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Frequency Resolution Quantification of Brillouin Distributed Optical Fibre Sensors

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***Abstract*— Noise analysis using Monte Carlo method is conducted in this paper to correct the relationship between the frequency resolution, the Q-factor, Signal-to-Noise Ratio (SNR), and frequency step in the Brillouin distributed optical fibre sensors. The quantification of the Brillouin gain spectrum is important in distributed Brillouin sensors in order to improve the Brillouin frequency resolution and the corresponding strain and temperature resolutions. Two analytical expressions are derived in order to estimate the error in the determination of the Brillouin central frequency with or without second order polynomial fitting.**

***Index Terms*— Brillouin scattering, Monte Carlo, SNR, frequency resolution.**

I. INTRODUCTION

DISTRIBUTED optical fibre sensors, based on Brillouin scattering, have attracted significant interest in the past few decades, as they can provide a convenient method for health and safety monitoring for large civil structures [1]–[3]. Brillouin scattering has been utilised to measure the distributed temperature and strain along the fibre by applying proportionality between the strain and temperature in the optical fibre and the Brillouin Frequency Shift (BFS) [4]. Based on this principle, Brillouin distributed optical fibre sensors, are widely used in civil engineering applications [1].

The BFS is obtained by measuring the peak-power frequency of the Lorentz-shaped Brillouin gain spectrum along the optical fibre. The quantification of the Brillouin gain spectrum is important in Brillouin distributed optical fibre sensors in order to improve the Brillouin frequency resolution and the corresponding strain and temperature resolutions.

The noise deteriorates system accuracy [5] and generates random spectral deviation of the measured BFS along the optical fibre. The analysis of the random process of this deviation is proposed and simulated based on the Monte Carlo method in this paper, which corresponds to the minimum detectable Brillouin frequency change and its dependence on the signal to noise ratio (SNR), quality factor (the Q-factor), and frequency step resolution.

The minimum detectable change in the Brillouin distributed optical fibre sensor was first declared in [4], centred at a Brillouin frequency of v_B shown as:

$$\sigma = \frac{FWHM}{\sqrt{2}(SNR_e)^{1/4}} \tag{1}$$

where $FWHM$ is the Full Width at Half Maximum (FWHM) and SNR_e is the electrical signal to noise power ratio. However, a detailed justification and estimation procedure were lacking, including the fitting method and frequency step resolution. Recently, the BFS was analysed by using the propagation of errors on the parameters obtained from a least-square parabolic fitting, as stated and demonstrated in [6], estimated as:

$$\sigma = \frac{1}{SNR_a}\sqrt{\frac{3}{4}\delta \cdot FWHM} \tag{2}$$

where δ is the frequency step resolution and SNR_a is defined as amplitude SNR (maximum gain to the noise). These two pioneering articles described the estimation of the potential error on the BFS for a given SNR, FWHM, and frequency step. The differences between these two equations are the exponential order of each variable and the numerical coefficient. However, there is an insufficient explanation about the difference between the two equations.

In this work, the minimum Brillouin detectable change is studied numerically, by using the Monte Carlo simulation to determine the statistical variation of the Brillouin centre frequency with and without polynomial fitting, in which the random noise is added to the spectrum amplitude of the simulated signal. This work offers an opportunity to examine the difference between [4] and [6]. A densely sampled Brillouin gain spectrum can contribute to the minimum detectable change with and without polynomial fitting. The results show that the minimum detectable change is inversely proportional to the Q-factor and SNR without fitting and can be further reduced by fitting.

II. MONTE CARLO METHOD AND FREQUENCY RESOLUTION

Brillouin signal noise is caused by the cumulative noise added to the spectrum, including thermal and shot noise in the photodetector, laser relative intensity noise, and quantisation error [4][7]. The Monte Carlo method is a collection of computational algorithms which depend on repeated random

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sampling in order to obtain numerical results [8]. These results can be utilised to determine the statistical variation of the peak-power frequency of a Lorentz-shaped Brillouin gain spectrum when a random noise is added to the signal. The SNR used in this study is defined as:

$$SNR = \frac{P_{signal}}{P_{noise}} \tag{3}$$

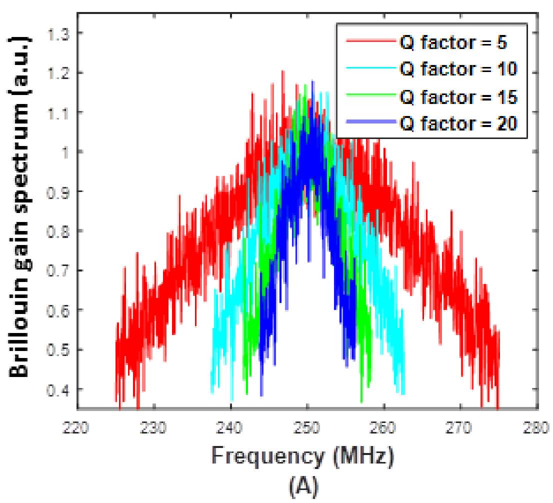
where the P_{signal} is the averaged signal power measured by its summed squared magnitude within FWHM, P_{noise} is noise power (regarded as the variance of the noise distribution within FWHM). The Q-factor is defined as the ratio of the Brillouin central frequency v_B to the FWHM shown as:

$$Q = \frac{v_B}{FWHM} \tag{4}$$

In order to evaluate the approximated governing equation for the standard deviation of the Brillouin central frequency, the Q-factor, SNR, and frequency step, the variables need to be investigated by assuming the Brillouin central frequency v_B is constant. Based on observing the similarity between Eqn. (1) and (2), it is assumed that the error in the estimated Brillouin central frequency with respect to the frequency step resolution δ, SNR, FWHM, and numerical coefficient Z can be expressed as:

$$\sigma = Z \cdot SNR^A FWHM^B \delta^C \tag{5}$$

In the Monte Carlo simulation, the spectrum with random noise is generated repeatedly, while the peak-power frequency is obtained for each simulation, resulting in the Brillouin profile in the iteration process. The peak signal amplitude is assumed to be 1 and the noise is controlled by the value of SNR. The spectral location of the peak power from the Brillouin gain spectrum is marked for each iteration and then statistically analysed, in order to generate the standard deviation.



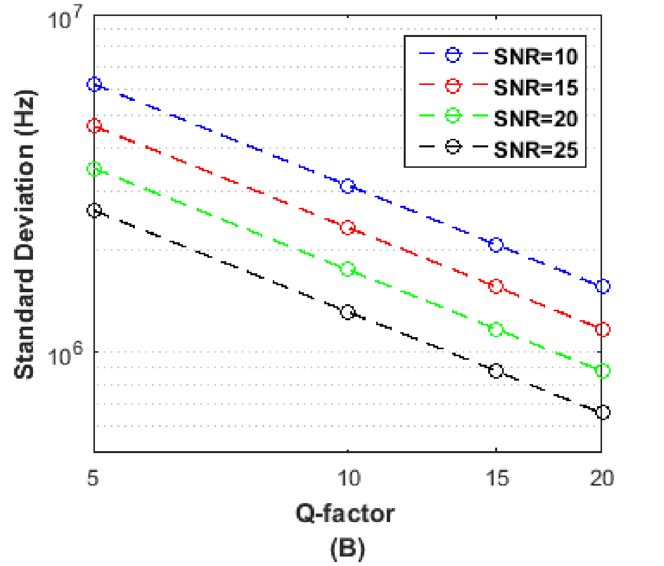


Fig. 1. (A). Lorentz-shaped Brillouin spectrum with 20 dB SNR and 0.05 MHz frequency step and various Q-factors ranging from 5 to 20. (B). Results of Monte Carlo simulations for the standard deviation of Brillouin central frequency detection over different SNRs from 10 to 25 and Q-factors from 5 to 20 with a fixed frequency step of 0.05 MHz.

The Lorentz-shaped Brillouin gain spectrum shown in Fig.1 (A) has a white Gaussian noise added to the signal, with varied Q-factors from 5 to 20 and a fixed SNR of 20dB and a fixed frequency step of 0.05 MHz. Fig.1 (B) shows the standard deviation of the Brillouin peak-power frequency with varied SNRs from 10 to 25dB and varied Q-factors from 5 to 20 and a frequency step of 0.05 MHz. The noise simulation results reveal that a high value of the Q-factor and/or a high value of SNR can reduce the frequency resolution of the Brillouin distributed optical fibre sensors.

In terms of estimating the influence of the variable SNR, the other variables, including the Q-factor and frequency step, are set as constant at 10 and 0.05 MHz respectively. The value of SNR varies: 5, 10, 15, 20, 25, and 30. Then, the standard deviations of 10,000 Monte Carlo simulations versus the corresponding SNR are shown in Fig.2 (A). Similarly, by keeping the SNR fixed at 10 dB and the frequency step fixed at 0.05 MHz and by varying the Q-factor from 3 to 20, the standard deviation versus the Q-factor is shown in Fig.2 (B). By maintaining the SNR and the Q-factor constant at 10 dB and 10, the frequency step changing from 0.001 to 0.3 MHz results in the plot shown in Fig.2 (C). The combination of the Q-factor and the Brillouin central frequency can be regarded as the FWHM as a single variable. The standard deviations for 10,000 Monte Carlo simulations and the corresponding fitted line with the parametric analysis of SNR, the Q-factor, and frequency step can be plotted as:

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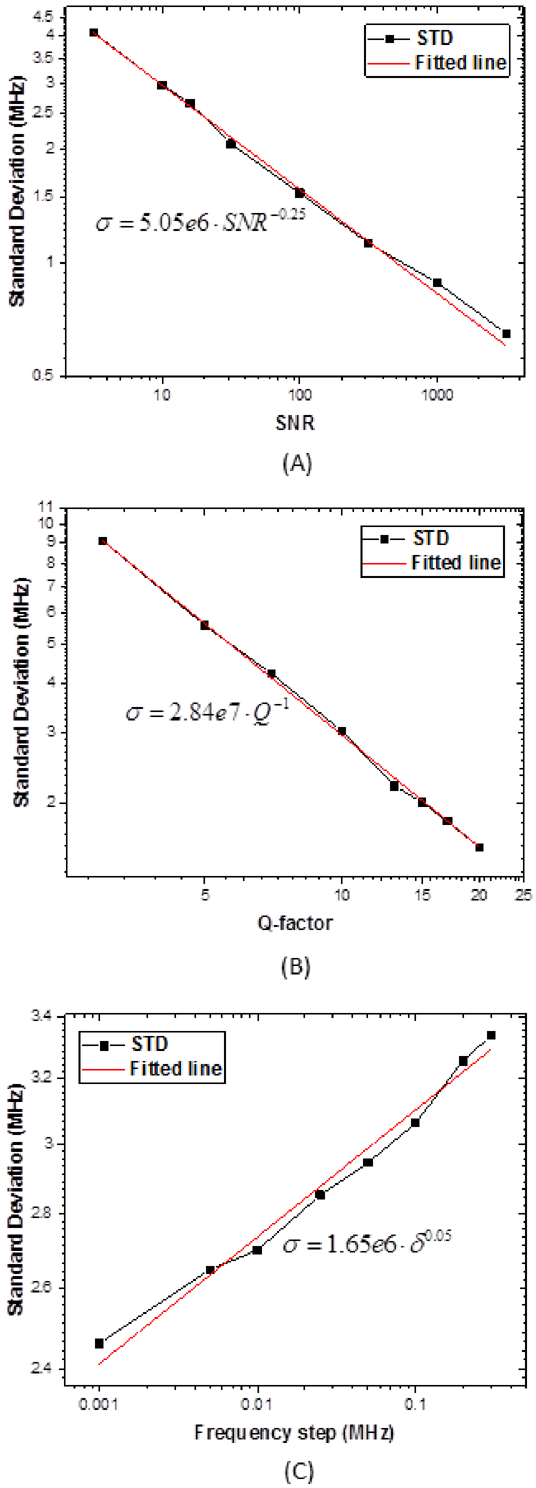


Fig. 2. The parametric study of the standard deviation of the Brillouin central frequency without fitting, (a) the standard deviation versus various SNR with a fixed Q-factor (10) and frequency step (0.05 MHz), (b) the standard deviation versus various Q-factors with a fixed SNR (10dB) and Brillouin step (0.05 MHz), (c) the standard deviation versus various frequency step with a fixed SNR (10dB) and the Q-factor (10).

The fitting coefficients A, B, and C for the variable SNR (in linear expression), the Q-factor, and frequency step resolution are -0.25, -1, and 0.05 respectively and the numerical coefficient is 1/8.5. The parametric investigation process are summarised in Table 1.

TABLE 1
PARAMETRIC INVESTIGATION WITHOUT FITTING

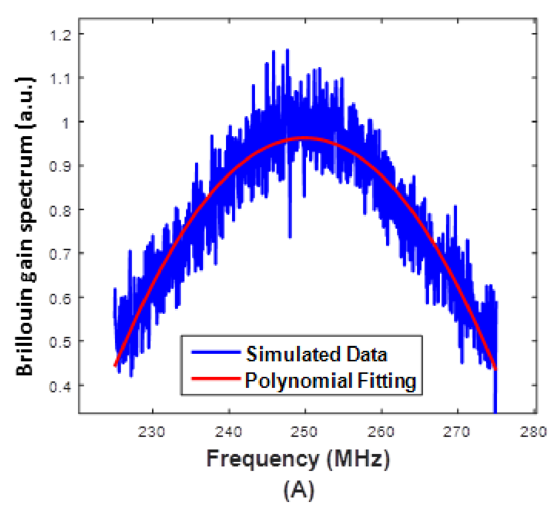
SNR (dB)	Q-Factor	δ (MHz)	Temporal Numerical Coefficient	Exponential Coefficient
5-30	10	0.05	5.05E+06	A = -0.25
10	03-20	0.05	2.84E+07	B = -1
10	10	0.001-0.3	1.65E+06	C = 0.05

Therefore, the Monte Carlo simulation approximates the relationship between the SNR, Q-factor, frequency step, and the corresponding spectrum variation of the peak-power frequency in the Brillouin distributed optical fibre sensors, which reveals a dependence of the standard deviation (σ) on the BFS, Q-factor, frequency step, and SNR. The approximated governing equation is:

$$\sigma \approx \frac{v_B \times \delta^{0.05}}{8.5 \times Q \times SNR^{0.25}} \tag{6}$$

where the *SNR* is shown in linear units in the expression. This expression confirms the equation format of [4], about the same exponential coefficient of SNR. The exponential coefficient 0.25 of SNR in [4] and Eqn. (6) is due to the direct error estimation of the BFS for a given SNR without applying any fitting process. The difference between the numerical coefficients (1/8.5 and $1/\sqrt{2}$) is due to the influence of the frequency step involved in this study in refining the error estimation. This governing equation can be used to calculate the minimum detectable change of BFS without any fitting processing.

The polynomial fitting can be applied to the noisy spectrum in order to enhance the peak-power frequency detection. In the fitting processing, the second order polynomial fitting is applied to the simulated data above the FWHM. Fig. 3(A) shows the polynomial fitting result of the corresponding Brillouin spectrum with a SNR of 25dB, a Q-factor of 5, and frequency step of 0.05 MHz. Fig.3 (B) plots the standard deviation of the Brillouin central frequency measured with second order polynomial fitting.



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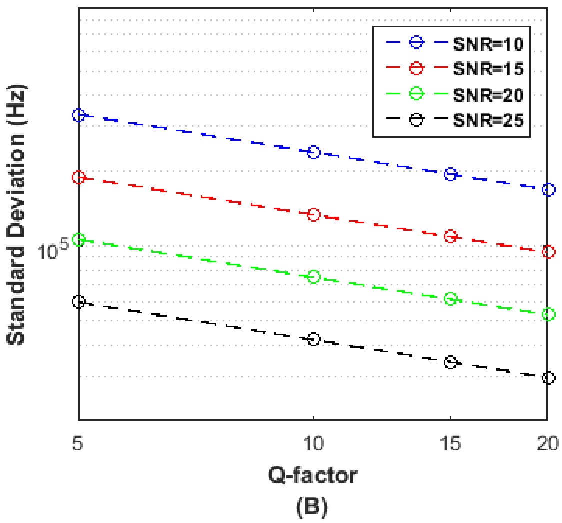

Fig. 3. (A). Lorentz-shaped Brillouin spectrum with a SNR of 25dB, a Q-factor of 5, and a 0.05 MHz frequency step and the corresponding second order polynomial fitting. (B). Results of the Monte Carlo simulations for the standard deviation of Brillouin central frequency detection with second order polynomial fitting over different SNRs from 10 to 25 and Q-factors from 5 to 20, with a fixed frequency step of 0.05 MHz

It is helpful to follow the parametric investigation in the same way as in the previous section, by keeping all of the other variables constant and only varying a variable, as depicted in Table 2.

TABLE 2
PARAMETRIC INVESTIGATION WITH FITTING

SNR (dB)	Q-Factor	δ (MHz)	Temporal Numerical Coefficient	Exponential Coefficient
5-30	10	0.05	7.50E+05	A = -0.5
10	03-20	0.05	7.49E+05	B = -0.5
10	10	0.001-0.3	1060	C = 0.5

By applying the fitting model (as in the previous section) the parameters A, B, and C for SNR, the Q-factor, and the frequency step are estimated as -0.5, -0.5, and 0.5 severally and the numerical coefficient is about 0.67. Therefore, by applying second order polynomial fitting, the approximated governing equation of the standard deviation of the Brillouin central frequency can be rewritten as:

$$\sigma_f \approx 0.67 \times \sqrt{\frac{\delta \times v_B}{Q \times SNR}} \qquad (7)$$

where σ_f is the standard deviation of the resultant spectrum with second order polynomial fitting. The exponential coefficients of the frequency step and the FWHM have the same format as [6]; the differences compared with [6] are in the exponential coefficient of SNRs and the overall numerical coefficients, which are due to different SNR definitions and fittings. The SNR defined in [6] is the ratio of maximum gain to the standard deviation of the Gaussian noise, which is a single amplitude SNR. The SNR defined in this study is the averaged power to the variance of the noise within the FWHM, which can be regarded as the power SNR within the FWHM compared with [6]. The definitions of the SNRs applied in this work and [6] explain the different exponential coefficients, -0.5 and -1. The amplitude SNR is the square root of the power SNR. Moreover, the variance between the numerical coefficient 0.67 and $\sqrt{3/4}$ is due to the influence of differently defined SNRs and fittings. As [6] mentioned, the numerical factor $\sqrt{3/4}$ can be reasonably made smaller for a different fitting method. This result indicates that the standard deviation of the peak-power frequency measurement error can be reduced by the reasonable fitting procedure, which is also confirmed by[9][10].

III. Conclusion

The Monte Carlo statistical simulation can characterise the frequency resolution of Brillouin distributed optical fibre sensors with varied Q-factors, SNRs, and frequency steps with and without second order polynomial fitting. This work examined the previously reported results in [4] and [6] and offered different coefficients in the resolution governing equations, which is caused by direct Brillouin central frequency detection and by the fitting approach. The second order polynomial fitting method can offer more accurate Brillouin central frequency measurement compared with direct detection without any fitting.

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