematicas pertinentium, 2 vols Bolognia 1757-62

J.H. Lambert: Hist. Acad. Berlin 2 1768 p.327

HYPERGEOMETRIC SERIES

Wallis originated the name 'hypergeometric' in his work on a similar series. The hypergeometric series itself was first studied by Euler who derived many of its properties in a formal way. Pfaff devoted a long chapter in his book to the derivation of many of its properties. Its convergence was first proved by Gauss in a famous paper in which he used the ratio test now named after him. He is sometimes considered as the originator of the hypergeometric series possibly owing to the influence of the title of 1857 paper of Riemann. Riemann had however himself published another short historical paper in which he noted the prior work of Euler and Pfaff. Gauss made other contributions which were published posthumously. Many of these results had been given in Kummer's paper of 1836.

J. Wallis: Opera Mathematica I, Oxford 1695

L. Euler: Specimen transformationis singularis serierum,
Nov. Acta Acad. Petrop. 12 (1794) 1801 58-70; Opera omnia ser.1 XVI 41-55
(cf. also comments in the prefaces to vols. 12 and 16 of Opera Omnia, ser.1)

J. Pfaff: Disquisitiones analyticae, Helmstadt 1797

K.F. Gauss: Disquisitiones generales circa seriem infinitam

$$1 + \frac{\alpha.\beta}{1.\gamma}.x + \frac{\alpha\,(\alpha+1)\,\beta.(\beta+1)}{1.2.\,\gamma\,(\gamma+1)}\,x^2 + \ldots$$

Comm. Roy. Soc. Gött II (1812) 1813; Werke III 125-162

E.E. Kummer: Ueber die hypergeometrische Reihe,
J. Math. (Crelle) 15 1836 39-83; 127-172

B. Riemann: Beiträge zur Theorie der durch die Gaussche Reihe, F(α, β, γ; x) darstellbaren Funktionen, Gött. Abh. 7 1857.; Werke 67-83;
Selbstanzeige der vorstehenden Abhandlung. Gött. Nachr. 1 1857; Werke 84-58

K.F. Gauss: Determinatio seriei nostrae per aequationem differentialem secundi ordininis, Werke III 207-230

I

I AS SQUARE ROOT OF MINUS ONE

Euler was first to use i for $\sqrt{-1}$ but the notation only came into general use after Gauss had used it consistently. (Cajori's Hist. Math. Notns.)

L. Euler: De formulis differentialibus, Mem. Acad. Sci. St. Petersb. 1777;
published posthumously in Inst. Calc. Int., 2nd ed. 1794 IV 184-

K.F. Gauss: Disquisitiones arithmeticae, Leipzig 1801; Werke I p.414

INDICES

The use of indices can be traced to Oresme in the 14th century. They were systematically developed by Stevin and DesCartes. Negative and fractional indices were used first by Wallis and Newton in connection with the binomial theorem.
(Source: Cajori, Hist. Math. Notns.)

S. Stevin: Œuvres, Leiden 1634

R. DesCartes: La Géométrie, Leiden 1637 (Transl. in Smith's Source Book)

I. Newton: Commercium Epistolicum 1712. Letters of June 13, October 24, 1676 to Henry Oldenburg, Sec. Royal Society; Transl. in Smith's Source Book I, 224-231

J. Wallis: Algebra, London 1685, chap.91:'The Doctrine of Infinite Series, further prosecuted by Mr Newton.' Transl in Smith's Source Book I 219-223

INFINITY

The concept of infinity in a philosophical and religious sense can be traced back to antiquity, especially to Aristotle. It found a clear statement in the writings of Giordano Bruno and the infinite was a primary concept in DesCartes' philosophy. Mathematically it was Kepler who in 1615 introduced for the first time the name and notion of infinity into geometry in connection with optics. The symbol ∞ for infinity was first used by Wallis in his 1655 book on conics. It may have originated from a Roman sign for 10,000 which more generally meant any very large number.
(Sources: Koyré, Enc. Brit. 11th ed., Oxford Eng. Dict., Cajori's Hist. Math. Notns.)

Aristotle: Physica Γ 6: The infinite,
In Greek and English in I. Thomas: Greek Mathematical Works (Loeb) I 424-429

G. Bruno: De l'infinito universo et mondi, Venice-London 1584,
Engl. transl: 'On the Infinite Universe and the Worlds' in D.W. Singer 'Giordano Bruno, His Life and Thought', NY 1950 (Abelard-Schuman)

J. Kepler: Nova Stereometria ..., Linz 1615

J. Wallis: De sectionibus conicis ..., Oxford 1655

J

JACOBIAN

Although previous indications of this idea can be found, the concept really clarified in the discussion of the formula for transformation of multiple integrals, in particular, in the paper of Catalan (although priority has been claimed for Ostrogradski). The theory was developed in detail by Jacobi in his 1841 paper and later repeated in his lectures on dynamics. The usual notation is due to Donkin. cf. Muir 1906, Cajori 1928.

M. Ostrogradski: Sur la transformation des variables dans les intégrales multiples,
Mém. Acad. Sci. St. Petersb. 1 1838 401- 407

E.C. Catalan: Sur la transformation des variables dans les intégrales multiples,
Mém. Acad. Bruxelles 14 (pt 2) 1839, 47 pp

C.G.J. Jacobi: De determinantibus functionalibus,
J. Math.(Crelle) 22 1841 319-59; Werke III 393-438

C.G.J. Jacobi: Vorlesungen über Dynamik, Königsberg 1842/3
Rpr. New York 1969 (Chelsea); Ges. Werke VIII, (See 30th lecture)

W.F. Donkin: Demonstration of a theorem of Jacobi relative to functional determinants,
Camb. & Dublin Math.J. 9 1854 161-163

JACOBI'S LAST MULTIPLIER

C.G.J. Jacobi: Sul principio dell'ultimo moltiplicatore, e suo uso come nuove principio generale de meccanica, J. Math.(Crelle) 10 1845 387-346

JENSEN'S INEQUALITY

J.L.W.V. Jensen: Sur les fonctions convexes et les inégalités entre les valeurs moyennes, Acta Math. 30 1906 175-193

JORDAN CURVE

C. Jordan: Cours d'Analyse de l'Ecole Polytechnique,
1st ed III 1887 593; 2nd ed I 1893 p.90 [cf Kline]

JORDAN DECOMPOSITION

Originally it was the decomposition of a function of bounded variation into monotone components. Riesz extended it to additive set functions. The name is due to Saks.

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C. Jordan: Cours d'Analyse de l'Ecole Polytechnique, 3rd ed.
Paris 1909-1915 (Gauthier-Villars) I p.54

F. Riesz: Sur la décomposition des opérations fonctionnelles,
Atti Congr. Bologna 3 1928 143-8

S. Saks: Théorie de l'intégrale,
Warsaw 1933 (Monografje Matematyczne)

JORDAN-HÖLDER THEOREM

C. Jordan: Commentaire sur Galois,
Math. Ann. 1 1869 141-160

C. Jordan: Traité des substitutions et des équations algébriques,
Paris 1870 (Gauthier-Villars); rpr. 1957

O. Hölder: Zurückführung einer beliebigen Gleichung auf eine Kette von Gleichungen,
Math. Ann. 34 1889 26-56

JULIA SETS

G. Julia: Mémoire sur l'itération des fonctions rationnelles,
J. Math. (Liouville) 1 1918 47-245

K

KÄHLER VARIETY

E. Kähler: Über eine bemerkenswerte Hermitische Metrik,
Abh. Math. Seminar. Hamb. 9 1933 173-186.

KEPLER'S LAWS

The first two laws were formulated in 1609 for Mars, the third law ten years later.
Ref. Dugas 1950

J. Kepler: Astronomia nova aitiologitis ..., Prague 1609.
(Law of areas: chap.XXXII, p.165; law of ellipticity: chap.LV p.285)

J.Kepler: Harmonice Mundi Linz 1619
(Third law: book V, chap.III, prop.VIII)

KEPLER'S EQUATION

J. Kepler: Epitomes astronomiae Copernicaniae,
Linz 1618-1622, book 5 694-696.

J.L. Chabert et al: Histoire d'Algorithmes, Berlin 1993

KILLING EQUATION

W. Killing: Ueber die Grundlagen der Geometrie,
Crelle 1892 121-186

KLEIN BOTTLE

It was described by Klein in terms of a rubber tube and so it was called "Kleinsch Schlauch" usually translated, though not quite correctly, as Klein bottle.

F. Klein: Über Riemann's Theorie der algebraischen Funktionen und ihre Integrale
Leipzig 1882 (Teubner), rpr. New York (Dover); Ges. Math. Abh. III 499-573.
(See section 23, case (3))

KOCH CURVE

H. von Koch: Sur une courbe continue sans tangente obtenue par une construction géométrique élémentaire, Arkiv Math. Astr. Fysik 1 1904 681-704

KOENIGSBERG BRIDGE PROBLEM

L. Euler: Solutio problematis ad geometriam situs pertinentis, [1735]
Comm. Acad. Sci. Petrop. 8 1736 128-40 published 1741; Engl. tr. in J.R. Newman, The World of Mathematics 1956 (Simon & Schuster) also in Struik 184-187

KRONECKER DELTA

Kronecker: Über bilineare Formen,
Preuss. Akad. Wiss. Monatsb. 1866; Crelle 1866 (cf. Jeffries, Bull. IMA 21 1985)

KRYLOV-BOGOLYUBOV METHOD

N. Krylov & N. Bogolyubov: Méthodes de la mécanique non-linéaire appliquées à l'étude des oscillations stationnaires (Ukrainian with a French summary) Kiev 1934, Ukrainska Akad. Nauk Inst. Mech., Report 8

N. Krylov & N. Bogolyubov: Introduction à la mécanique non-linéaire; les méthodes approchées et asymptotiques, Kiev 1937, Ukrainska Akad Nauk, Inst. Mech. Annales 1-2. Abridged, partial English translation by S. Lefschetz: 'Introduction to Non-linear Mechanics' Princeton 1943 (Annals of Math. Studies)

L

LAGRANGE'S DYNAMICAL EQUATIONS

The use of his dynamical equations in generalized coordinates was a central theme in Lagrange's treatise 'Mecanique analytique'. The usual statement of these equations using what is now known as the Lagrangian function is, however, due to Routh.

J.L. Lagrange: Mécanique Analytique, Paris 1797
(See I 290-292 in the 3rd. reprinted edition)

E.J. Routh: Stability of a given State of Motion, London 1877 (MacMillan)
rpr. 'Stability of Motion' ed. A.F. Fuller, London 1974 (Taylor & Francis) (See p.47 for the mathematical definition, the term 'Lagrangian function' being subsequently used)

E.J. Routh: Dynamics of a System of Rigid Bodies,
London & Cambridge 1860 (MacMillan) (cf. I p.357)

LAGRANGE INTERPOLATION FORMULA

Waring stated this formula explicitly in 1779 and had a clear priority over Lagrange who gave the formula in lectures at the Ecole Normale publishing it in 1795. Euler in 1783 had given what is now called the Lagrange formula for equidistant coordinates. (cf. Kopal: Numerical Analysis p.79)

E Waring: Problems concerning interpolations,
Phil. Trans. 69 1779 59-67

L. Euler: Opuscula Analytica 1783 vol.1

J. L Lagrange: J. Ec. Polytech. 2 1795 274-

LAGRANGE'S THEOREM ON GROUPS

J.L. Lagrange: Réflexions sur la résolution algébrique des équations
Mém. Acad. Berlin 3 1770-71; Œuvres III 205-424

LAGRANGE PROBLEM IN THE CALCULUS OF VARIATIONS

This was added in the 2nd edition of Lagrange's lessons on the calculus of functions.

J.L. Lagrange: Leçons sur le calcul des fonctions. 2nd ed. Paris 1806; Œuvres X;
rpr: J. Ec. Polytech 1808. (See lesson 22 p.416 in Œuvres)

LAGUERRE'S PROJECTIVE DEFINITION OF ANGLE

E. Laguerre: Note sur la théorie des foyers,
Nouv. Ann. Math. 11 1852 290-292; 12 1853 57-66; 225-236, Œuvres II 4-15

LAPLACE EQUATION

The 2-dimensional form of the equation first occurred in the theory of 2- dimensional fluid motion due to d' Alembert and Euler, being implicit in d' Alembert's paper and explicit in Euler's paper. Lagrange's paper of 1781 on fluid motion gave the 3-dimensional equation for the potential function when it exists. The 3 dimensional form of the equation then occurred in spherical coordinates in Laplace's paper on the attraction of spheroids and in Cartesian coordinates in his 1787 essay on Saturn's rings. Laplace in his Mécanique Céleste gave the equation central importance, it always being quoted in the form

$$0 = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Here V is what was later called the gravitational potential. On the initial occurrence of the equation Laplace says:

> *'cette équation remarquable nous sera de la plus grande utilité dans la théorie de la figure des corps célestes.'*

cf. Birkhoff's Source Book

J.L. Lagrange: Mémoire sur la théorie du mouvement des fluides,
Nouv. Mém. Berlin 1781; Œuvres IV 695-748
(The 3-dimensional Laplace equation for fluid potential is on p.713)

P.S. Laplace: Théorie des attractions des sphéroides et de la figure des planètes,
Mém. Acad. Sci. Paris [1782] 1785; Œuvres X 339-419 (cf. Birkhoff's Source Book)

P.S. Laplace: Mémoire sur la théorie de l'anneau de saturne, [1787],
Mém. Acad. Sci. 1789; Œuvres XI 273-292 (see p. 252) (cf. Birkhoff's Source Book)

P.S. Laplace: Traité de Mécanique Céleste 1799-1825
(See: Premiere Partie, Livre II, chap.II; Livre III, chap.II; chap.VI on Saturn's rings)

LAPLACE TRANSFORM

The ascription to Laplace comes from his use of the technique of generating functions, e.g. he says (1812):

> *'The passage from the functions to their generating functions and back constitutes a calculus of generating functions.'*

Laplace's use of generating functions resembles the modern z-transform and he used it in a similar way to solve difference equations. But he only indicated the possibility of the continuous analogue by a brief comment. The continuous analogue was subse-

quently well described by Abel who very clearly stated the transformation idea of what is now called the Laplace transform, demonstrating its characteristic properties. He did not however apply it to concrete problems. This was first done by Heaviside who, contrary to what is often thought, was well aware of the Laplace transform method although he considered his own operator technique to be superior since it went directly to the answer. The Laplace transform method was rediscovered in its connection with the theory of functions of a complex variable by Lerch and Bromwich. Its gradual adoption came about through a reaction to the lack of mathematical rigour in the Heaviside operator approach, the book of Doetsch being especially influential. The method owes its current popularity to its use in electrical, control, and systems engineering. These applications happened largely in and after World War II.
(see e.g. Gardener & Barnes).

P.S. Laplace: Théorie analytique des probabilités, Paris 1812, 1825
(Chap.I, sect. 2-20, On generating functions)

P.S. Laplace: Essai philosophique sur les probabilités, 1814 etc,
Book 1, Calculus of generating functions

N.H. Abel: Sur les fonctions génératrices et leurs déterminantes, [1820]
Œuvres Complètes, B. Holmboe ed. 1839 II 77-88

O. Heaviside: The solution of definite integrals by differential transformation,
Electromagnetic Theory, London 1893/1912 (see section 526)

M. Lerch: Sur un point de la théorie des fonctions génératrices d'Abel, Rosprovy ceské Akademie 1892; Acta Math. 27 1903 9-351 (Proof of the inversion formula)

T.J. Bromwich: Normal coordinates in dynamical systems,
Proc. London Math. Soc. 15 1916 401-448 (cf. 412) (Bromwich integral)

G. Doetsch: Theorie und Anwendung der Laplacesche Transformation,
Berlin 1937 (Springer), Engl. tr. New York 1943

M.F. Gardner & J.L. Barnes: Transients in Linear Systems studied by the Laplace Transform. New York 1942 (Wiley)

LAURENT SERIES

M Laurent: Extension du théorème de M. Cauchy relatif à la convergence du développement d'une fonction suivant les puissances ascendantes de la variable x.
C. R. Acad. Sci. Paris 17 1843 365- .

M Laurent: Théorie des Séries, Paris 1862

LEAST ACTION PRINCIPLE

This originated in Maupertius' attempt to obtain, for the corpuscular theory of light, a principle analogous to Fermat's Principle of Least Time. It was more correctly established for the case of a single particle under the attraction of a central force by Euler and then treated more generally by Lagrange. cf. Whittaker 1904

P.L.M. de Maupertius: Accord de différentes lois de la Nature qui avaient jusqu'ici paru incompatibles, Mém. de l'Acad. 1744 (cf. p.417)

L. Euler: Methodus inveniendi ...   Lausanne 1744 (Addit. II p.309);
Dissertatio de principio minimae actionis 1753;

J.L. Lagrange: Application de la méthode exposée dans la Mémoire précédent à la solution de différents Problèmes de Dynamique,
Misc. Taurin. II 1760-1; Œuvres I 365-470

J.L. Lagrange: Mécanique Analytique, Paris
2nd part, sect.VI, para.39; Œuvres XI p.315

LEAST SQUARES INTERPOLATION

Approximation and interpolation of functions by polynomials using least squares was developed fully by Chebyshev in papers normally overlooked. The reference quoted is his most important contribution.

P. L. Chebyshev: On interpolation by the method of least squares,
(Russian) Acad. Sci. St Petersburg 1859; Coll. Works vol.I (1859)

LEAST SQUARES METHOD

The first published account of the classical least squares method was that of Legendre in 1806 though without any probability interpretation. Gauss' first published account of 1809 made use of the normal law of error and was based on effective use of the method in astronomy. Gauss mentioned (section 187) the work of Legendre remarking that he himself had used the method since 1795, a remark which gave rise to bad feeling between him and Legendre. There is not known any written record of such prior use by Gauss. The probability interpretation was further discussed by Laplace in 1812. To Gauss must go most credit for the method both for his clear and complete mathematical description and also for his extensive practical application of the method in astronomy, surveying, and geodesy (e.g. his paper of 1826 was a direct outcome of work in geodesy).

A.M. Legendre: Nouvelles méthodes pour la détermination des orbites des comètes. Appendice sur la méthode des moindre carrés, Paris 1806,
Engl. tr. in Smith's Source Book II

C.F. Gauss:Theoria Motus Corporum Coelestium in Sectionibus Conicis Solem Ambientium. Hamburg l809 (Perthes & Besser), Werke VII; (book II, section III, Bayes' method is used) C.H. Davis (tr.): Theory of the Motion of Heavenly Bodies Moving about the Sun in Conic Sections, 1857 (Little - Brown Co) rpr. 1963 (Dover)

A.M. Legendre: Méthode des moindres carrés pour trouver le milieu le plus probable entre les résultats de différentes observations, Mém. Inst. France 1810 149-154

P.S. Laplace: Théorie analytique des probabilités, Paris 1812

C.F. Gauss: Theoria combinationis observationum erroribus minimis obnoxiae, pars prior 1821, pars posterior 1823 (least squares is in pars posterior)
Supplementum theoriae combinationis observationum erroribus minimis obnoxiae Roy. Soc. Göttingen 1826; Werke IV 1-90

LEBESGUE INTEGRAL

Lebesgue first gave the theory in his 1902 thesis developing it in greater detail in his 1904 book.

H. Lebesgue: Intégrale, Longueur, Aire, Thesis Paris Univ. 1902; rpr. Annali Mat. pura appl, 7 1902 231-359

H. Lebesgue: Leçons sur l'intégration et la recherche des fonctions primitives, Paris 1904, 1928 (Gauthier-Villars)

LEGENDRE POLYNOMIALS

These polynomials occurred almost simultaneously in works Laplace and Legendre. Legendre however discussed their special properties.

P.S. Laplace: Théorie de Jupiter et de Saturne. [1782], Mém. Acad. sci. Paris 1785 (differential equation for Legendre and associated Legendre polynomials on p.137)

A.M. Legendre: Sur l'attraction des spheroïdes homogènes, Mém. pres. div. savants. 10 1785 411-434 (Extract in Birkhoff's Source Book)

LINEAR PROGRAMMING

What later became known as linear programming was developed in 1947 by Dantzig to solve problems of the United States Air Force. His papers had restricted circulation for some years before being printed in Koopman's book. Similar techniques had previously been developed in 1940 by Kantorovich who was the discoverer of the method.

L.V. Kantorovich: A new method for solving some classes of extremal problems, Doklady Acad. Sci. USSR 28 1940 211-14

G.B. Dantzig: Maximization of a linear function of variables subject to linear inequalities 1947 (unpublished)

T.C. Koopmans (ed.) Activity Analysis of Production and Allocation, New York 1951 (Wiley-Chapman-Hall) Danzig's article is on pp.339-347

LIOUVILLE'S THEOREM

i.e. the conservation of 2n dimensional volume in the phase space of a dynamical system of n degrees of freedom. This theorem was used by Maxwell in the kinetic theory of gases and by Poincaré in a proof of his Recurrence Theorem (qv.)

J. Liouville Note sur la théorie de la variation des constantes arbitraires
J. Math (Liouville) 3 1838 342-349

J.C. Maxwell: On Boltzmann's Theorem on the average distribution of energy in a system of material points. Camb. Phil. Soc. Trans. 12 1869; Sci. Papers II 713-741

LIPSCHITZ CONDITION

R. Lipschitz: Disamin della possibilita… d'integrare completamente un dato sistema di equazioni differenziali ordinarie, Ann Mat. pura appl. 2 1868/69 288-302; Sur la possibilité d'intégrer complètement un système donné d'équations différentielles, (French) Bull. Sci. Math. 10 1876 149-159 (Extract in Birkhoff Source Book)

LOGARITHMS

They were discovered independently by Bürgli and Napier whose logarithms were equivalent to those to base e. The logarithms developed soon after by Briggs and Vlacq were equivalent to logarithms to base 10. The analytical theory of logarithms and the relations with the exponential function was worked out in detail by Euler.

Napier: Mirifici logarithmorum canonis constructio seu arithmeticarum supputationum mirabilis abbreviatio. [1614], published posthumously, Edinburgh 1620 (Barth. Vincentium)   An English translation was produced by Edward Wright in 1616 before it was published. Excerpts are given in Smith's Source Book I 149-155 from the translation of W.R. MacDonald, Edinburgh 1889 (Blackwood)

J. Bürgli: Arithmetische und geometrische Progress Tabulen, sambt grundlichen unterricht wie solche nutzlich in allerley Rechnung zugerbrauchen un verstanden werden sol, Prague 1620

E. Wright:Description of the Admiralty Table of Logarithms, London 1616

H. Briggs: Arithmetica logarithmica sive logarithmorum chiliades triginta, London 1624

A. Vlacq: Arithmetica logarithmica sive logarithmorum chiliades centrum, 2nd ed. Gouda 1628

L. Euler: De quantitatibus exponentialibus ac logarithmis, Analysin Infinitorum, Lausanne 1748, chapters 6,7; Opera Omnia ser.1 VIII 103-132

LORENZ ATTRACTOR

E.N. Lorenz: Deterministic nonperiodic flow, J. Atmos. Sci. 20 1963 130-141

M

MACLAURIN'S THEOREM

This theorem had been previously stated by Stirling. It is, of course, a special case of Taylor's Theorem (qv.) known to Taylor and Gregory before MacLaurin. Cauchy's later exposition of the theorem was the most complete; he illustrated it with examples, found the well-known Cauchy form for the remainder and also derived Taylor's theorem. cf. Boyer.

J. Stirling: Methodus Differentialis, Edinburgh 1730 (See p.102)

C. MacLaurin: A Treatise of Fluxions, Edinburgh 1742 (See art 751, p.610)

A.L. Cauchy: Leçons de calcul différentiel, Paris 1829; Œuvres ser.2 IV
(Cauchy remainder: 9th lesson. See extract in Birkhoff's Source Book)

MARKOV CHAINS

The theory of Markov chains arose from Markov's interest, together with Lyapunov, in the central limit theorem for independent events. He then extended this theory to dependent events e.g. in the 1912 paper quoted below. Other papers are conveniently seen in his Selected Works 1951.

A.A. Markov: Ob ispytaniyakh, svyazannykh b tsep' nenablyudaemymi sobytiyami, (On trials associated with a chain of nonindependent events) Izv. Akad. Nauk Sb. 1912 551-572; Izbr. Trudy 1951 437-507

MATRIX

The word 'matrix' was introduced to mathematics by Sylvester in 1850. Initially the word was used to describe an array of numbers used for calculations with determinants. Sylvester says:

> *' For this purpose we must commence, not with a square, but with an oblong arrangement of terms consisting, suppose of m lines and n columns. This will not in itself represent a determinant, but is as it were, a Matrix out of which we may form various systems of determinants...'*

Sylvester soon realized the general importance of the idea as in his 1851 paper below. Cayley then briefly described the algebra of matrices (with a notation as nowadays understood) in the paper of 1855 and developed their theory in some detail in the 1858 paper (including the definition of the unit matrix and Cayley's statement of the Cayley-Hamilton theorem). The theory was further developed by Sylvester who, in his 1867 paper quoted below, defined inverse and orthogonal matrices, and in the 1883 paper defined latent roots of a square matrix and stated the Sylvester interpolation theorem. Various notations continued to be used for matrices. Cullis appears to have been the first to use the most common modern notation for matrices, viz. a doubly suffixed array enclosed by straight brackets.

J.J. Sylvester: Addition to the articles 'On a New Class of Theorems' and 'On Pascal's Theorem' Phil. Mag. 37 1850 363-370,
Coll. Math. Papers I 145-151 (see last paragraph of p.150)

J.J. Sylvester: On the relation between the minor determinants of linearly equivalent quadratic forms, Phil. Mag. 1 1951 295-305 p.247; Coll. Math. Papers I p.37

A. Cayley: Remarques sur le notation des fonctions algébriques,
J. Math.(Crelle) 50 1855 282-285; Math. Papers II 185-188

A. Cayley: A memoir on the theory of matrices,
Phil. Trans. 148 1858 17-37; Math. Papers II 475-496

J.J. Sylvester: Thoughts on inverse orthogonal matrices,
Phil. Mag. 1867 34 461-475; Coll. Math. Papers II 615-628

J.J. Sylvester: On the equation to the secular inequalities in the planetary theory,
Phil. Mag. 16 1883 267-269; Coll. Math. Papers IV 110-111

C.E. Cullis: Matrices and Determinoids, 2 vols, Cambridge 1913 (Univ. Press).

MAXWELL'S DEMON

Maxwell referred to 'an intelligent being' who was then designated a (non-malevolent) demon by Lord Kelvin in a public lecture. A connection with information theory was first discussed by Szilard.

J.C. Maxwell: Theory of Heat, 1871
(See p.328 Chap. 22, 'Limitation on the Second Law of Thermodynamics')

W. Thomson: The sorting demon of Maxwell,
Proc. Roy. Inst. 9 1899 113-; Math. & Phys. Papers V 21-23

L. Szilard: On the decrease of entropy in a thermodynamic system by the intervention of intelligent beings, (German) Z. Phys. 53 1929 840-856

MAXWELL'S EQUATIONS

These were developed in the papers referenced below and in Maxwell's text on electricity and magnetism. Maxwell's form was different to that now in use which is due to Heaviside and Herz. cf. Lorentz, Enz. Math. Wiss. V.2 p.68

J.C. Maxwell: A dynamical theory of the electromagnetic field,
Proc. Roy. Soc. 13 1864 531-536; Phil. Trans. 55 pt 3 1865 459-512;
Phil. Mag. 24 1865 152-157; Sci. Papers I 526-597

J.C. Maxwell: A Treatise on Electricity and Magnetism, Oxford 1873 (Univ. Press)

O. Heaviside: Electromagnetic induction and its propagation,
The Electrician 1885 pp.206, 306; Electrical Papers I 429-

H. Herz: Über die Grundgleichungen der Elektrodynamik für ruhende Körper,
Ann. Phys. 40 1890 577- (Elektrische Wellen 195-240)

H. Herz: Über die Grundgleichungen der Elektrodynamik für bewegte Körper,
Ann. Phys. 41 1890 369- Elektrische Wellen 241-268

H. Herz: Elektrische Wellen, Leipzig 1892 (Teubner)

O. Heaviside: On the forces, stresses, and fluxes of energy in the electromagnetic field, Phil. Trans.183A 1893 423-480; Electr. Papers II 521

MERCATOR PROJECTION

Called after Gerhardus Mercator, the latinized form of Gerhard Krämer (b. Flanders 1512), a cartographer whose major work was done in Duisberg, Germany. He made his reputation in 1554 with a map of Europe and subsequently in 1569 with a map of the world. His major work 'Atlas' where the Mercator projection was explained was only published posthumously. His results were only approximate, the correct principles for his projection being laid down by Edward Wright of Cambridge. The underlying idea of the Mercator projection can be traced back to China.
[Dict. Sci. Biog.; Needham: Sci. Civ. China; Enz. Math. Wiss. III]

G. Krämer: Atlas- or Cosmographic Meditations on the Structure of the World, 1595

E. Wright: Certain Errors in Navigation, London 1599 (Sims)

METRICALLY TRANSITIVE

Metrical transitivity was first defined in the paper quoted below.

G.D. Birkhoff & P.A. Smith: Structure analysis of surface transformations
J. Math. (Liouville) 7(4) 1928 348-379; Birkhoff's Coll. Math. Papers II 360-394.
(cf. p.365 in original paper, p. 380 in Coll. Works)

MINKOWSKI INEQUALITY

The inequality for integrals was proved by Riesz in connection with the theory of spaces of functions absolutely Lebesgue integrable to power p. It became known as Riesz's inequality before Riesz himself pointed out that the discrete form had been previously proved by Minkowski. cf. Dudley 1989

H. Minkowski: Geometrie der Zahlen,
Leipzig 1896 I 115-117

H. Minkowski: Diophantische Approximation,
Leipzig 1907 (Teubner) p. 95

F. Riesz: Untersuchungen über Systeme integrierbarer Funktionen,
Math. Ann. 69 1910 449-497; Œuvres Complètes I 441-489

MÖBIUS STRIP

The discovery of the one-sided surface was made independently by Möbius and Listing in 1858, neither publishing the result immediately. The recognition that the projective plane is one-sided came out of correspondence between Schläfli and Klein. cf. Staeckel 1899, Klein 1926, Fauvel 1993.

A.F. Möbius: Einseitige Polyëder (Posthumous)
See C. Reinhardt ed: Möbius' Nachlass, Ges. Werke II 519-521

A.F. Möbius: Über die Bestimmung des Inhaltes eines Polyëders,
Verh. königl. Sächs. Ges. Wiss. 17 1865 31-68; Ges. Werke II 473-512,519

J.B. Listing: Der Census räumlicher Complexe oder Verallgemeinerung des Eulerschen Satzes von den Polyëdern, Gött. Abh. 10 1861/2 97-180; Gött. Nachr. 1861 352-358.

F. Klein: Bemerkungen über den Zusammenhang der Flächen
Math. Ann. 7,9 1874; Ges. Abh. 1922,II 63-77 (cf. pp. 65,76)

F. Klein: Vorlesungen über Nicht-Euklidische Geometrie. [1892]
ed. W. Rosemann, Berlin 1928 (Springer); rpr. 'Nicht-Euklidische Geometrie',
New York 1959 (Chelsea)

P. Staeckel: Die Entdeckung der einseitigen Fläche,
Math. Ann. 52 1899 598-600

J. Fauvel et al: Möbius and his Band, Oxford 1993 (Univ. Press)
(See especially the article of N.Biggs 'The development of topology')

MOORE-SMITH CONVERGENCE

E.H. Moore & H.L. Smith: A general theory of limits,
Amer. J. Math. 49 1922 102-121.

N

NABLA

The operator was introduced in quaternionic form by Hamilton. The name nabla was suggested by Robertson Smith in 1870 and comes from the resemblance of the symbol for the operator to an Assyrian harp.

C.G. Knott: Life and Scientific Work of Peter Guthrie Tait, Cambridge 1911 p.143

NAVIER-STOKES' EQUATIONS

The equations of fluid motion were first derived by Navier by considering interaction of molecular forces. Stokes showed the same equations could be derived on the sole assumption that the internal stresses depend linearly on the rates of shear, an assumption similar to that made for the displacements of elastic solids.

C.L.M.H. Navier: Mémoire sur les lois du mouvement des fluides,
Mém. Acad. Sci. Paris 6 1823 389-440

G.G. Stokes: On the theories of the internal friction of fluids in motion and of the equilibrium and motion of elastic solids,
Trans. Camb. Phil. Soc. 8 1845 247-; Math. & Phys. Papers I 75-129

NEUMANN PROBLEM OF POTENTIAL THEORY

C. Neumann: Untersuchungen uber das Logarithmische und Newton'sche Potential,
Math. Ann. 13 1878 255-300

NEWTON-COTES FORMULAE

I. Newton: Principia .., London 1687
(See book III, lemma V 499-500 in Cajori's Motte Translation)

R. Cotes: Harmonia mensurarum, Cambridge 1722 (See appendix)

NEWTON'S FORMULA

i.e. the formula for the sums of powers of the roots of an equation.

I. Newton: Arithmetica Universalis, London 1707

NEWTON'S INTERPOLATION FORMULAE

The principal reference here is to Newton's Principia where the subject is discussed geometrically. For other references see the appendix to the Motte-Cajori edition of the Principia. These contributions were collected and reprinted by Fraser.

I. Newton: Researches in the theory of equations [1665-1666]
Math. Papers I 517-539 (see 519)

I. Newton:.Principia .., 1687, book III, lemma V
(p.499 in the 1947 revision of Motte's translation)

D.C. Fraser: Newton's Interpolation Formulas,
J. Inst. Actuaries 51 1918 77-106, 1919 211-232; 58 1927 53-95; reprinted as a book under the same name for the 8th Intern. Congr. Actuaries, London 1927 (Layton)

NEWTON-RAPHSON METHOD

Newton first used a similar technique for solution of polynomial equations. According to Jeffreys & Jeffreys, what is nowadays called the Newton-Raphson method is related to that used by Newton (1669) which has now become known as Horner's method. Newton subsequently used this method in Principia in a geometrical form for solution of Kepler's equation. Wallis commented on its use and referred to it as Newton's method. Subsequently, Raphson modified the method for the solution of algebraic equations. The currently used version, applicable to any function, may have originated from Adams' analytical formulation of the method as applied to Kepler's equation.

I. Newton: De analysi, [1669] London 1711

J. Wallis: De algebra tractatus, Oxford 1685,
(includes a summary of Raphson's method apparently written by himself - see p.338)

I. Newton: Principia…., London 1687 Book I, Prop. 23

J. Raphson: Analysis aequationum universalis, London 1690 (Swalle);
2nd ed. 1697 (Taylor) (see prop.II)

J.C. Adams: On Newton's solution of Kepler's equation, Mon. Not. Roy. Astr. Soc., London 43 1882 43-49 (cf. appendix to Motte-Cajori Principia)

H. Jeffreys & B.S. Jeffreys: Methods of Mathematical Physics, Cambridge UP 1946

NORMAL DISTRIBUTION

This distribution first occurred in the 1733 work of de Moivre as a limiting form of a binomial distribution for a large number of events. It arose in a similar context in Laplace's 1774 memoir on inverse probability as well as in two further memoirs of 1783. It was then used almost simultaneously by Gauss and Laplace in the theory of errors from which the distribution became known in France as the law of Laplace and in Germany and in English-speaking countries as the Gaussian distribution. The term 'normal distribution', proposed by Pearson in 1924, avoids the question of priority and so came into general use in the mathematical literature. However, recently, the term 'Gaussian distribution' has come back, especially in the technical literature.

A. de Moivre: Approximatio ad summan terminorum binomii $(a+b)^s$ in seriem expansi, London 1733 8 pp; rpr. R.C. Archibald (ed.) Isis 8 1926 671-683;
Engl. tr. in last two editions of Doctrine of Chances

P.S. Laplace: Mémoire sur la probabilité des causes par les événements,
Mém. Acad. Sci. Paris 6 1774 (Rpr. Dales' Hist. Inverse Prob.)

P S Laplace: Mémoire sur les approximations des formules qui sont fonctions de très grands nombres, Mém. Acad. Sci. Paris 1783 423-467 (Rpr. Dales' Hist. Inverse Prob.)

P.S. Laplace: Sur les naissances, les mariages et les morts à Paris depuis 1771 jusqu'en 1784... Mém. Acad. Sci. Paris 1783 693-702 (Rpr. Dales' Hist. Inverse Prob.)

C.F. Gauss: Theoria Motus Corporum Coelestium in Sectionibus Conicis Solem Ambientium. Hamburg l809 (Perthes & Besser); C.H. Davis (tr.): Theory of the Motion of Heavenly Bodies Moving about the Sun in Conic Sections, 1857 rpr. New York 1963

(Dover) (The Bayes' method is used for least squares in book II, section III)

P.S. Laplace: Mémoire sur les approximations des formules qui sont fonctions de très grands nombres et sur leur application aux probabilités.
Mém. Acad. Sci. Paris 10 1809 353-415

K. Pearson: Historical note on the origin of the normal curve of errors,
Biometrika 16 1924 402-404

O.B. Sheynin: Laplace's theory of errors,
Arch. Hist. Exact Sci. 17 1977 1-61

A.I. Dale: A History of Inverse Probability: NY-Berlin 1991 (Springer)

O

ORDERS OF MAGNITUDE

Use of the symbols O and o to denote orders of magnitude is originally due to Bachmann but it became more widespread after Landau's work.

P. Bachmann: Die analytische Zahlentheorie, Leipzig 1894 (Teubner)

E. Landau: Vorlesungen über Zahlentheorie, Leipzig 1927 (Hirzel)

ORLICZ SPACE

W. Orlicz: Über eine gewisse Klasse von Raümen von Typus B,
Bull. Intern. Acad. Polon. 8A 1932 207-220

ORTHOGONAL FUNCTIONS

These were first studied in a systematic way by Murphy.

R. Murphy: On the inverse method of definite integrals, with physical applications,
Camb. Phil Trans. 1833 253-408; 1835 113-148; 315-394

P

PARALLEL TRANSPORT

T. Levi-Cività: Nozione di parallelismo in una varietà qualunque,
Rend. Circ. mat. Palermo fasc. 42 1917 173-215

PARSEVAL'S THEOREM

Parseval stated the theorem for Fourier series with restrictive conditions on the function to be expanded. The theorem was extended to all Riemann integrable functions by Lyapunov and in consequence often called the Parseval-Lyapunov theorem in Russia. It was then extended to general orthogonal functions by Riesz (see 'Riesz-Fischer theorem').

M.A. Parseval: Mémoire sur les séries et sur l'intégration complète d'une équation aux différences partielles linéaires du second ordre à coefficients constants,
Mém. pres. par divers savants, 1 1806 638-648

M.A. Lyapunov: On a question concerning linear differential equations of the second order with periodic coefficients. (Russian) Comm. Kharkov Math. Soc. 1896 5 190-254; French reprint in abbreviated form in C.R. Acad. Sci. Paris 123 1896 1248-1252

PARTITIONS

The theory of partitions is due to Euler who devoted one chapter to it in his book 'Analysis Infinitorum'.

L. Euler: Introductio in Analysin Infinitorum, Lausanne 1748, Chap.16, De Partitione Numerorum; Opera Omnia ser.1 VIII 313-338

PASCAL'S TRIANGLE

The arithmetic triangle of the binomial coefficients was known from medieval times in China, Persia, the Arabic world and India. It was later described in Europe by Cardan, Tartaglia, Briggs and Oughtred. Pascal's treatise on the arithmetic triangle written in 1654 was at a somewhat higher theoretical level and attracted attention at the time because of the use of the triangle in calculating probabilities.

Yang Hui: Hsiang-chieh suan-fa (Chinese) 1261 (cf. Biot, Needham SCC, Lam Lay-Yong. This refers to an earlier occurrence)

Chu Shih-chieh: (Ssu-yan Y-chien (Precious Mirror of the Four Elements) (Chinese) 1303; The Old Method Chart of the Seven multiplying Squares and Lower Powers (Chinese) (rpr. in Needham SCC). The triangle to the 6th power is given at the beginning of the book

Al-Kashi: Key to Arithmetic (Persian) 1425
(Binomial triangle up to 9th power)

P. Apianus: Rechnung, Nürenberg 1527
(The triangle is given on the title page)

B. Pascal Traité du triangle arithmétique, avec quelques autres petits traités sur la même manier. (posth.) [1654] Paris 1665 (Desprez); Œuvres p. 243 - (Extracts in Source books of Smith and Struik)

E. Biot: On Pascal's triangle in the Suan Fa Thung Tsung, J. Savants Etrang. 1835 [Needham: SCC Bibl. 747]

Lam Lay-Yong: The Chinese connection between the Pascal triangle and the solution of numerical equations of any degree, Hist. Math. 7 1980 407-424.

A. W. F. Edwards: Pascal's Arithmetic Triangle, London & New York 1987 (Griffin & Oxford Univ. Press)

PEANO'S AXIOMS FOR THE INTEGERS

G. Peano: Sul concetto di numero, Rivista di Matematica 1 1891 87-102, 256-67; Opera Scelte III 80-109

G. Peano: Arithmetices Principia Nova Methodo Esposti, Torino 1889 (Bocca)

PEANO-BAKER SERIES

This provides a matrix series solution of a system of linear differential equations with coefficients depending on the independent variable. The paper by Peano is noteworthy as the earliest example of the use of n-vectors and nxn matrices in the modern sense. Baker also applied matrix notation to the problem.

G. Peano: Integrazione per serie delle equazioni differentiali lineari, Atti Accad. Sci. Torino 22 1887 437-446; Math. Ann. 32 1888 450-456 (French); Opere Scelte I 83-90

H.F. Baker: On the integration of linear differential equations, Proc. Lond. Math. Soc. 35 1902 333-378

H.F. Baker: Further applications of matrix notation to integration problems, Proc. Lond. Math. Soc. 34 1902 347-360

PEANO CURVE

G. Peano: Sur une courbe, qui remplit toute une aire plane Math. Ann. 36 1890 157-160; Opere Scelte I 110-114

PEARSON FAMILY OF PROBABILITY DISTRIBUTIONS

K. Pearson: Mathematical contributions to the theory of evolution, Phil. Trans. 186A 1895 343-414; 197A 1901 443-459; 216A 1916 429-457

PERRON INTEGRAL

O. Perron: Über den Integralbegriff,
Sitzber. Heidelberg Akad. Wiss. 16 1914 1-16

PICARD'S METHOD OF SUCCESSIVE APPROXIMATIONS

Picard first used this method in 1890 for partial differential equations in the two papers quoted below, the method being briefly described in the first and dealt with at length in the second. The second paper goes on to apply the method to ordinary differential equations, a subject which is developed in detail in the succeeding papers of 1892 and 1893. Peano had already used the same method for a system of linear ordinary differential equations in 1887 (see 'Peano-Baker series')

E. Picard: Sur l'emploi des approximations successives dans l'étude de certaines équations aux dérivées partielles,
C.R. Acad. Sci. Paris 110 1890 61-67; Œuvres II 377-383

E. Picard: Mémoire sur la théorie des équations aux dérivées partielles et la méthode des approximations successives, J. Math. (Liouville) 6(4) 1890 145-210; Œuvres II 385-450 (For ordinary differential equations, see Chapter V)

E. Picard: Sur l'application aux équations différentielles ordinaires de certaines méthodes d'approximations successives,
C.R. Acad. Sci. 115 1882 543-54; Œuvres II 167-173

E. Picard: Sur l'application des méthodes d'approximations successives a l'étude de certaines équations différentielles ordinaires,
J. Math. (Liouville) 9 1893 217-271; Œuvres II 177-231

PICARD'S THEOREM

E. Picard: Sur le propriété des fonctions entières,
C.R. Acad. Sci Paris 88 1879 1024-1027 (Extract in Birkhoff's Source Book)

PLANCHEREL'S THEOREM

This theorem relates the integral-squared value of a function to the integral-squared value of its Fourier transform. It has a physical interpretation in terms of energy of waves, first demonstrated by Lord Rayleigh. The later statement by Plancherel made it mathematically more precise.

Lord Rayleigh: On the character of the complete radiation at a given temperature,
Phil. Mag. 27 1889 460-469; Sci. Papers 260-276

M. Plancherel: Contribution a l'étude de la représentation d'une fonction arbitraire par les intégrales définies, Rend. Circ mat. Palermo 30 1910 289-335

PLATEAU'S PROBLEM

The name derives from the experimental study of Plateau (1801-1883), Professor at the University of Gand (Ghent)

J.A.F. Plateau: Statique expérimentale et théorique des Liquides soumis aux seules Forces moléculaires,
Paris (Gauthier-Villars); London (Trübner & Co.) Gand et Leipzig (F. Clemm) 1873

POINCARE CONJECTURE ON 3-MANIFOLDS

Poincaré H: Cinquième complément à l'Analyse Situs, Rend. Circ. Mat. Palermo 18 1904 45-110; Œuvres VI 435-498 (See the final paragraph)

POINCARE RECURRENCE THEOREM

The problem was raised and discussed by Poincaré. It was solved using measure theory by Carathéodory.

H. Poincaré: Sur le problème des trois corps et les équations de la dynamique,
Acta Math. 13 1890 1-270; Œuvres VII 262-490 (See theorem 1 of section 8)

C. Carathéodory: Über den Wiederkehrsatz von Poincaré. Berl. Sitzungsber. 1919, 580-584; Ges. math. Schr. IV 296-301

POINCARE'S REPRESENTATIONS FOR THE HYPERBOLIC PLANE

Poincaré is credited with two representations of the hyperbolic plane: the half-plane representation and the representation by orthogonal arcs within a circle. The first was briefly described in the final remarks in his 1882 work on bilinear transformations and Fuchsian functions. The second is in his 1883 memoir also on bilinear transformations, a connection with Lobachevsky geometry being observed only in the final paragraph. The three-dimensional version of the second was sketched in his 1891 paper. Both his representations are described in general terms in later popular expositions of 1902 and 1909 giving philosophical views on non-Euclidean geometries. The 1882 and 1883 memoires are reproduced with English translations and commentary in the book of Stillwell who also observes the priority of Beltrami (1868) for these representations.

H. Poincaré: Théorie des groupes fuchsiens,
Acta Math. 1 1882 1-62; Œuvres II 108-168 (cf. p.114)

H. Poincaré: Sur les groupes kleiniens,
Acta Math. 3 1883 49-92; Œuvres II 258-299. (pp. 55, 56 : 'On reconnaîtra alors ... la géométrie non-euclidien de Lobachewski...')

H. Poincaré: Les géométries non-euclidiennes,
Revue générale des sciences 2 1891 769-774; Engl. tr. Nature 45 1892 404-7

H. Poincaré: La Science et l'Hypothèse, Paris 1902 (Flammarion)
Engl. tr. 'Science and Hypothesis', New York 1952 (Dover) cf. pp. 35-88

H. Poincaré: Science et Méthode, Paris 1909;
Engl. tr. Science and Method, London 1914 (Nelson)

Stillwell.J: Sources of Hyperbolic Geometry,
Amer. Math. Soc. & London Math. Soc. 1996

POISSON BRACKETS

Poisson S.D: Mémoire sur la variation des constants arbitraires dans les questions méchaniques, Bull. Soc. philom. 14 1808 422-426; J. Ecole poly. 8 (15) 1809 266-

POISSON'S DISTRIBUTION LAW

S.D. Poisson: Recherches sur la probabilité des jugements en matière criminelle et en matière civile précédées des règles générales du calcul des probabilités, Paris 1837 (Bachelier)

S.M. Stigler: Poisson on the Poisson distribution, Statistics and Probability letters, 1 1982 33-35

POISSON'S EQUATION FOR POTENTIAL

Stated by Poisson (gravitational potential) and Green (electrical potential)

S.D. Poisson: Remarques sur une équation qui se présente dans la théorie de l'attraction des sphéroïdes, Bull. Soc. Philomath. Paris 3 1813 388-392

G. Green: An Essay on the Application of Mathematical Theories to Electricity and Magnetism, Nottingham 1828, sect. 1

POTENTIAL

The idea of a potential force function first occurs in the work of Clairaut and Lagrange and the gravitational potential, denoted by V, was much used by Laplace though not called so. Laplace's mathematics were used in electricity by Green who then referred to V as the potential. The function V was later called potential by Gauss in a mechanical context.

A-C Clairaut: Théorie de la figure de la terre, Paris 1743

P.S. Laplace: Mécanique Céleste, Paris 1799

J.L. Lagrange: Mécanique analytique, Paris 1788 (vol.I p.290 in 3rd ed.)

G. Green: An Essay on the Application of Mathematical Theories to Electricity and Magnetism, Nottingham 1828

C.F. Gauss: Ueber ein neues allgemeines Grundgesetz in der Mechanik,

J. Math.(Crelle) 4 1829 232-235

C.F. Gauss: Allgemeinen Lehrsätze in Beziehung auf die im verkehrten Verh, J. Math.(Liouville) 7 1842 273-324; Werke V 194-242, (cf. p.200)

H. Geppert: Über Gauss' Arbeiten zur Mechanik und Potentialtheorie, Gauss' Werke X.2 Abh.7

PROJECTIVE GEOMETRY

This originated in Desargues' essay but its detailed theory is due to Poncelet after 1813 who developed it while in prison in Saratov in Russia after being captured during the Napoleonic campaign.

G. Desargues: Brouillon projet, Paris 1639

J.V. Poncelet: Traité des propriétés projectives des figures, Paris 1822

PYTHAGORAS' THEOREM

Special cases of the theorem, especially the 3, 4, 5 triangle, were widely known in antiquity e.g. in Babylon, Egypt, China and India. It was first stated in the form we know it by Euclid in 3 B.C. and by tradition generally ascribed to Pythagoras (fl. 6 B.C.) although the evidence is not clear. What is new in the theorem, as stated by Euclid, appears to be the method of proof. See Heath for history of the theorem and Neugebauer for its Babylonian connections.

Chiu Chang Suan Shu: Nine Chapters on the Mathematical Art (Chinese) c.250 B.C.

Chou Pei Suan Ching: The Arithmetic Classic of the Gnome and the Circular Paths of Heaven, (Chinese) c.300 B.C.

Euclid: Elements I, 351-366 in Heath's translation

T.H. Heath: A Manual of Greek Mathematics, Oxford 1919 3 vols.

O. Neugebauer: Zur Geschichte des Pythagoreischen Lehrsatzes, Gött. Nachr. 1928

PYTHAGOREAN TRIPLES

A Pythagorean triple is a set of three integers satisfying the Pythagoras' theorem relation. Knowledge of at least the 3,4,5 triple was widespread in antiquity e.g. in Egypt and China. According to Neugebauer's analysis of the Plimpton tablets at Columbia University, the Babylonians already had a detailed knowledge of the Pythagorean triples. General formulae for the triples were attributed by Proclus to Pythagoras and Plato. These formulae were generally known in Hellenistic times and, somewhat later, in India.

Chiu-Chang Suan Shu (Nine Chapters on the Mathematical Art), 250 B.C.; Chou Pei Suan Ching, circa 300 B.C. (See Needham's Science and Civilization in China)

Proclus: Commentary on Euclid I, 428-487 (Heath: Greek Math. I 80,81)

O. Neugebauer Zur Geschichte des Pythagoreischen Lehrsatzes, Gött. Nachr. 1928.

O. Neugebauer The exact Sciences in Antiquity, Princeton 1952 (Univ. Press); Brown 1957 (Univ. Press); New York 1962 (Harper)

J. Friberg: Methods and traditions of Babylonian mathematics, Hist. Math. 8(3) 1981 277-318 (Plimpton 322)

Q

QUATERNIONS

The story of Hamilton's discovery of quaternions on the way to Dunsink Observatory from Dublin on Oct 16th 1843 has been frequently told. He described that discovery next day in a letter to John Graves and then developed the theory in a series of papers starting with the one quoted below. He later treated the subject in detail in his 1853 book. A readable account is given in the preface to Lectures on Quaternions which also includes interesting historical information on complex numbers. (Smith: Source Bk II quotes extracts)

W.R. Hamilton: Letter to Graves on quaternions; or on a new system of imaginaries in algebra, Phil. Mag. 25 1844 489-95; Math. Papers III 106-110

W.R. Hamilton: On a new species of Imaginary Quantities connected with the Theory of Quaternions, Proc. Roy. Irish Acad. 2 1846 424-34; Math. Papers III 111-116

W.R. Hamilton: Lectures on Quaternions, Dublin 1853

W.R. Hamilton: Elements of Quaternions, (posth, ed. W.E. Hamilton) London 1866

QUINTIC EQUATION – PROOF OF INSOLVABILITY BY RADICALS

In proving the insolvability of the quintic equation by radicals Abel was anticipated by Ruffini and the result is sometimes referred to as that of Ruffini – Abel.

P. Ruffini: Teoria generale delle equazioni in cui si dimonstra impossibile la soluzione algebrica delle equationi generali di grado superiore al quarto, 2 vols. Bologna 1799

H. Abel: Mémoire sur les équations algébriques ou l'on démontre l'impossibilité de la résolution de l'équation générale du cinquième degré Oslo 1824 (privately printed); Œuvres Complètes 1881 I 28-33; Engl. tr. in Smith's Source Book I 261-266

R

RADIAN

A term introduced in 1873 by James Thomson, the brother of William Thomson (Lord Kelvin). (Source: Cajori Hist. Math. Notations)

RADON-NYKODYM THEOREM

Apart from the papers of Radon and Nikodym, Daniell's work also deserves attention.

J. Radon: Theorie und Anwendungen der absolut additiven Mengenfunktionen, Sitzber. Akad. Wiss. Wien 122 1913 Abt.IIa 1295-1438

P.J. Daniell: Stieltjes derivatives, Bull. Amer. Math. Soc. 26 1920 444-448

O.M. Nykodym: Sur une généralisation des mesures de M.J. Radon, Fund. Math. 15 1930 131-179

RANDOM WALK, FLIGHTS

The name dates from Karl Pearson's statement of the problem in 1905 which was as follows:

> *'A man starts from a point O and walks 1 yard in a straight line, he then turns through any angle whatever and walks another 1 yard in a second straight line. I require the probability that after n stretches he is at a distance between r and r+dr from his starting point O.'*

Pearson only solved the problem for two steps but in the same journal Lord Rayleigh observed that the problem is that of combining n vibrations of equal amplitude and period but of random phase, a problem which had many years previously occurred in his work in acoustics and optics. He had then shown that when n is large the asymptotic probability density approximates to the symmetrical Gaussian distribution from which he had deduced the probability distribution of the radius vector was what is now known as the Rayleigh distribution.

Lord Rayleigh: On the resultant of a large number of vibrations of the same pitch and of arbitrary phase, Phil. Mag. 10 1880 73-78; Sci. Papers I 491-496

Lord Rayleigh: The wave theory of light, Enc. Brit. 1888

Lord Rayleigh: Theory of Sound, 2nd ed. 1894
(See chap. II: 'Harmonic Motions', para. 42a)

K. Pearson: The Problem of the Random Walk. Nature 72 1905 p.294

Lord Rayleigh: The Problem of the Random Walk. ibid. p.318

Lord Rayleigh: On the problem of random vibrations and of random flights in one, two or three dimensions, Phil Mag. 37 1919 321-347

RAYLEIGH'S QUOTIENT AND RAYLEIGH-RITZ METHOD

Lord Rayleigh's contributions here are numerous. Rayleigh's Principle concerning the extreme value of the Rayleigh quotient is described very clearly in his 1873 article. Commenting in 1911 on Ritz's 1908 paper, Lord Rayleigh said that he considered it surprising that Ritz should have considered this method to be a new one since he himself had previously used it on several occasions in the Theory of Sound and elsewhere. Rayleigh was however primarily concerned with the use of the Rayleigh quotient as a method of estimating the frequency in the fundamental mode. Ritz's method substituted a few terms of an appropriate orthogonal expansion directly into the expression for the integral whose extreme value expressed the solution of the problem. Then by minimising or maximising this as a function of a few unknown coefficients an approximate solution is found. The Rayleigh quotient idea was not used. He used the method primarily for static boundary value problems although it is also applicable to the least action integral and in the linear-quadratic case it is effectively the same as Rayleigh's method.

Lord Rayleigh: On the theory of resonance,
Phil. Trans. 161 1870 77-118; Sci. Papers I 33-67 (see p.57 et seq.)

Lord Rayleigh: Some general theorems relating to vibrations,
Proc. Lond. Math. Soc. 4 1873 357-368; Sci. Papers I 170-181
(rpr. in Birkhoff's Source Book 387-390)

Lord Rayleigh: Theory of Sound, London 1877, 1894; rpr. NY 1945 (Dover)
(See chapter IV: 'Vibrating systems in general', section 88. In his 1911 paper Rayleigh refers to the method in paragraphs 88, 89, 90, 91,182,209,210,265 and calls the example in vol. II appendix A a very significant application which was republished in 1899 – see below)

Lord Rayleigh: On the Calculation of the Frequency of Vibration of a System in its Gravest Mode, with an Example from Hydrodynamics,
Phil. Mag. 47 1899 556; Sci. Papers IV 407

W. Ritz: Über eine neue Methode zur Lösing gewisser Variations-probleme der mathematischen Physik, Habilitationsschrift,
J. Math (Crelle) 135 1908 1-61; Ges. Werke 192-250

W. Ritz: Über eine neue Methode zur Lösing gewisser Randwertaufgaben,
Gött. Nachr. 1908 236-248; Ges. Werke 251-264

W. Ritz: Theorie der Transversalschwingungen einer quadratischen Platte mit freien Rändern, Ann. Phys. 28 1909 737-786; Ges. Werke 265-316

Lord Rayleigh: On the calculation of Chladni's figures for a square plate,
Phil Mag. 22 1911 225-229; Sci. Papers 6 47-50

REDUCTIO AD ABSURDUM

The first clearly stated use of this method is that of Euclid where he proves that there are infinitely many primes.

Euclid: Elements book IX, prop. 20

RIEMANN INTEGRAL

B. Riemann: Üeber die Darstellbarkeit einer Function durch eine trigonometrische Reihe, Habilitationsschrift Göttingen 1854, Gött. Abh. 13; Ges. math. Werke 227-264 (See section 4 et seq.in Dover reprint)

RIEMANN HYPOTHESIS

B. Riemann: Über die Anzahl der Primzahlen unter einer gegebener grösse, Monatsb. Berlin Akad., Nov 1859; Werke 145-155 (See pp.148, 154 in Werke)

RIEMANN SPACE

G.F.B. Riemann: Über die Hypothesen, welche die Geometrie zu Grund liegen. Habilitations Vertrag 1854, Published posthumously by Dedekind in Göttinger Abh. Bd. 13 1868, Werke 272-, Republished by H. Weyl, Berlin 1919, English translation by W. K. Clifford "On the Hypotheses which lie at the Bases of Geometry" Nature, vol 8, 1873 pp.14-17

RIEMANN SPHERE

This is a form of stereographic projection (which has a long history qv.) with the difference that the sphere is projected on to the equatorial plane instead of a polar plane as in the usual stereographic projection. The name first occurred in Carl Neumann's tract.

B. Riemann: Theorie der Abel'sche Funktionen,
J. Math. (Crelle) 1857 54-110; Werke 88-144

C. Neumann: Vorlesungen über Riemanns Theorie der Abelsche Integrale,
Leipzig 1865 (Teubner)

RIEMANN SURFACE

G.F.B. Riemann: Grundlagen für eine allgemeine Theorie der Funktionen einer veränderlichen complexen Grösse, Inaugural diss. Göttingen 1851;

G.F.B. Riemann: Theorie der Abel'sche Funktionen, J. Math. (Crelle) 1857 54-110;

Werke 88-144 (The relevant extract is in English in Smith's Source Book, vol. II)

RIEMANN-CHRISTOFFEL TENSOR

Riemann only gave brief indications, the details being worked out by Christoffel. Riemann's calculations were published in the first edition of his collected works after those of Christoffel.

G.F.B. Riemann: Commentatio mathematica,qua respondere tentatur quaestioni ab Academia Parisiensi propositae; submitted as prize essay, Paris 1861; published posthumously, Ges. math. Werke p.402

E.B. Christoffel: Über die Transformation der homogenen Differentialausdrücke zweiten Grades, J. Math. (Crelle) 70 1869 46-70, [See 48-49]; Ges. Math. Abh. I 352-

RIEMANN-LIOUVILLE FRACTIONAL INTEGRAL

J. Liouville: Mémoire sur quelques Questions de la Géométrie et de Mécanique, et sur un nouveau genre de Calcul pour résoudre ces Questions, J. Ecole poly. 8 cahier 11 1832 1-69

B. Riemann: Versuch einer allgemeinen Auffassung der Integration und Differentiation [1847] Nachlass; Ges. math. Werke 354-66

RIEMANN-ROCH THEOREM

B. Riemann: Theorie der Abel'sche Funktionen, J. Math. (Crelle) 1857 54-110; Ges. math. Werke 1892 88-144 (cf. pp.101,114,120)

G. Roch: Über die Anzahl der Willkürlicher Constanten in algebraischen Funktionen, J. Math. (Crelle) 64 1864 372-376 (Rpr. in Birkhoff)

RIESZ-FISHER THEOREM

Fischer's paper was also the origin of the idea of convergence in mean.

F. Riesz: Über orthogonale Funktionensysteme, Gött. Nachr. 1907 116-122; Œuvres I 389-395

F. Riesz: Sur les systèmes orthogonaux de fonctions, C.R. Acad. Sci. Paris 144 1907 615-619; Œuvres I 378-381

E. Fischer: Sur la convergence en moyenne, ibid. 1022-1024

RIESZ REPRESENTATION THEOREM

This was published by Riesz and Fréchet in the same volume of Comptes Rendus and is sometimes called the Fréchet-Riesz theorem.

F. Riesz: Sur les systèmes orthogonaux de fonctions,
C.R. Acad. Sci. Paris 144 1907 615-619; Œuvres I 378-381

M. Fréchet: Sur les ensembles de fonctions et les opérations linéaires
ibid. 1414-1416

RODRIGUES PARAMETERS

O. Rodrigues: Des lois géométriques qui régissent les déplacements d'une système solide dans l'espace, et de la variation des coordonnés provenance de ces déplacements considérées indépendamment des causes qui peuvent les produire,
J. Math. 5 1840 380-440 (cf. 404-405)

ROLLE'S THEOREM

The origin is traced in Smith's Source Book vol. I. Briefly, it was published in 1691 by Rolle to justify a method he had earlier used to solve equations in his popular Traité d'algebre 1690. The name 'Rolle's Theorem' was, it seems, first used by G. Bellavitis in 1846.

ROUTH-HURWITZ STABILITY CRITERION

E.J. Routh: A Treatise on the Stability of a Given State of Motion, London l887 (MacMillan); rpr. A.T. Fuller 'Stability of Motion' London 1975 (Taylor & Francis)

A. Hurwitz: Über die Bedingungen unter welchen eine Gleichung nur Würzeln mit negativen reellen Teilen besitzt. Math. Ann. 4 l895 273-284

RUNGE-KUTTA METHOD

Using a simple geometrical idea, Runge produced a scheme accurate to third order. Kutta made a modification accurate to higher order, the geometrical interpretation, however, becoming lost.

C. Runge: Über die numerische Auflösing von Differentialgleichungen,
Math. Ann. 46 1895 167-178

W. Kutta: Beitrag zur näherungsweisen Integration totaler Differentialgleichungen,
Z. Math. Phys. 46 1901 435-453

S

SCHAUDER'S FIXED POINT THEOREM

P.J. Schauder: Der Fixpunktsatz in Functionalräumen,
Studia Math. 2 1930 171-180; Œuvres, Warsaw 1978 (Acad. Pol. Sci.) 168-176

SCHWARZ-CHRISTOFFEL TRANSFORMATION

H.A. Schwarz: Ueber einige Abbildungsaufgaben,
J. Math. (Crelle) 70 1869 105-120; Ges. Math. Abh. II 65-83

E.B. Christoffel: Sul problema delle temperature stationarie e la rappresentazione
di una data superficie, Annali di Mat. 1 1867-68 89-103; Ges. Werke I 255-

SCHWARZ'S INEQUALITY

i.e. the Cauchy inequality for integrals. It was designated the 'Schwarz inequality' by
Schmidt in his 1908 paper which laid the geometrical foundations for Hilbert space.
Schwarz's statement of the inequality was an indirect one made in connection with
the calculus of variations. Later the name was amended to Cauchy-Schwarz inequality.
Nowadays it is fairly well known that this inequality originated with Bunyakovski.

V.Y. Bunyakovski: Sur quelques inégalités concernant les intégrales ordinaires et les
intégrales aux différences finies, Mém. Acad. St. Petersb. 1(9) 1859

H.A. Schwarz: Über einen die Flächen kleinsten Flächeninhalts betreffendes Problem
der Variationsrechnung, Acta Soc. sci. Fenn. 1885 15 315-362; Abh. I 224-269

E. Schmidt: Über die Auflösung linearer Gleichungen mit abzählbar unendlich vielen
Unbekannten, Rend. Circ. Mat. Palermo 25 1908 53-77

SIMPSON'S RULE

This had previously been proposed by Cavalieri and Gregory. It became known as
Simpson's Rule after being described in Simpson's popular textbooks, e.g. the one
in 1743 cited below.

B. Cavalieri: Centuria di Varii Problemi, Bologna 1639 p.446

J. Gregory: Excercitationes Geometricae, London 1668

T. Simpson: Mathematical Dissertations on a Variety of physical and analytical
Subjects. London 1743 (cf. pp. 109-119)

SPHERICAL HARMONICS

J.C. Maxwell: Treatise on Electricity and Magnetism,
Oxford 1892 vol I, p.198, arts. 130-1

STEREOGRAPHIC PROJECTION

This projection has a long history going back to antiquity. It originated for the purpose of representing the constellations in the sky onto a plane. Its first use was closely associated with the astrolabe (e.g. by Ptolemy) particularly by Arabic commentators (e.g. Al-Biruni) who also used the projection for map-making. The name is due to d'Aguillon of Antwerp. In mathematics it came into use with Lambert and especially Euler who used it to make a map of Russia and supplied the theory relating the projection to complex numbers.(quoted from Rosenfeld)

Ptolemy: Representation of the sphere in the plane (Greek)
Commanctius (tr.): Ptolemaei Planisphaerum, Venice 1558, 295-

Al-Biruni: Treatise on projection of constellations and on the representation of countries on a map (Arabic) c.1000

F D'Aguillon: Six Books on Optics, Antwerp 1613

J. H. Lambert: Anmerkungen und Zusätze zur Entfernung der Land- und Himmels-karten, 1772; Ostwald's Klassiker No.54

L. Euler: De projectione geographica superficiei sphaericae (On geographic projection of a spherical surface) Novi Comm. Acad. Sci. Petrop. 1778; Opera Omnia ser.1 XXVIII 133-141

L. Euler: De repraesentatione superficiei sphaericae super plano (On representation of a spherical surface on the plane) ibid; Opera Omnia ser.1 XXVIII 228-235

B. A. Rosenfeld: A History of Non-Euclidean Geometry, Moscow 1976 (Russian); transl. New York etc. 1988 (Springer)

STIELTJES INTEGRAL

J. Stieltjes: Recherches sur les fractions continues,
Ann. Fac. Sci. Toulouse 8 1894 1-122; Œuvres II 402-566. (Extract in Birkhoff's Source Book)

STIRLING'S INTERPOLATION FORMULA

Due to Newton as well as Stirling

I. Newton: Methodus Differentialis, London 1711 (Prop. iii case 1)

J. Stirling: Methodus Differentialis, London 1730 (Prop. xx)

STIRLING'S FORMULA FOR FACTORIALS

J. Stirling: Methodus Differentialis, Edinburgh 1730 (See p.137)

STOKES' THEOREM

This first appeared in a postscript to a letter from Lord Kelvin to Stokes in 1850. Subsequently, in 1854, Stokes set it as an examination question at Cambridge University. After its publication by Hankel in 1861, Kelvin published it with proof in the 1867 Thomson-Tait treatise. In his 1869 paper on vortex motion he ascribed the theorem to Stokes and referred to his 1867 treatise. According to Bell, it only later became known as 'Stokes' Theorem' after a paper of W.H.Young in 1932.

Lord Kelvin: Letter to Stokes of 2 July 1850 (postscript);
D.B. Wilson: The correspondence between Sir George Gabriel Stokes and Sir William Thomson, Baron Kelvin of Largs. vol 1, Cambridge 1991 (Univ. Press) pp.96-97

G.G. Stokes: Smith's Prize Examination Papers, Cambridge University Calender 1854

H. Hankel: Zur allgemeinem Theorie der Bewegung der Flüssigkeiten, Göttingen 1861

W. Thomson & P.G.Tait: Treatise on Natural Philosophy, Cambridge 1867 (Univ. Press)
(See I sect.190, p.143)

W. Thomson: On vortex motion,
Trans. Roy. Soc. Edin. 25 1869 217-260; Papers IV 13-66 (cf. p.54)

STREAM FUNCTION

This is also known as Stokes' Stream Function. Its kinematic interpretation was later given by Rankine.

G.G. Stokes: On the steady motion of incompressible fluids,
Trans. Camb. Phil. Soc. 7 1842 439-454; Papers I 1-16
(See 'Steady motion in Two Dimensions', Papers I p.4)

W.J.M. Rankine: On plane water lines in two dimensions,
Phil. Trans. 1864; Misc. Sci. Papers, London 1881 495-

STURM-LIOUVILLE EQUATION

G. Sturm: Mémoire sur les équations différentielles linéaires du seconde ordre,
J. Math. (Liouville) 1 1836 253-265

J. Liouville: Mémoire sur le développement des fonctions ou parties de fonctions en série dont les divers termes sont assujettis à satisfaire à une mémé équation différentielle du seconde ordre contenant un paramètre variable, ibid. 1 1836 253-265; 2 1837 16-35 (cf. Birkhoff's Source Book)

SUMMATION CONVENTION

It was introduced by Einstein into the tensor calculus and so is often called the Einstein summation convention.

A. Einstein: Grundlage der allgemeinen Relativitätstheorie,
Ann. Phys. 49 1916 769-822; (See section 5)

SUMMATION SIGN

The capital sigma sign for summation first occurred in Euler's work but it only slowly came into general use. (Source: Cajori: Hist. Math. notations)

L. Euler: Inst. Calc. Diff., St Petersb. 1755; 'De differentiis finitiis', Chap.1, sect.26; Opera Omnia ser.1 X p.32

T

TAYLOR SERIES

Deduced from the Gregory-Newton interpolation formula by Taylor in his book of 1715. Gregory had previously published the theorem in 1668. The idea of the theorem can be traced back even further.

J. Gregory: Exercitationes geometricae, London 1668

B. Taylor: Methodus Incrementorum directa et inversa, London 1715;
Engl. tr. in Struik 328-339

A. Pringsheim: Zur Geschichte des Taylorschen Lehrsatzes,
Bibliotheca Mathematica 1900 I 433-479

TENSOR

The word was invented by Hamilton in connection with quaternions. Multiplication by a quaternion turns and stretches a vector. The tensor measures the stretching. The use of a 3x3 system of quantities to denote the same idea appears to be due to Kelvin and Rankine. Throughout the 19th century the term was only used for the stress tensor as it has arisen in the analysis of the deformation of an elastic solid, an alternative way of dealing with the same problem being provided by the dyadic of Gibb's vector analysis. Voigt used this subsequently in his work on crystals and in his later paper used the description of tensor. From there the term came into use for the electromagnetic stress tensor in relativity which gave rise with Sommerfeld to the generalization of the idea to 4 and 6 dimensions. The general n-dimensional tensor was used by Grossmann & Einstein (cf. Enz. Mat. Wiss., Whittaker 1910, Crowe 1967)

W.R. Hamilton: Lectures on Quaternions, Dublin 1853

W. Thomson: Elements of a mathematical theory of elasticity,

Phil. Trans. 146 1856 481-498.

W.J.M. Rankine: On the axes of elasticity and crystalline forms, ibid. 261-.

M. Voigt: Theoretischen Studien über die Elasticitätsverhältnisse der Krystalle, Gött. Abh. 34 1887 3-52; 53-100

M. Voigt: Der gegenwärtige Stand unsere Kenntnisse der Kristallelastizität, Gött. Nachr. 2 1900 117-176

A. Sommerfeld: Zur Relativitätstheorie, Ann. Phys. 32 1910 749-776; 33 1910 649-689

A. Einstein & M. Grossmann: Entwurf einer verallgemeinerten Relativitätstheorie und eine Theorie der Gravitation, Z. Math. Phys. 62 1913 225-261; Leipzig 1913 (Teubner)

TOPOLOGY

The word originates from the title of book published in 1847 by Listing who had worked as Gauss' assistant. The book does not develop many ideas in topology which mainly occur in his later 1861 work where Listing analyses different forms of multiply connected manifolds in connection with Euler's formula (qv.) The subject was further developed by Betti and then by Poincaré in his major memoirs on analysis situs starting with that cited below.

J.B. Listing: Vorstudien zur Topologie, Gött. Studien 1847 811-875

J.B. Listing: Der Census räumlicher Complexe oder Verallgemeinerung des Euler'schen Satzes von den Polyëdern, Gött. Abh. 10 1861-2 97-180; Gött. Nachr. 1861 352-358

E. Betti: Sopra gli spazi di un numero qualunque di dimensionali, Annali di Mat. 4 1871 140-158; Opere 2 273-290

H. Poincaré: Analysis situs J. Ec. Poly. 1 1895 1-121; Œuvres VI 193-288

TRANSCENDENCE OF PI

C.L.F. Lindemann: Ueber die Zahl π, Math. Ann. 20 1882 213-225

TRIGONOMETRY

Although the origins of trigonometry can be traced back to ancient uses in astronomy (in particular in India) trigonometry in its modern form evolved slowly in the 16th and 17th centuries through the necessity of solving spherical triangles on the Earth's surface for navigation. A significant early contribution is the proof of the spherical sine formula by Regiomontanus in 1533. The earliest known use of abbreviations for trigonometric functions sin, tan, and sec is that of Oughtred in 1657. A variety of notations continued in use until the 19th century. (cf. Cajori 1909)

Regiomontanus: De triangulis omnimodis, Nürnberg 1533
(Translated extract in Smith's Source Book II)

Oughtred: Trigonometria, London 1657,
Reprinted in English: Trigonometrie, London 1657

TURING MACHINE

A.M. Turing: On computable numbers with an application to the Entscheidungs-problem, Proc. Lond. Math. Soc. 24 1936 230-265

U

UNIFORM CONVERGENCE

Weierstrass was the first to use the concept of uniform (gleichförmig) convergence by proving the absolute and uniform convergence of multiple power series. Stokes and Seidel subsequently defined the concept for oscillatory series. cf. Hardy 1918, Bromwich 1942, Gratton-Guiness 1970

K. Weierstrass: Zur Theorie der Potenzreihen,
Manuscript 1841; Ges. Werke I 67-74

G.G. Stokes: On the critical values of the sums of periodic series,
Trans. Camb. Phil. Soc. 8 1847 533-583; Math. Phys. Papers I 236-313

P.L. Seidel: Note über eine Eigenschaft der Reihen, welche discontinuirliche Funktionen darstellen, Abh. Akad. Wiss München 7 1847-49 381-393;
Leipzig 1900 (Engelman); Ostwald's Klassiker no. 116

G.H. Hardy: Sir George Stokes and the concept of uniform convergence,
Proc. Camb. Phil. Soc. 19 1918 148-156

V

VECTOR

The word 'vector' was originally used in the sense of 'radius vector' to denote the displacement of the Earth from the Sun. It was used by Hamilton to denote that part of a quaternion which depended in the quaternionic quantities i, j, k (the remaining part being the 'scalar' part). From there it passed into modern use via Gibbs' form of vector calculus and became identified with what is now thought of as a geometrical vector familiar from forces and velocities in mechanics. Hamilton gave a new proof for the parallelogram rule for the composition of vectors, previously stated less clearly by Newton and Laplace. He claimed that he did this for the first time, the previous proof due to Laplace having dealt only with the magnitude of the forces. See Crowe 1967 for further references.

I. Newton: Principia .., London 1687
(See p.14 in the 1947 revised Motte version)

P.S. Laplace: Traite de Mécanique Céleste, vol. I Paris 1799

W.R. Hamilton: On the composition of forces,
Proc. Roy. Irish Acad. 2 l844 l66-168; Math. Papers II 284-

M.J. Crowe: History of Vector Analysis, Indiana 1967 (Univ. Notre Dame Press)

VECTOR CALCULUS

The basic ideas of vector analysis originated in 19th century physics, especially in the theory of potential, 3 dimensional fluid dynamics and electromagnetism. In this context arose the quantities (qv.) now called div, grad and curl. The earlier work was closely associated with the use of Hamilton's quaternions and in particular, with the del operator (qv.) This line of thought was developed in its application to physics by Tait and Maxwell. Vector analysis as such is due to Gibbs who combined ideas from the quaternionic representation with others taken from Grassman's Ausehnungslehre. His techniques were initially presented in lectures to his students and are recorded in a teaching pamphlet on the subject which, though not published, became widely circulated. Gibbs' vector analysis was taken up by Heaviside and applied by him to give the now familiar statement of Maxwell's equations. Subsequently Gibb's vector analysis, as presented in his lectures, was published as a book by Wilson. A variety of notations were used in the early forms of vector analysis and the notation did not settle down until the mid-twentieth century. cf. Abraham EMW, Crowe 1967

J.W. Gibbs: Elements of Vector Analysis, arranged for the Use of Students of Physics, New Haven 1881-84 Lithographed

O. Heaviside: On the forces, stresses and fluxes of energy in the electromagnetic field [1891] Phil. Trans. 183A 1893 423-480 See p.550

E.B. Wilson: Vector Analysis – Founded on the Lectures of J. Willard Gibbs, New York 1901, 1909 (Scribner); rpr. New York 1960 (Dover)

M. Abraham: Geometrische Grundbegriffe, Enz. Math. Wiss. IV 3 art 14

M.J. Crowe: History of Vector Analysis, Indiana 1967, (Univ. Notre Dame Press)

VECTOR SPACE IN N-DIMENSIONS

First clearly axiomatically set out by Peano. He applied it to the solution of linear differential equations (see Peano-Baker series)

G. Peano: Calcolo geometrico secundo l'Ausdehnungslehre di H. Grassman e precedutto dalle operazioni della logica deduttiva, Turin 1888 (Fratelli Bocca Editori)

J-L Dorier: A general outline of the genesis of vector space theory, Hist. Math. 22 1995 227-261.

VENN DIAGRAM

J. Venn: On the diagrammatic and mechanical representation of propositions and reasonings, Phil. Mag. 10 1880 1-18

J. Venn: Symbolic Logic, London 1881

VOLTERRA INTEGRAL EQUATION

The name appears to be due to Lalesco in the cited 1908 paper.

V. Volterra: Sulla inversione degli integrali definiti,
Atti Acc. Sc. di Torino 31 1896 311-323;400-408;557-567;693-708;I 216-254
R.C. Accad. Lincei (5)5 1896 177-185; Opere Mat. II 216-262

T. Lalesco: Sur l'équation de Volterra, J. Math. (Liouville) 4 1908 127-

V. Volterra: Leçons sur les Équations Intégrales, Paris 1913 (Gauthier Villars)

W

WEIERSTRASS COORDINATES

Weierstrass introduced his coordinates at a seminar at Berlin University in 1870. There is apparently no written account of his lecture. They were mentioned by Killing, who attended, as a footnote in Crelle's journal in 1880 and in 1885 Klein, who also attended, gave the lecture title as below in his 'Entwicklung der Mathematik ..' vol I, chapter 4 p.152. The coordinates were later described by Sommerville.

K Weierstrass: 'Vortrag über Cayleys Massbestimmung',
Seminar at Berlin University, Feb 1970

W Killing: Die Rechnung in den Nicht-Euklidischen Raumformen,
Crelle J. vol.89 (4) 1880 265-287 (see footnote p.273)

W Killing: Die nicht-euklidischen Raumformen in analytischen Behandlung,
Crelle 1885 vol 26 157-

D.M.Y Sommerville: Non Euclidean Geometry,
London 1914 (Bell); Dover 1958, 2005

WEIERSTRASS-STONE THEOREM

The theorem that a continuous function on a closed interval may be approximated arbitrarily closely by a polynomial. Stone gave the version usually quoted.

K. Weierstrass: Über die analytische darstellbarkeit sogenannter willkürlicher functionen reeller argumente,
Sitzb. kgl. Preuss. Akad. Wiss. Berlin 1885 633-639; 789-805; Werke I 37-, II 1-

M.H. Stone: The generalized Weierstrass approximation theorem,
Math. Mag. 21 1948 167-183; 237-254

Z

ZENO'S PARADOXES

Zeno of Elea (495 BC? - 435 BC?) was a pupil of Parmenides. His four paradoxes were described and analysed by Aristotle.

Aristotelis: Physica 239b; versions are given by:
I. Thomas (ed.): Greek Mathematical Works I, (Loeb) I 366-374; (Greek and English)
Heath Greek Mathematics, I chap.8

F. Cajori: The History of Zeno's arguments on motion,
Amer. Math. Monthly 22 1915 (8 articles)

ZETA FUNCTION

It is commonly referred to as Riemann's zeta function owing to its use by Riemann in his investigation into the distribution of prime numbers. But, as Riemann himself made clear, he was merely giving a name and notation to an expression previously introduced by Euler who had proved a variety of results concerning it. The function had also been used before Riemann by Chebyshev in his investigations on primes.

L. Euler: Variae observationes circa series infinitae, Comm. Acad. Sci. Petrop. 9 1737 160-188. theorem 8, p.174; Opera Omnia ser.1 XIV 216-244, (see p.230)

P.L. Chebyshev: Sur la totalité des nombres premiers inférieurs à une limite donnée,
Mém. pres. Acad. Sci. St. Petersb. par divers savants 6 1851 141-157
J. Math.(Crelle) 17 1852 341-365 (cf. Smith's Source Book)

B. Riemann: Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse,
Monatsb. Berliner Akad. 1859 671-680; Ges. math. Werke 145-153

PRIMARY SOURCES

ABEL Niels Hendrik (1802-1829)

Œuvres complètes, 2 vols. (ed. B. Holmboe) Christiana 1839 (Grondahl)

Œuvres complètes, 2 vols. (eds. L. Sylow & S. Lie) Christiana 1881 (Grondahl);
reprinted 1964 New York (Johnson Reprint Corp.); New York 1965 (Chelsea)

APOLLONIUS (260? - 200? BC)

Heath T.L. Apollonius' Treatise on Conic Sections, Cambridge 1896
rpr. 1961 (Univ. Press)

Heiberg I.L: Apolloni Pergasi quae graece existant cum commentariis antiquis,
Leipzig 1891-93 (Teubner) rpr. 1974

ARCHIMEDES (287? - 212 B.C.)

J.L. Heiberg (ed.): Archimedis Opera Omnia, 3 vols. Leipzig 1880-81;
2nd ed. 1913-1915 (Teubner); rpr. E.S. Stamatis, Stuttgart 1972

T.L. Heath (ed.): The Works of Archimedes edited in modern notation with
introductory chapters, Cambridge 1897 (Univ. Press);
rpr. with supplement: The Method of Archimedes, New York 1950 (Dover)

Eycke P.V: Les Œuvres complètes d'Archimède, Paris 1921 (Desales Brouwer)
Liege 1960 (Vaillant-Carmonne)

BELTRAMI Eugenio (1835-1890)

Opere Matematiche 4 vols Milan 1902-20 (Ulrico Hoepli)

BERNOULLI James (Jacob I) (1654-1705)

Opera 2 vols. Geneva 1744 (Cramer-Philibert);
1968 (Birkhãuser)

Ars Coniectandi, Basle 1713,
Ostwald's Klassiker Nr 169, New York (Chelsea)

Die gesammelte Werke der Mathematiker und Physiker der Familie Bernoulli, Basel etc. 1969-1975 (Birkhãuser)

BERNOULLI John (Johannis I) (1667-1748)

Die Erste Integralrechnung, Leipzig-Berlin 1914 (Engelmann);
Ostwald's Klassiker no.194

Die Differentialrechnung Leipzig 1924 (Akad. Verlag);
Ostwald's Klassiker no.211

Opera Omnia 4 vols. Lausanne-Geneva 1742 (Bousquet);
Hildesheim 1968 (Georg Olhms)

Der Briefwechsel von Johann Bernoulli (Bernoulli-Kommision der Naturforschernden Gesellschaft, Basel) Basel-Stuttgart (Birkhãuser)

Die gesammelte Werke der Mathematiker und Physiker der Familie Bernoulli, Basel etc. 1969-1975 (Birkhãuser)

BERNOULLI Daniel (1700-1782) b. Groningen, d. Basel

Hydrodynamica, sive de viribus et motibus fluidorum commentarii,
Strasbourg 1738

Die Werke von Daniel Bernoulli, in Die gesammelte Werke .... (next entry)

Die gesammelte Werke der Mathematiker und Physiker der Familie Bernoulli,
Basel etc. 1969-1980 (Birkhãuser)

BESSEL Friedrich Wilhelm (1784-1846)

Abhandlungen von Friedrich Wilhelm Bessel. 3 vols. (Rudolf Engelmann ed.)
Leipzig 1875 (W. Engelmann)

BIRKHOFF George David (1884-1944)

Collected Mathematical Papers, 3 vols. Amer. Math. Soc, New York 1950

BOLZANO Bernard P. J. N. (1781-1848)

Schriften, 5 vols. Prague 1930-1948 (Kõnigl. Bõhmische Ges. Wiss.)

BOOLE George (1815-1864)

A list of Boole's writings is included in his book on differential equations.

The mathematical analysis of logic; being an essay towards a calculus of deductive reasoning, Cambridge 1847; Oxford 1948

An investigation of the laws of thought, on which are founded the mathematical theories of logic and probability,
London 1854 (Walton & Mobeley) & (MacMillan); Coll. Logical Works II

A Treatise on Differential Equations, London 1859 (MacMillan);
repr. Chelsea

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Annalen der Physik = Pogg. Ann. 1824-1876; Wied. Ann. 1877-1899; Drude Ann. 1900-1928

Bull. Sci. math. (Férussac)   = Bull. Sci. math. phys. chem. (Bulletin de Férussac)

Comm. Acad. Sci. Petrop. = Commentarii Academiae Scientiarum Petropolitanae

C. R. Acad. Sci. Paris = Comptes rendus de l'academie des sciences, Paris

Exerc. d' Anal. = Exercices d'Analyse et de physique mathématique

Gött. Abh.   or   Abh. König. Ges. Wiss. Gött.   =   Abhandlungen von der Königlichen Gesellschaft der Wissenschaften zu Göttingen

Gött. Nachr. or Nachr. König. Ges. Wiss. Gött. = Nachrichten von der Königlichen Gesellschaft der Wissenschaften zu Göttingen

Hist. Acad. Sci. Berlin =
Histoire de l'Academie Royale des Sciences et des Belles Lettres de Berlin

Hist. Acad. Sci. Paris = Histoire de l'Académie Royale des Sciences de Paris avec les Mémoires de Mathématique et de Physique

Crelle = J. Math. (Crelle) = Crelle's journal = J. f. d. reine u. angew. Math. (Berlin)

J. Math (Liouville) or Liouville's journal or Jour. de Math. =
Journal de Mathématiques pures et appliquées

Lettres de Berlin = Nouveaux Mémoires de l'académie royale des Sciences et Belles-Lettres de Berlin

Mem. Acad. Sci. = Mémoires de l'académie royale des Sciences de l'Institut de France

Mem. pres. div. sav. = Mémoires présentées par divers savants à l'académie royale des Sciences de l'Institut de France

Misc. Berolin or Miscellanea Berolinensis = Histoire de l'académie, Berlin

Misc. Taur. = Philosophica-Mathematica Societatis Privatae Taurinensis (publishers: Accademia della Scienze di Torino)

Rend. Lincei = Atti Acad. naz. Lincei, Rendiconti =
Atti della reale Accademia nazionale dei Lincei, Rendiconti

Berlin Sitzb. = Sitzb. Akad. Wiss. Berlin = Sitzungsberichte der Königlichen Preussischen Akademie der Wissenschaften zu Berlin