Active structural acoustic control using the remote sensor method

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Abstract. Active structural acoustic control (ASAC) is an effective method of reducing the sound radiation from vibrating structures. In order to implement ASAC systems using only structural actuators and sensors, it is necessary to employ a model of the sound radiation from the structure. Such models have been presented in the literature for simple structures, such as baffled rectangular plates, and methods of determining the radiation modes of more complex practical structures using experimental data have also been explored. A similar problem arises in the context of active noise control, where cancellation of a disturbance is required at positions in space where it is not possible to locate a physical error microphone. In this case the signals at the cancellation points can be estimated from the outputs of remotely located measurement sensors using the "remote microphone method". This remote microphone method is extended here to the ASAC problem, in which the pressures at a number of microphone locations must be estimated from measurements on the structure of the radiating system. The control and estimation strategies are described and the performance is assessed for a typical structural radiation problem.

1. Introduction

Active vibration control has significant benefits compared to passive vibration control when there are size and weight restrictions on the control treatment. These benefits become particularly useful at lower frequencies where achieving high performance with passive noise control methods is often impractical due to its necessary size and weight [1]. Another advantage of active control methods is the ability to more flexibly manipulate the sound or vibration response. For example, in the context of active noise control it is often necessary to control the sound field at a position remote from the physical error microphones. This sound field manipulation requires the controller to minimise an estimate of the sound field at a remote location and this has been achieved using either virtual [2] or remote [3] microphone techniques. In the virtual microphone technique it is assumed that the primary pressure is equal at the physical error sensor and the control location, whereas in the remote microphone technique a transfer response between the physical sensor and control location is used to predict the primary disturbance at the control location.

A similar remote problem can also occur in structural vibration control, where it may be necessary to minimise the vibration at a position on the structure at which it is not possible to locate an error sensor or actuator and, therefore, it is necessary to use a remote control technique [4]. In this paper the problem of controlling a structure to minimise the sound

radiation is investigated. This type of control problem is referred to as active structural acoustic control (ASAC) and in its simplest form can be achieved by using structural control actuators to minimise the sound pressure measured at error microphones located in the sound field produced by the vibrating structure [5]. However, a more practically useful and cost effective ASAC system aims to control the radiated sound field by using only structural actuators and sensors [6].

ASAC requires a model of the radiation properties of the structure, such that given a number of structural measurements the radiated sound can be estimated. A number of methods of deriving such models have been proposed in the literature for tractable structures such as baffled rectangular plates [7], however, the applicability of these methods when more realistic structures are considered is rather limited. Therefore, there has also been an interest in determining models of structural radiation from experimental data. For example, Berkhoff [8] proposed a method of determining the radiation modes of a structure from experimental data and a number of other ASAC systems have been proposed based on modal representations [6, 9, 10].

In this paper an alternative to the modal-based ASAC strategies is proposed based on the remote microphone method [3] previously employed in active noise control applications [3, 11, 12]. In particular, the remote microphone method is applied to the problem of implementing an adaptive tonal controller for the control of structural radiation. In Section 2 the basic formulation of an adaptive tonal active vibration control system is reviewed and then in Section 3 the remote microphone method is presented in terms of the structural radiation control problem. The optimal frequency domain solution to this problem is first presented, before the iterative algorithm is derived. In Section 4 a practical structural radiation control problem is described and the performance of the ASAC system using the remote microphone method is compared to traditional active vibration control. Finally, in Section 5 conclusions are drawn.

2. Active Vibration Control

In general, AVC is implemented by driving a number of structural control actuators with signals calculated to minimise the response at a number of structural error sensors [1]. The most common cost function to minimise in this case is given by the sum of the squared structural error signals, which can be expressed at a single frequency as

$$J_s = \mathbf{e}_s^H \mathbf{e}_s, \tag{1}$$

where $\mathbf{e}_s$ is the vector of L_s structural error signals. For notational convenience, the dependency on angular frequency, ω, will be omitted in the rest of this paper as only tonal excitation is considered. The vector of structural error signals, $\mathbf{e}_s$, can be expressed as the linear summation of the structural disturbances produced by the primary source, $\mathbf{d}_s$, and the structural responses due to the control signals, $\mathbf{G}_s \mathbf{u}$, such that

$$\mathbf{e}_s = \mathbf{d}_s + \mathbf{G}_s \mathbf{u}, \tag{2}$$

where $\mathbf{G}_s$ is the matrix of plant responses between the inputs to the M control actuators and the L_s structural error sensors, and $\mathbf{u}$ is the vector of M control signals. Substituting eq. (2) into (1) gives the cost function as

$$J_s = \mathbf{u}^H \mathbf{G}_s^H \mathbf{G_s} \mathbf{u} + \mathbf{d}_s^H \mathbf{G}_s \mathbf{u} \mathbf{u}^H \mathbf{G}_s^H \mathbf{d}_s + \mathbf{d}_s^H \mathbf{d}_s. \tag{3}$$

If the matrix $\mathbf{G}_s^H \mathbf{G}_s$ is positive-definite, which is generally the case in practice when $L_s > M$, the vector of control signals which minimises the cost function J_s is given by setting the derivative of (3) with respect to the real and imaginary parts of $\mathbf{u}$ to zero [13]. The optimal vector of control signals is then

$$\mathbf{u}_{opt} = - \left[\mathbf{G}_s^H \mathbf{G}_s \right]^{-1} \mathbf{G}_s \mathbf{d}_s. \tag{4}$$

2.1. Iterative Implementation

In practice, the vector of disturbance signals at the structural error sensors is not known in advance and, therefore, it is necessary to employ an iterative algorithm which updates the vector of control signals and converges towards the optimal solution given by eq. (4). This can be achieved using the method of steepest-descent and in this case the update algorithm is given by

$$\mathbf{u}(n+1) = \mathbf{u}(n) - \alpha \mathbf{G}_s^H \mathbf{e}_s(n), \tag{5}$$

where n denotes the current iteration step of the algorithm and α is the convergence gain. The convergence of this iterative update equation has been shown to be dependent on the eigenvalue spread of the matrix $\mathbf{G}_s^H \mathbf{G}_s$ [13, 14], such that when the eigenvalue spread, or condition number of the matrix is large the convergence will be slow. This standard feedforward control algorithm for AVC will be used as a baseline to assess the performance and the need for the ASAC algorithm presented in the following section.

3. Formulation of ASAC using the Remote Microphone Method

ASAC systems have generally relied on some form of modal controller to control the sound radiated from a structure [6, 7, 8, 9, 10]. However, the ASAC problem strongly relates to the challenges solved by remote sensing methods in vibration control [4, 15], where control is required at locations at which it is not possible to locate sensors or actuators. In the context of active noise control, remote methods have been used when cancellation of the primary disturbance is desired at a point in space where it is not possible to position error microphones [11, 12]. In this case the signals at the cancellation points must be estimated from the outputs of remotely located measurement sensors. This estimation was originally achieved using the virtual microphone method, in which it is assumed that the primary disturbance is equal at the measurement sensor and the cancellation position [2]. Subsequently, the more advanced remote microphone method was proposed, which estimates the primary disturbance at the cancellation position using a transfer response between the measurement and cancellation positions [3]. This remote microphone method will be extended here to the ASAC problem, in which the pressures at a number of microphone locations must be estimated from measurements on the radiating structure.

The aim of the ASAC system in this case is to minimise the sum of the squared pressures measured by L_a microphones. The cost function to minimise can therefore be defined as

$$J = \mathbf{e}_a^H \mathbf{e}_a \tag{6}$$

where $\mathbf{e}_a$ is the vector of L_a error signals measured at the microphone locations. This is given by the linear superposition of the primary disturbance measured at the acoustic error sensors, $\mathbf{d}_a$, and the contribution due to the controller, $\mathbf{G}_p \mathbf{u}$. However, in the ASAC system these error signals will not be directly available to the controller and, therefore, must be estimated from the error signals measured on the vibrating structure. In this case, assuming perfect knowledge of the plant response matrix, the estimated acoustic error signals can be expressed as

$$\hat{\mathbf{e}}_a = \hat{\mathbf{d}}_a + \mathbf{G}_p \mathbf{u} \tag{7}$$

where $\hat{\mathbf{d}}_a$ is the primary disturbance estimated at the acoustic error sensors, $\mathbf{G}_p$ is the matrix of transfer responses between the inputs to the structural control actuators and the resulting pressures measured at the acoustic error sensors, and $\mathbf{u}$ is the vector of M control signals.

The primary disturbance at the acoustic error sensors can be estimated as a linear function, $\mathbf{O}$, of the disturbance signals measured on the structure, $\mathbf{d}_s$, such that

$$\hat{\mathbf{d}}_a = \mathbf{O} \mathbf{d}_s. \tag{8}$$

In practice, the vector of structural disturbances can be calculated from the structural error signals as

$$\mathbf{d}_s = \mathbf{e}_s - \mathbf{G}_s \mathbf{u}. \tag{9}$$

The optimal observation filter, $\mathbf{O}$, can then be calculated by minimising the cost function

$$J_O = \text{trace}\left[(\mathbf{d}_a - \mathbf{O}\mathbf{d}_s)(\mathbf{d}_a^H - \mathbf{d}_s^H \mathbf{O}^H) \right] \tag{10}$$

and the optimal observation filter is given by [12]

$$\mathbf{O}_{opt} = \mathbf{d}_a \mathbf{d}_s^H \left[\mathbf{d}_s \mathbf{d}_s^H \right]^{-1}. \tag{11}$$

The ASAC system can now be optimised to minimise the cost function given by the sum of the squared estimated acoustic error signals given by eq. (7). The cost function in this case is

$$\mathbf{J} = \hat{\mathbf{e}}_a^H \hat{\mathbf{e}}_a \tag{12}$$

and substituting eq. (7) for $\hat{\mathbf{e}}_a$ and eq. (8) for $\hat{\mathbf{d}}_a$ gives

$$\mathbf{J} = (\mathbf{d}_s^H \mathbf{O}^H + \mathbf{u}^H \mathbf{G}_p^H)(\mathbf{O}\mathbf{d}_s + \mathbf{G}_p \mathbf{u}) \tag{13}$$

$$= \mathbf{d}_s^H \mathbf{O}^H \mathbf{O} \mathbf{d}_s + \mathbf{u}^H \mathbf{G}_p^H \mathbf{G}_p \mathbf{u} + \mathbf{d}_s^H \mathbf{O}^H \mathbf{G}_p \mathbf{u} + \mathbf{u}^H \mathbf{G}_p^H \mathbf{O} \mathbf{d}_s. \tag{14}$$

This is a standard Hermitian quadratic form and the optimal vector of control signals is thus given by

$$\mathbf{u}_{opt} = -\left[\mathbf{G}_p^H \mathbf{G}_p \right]^{-1} \mathbf{G}_p^H \mathbf{O} \mathbf{d}_s. \tag{15}$$

It is interesting to highlight that the observation filter, $\mathbf{O}$, acts in a similar way to a error sensor weighting matrix [13], however, in comparison to a standard sensor weighting matrix, which is diagonal, the observation matrix is fully populated.

3.1. Iterative Implementation

Once again, it is important in practice to obtain an iterative algorithm that converges towards the optimal solution given by eq. (15) since the disturbance signals at the structural error sensors will in practice not be known in advance. The iterative update algorithm in the case of the remote sensing ASAC system is given by

$$\mathbf{u}(n+1) = \mathbf{u}(n) - \alpha \mathbf{G}_p^H \hat{\mathbf{e}}_a(n). \tag{16}$$

Substituting eq. (7) for the estimated acoustic error signal vector gives

$$\mathbf{u}(n+1) = \mathbf{u}(n) - \alpha \mathbf{G}_p^H (\hat{\mathbf{d}}_a + \mathbf{G}_p \mathbf{u}). \tag{17}$$

and using the acoustic disturbance estimated according to eq. (8) and the structural disturbance calculated according to eq (9) gives

$$\mathbf{u}(n+1) = \left[\mathbf{I} - \alpha \mathbf{G}_p^H \mathbf{G}_p + \alpha \mathbf{G}_p^H \mathbf{O} \mathbf{G}_s \right] \mathbf{u}(n) - \alpha \mathbf{G}_p^H \mathbf{O} \mathbf{e}_s(n). \tag{18}$$

From this update equation it can be seen that the remote microphone method results in a control signal weighting matrix, given by the term in square brackets, and a sensor weighting matrix, given by the observation filter $\mathbf{O}$.

For the case when the plant models are perfect, the convergence of the iterative algorithm given by (18) will be dependent on the eigenvalues of the matrix $\mathbf{G}_p^H \mathbf{G}_p$. However, when there are

differences between the physical plant responses, $\mathbf{G}_s$ and $\mathbf{G}_p$, and the modelled plant response, $\widehat{\mathbf{G}}_s$ and $\widehat{\mathbf{G}}_p$, then the convergence and stability of the adaptive algorithm will be dependent on the eigenvalues of the matrix [16]

$$\widehat{\mathbf{G}}_p^H \widehat{\mathbf{G}}_p + \widehat{\mathbf{G}}_p^H \mathbf{O} \left(\mathbf{G}_s - \widehat{\mathbf{G}}_s \right). \tag{19}$$

Specifically, the algorithm will be unstable if the real parts of the eigenvalues of this matrix are negative. In the following section, the influence of errors in the plant responses will not be investigated, as this is likely to depend on the specific characteristics of the plant responses $\mathbf{G}_p$ and $\mathbf{G}_s$ and the practical modelling errors, but this will form the basis of future work.

4. Performance Assessement

To evaluate the performance of the ASAC strategy based on the remote microphone method described in the previous section, a practical control system has been implemented, as shown in Figure 1. The noise radiating structure considered is a thin aluminium plate with dimensions of 414 mm × 314 mm. The plate is mounted on a perspex enclosure and the primary disturbance is provided by a loudspeaker inside the enclosure. The control system consists of 4 electromagnetic structural actuators and 18 accelerometers and the acoustic pressure has been measured over a planar grid of 66 positions at a distance of 7 cm from the surface of the plate suing a single microphone in a reverberant laboratory environment. The acoustic and structural responses from the structural actuators, $\mathbf{G}_s$ and $\mathbf{G}_p$, and the primary loudspeaker, $\mathbf{P}_s$ and $\mathbf{P}_p$, have been measured using a sampling frequency of $Fs = 4$ kHz and the appropriate anti-aliasing and reconstruction filters.
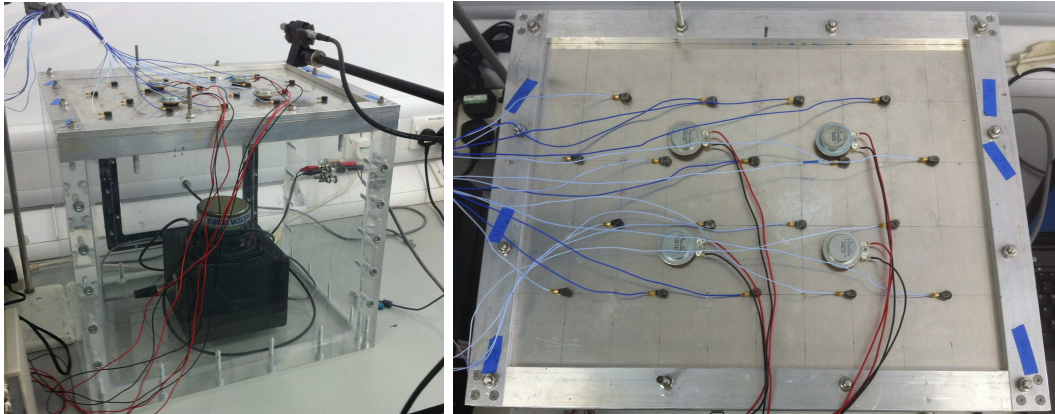

Figure 1: The radiating plate system showing the four structural actuators, 18 accelerometers, one of the 66 acoustic measurement positions and the primary disturbance loudspeaker.

The measurements have been taken in close proximity to the plate to reduce the influence of the reverberant environment in this case. However, it should be highlighted that since these measurements are within the near field in the considered frequency range, depending on the excited radiation modes, the implemented controller may not achieve optimal control of the far field radiation. The following results are intended to demonstrate the effectiveness of the remote sensor technique and this is achieved whether or not the far field levels are most effectively controlled. To achieve control of the far field radiation control microphone locations in the far field could be employed. Additionally, it is worth highlighting that the number of structural sensors could be reduced if only low frequency control was required, since the spatial complexity of the radiating modes is low at low frequencies.

4.1. Structural Acoustic System Response

In order to characterise the structural acoustic response of the plate, it has first been excited by driving the loudspeaker positioned in the enclosure with broadband white noise. Figure 2 shows the resulting structural response in terms of the cost function given by eq. (1), which is the sum of the squared signals measured by the structural error sensors. Figure 3 shows the resulting acoustic response in terms of the cost function given by eq. (6), which is the sum of the squared pressures measured at the microphone locations. From these two figures it can be seen that the radiated sound pressure measured at the microphone locations and the overall structural response are characterised by a number of lightly damped resonances up to around 1 kHz. At 1 kHz the response of the primary loudspeaker begins to roll-off and, as a result, the response above this frequency is low. By comparing the acoustic and structural responses shown in Figures 3 and 2 it can be seen that although a number of structural resonances result in peaks in the radiated sound field, a number of structural resonances either do not radiate or do not radiate efficiently. If the structure is excited by a single frequency tone at a radiating structural resonance, for example at the 181 Hz resonance, then suppressing the vibration due to this single resonance is also generally considered an effective means of controlling the radiated sound field. This will be explored using the proposed ASAC algorithm in the following section.
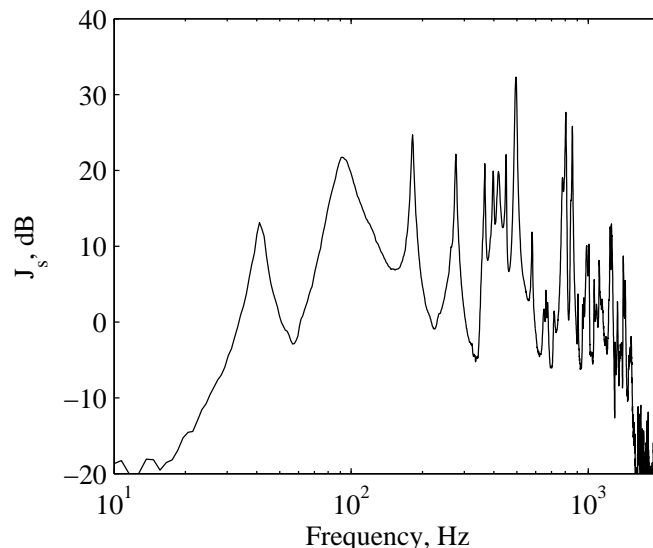

Figure 2: The sum of the squared structural error signals measured by the 18 accelerometers when the structure is excited by broadband white noise produced by the internal loudspeaker.

4.2. ASAC Performance

The performance of the AVC and ASAC systems has been simulated in the time-domain using the responses measured for the radiating system shown in Figure 1. The primary and secondary path responses have been modelled using 1024 coefficient finite impulse response (FIR) filters and the primary disturbance has been defined as a single sinusoidal tone. The performance of the iterative implementations of the two control systems has been evaluated for a tonal primary disturbance at 181 Hz, which coincides with a strong structural resonance that also radiates. The convergence gain for each controller has been set to achieve the maximum convergence speed with respect to the algorithms' respective cost functions. Figure 4 shows the convergence of the structural cost function, J_s, for the two controllers and Figure 5 shows the convergence of the acoustic cost function, J_p, for the two controllers.
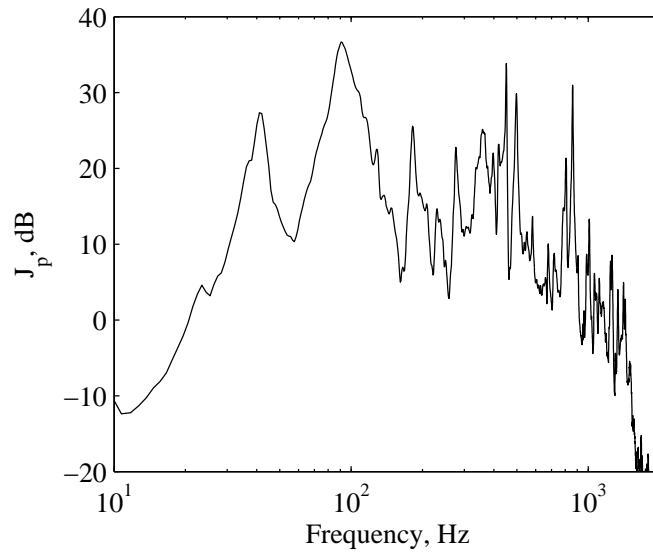
Figure 3: The sum of the squared pressures measured at the 66 microphone locations when the structure is excited by broadband white noise produced by the internal loudspeaker.
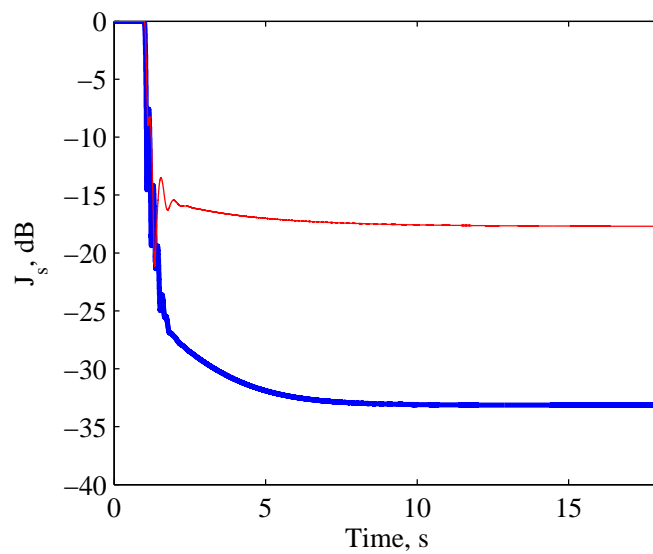

Figure 4: The convergence of the sum of the squared structural error signals when the system is excited by a primary tonal disturbance at 181 Hz for the AVC system (thick blue line) and the ASAC system (thin red line).
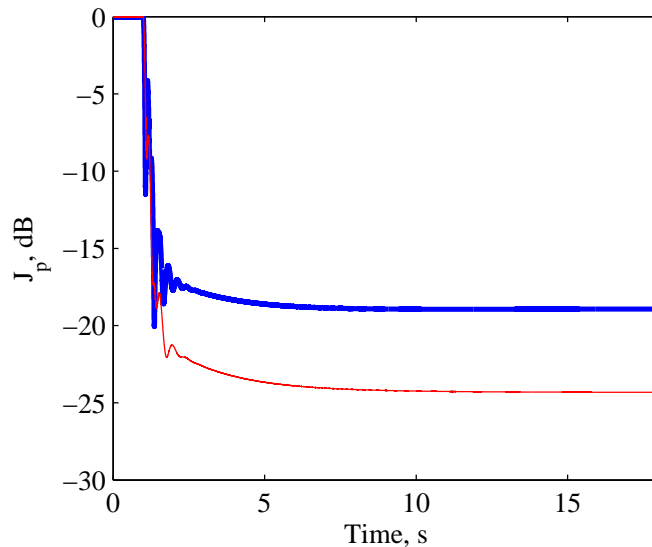
Figure 5: The convergence of the sum of the squared pressures when the system is excited by a primary tonal disturbance at 181 Hz for the AVC system (thick blue line) and the ASAC system (thin red line).

From Figure 4 it can be seen that the AVC system achieves around 33 dB attenuation in the structural cost function, while the ASAC system only attenuates the structural response by around 18 dB. However, from Figure 5 it can be seen that the AVC system attenuates the acoustic cost function, J_p, by 18 dB, while the ASAC system achieves a further 6 dB of attenuation in the acoustic cost function. From these results it is clear that the ASAC system employing the remote microphone method is effective at controlling the pressure at the microphone locations and improves the acoustic control performance by 6 dB compared to the AVC system. However, as a result, the ASAC system also loses 15 dB of structural control performance. It is interesting to highlight the benefit of controlling the acoustic pressures directly using the proposed ASAC algorithm rather than using the AVC strategy, even though the structure is excited on resonance.

5. Conclusions

ASAC algorithms are often required when the noise and vibration performance of a structure is evaluated in terms of its acoustic response rather than its structural response. Previous ASAC algorithms have focused on employing methods of controlling radiating modes. This can be difficult to implement for complex structures away from resonances and, therefore, in this paper an alternative ASAC strategy has been proposed based on the remote microphone method previously employed in active noise control systems.

The optimal solution for the ASAC algorithm based on the remote microphone method has first been derived and then an iterative, steepest-descent based implementation has been presented. This highlights that the ASAC algorithm based on the remote microphone method results in an update equation that employs both source and sensor weighting matrices in order to achieve the required control performance. The behaviour of the ASAC algorithm based on the remote microphone method has then been investigated using a practical structural radiation control problem. Through time domain simulations of the AVC and ASAC algorithms it has been shown that the remote microphone method is effective at minimising the sum of the squared pressures at the microphone locations. This ASAC strategy avoids the need for microphone

signals to be available to the controller during operation and also avoids the need for a radiation mode analysis of the structure to be conducted.

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