Performance of Cognitive Hybrid Automatic Repeat reQuest: Go-Back-N

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***Abstract*—In this paper, we propose a cognitive Go-Back-N Hybrid Automatic Repeat reQuest (CGBN-HARQ) scheme for a cognitive radio (CR) system to opportunistically transmit data over a primary radio (PR) channel. We model the activity of PR users (PRUs) occupying the PR channel as a Markov chain with two states: 'ON' and 'OFF'. In order to use the PR channel, the CR system first senses the availability/unavailability of the PR channel. Once it finds that the PR channel is free, the CR system transmits data packets over the PR channel's spectrum, whilst relying on the principles of GBN-HARQ. In this paper, we investigate both the throughput and delay of CGBN-HARQ, with a special emphasis on the impact of various system parameters involved in the scenarios of both perfect and imperfect spectrum sensing. Our studies demonstrate that the activity of PRUs, the transmission reliability of the CR system as well as the number of packets transmitted per time-slot may have a substantial impact on both the throughput and the delay of the CR system.**

I. Introduction

Recent studies conducted by the Federal Communication Commission (FCC) in the USA and by the European Telecommunications Standards Institute (ETSI) in Europe [1, 2] reveal that, under the static spectrum allocation policy, the reserved electromagnetic spectrum is heavily underutilized, whilst at the same time the remaining spectral resources become insufficient for new wireless [2, 3]. The predicament of inefficient spectrum utilization motivates the research of dynamic spectrum access, which allows cognitive radio users (CRUs) to opportunistically access the earmarked, but momentarily unoccupied spectrum, which was originally assigned to primary radio users (PRUs) [1, 3, 4].

In [5], we have proposed a Cognitive Stop-and-Wait Hybrid Automatic Repeat reQuest (CSW-HARQ) scheme for opportunistically accessing a PRU's channel, and studied both the attainable throughput and the delay imposed by cognitive Stop-and-Wait-HARQ (CSW-HARQ) scheme. Following on from our studies in [5], in this paper, we propose a Cognitive Go-Back-N HARQ (CGBN-HARQ) scheme and investigate both its throughput and delay. As in [5], we assume that PRUs activate a PR channel according to a two-state Markov chain with the states 'ON' and 'OFF' [6]. In order to access the primary radio (PR) channel, the CR system first senses the channel and it is allowed to utilize this channel, if it is found in the 'OFF' state. Otherwise, the CR system waits and keeps on sensing the PR channel. Furthermore, we assume that the PR channel is partitioned divided into time-slots (TS) of duration T. For our CR system, we assumed that every TS is further divided into two segments, namely the sensing time of duration T_s and the data transmission time of duration $T_d = T - T_s$ [7, 8], whilst relying on the GBN-HARQ principles [9, 10], provided that the PR channel is sensed to be free.

Both the throughput and delay of the conventional ARQ schemes have been widely studied [9–11]. Specifically, for the conventional GBN-ARQ, the throughput and delay have been investigated, for example in [12–17], based on the classic Markov chain models. In more detail, the authors of [12, 13] have analyzed the throughput of the GBN-ARQ based on a hidden Markov model, when assuming either reliable or unreliable feedback. In [14, 15], the authors analyzed the delay of GBN-ARQ, while both the average packet delay and its distribution have been studied in [16]. A hybrid of the interweave and underlay paradigms has been proposed in [18, 19] for improving the performance of CRU. The authors of [19] have also used the ARQ approach for improving the reliability.

In this paper, we specifically investigate the achievable throughput and the average packet delay of the proposed CGBN-HARQ scheme in perfect and imperfect sensing environment. Furthermore, we study the end-to-end packet delay, by investigating its probability distribution. Note that in this paper the average packet delay is defined as that of the total time required for successfully transmitting a packet. By contrast, the end-to-end packet delay is defined as the time elapse from starting the transmission of a packet to the successful reception of the packet. In this paper, the throughput and delay of the CGBN- HARQ protocol is characterized from different perspectives. Our studies demonstrate that both the activity of the PR system and the transmission reliability of the CR system impose a substantial impact on the achievable performance of CGBN-HARQ, where the parameters have to be carefully adapted according to the specific communication environment encountered, in order to maximize its performance.

Against the above background, the contribution of this paper can be summarized as follows,

1) *A novel cognitive protocol is proposed, which intrinsically amalgamates CR with the conventional GBN-HARQ protocol for the sake of achieving reliable communication, where both the perfect and imperfect sensing scenarios are considered. Furthermore, the CR transmitter may receive the required feedback flag within the transmission duration of a single TS, regardless of the PR and/or CR activity. Therefore, significant advances are required for reformulating the transmission principles of the conventional GBN-HARQ in order to intrinsically amalgamate it with CR systems.*
2) *The performance of the proposed scheme is quantified in terms of diverse performance metrics, such as its throughput, average packet delay, probability distribution of end-to-end packet delay.*

The rest of this paper is organized as follows. The models for the PR and CR systems are described in Section II. The principles of CGBN-HARQ are stated in Section III. Sections III-A and III-B consider the operations of the CR transmitter and receiver. Our performance results are provided in Section IV, whilst our conclusions are offered in Section V.

II. System Model

In this section, we describe both the PR as well as the CR systems and detail our assumptions used for obtaining our provided results in Section IV.

A. Modeling of Primary Radio

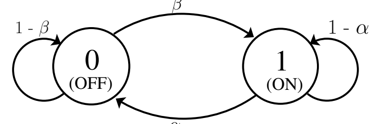


Fig. 1. Two-state discrete-time Markov chain model of the PR system.

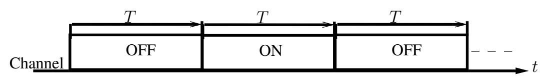
As in [5], we consider a PR channel, which is used by the PRUs accessing it based on TSs of duration T. We assume that the activation of each TS is independent of that of the others and has the same probability. The activation of the PR channel by PRUs is modeled as a two-state Markov chain with the state transitions shown in Fig. 1. The state 'OFF' represents that the channel is free for CR to use, whereas state 'ON' indicates that the channel is occupied by the PR; β and α represent the transition probabilities from 'OFF' and ON' to 'ON' and 'OFF' state respectively. Furthermore, the probabilities of the PR channel being 'ON' and 'OFF' are expressed as P_{on} and $P_{off} = 1 - P_{on}$. Under these assumptions, it can be shown that when the Markov chain is in its steady state, we have [10]

$$P_{on}\alpha = P_{off}\beta, \tag{1}$$

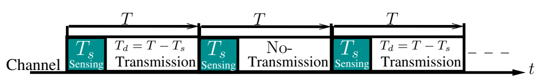
which gives

$$P_{on} = \frac{\beta}{\alpha+\beta}, \;\; P_{off} = \frac{\alpha}{\alpha+\beta}. \tag{2}$$

Furthermore, as shown in Fig. 2(a), we assume that if the PR channel is found in the 'ON' state at the start of a TS, it remains busy until the end of that TS and the TS should not be used by the CR. On the other hand, if a TS is detected to be free from the PR, then, it is free for use by the CR [6].



(a) Pattern of channel usage by PR



(b) Pattern of channel usage by CR

Fig. 2. Time-slot structure of PR and CR systems, where a CR TS consists of a sensing duration of T_s and a transmission duration of $T_d = T - T_s$, when given the total duration T of a time-slot.

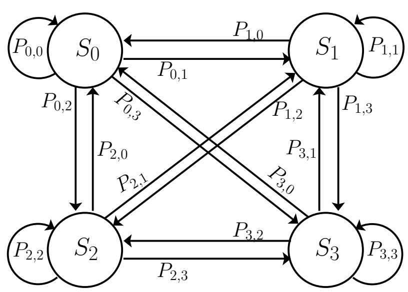


Fig. 3. Discrete-time Markov chain for modeling the CR system where $S_0 = 00$ illustrates that the respective TS is free and also the CRU detected it free, $S_1 = 01$ represents that the channel is free but the CRU finds it busy. Likewise $S_2 = 10$ and $S_3 = 11$.

B. Modeling of Cognitive Radio

As mentioned above, in the CR system, each TS is divided into two phases: sensing with a duration T_s and data transmission having a duration of $T_d = T - T_s$, as shown in Fig. 2(b). Hence, when the PR channel is detected to be in the 'OFF' state within the sensing period T_s, the CR uses the time T_d for its data transmission based on the principles of GBN-HARQ [1]. In practice, the CRU may not always be capable of perfectly detecting the 'ON' and 'OFF' activity of the PRUs due to the channel induced shadowing and fading. Firstly, it is possible that the CRU senses the PR channel to be in 'ON' state, while it is actually in the 'OFF' state. Secondly, it is possible that the channel is detected to be in the 'OFF' state, while it is actually in the 'ON' state. The former and latter scenarios are known as false-alarm (P_{fa}) and missed-detection (P_{md}), respectively. Similarly, there are other scenarios, as seen in Fig. 3.

[1]Note that during miss-detection scenario, the collision between CRU and PRU may happen which affects the performance of CRU and PRUs. Affect on the performance of PRUs is out of the scope of this paper.

Let us define the states reflecting the above situations as (R_T, S_T), where R_T is the real status of the PR channel and S_T represents the status of the PR channel sensed by the CR system. Specifically,

$$R_T = \begin{cases} 0, & \text{PR channel is free} \\ 1, & \text{PR channel is busy.} \end{cases}$$

$$S_T = \begin{cases} 0, & \text{the PR channel is sensed to be free} \\ 1, & \text{the PR channel is sensed to be busy.} \end{cases}$$

Let the transition probabilities between state i and state j be represented by $P_{i,j}$. Then, we have a transition matrix given by

$$\boldsymbol{P} = \begin{bmatrix} P_{0,0} & P_{0,1} & P_{0,2} & P_{0,3} \\ P_{1,0} & P_{1,1} & P_{1,2} & P_{1,3} \\ P_{2,0} & P_{2,1} & P_{2,2} & P_{2,3} \\ P_{3,0} & P_{3,1} & P_{3,2} & P_{3,3} \end{bmatrix}$$

According to Fig. 3, it can be shown that

$$\begin{array}{l} P_{0,0} = P_{1,0} = (1-\beta)(1-P_{fa}) \\ P_{0,1} = P_{1,1} = (1-\beta)P_{fa} \\ P_{0,2} = P_{1,2} = \beta P_{md} \\ P_{0,3} = P_{1,3} = \beta(1-P_{md}) \\ P_{2,0} = P_{3,0} = (\alpha)(1-P_{fa}) \\ P_{2,1} = P_{3,1} = (\alpha)(1-P_{fa}) \\ P_{2,2} = P_{3,2} = (1-\alpha)(P_{md}) \\ P_{2,3} = P_{3,3} = (1-\alpha)(1-P_{md}) \end{array} . \tag{3}$$

Let the steady state probabilities of the Markov chain be expressed as $\boldsymbol{\pi} = [\pi_0, \pi_1, \pi_2, \pi_3]^T$. Then, we have

$$\boldsymbol{\pi} = \boldsymbol{P}^T\boldsymbol{\pi}, \tag{4}$$

which shows that $\boldsymbol{\pi}$ is the right eigenvector of $\mathbf{P}^T$ corresponding to the eigenvalue of 1. Therefore, after solving Equation (4), we obtain

$$\begin{aligned} \boldsymbol{\pi} &= [\pi_0 \;\; \pi_1 \;\; \pi_2 \;\; \pi_3]^T \\ &= m \times \left[\frac{\alpha(1-P_{Fa})}{\beta(1-P_{md})} \;\; \frac{\alpha(P_{Fa})}{\beta(1-P_{md})} \;\; \frac{(P_{md})}{(1-P_{md})} \;\; 1\right]^T, \end{aligned} \tag{5}$$

where $m \in \mathbb{R}$, which can be determined by the

$$\sum_{i=0}^{3} \pi_i = 1 \tag{6}$$

giving

$$m = \frac{\beta(1-P_{md})}{\alpha+\beta}. \tag{7}$$

Consequently, the steady state probabilities are

$$\pi_0 = \frac{\alpha(1-P_{fa})}{\alpha+\beta}, \pi_1 = \frac{\alpha(P_{fa})}{\alpha+\beta} \tag{8}$$

$$\pi_2 = \frac{\beta(P_{md})}{\alpha+\beta}, \qquad \pi_3 = \frac{\beta(1-P_{md})}{\alpha+\beta}. \tag{9}$$

III. Principles of Cognitive Go-Back-N Hybrid Automatic Repeat Request

In our CGBN-HARQ system, the data is encoded using a Reed-Solomon (RS) code RS(N_d, K_d) [9], defined over the Galois Field of GF(q)=GF(2^m), where K_d and N_d represent the number of information and coded symbols, respectively and m is the number of bits/symbol. We assume that every packet consists of a RS codeword, which is transmitted within T_p seconds. Let $N = T_d/T_p$. Then, within a free TS, the CR transmitter can transmit N packets. Additionally, we assume that the RS code is capable of correcting t number of m-bit symbol errors and that its error-detection is perfect, when there are uncorrectable errors in a received packet, i.e. when more than t m-bit symbol errors were imposed by the channel.

Based on the above assumptions, data is transmitted between two CR users over the free PR channel, based on the principles of CGBN-HARQ, which is depicted in Fig. 4. This protocol allows the CR

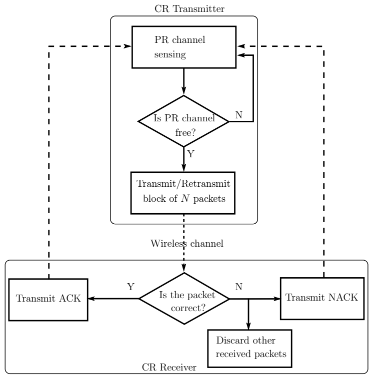


Fig. 4. Flow chart showing the operations of propose CGBN-HARQ scheme.

transmitter to continuously transmit N packets without waiting for the corresponding feedback flags. Here, N represents the maximum number of packets transmitted during a single round-trip time (RTT), which is assumed to be equal to $(T_d + T_s)$. Note that the RTT is defined as the time interval between the transmission of a packet and the reception of a feedback flag for that packet.

The operational principles of the CGBN-HARQ protocol can be described by the algorithm of Fig. 4 as detailed below.

Algorithm 1 : CGBN-HARQ Algorithm

1: **Initialization**: M_c = number of packets, $T_d = N$, $T_s = k$, $i = 1$, $c = 0$, TS$= 1$.
2: **Input:** N, k, packets.
3: **while** $i \leq M_c$ **do**
4: CR transmitter senses a TS.
5: **if** TS is free **then**
6: transmits packets from i to $i + N - 1 - c$.
7: TS $=$ TS$+1$, $j = i$.
8: Transmitter starts sensing TS immediately.
9: **while** $j \leq i + N - 1 - c$ **do**
10: **if** the jth packet is received error-free **then**
11: receiver transmits ACK for the jth packet.
12: $j = j + 1$.
13: **if** TS is free && ACK is received **then**
14: transmit the $(N + j)$th packet.
15: $c = c + 1$; ▷ Counter of packets transmitted
16: **else**
17: TS is busy && ACK is received
18: No transmission and wait.
19: **end if**
20: **else**
21: receiver transmits NACK for the jth packet and discard the following packets.
22: **if** TS is free **then**
23: **Goto** Line 6.
24: **else**
25: TS is busy.
26: **Set** $c = 0$,
27: **Break**.
28: **end if**
29: **end if**
30: **end while**
31: **else**
32: Waits until the next TS.
33: **end if**
34: TS=TS+1.
35: **end while**

A. Operation of the CR Transmitter

In the traditional GBN-HARQ protocol, the transmitter continuously transmits its packets, until a NACK signal is received. In this case, both the erroneous packet as well as the other packets transmitter sent after the erroneous packet will be retransmitted. By contrast, in our proposed CGBN-HARQ scheme formulated in Algorithm 1 and in Fig. 4, the CR transmitter is only allowed to transmit packets in the TSs free from the PR. The transmitter first has to sense the PR channel before the transmission or retransmission of packets. If the PR channel is found in the 'OFF' state, the CR transmitter can transmit N packets, which may include both new packets and the packets requiring retransmission. Otherwise, its waits for the next TS and senses the channel again. In our scheme, we assume that all packets are of the same length and the CR transmitter is always ready to transmit these packets in free TSs. Furthermore, each CR packet consists of a RS coded codeword, which is transmitted within the duration of T_p seconds, as shown in Fig. 5. The feedback of each packet is received after the RTT of $T_d + T_s = (N + k)T_p$ seconds, where $T_s = kT_p$ is assumed.

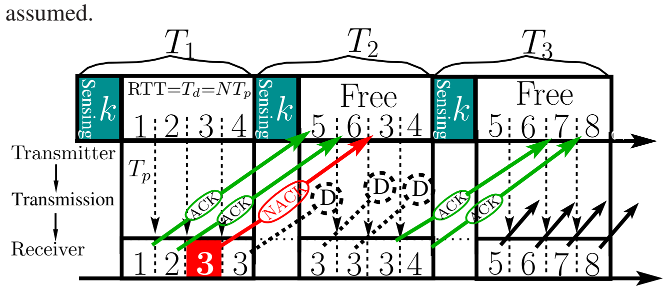


Fig. 5. Transmission flow of CGBN-HARQ scheme, when $N = 4T_p$ and $k = 1T_p$.

It is worth mentioning that for implementing the CGBN-HARQ protocol, the CR transmitter is assumed to have a buffer size of N packets, which follows the FIFO principle [9–11]. The transmitter stores the copies of the transmitted packets in its buffer until they are positively acknowledged (ACK). The buffer is updated according to the feedback flags received from the CR receiver. In detail, as shown in Fig. 5, if the ACK for the ith packet, $i = 1, 2$ in this example, is received, the copy of the ith packet is then deleted from the buffer and a new packet is appended at the end of the buffer. Furthermore, if the next TS is sensed to be free, which is the case in Fig. 5, this newly appended packet will be transmitted accordingly. On the other hand, if a NACK flag is received for the ith packet, such as $i = 3$ in our example, then the transmitter retains both the ith packet as well as the following packets, which are retransmitted in the next free TS, as presented in Fig. 5.

Additionally, in the proposed CGBN-HARQ scheme, we assume that the CR transmitter may receive feedback flags both during free and busy TSs. This assumption is reasonable, because the ACK and NACK flags only contain a few bits, hence they do not impose a grave impact on the transmission of PR signals.

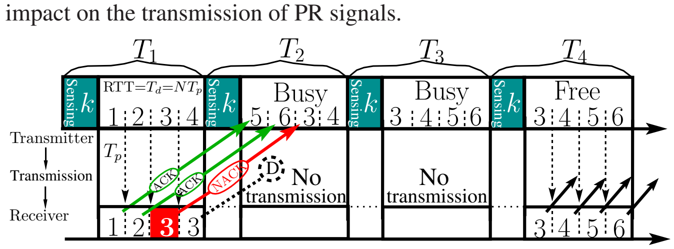


Fig. 6. Illustration of transmission of an erroneous packet followed by a busy TS, when $N = 4T_p$ and $k = 1T_p$.

In Fig. 6, we portray the scenario, where a free TS is followed by a busy TS. As shown in Fig. 6, the first 2 packets are correctly received, whereas the 3rd is received in error. Therefore, the 2 copies of the correctly received packets are deleted and 2 new packets are appended to the end of the buffer, which are packets 5, and 6. Since packet 3 is in error, packet 3 and 4 are kept in the buffer. Furthermore, since the second TS is busy, the CR transmitter has to wait until a free TS is sensed. Then, all these 4 packets are transmitted in the free TS, as shown in Fig. 6.

B. The operation of the CR Receiver

The CR receiver operates similarly as in the conventional GBN-HARQ. In the CGBN-HARQ, the CR receiver is assumed to have a buffer for storing the index of received packets [10, 11], which increases only when an error-free packet is received, as presented in

Algorithm 1. In detail, the operations of CR receiver can be explained in terms of *normal states* and *erroneous states* as follows.

1) Normal States: The CR receiver is considered to be in the normal state, when the sequence index of a received packet matches the sequence index stored in the receiver buffer. If this is the case, the CR receiver first carries out RS decoding and, then, generates a corresponding ACK or NACK flag, to be sent to the CR transmitter. Furthermore, if the packet becomes error-free after RS decoding, the CR receiver increases the sequence index by one, as stated on lines 11 and 12 of Algorithm 1. Then, the CR receiver waits for the reception of the next packet.

2) Erroneous States: When the CR receiver is in the normal state but an erroneous packet is output after RS decoding, the CR receiver changes to the erroneous state. During this state, the CR receiver discards the erroneous packet and transmits the NACK flag. Furthermore, the sequence index in the CR receiver buffer remains unchanged. Additionally, the CR receiver discards the subsequent packets received following the erroneous packet, regardless whether they are correct or not, as stated on line 21 of Algorithm 1. Following the above actions, the CR receiver enters into its normal state and waits for receiving the retransmission of the erroneous packet.

As the example seen in Fig. 5 shows, the CR receiver discards the erroneous *packet* 3 as well as the subsequent *packets* 4, 5 and 6, since it has to first correctly receive *packet* 3. As shown in Fig. 5, after the reception of a NACK flag for *packet* 3, the CR transmitter stops transmitting new packets and immediately retransmits *packet* 3 as well as the other packets already transmitted. The above process continues until all packets are successfully received.

IV. Performance Results

In this section, we provide our performance results for characterizing the proposed CGBN-HARQ system, which were obtained from Matlab based simulations. Both the throughput and the delay are quantified. In our simulations, the throughput is formulated as

$$R_S = \frac{N_s}{N_t(k+N)} \text{ (packets per } T_p\text{)}, \tag{10}$$

where N_t represents the total number of TSs used for the successful transmission of N_s packets, while $(k+N)$ is the number of T_p's per TS. Note that N_t includes both the free TSs and the busy TSs from the start of transmission to the successful reception of the N_s packets. The average packet delay is formulated as

$$T_{DS} = \frac{N_t(k+N)}{N_s} \times T_p \text{ (seconds)}. \tag{11}$$

However, the results provided in the following figures were the average packet delay normalized by T_p, which is given by $T'_{DS} = T_{DS}/T_p$ in terms of the unit of T_p.

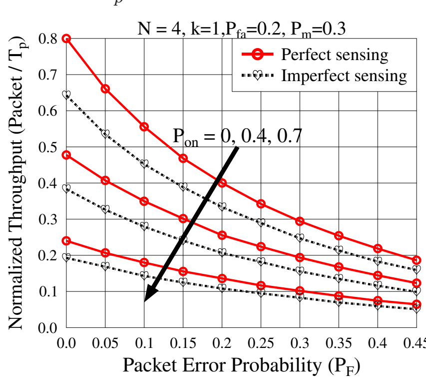


Fig. 7. Throughput of the CGBN-HARQ versus packet error probability in terms of various channel occupancy probabilities, for $k = 1$ and $N = 4$.

Figs. 7 and 8 characterize the attainable throughput performance of CGBN-HARQ versus the packet error probability for the scenario perfect and imperfect sensing. For a given value of P_{on}, it is observed from Fig. 7 that the throughput of CGBN-HARQ is maximum, when the channel is error-free, i.e. for $P_F = 0$. As P_F increases, implying that the CR channel becomes less reliable, the achievable throughput is reduced, due to the increased fraction of packet retransmissions. As seen in Fig. 7, at a given value of P_F, the throughput of CGBN-HARQ is maximum, when the PR channel is always free, i.e. for $P_{on} = 0$. As the probability P_{on} of PR utilization increases, the attainable throughput of the CR system drops significantly, since it has less opportunity to transmit data on the PR channel.

Moreover, we compare the throughput of the CGBN-HARQ scheme in the scenario of both perfect and imperfection sensing. We can observe from Fig. 7 that the throughput is higher in the perfect sensing environment. This is due to the fact that the probability of false-alarm and miss-detection is zero. Hence, no free TS is wasted, and no transmission takes place during a busy TS. On the other hand, in an imperfect sensing environment, the throughput reduces compared to that of the perfect sensing scenario, because a free TS may be falsely sensed as being busy, which decreases the number of TSs available for transmission. Additionally, the decrease in throughput may also be the result of transmitting packets in a busy TS, which always results in erroneous reception of packets due to collision with the packets transmitted by the PRUs.

Given the other system parameters, when the value of N increases from $N = 1$ to 15 in perfect sensing environment, we can observe that the highest achievable throughput is dependent on the CR channel's reliability. Specifically, when the channel is relatively less reliable yielding a large P_F, a smaller N should be used for attaining a higher throughput. By contrast, if the CR channel is reliable, yielding a low value of P_F, the highest throughput may be attained by applying a relatively high N value. From these observations we may conclude that when the other system parameters as well as the channel reliability are given, there is an optimum value for N, which results in the highest throughput for CGBN-HARQ.

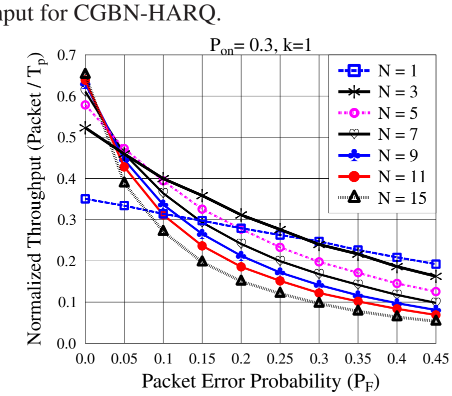


Fig. 8. Throughput performance of the CGBN-HARQ scheme versus packet error probability in perfect sensing environment for various values of N, when $P_{on} = 0.3$ and $T_s = T_p$.

Fig. 9 and 10 quantify the average packet delay of CGBN-HARQ systems, where the average packet delay is at its minimum, when P_F and/or P_{on} is zero. Naturally, the average packet delay is increased with the increase of P_F and/or P_{on}. Furthermore, similar to the above the discussions, the proposed CGBN-HARQ scheme imposes a lower delay in a perfect sensing than imperfect sensing. Additionally, as shown in Fig. 10, the average packet delay depends on the value of N. For given values of P_F and P_{on}, there is a corresponding value for N, which results in the lowest packet delay. However, if the packet error rate is high, a longer delay is imposed upon using a higher value of N.

In Fig. 11, the probability mass function (PMF) of the *end-to-end packet delay* is characterized in the context of $N = 7, 9$ and 11, when sensing results are perfect. Here, the end-to-end packet delay

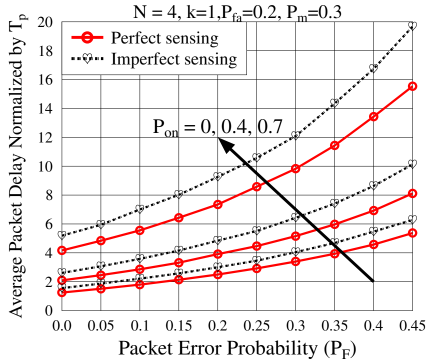


Fig. 9. Average packet delay of the CGBN-HARQ system versus packet error probability for perfect and imperfect sensing, when $T_s = kT_p$ and $N = 4$.

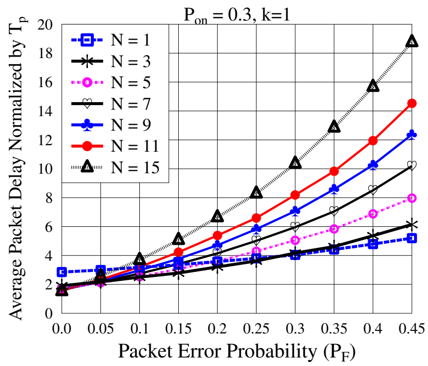


Fig. 10. Average packet delay of the CGBN-HARQ system versus packet error probability in perfect sensing environment, when $P_{on} = 0.3$ and $T_s = T_p$.

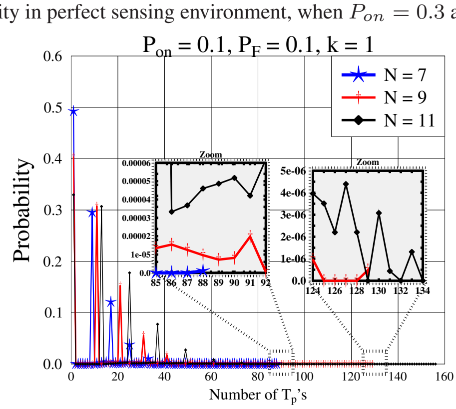


Fig. 11. Probability of end-to-end packet delay in the CGBN-HARQ systems, when $P_{on} = 0.1$, $P_F = 0.1$, $T_s = T_p$ and $N = 7, 9$ and 11.

is defined by the time duration spanning from starting to transmit a packet to the instant, when it is successfully received. Specifically, in our simulations, let $\boldsymbol{d}$ be a N_s-length vector with the jth element $\boldsymbol{d}(j)$ representing the end-to-end delay of the jth packet, where N_s is the total number of packets successfully transmitted. Then, the PMF of the end-to-end packet delay presented in Fig. 11 is evaluated as [5]

$$\boldsymbol{P}_d(i) = \frac{\sum_{j=1}^{N_s} \delta\left(d(j) - i\right)}{N_s}, \quad 1 \leq i \leq \max(\boldsymbol{d}), \qquad (12)$$

where $\max(\boldsymbol{d})$ denotes the maximum delay of the N_s packets.

We can observe from Fig. 11 that for the given parameter values, in the case of $N = 7$, 49.2% of the packets are successfully received during their first transmission, 29.5% packets have a delay of $9T_p$, which corresponds to two transmissions for successfully receiving a packet, and 12% of the packets are successfully received with a delay of $17T_p$, which corresponds to requiring three transmissions for successfully receiving a packet or it may be due to the occurrence of a busy TS in between the erroneous and successful transmission of a packet. Similarly, we can find the probabilities for the other cases of $N = 9$ and 11. Moreover, Fig. 11 also illustrates that the tail of the distribution curves shift towards the righthand side, as the value of N increases, which implies that a longer delay is experienced by a packet.

V. Conclusions

We have proposed a CGBN-HARQ scheme for a CR system. Our results demonstrate that the channel utilization of the PRUs as well as the reliability of both CR communications and of spectrum sensing have a substantial impact both on the throughput and the delay of CGBN-HARQ. When the CR communication becomes less reliable or when the PR has a high probability of occupying the channel, the CR system's throughput may drop significantly, which also implies having a longer transmission delay. Furthermore, our results show that when the communication environment varies, the number of packets transmitted within a time-slot should be accordingly adapted, in order to attain the highest throughput and the shortest average transmission delay.

References

[1] FCC, "Second Report and Order", "Washington, DC, (ET Docket no. 08-260)," Nov 2008.
[2] G. Staple and K. Werbach, "The end of spectrum scarcity," *IEEE Spectrum*, vol. 41, no. 3, pp. 48–52, 2004.
[3] E. Hossain, D. Niyato, and Z. Han, *Dynamic spectrum access and management in cognitive radio networks*. Cambridge University Press, 2009.
[4] Q. Zhao and B. Sadler, "A survey of dynamic spectrum access," *Signal Processing Magazine, IEEE*, vol. 24, no. 3, pp. 79–89, May 2007.
[5] A.-U. Rehman, L.-L. Yang, and L. Hanzo, "Performance of cognitive hybrid automatic repeat request: Stop-and-wait," in *IEEE 81st Vehicular Technology Conference (VTC Spring)*, May 2015, pp. 1–5.
[6] G. Ozcan and M. Gursoy, "Throughput of cognitive radio systems with finite blocklength codes," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 11, pp. 2541–2554, November 2013.
[7] S. Stotas and A. Nallanathan, "Enhancing the capacity of spectrum sharing cognitive radio networks," *IEEE Transactions on Vehicular Technology,*, vol. 60, no. 8, pp. 3768–3779, 2011.
[8] Y.-C. Liang, Y. Zeng, E. Peh, and A. T. Hoang, "Sensing-throughput tradeoff for cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol. 7, no. 4, pp. 1326–1337, April 2008.
[9] S. Lin and D. J. C. Jr., *Error Control Coding: Fundamentals and Applications*, 2nd ed. Upper Saddle River, NJ: Prentice-Hall, 1999.
[10] D. Bertsekas and R. Gallagher, *Data Networks*, 2nd ed. Englewood Cliffs, NJ: Prentice Hall, 1991.
[11] A. Leon-Garcia and I. Widjaja, *Communication Networks*. McGraw-Hill Education, 2004.
[12] W. Turin, "Throughput analysis of the go-back-n protocol in fading radio channels," *IEEE Journal on Selected Areas in Communications,*, vol. 17, no. 5, pp. 881–887, 1999.
[13] K. Ausavapattanakun and A. Nosratinia, "Analysis of go-back-n arq in block fading channels," *IEEE Transactions on Wireless Communications*, vol. 6, no. 8, p. 2793, 2007.
[14] Y. Hayashida and M. Komatsu, "Delay performance of go-back-n arq scheme with markovian error channel," in *2nd International Conference on Personal Communications: Gateway to the 21st Century*, vol. 1, Oct 1993, pp. 448–452 vol.1.
[15] M. Zorzi and R. Rao, "Performance of arq go-back-n protocol in markov channels with unreliable feedback: delay analysis," in *Fourth IEEE International Conference on Universal Personal Communications*, Nov 1995, pp. 481–485.
[16] C. Dong, L.-L. Yang, and L. Hanzo, "Performance analysis of multihop diversity aided multi-hop links," *IEEE Transactions on Vehicular Technology*, vol. 61, no. 6, pp. 2504 –2516, 2012.
[17] J. Zuo, C. Dong, H. V. Nguyen, S. X. Ng, L.-l. Yang, and L. Hanzo, "Cross-layer aided energy-efficient opportunistic routing in ad hoc networks," *IEEE Transactions on Communications*, vol. 62, no. 2, pp. 522–535, February 2014.
[18] W. Liang, S. X. Ng, J. Feng, and L. Hanzo, "Pragmatic distributed algorithm for spectral access in cooperative cognitive radio networks," *IEEE Transactions on Communications*, vol. 62, no. 4, pp. 1188–1200, April 2014.
[19] J. Hu, L. liang Yang, and L. Hanzo, "Maximum average service rate and optimal queue scheduling of delay-constrained hybrid cognitive radio in nakagami fading channels," *IEEE Transactions on Vehicular Technology*, vol. 62, no. 5, pp. 2220–2229, Jun 2013.