PARAMETRICALLY EXCITED MEMS VIBRATION ENERGY HARVESTERS

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Abstract: Resonant-based vibration harvesters have conventionally relied upon accessing the fundamental mode of resonance to maximise the conversion efficiency of mechanical-to-electrical power transduction. This paper explores the use of parametric resonance, which is not limited by linear damping and can potentially offer higher and broader nonlinear peaks. Despite the promising potential, a damping-dependent initiation threshold amplitude has to be overcome first. Design approaches have been explored to resolve this limitation. A numerical model has been constructed to analysis the improvements over the convention. An out-of-plane (to accommodate large displacements) electrostatic MEMS prototype (~ 0.147 mm^3), driven at 4.2 ms^{-2}, has demonstrated a peak power of 0.011 µW at the fundamental mode of resonance and 0.16 µW at the principal parametric resonance. A two fold increase in frequency bandwidth was also observed for the parametrically excited device.

Keywords: MEMS, parametric resonance, vibration, energy harvesting, initiation threshold

INTRODUCTION

In recent years, there has been increased interest in vibration energy harvesting, especially to enable self-powered wireless sensor networks for structural health monitoring [1]. While some early commercial solutions have witnessed increasing deployment [2], two of the key technical limitations still persist; namely, the low power density relative to conventional power supply and the confined operational frequency bandwidth while faced with the wideband nature of real vibrations.

This paper attempts to address these issues through employing parametric resonance instead of the convention of accessing the fundamental mode of resonance. Parametric resonance is a type of self-excited nonlinear resonant phenomenon, which unlike the conventional approach, does not exhibit amplitude saturation due to linear damping. Therefore, theory [3] predicts the potential for an order higher in power output for such devices. Additionally, the onset of nonlinearity that eventually saturates its growth helps moderately widen the operational frequency bandwidth. Despite these promising advantages, an early experimental investigation [4] failed to report any notable performance enhancements over the current paradigm. One of the key limiting factors is the presence of a damping-dependent initiation threshold amplitude, which the excitation amplitude must attain in order to activate parametric resonance.

Following sections numerically and experimentally analyse the potential enhancements of parametric resonance over the fundamental mode of resonance, as well as exploring design approaches to overcome the limitation of initiation threshold amplitude in order to practically realise the promising potential of parametrically excited vibration energy harvesters.

THEORY AND SIMULATION
Parametric resonance

The motivation for employing parametric resonance for vibrational energy harvesting can be summarised as,

- Maximises power output by mechanically amplifying displacement amplitude.
- Broadens operational frequency bandwidth with the nonlinear characteristics of its resonant peak.

This resonant phenomenon is characterised by the presence of a time-dependent modulation in one of its system parameters as described by the Mathieu equation below [5].

$$\ddot{x} + c\dot{x} + (\delta + 2\varepsilon\cos(2t))x = 0 \qquad (1)$$

Where x is displacement, c is damping, t is time domain, ε is a generic amplitude parameter and δ is the generic frequency parameter.

Parametric excitation, unlike direct excitation, is usually perpendicular and not parallel to the direction of displacement. In order to achieve parametric resonance, it can be shown that the excitation frequency ω needs be approximately $2\omega_0/n$; where ω_0 is the natural frequency of the resonator and n is an integer defining the order of parametric resonance. Therefore, the first order (principal) parametric resonance can be attained when the excitation frequency is twice the natural frequency. Additionally, there is a prerequisite of a non-zero initial displacement in order to `push' the system out of stable equilibrium.

Apart from these requirements, the excitation amplitude needs to overcome a damping-dependent initiation threshold prior to accessing the parametric

resonant region as illustrated in the bifurcation diagram in Fig. 1. This appears counterproductive in the context of vibration energy harvesting, since the essence of the art relies on electrically damping the mechanical resonator in order to extract electrical energy. However, once parametric resonance is activated, the oscillatory amplitude growth is not limited by linear damping and can only be saturated by either physical limits or the onset of nonlinearity.

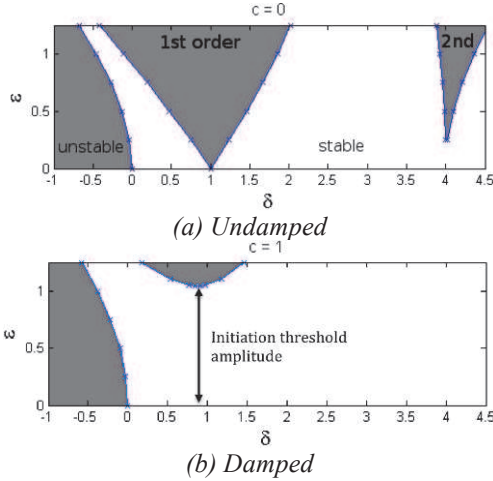*(a) Undamped*

(b) Damped

Fig. 1: Bifurcation diagrams for systems described by undamped and damped Mathieu equations. Grey (unstable) regions are where parametric resonance is achieved. With increasing damping 'c', higher excitation amplitudes are required to overcome the initiation threshold amplitude prior to accessing the unstable regions.

Cantilever resonators

The cantilever resonator is a common design choice in the art [2]. It is able to achieve large displacements near the free end (ideal for electromagnetic and electrostatic transducers) and experience high levels of stress near the clamped end (ideal for piezoelectric transducers). Conventionally, the cantilevers are driven with a direct excitation parallel to the displacement as summarised below.

$$\ddot{x} + 2c_1\dot{x} + c_2\dot{x}|\dot{x}| + \mu x^3 + \omega_0^2 x = \frac{A}{ml}\cos(\omega t) \quad (2)$$

$$\omega_0^2 = \frac{k}{m} = \frac{3EI}{ml^3} \quad (3)$$

Where, c_1 is the linear damping parameter, c_2 is a quadratic damping representing the dominant higher order damping nonlinearity, μ is a parameter representing cubic geometric non-linearities, A is the excitation displacement amplitude, k is the spring stiffness, m is the effective mass, l is the effective

beam length, E is the modulus of elasticity and I is the area moment of inertia.

When a parametric excitation is applied perpendicular to the direction of the displacement, the following equation of motion (a version of the damped Mathieu equation (Eq. 1)) can be observed.

$$\ddot{x} + 2c_1\dot{x} + c_2\dot{x}|\dot{x}| + \mu x^3 + (\omega_0^2 - \frac{A}{ml}\cos(\omega t))x = 0 \ (4)$$

Fig. 2 numerically (using ode45 solver in MATLAB with `Relative tolerance' of 1E-05) contrasts the resonant peaks of a micro-cantilever when subject to either type of excitation at varying acceleration levels. While direct excitation always yields a response regardless of the excitation level, its parametric counterpart has a zero steady-state response below the initiation threshold amplitude and a small non-zero initial displacement condition is required. However, with increasing excitation amplitudes beyond this threshold, parametric resonance rapidly outperforms the fundamental mode of resonance.
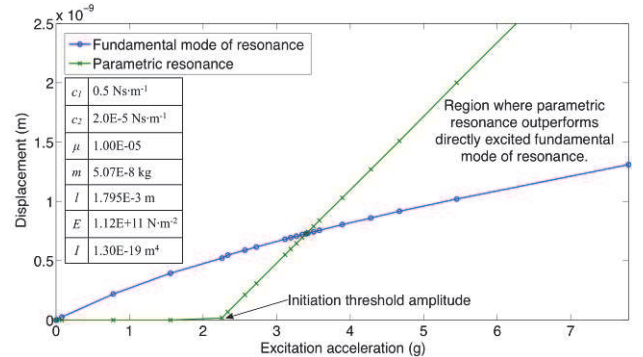

Fig. 2: Numerical comparison of the steady-state resonant peaks between direct and parametric excitations at various excitation acceleration levels.

Both this numerical simulation and an existing experimental study in the literature [4] have ascribed large values for this initiation threshold amplitude; whereas in practice, the ambient vibration available for harvesting is usually very small. Therefore, the gap of this threshold and the base axis shown in Fig. 1b needs to be minimised in order to practically exploit the merits of parametric resonance.

DESIGN AND FABRICATION

Other applications, such as MEMS gyroscopes [6] have already employed parametric resonance as a means of mechanical amplification, typically incorporating active excitation to overcome the limitation of initiation threshold amplitude. However, energy harvesting cannot afford the additional power

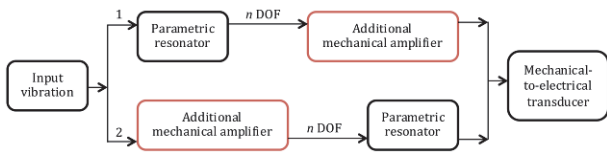expenditure. Therefore, a passive solution to fulfil this critical criterion is needed.

Fig. 3: Design approaches to passively minimise initiation threshold amplitude. 'Additional mechanical amplifiers' encompasses direct and parametric resonators as well as non-resonating amplifiers such as levers; and 'n DOF' represents n degrees-of-freedom.

Fig. 3 presents the proposed design approaches in an attempt to address this issue. The first design route employs an electrically undamped parametric resonator that is coupled to an additional electrically damped mechanical amplifier. Here, the parametric resonator has intrinsically lower initiation threshold amplitude. The second design route introduces an electrically undamped mechanical amplifier prior to the electrically damped parametric resonator. In this scenario, the base vibration amplitude is amplified and brought up towards the threshold. Referring back to Fig. 1b, the first approach essentially lowers the position of unstable region while the second approach raises the horizontal base axis.
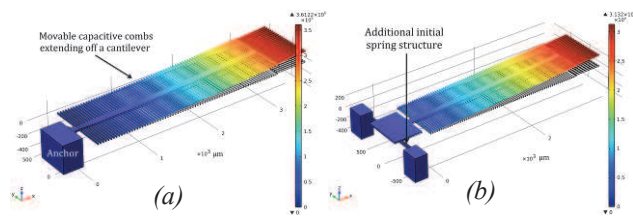

Fig. 4: COMSOL designs of the MEMS cantilevers. Direct excitation equates to out-of-plane driving force, whereas an in-plane excitation can potentially induce parametric resonance in (a) and auto-parametric resonance in (b). Capacitive comb fingers extend off the cantilever beam. Accompanying fixed comb fingers are not shown here.

A design model of a micro-cantilever with capacitive combs is shown in Fig. 4a. Out-of-plane vibration is equivalent to direct excitation while an in-plane excitation force applied along the long axis of the cantilever can potentially induce parametric resonance at the right frequency and amplitude conditions. Fig. 4b is an iteration of the second design route from Fig. 3 by adding a double beam between the anchor and the otherwise anchored end of the cantilever. This additional initial spring structure acts

as the electrically undamped additional mechanical amplifier (direct resonator) and enables the possibility of observing auto-parametric resonance.
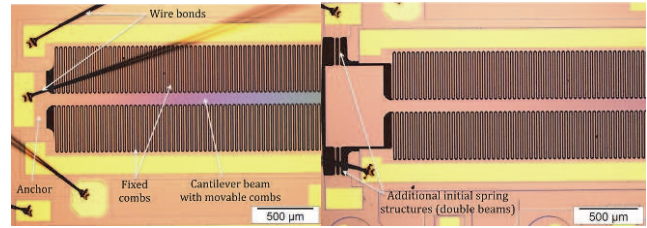
(a) Parametric harvester.   (b) Auto-parametric harvester.
Fig. 5: SOIMUMPS realization of the parametrically and auto-parametrically excitable MEMS designs as seen in Fig. 4a and 4b respectively.

The designs from Fig. 4 were fabricated using the MEMSCAP foundry's Silicon-On-Insulator (SOI) Mutli-User MEMS Processes (MUMPs) and sample devices can be seen in Fig. 5. The silicon thickness is 25 μm and the total device volume is ~ 0.147 mm^3. For each movable comb finger extending from the cantilever beam, a corresponding fixed capacitive comb finger is placed beside it with 10 μm gap spacing; thus, forming an electrostatic transducer.

RESULTS

Experimental tests were carried out using the set up shown in Fig. 6 with measurement circuit based on [7]. All tests were undertaken at normal atmospheric pressure and cantilevers were mounted with free ends upright to overcome the non-zero initial displacement criterion. This upright arrangement is equivalent

Fig. 6: Test set up.

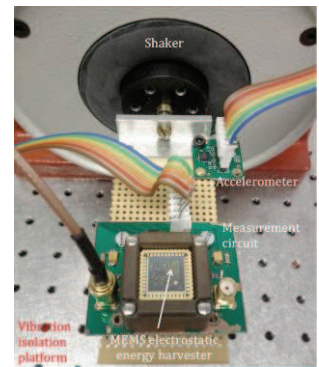to an inverted pendulum and the cantilever tip rests in an unstable equilibrium.

COMSOL simulations (Fig. 4) and frequency sweeps using a Polytec laser vibrometer of the MEMS devices revealed no resonant modes at either twice or half the natural frequencies. Therefore, the possible presence of other resonant peaks within the vicinity of these frequency ranges can be excluded.

Through mechanical excitation by the shaker, 1st and 3rd order parametric resonances were recorded for the auto-parametric harvester in Fig. 5b. Although onsets of 2nd order parametric resonance was also observed around f_n, steady-state response always converged towards the fundamental mode of

resonance. As predicted, this design had lower initiation threshold amplitude than the Fig. 5a device:

- Parametric harvester: ~ 30 ms^{-2}.
- Auto-parametric harvester (with additional spring)
  - 1st order: ~ 1 ms^{-2}.
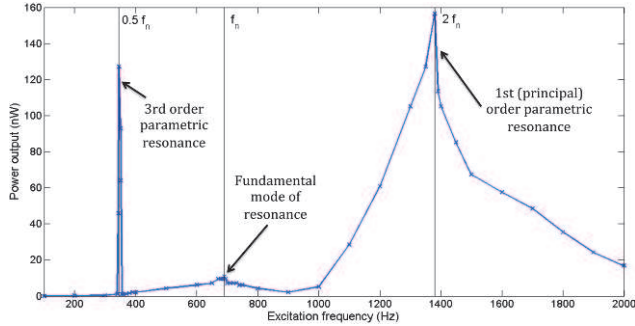 - 3rd order: ~ 3 ms^{-2}.

Fig. 7: Experimental power response of auto-parametric harvester at input acceleration of 4.2 ms^{-2}. 1st and 3rd order parametric resonances can be seen at twice and half of the natural frequency f_n respectively.

Table 1: Comparing the results with selected counterparts from the literature in terms of power density normalised against acceleration squared.

Reference	$\mu Wcm^{-3}m^{-2}s^4$
Parametric (1st order)	*61.7*
Parametric (3rd order)	*50.1*
Roundy *et al.* (2002) [8]	22.9
Wong *et al.* (2009) [7]	19.0
Fundamental mode	*4.24*

In fact, an order of magnitude higher power response can be observed for parametric resonance well below 1g of input acceleration as is demonstrated in Fig. 7. At 4.2 ms^{-2}, power peaks of 0.011 μW, 0.156 μW and 0.127 μW were recorded for the fundamental mode, first order parametric and third order parametric resonances respectively. Table 1 briefly contrasts these results with a few selected electrostatic harvesters in the literature. Additionally, the operational frequency bandwidth (measured from half power points) of the principal parametric resonance is approximately twice that for the fundamental mode of resonance.

FUTURE WORK

Further design iterations based on Fig. 3 are being explored to better address the issues of initiation threshold amplitude and the non-zero initial displacement requirement. MEMS, thin/thick-film and macro-scale devices are also being developed to investigate the power efficiency of this novel technique in contrast to directly excited harvesters when subjected to ambient vibrations in real scenarios.

CONCLUSION

This paper presents a novel study of incorporating parametric resonance as a means of mechanical amplification to maximise the electrical power output of MEMS vibration energy harvesters. While this resonant technique theoretically promises better power and frequency response in contrast to the conventional directly excited harvesters, a list of strict initiation criteria, such as a minimum excitation amplitude threshold, needs to be fulfilled. A passive design approach to address this limitation has experimentally shown a reduction of the excitation amplitude threshold by nearly 30 times. Over an order of magnitude higher power response and approximately twice the operational frequency bandwidth have been recorded for the parametrically excited scenario in contrast to the same harvester driven at fundamental mode of resonance. The results also compare favourably with the literature.

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