INTENSITY THRESHOLDS OF OPTICAL PARAMETRIC OSCILLATORS

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G.D. Boyd and D.A. Kleinman have published a comprehensive analysis of the 'Parametric Interaction of Focused Gaussian Light Beams' (Ref.1., hereinafter referred to as BK). In this paper they derived the 'optimum' focusing and 'optimum' crystal length which minimized the pump power, P required to achieve optical parametric oscillation (OPO) in a given system. This optimization has proved very valuable in obtaining OPO in the visible and near infra-red. (The first CW oscillator [2] based on $Ba_2NaNb_5O_{15}$ provides an excellent example of the application of BK power optimization).

Unfortunately, the price to be paid for decreasing the power threshold is an increase in intensity (power density) at the crystal. In many cases of practical interest for infra-red parametric oscillators (examples: tellurium pumped by 10.6μm or 5.3μm, proustite pumped by 1.06μm) the threshold powers are very much lower than the available laser power, but the intensities approach, or even exceed the damage limit of the non-linear material [3]. We derive here, from BK's theory, the pump intensity thresholds, I, for OPO and from our results it is clear that BK's 'optimum' focusing and crystal length should not be used with damage-prone materials. We give compromise values of length and focusing which result in much reduced intensity threshold with only a modest increase in power threshold. Our results are only applicable to critically-matched oscillators ($B > 1.68$), but this covers currently available infra-red non-linear materials.

In what follows the symbols* and equation numbering are consistent with BK. Re-stated here are the major parameters:

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* Our only departure is to drop the subscript 3 for quantities relevant to the pump; e.g. $P \equiv P_3$.

the double-refraction parameter,

$$B \simeq \rho\ (\ell\ k_o)^{\frac{1}{2}}/2 \qquad [BK\ 3.35],$$

where ℓ is the physical length of the non-linear crystal, ρ the double-refraction walk-off angle, and k_o the signal/idler degenerate wave-constant;

the focusing parameter,

$$\xi = \ell/b \qquad [BK\ 1.3],$$

where b is the pump (same for signal and idler) confocal parameter inside the crystal;

the reciprocal pump threshold function,

$$\bar{h}_m(B,\xi) \qquad [BK\ 3.37].$$

This function includes the effects of double-refraction and diffraction. The subscript m represents optimization (maximization) with respect to phase mis-match. A subscript mm indicates a second optimization with respect to ξ, yielding:

$$\bar{h}_{mm}(B) \qquad [BK\ 3.38],$$

and consequently the minimum power threshold, P_{opt}.

Using the approximate forms of $\bar{h}_m(B,\xi)$ for weak focusing ($\xi < 0.4$, [BK 3.51]), and $\bar{h}_{mm}(B)$ for strong double-refraction ($B > 1.68$, [BK 3.48]) it is readily shown that:

$$\frac{P}{P_{opt}} = \frac{\bar{h}_{mm}(B)}{\bar{h}_m(B,\xi)} \simeq \frac{1}{[\mathrm{erf}\ (B\sqrt{\xi})\ ]^2} \quad . \qquad (1)$$

Pump intensity is defined as that at the centre of Gaussian beam:

$$I = P/\tfrac{1}{2}\pi W^2 ,$$

where W is the e^{-1} radius of the electric field at the focal plane. By use of the asymptotic value of $\bar{h}_m(B,\xi)$ for very weak focusing ('near-field'),

$$\bar{h}_m(B,\xi) \to \xi \qquad [\mathrm{BK}\ 3.52],$$

and $b = kW^2$ gives

$$\frac{I}{I_{opt}} = \frac{4B^2}{\pi} \cdot \frac{\xi}{[\mathrm{erf}\ (B\sqrt{\xi})]^2} \qquad (\xi < 0.4) \quad . \qquad (2)$$

I_{opt} is now the minimum <u>intensity</u> threshold.

Re-normalization of the focusing parameter allows the removal of B as an explicit parameter and presentation of (1) and (2) as a general pair of curves:

$$A = \xi/\xi_c, \qquad \xi_c = \pi/4B^2,$$

$$\frac{P}{P_{opt}} = \frac{1}{[\mathrm{erf}(\sqrt{\pi A}/2)]^2} , \qquad (3)$$

$$\frac{I}{I_{opt}} = \frac{A}{[\mathrm{erf}(\sqrt{\pi A}/2)]^2} . \qquad (4)$$

Expressions (3) and (4) are plotted in Fig.1. against the normalized parameter A. The curves cross at A = 1, that is where $\xi = \xi_c$, the two ratios then having the value 1.603. As expected $I/I_{opt} \to 1$ as $\xi \to 0$.

P/P_{opt} is also asymptotic to unity, for large ξ. The latter, of course, just represents the approach to BK's optimum focusing condition. The failure of this curve to rise again for very large ξ (see BK Fig.12) is simply due to the nature of the approximation used for $\bar{h}_m(B,\xi)$.

ξ_c thus represents a compromise value of focusing at which both the power and intensity thresholds are only 60% above their minimum values. Using $\xi < \xi_c$ decreases the intensity threshold at the expense of the power threshold; for $\xi > \xi_c$ the opposite applies. The confocal parameter required to obtain BK 'optimum' focusing is generally very short and difficult to achieve in practise; ξ_c always requires a much larger value of b.

A further optimizable parameter is the length of the non-linear crystal. Here again the results from considering intensity threshold differ from those for power threshold. The one-way losses of the oscillator are defined [BK 3.74] as:

$$\varepsilon = \varepsilon_o + \alpha_o \ell \qquad\qquad (\alpha_o \ell << 1)$$

and also $x = \ell/\ell_o$, $\ell_o = \varepsilon_o/\alpha_o$. By working at $\xi = \xi_c$ the length dependence of the intensity and power thresholds are readily obtained from (1) and (2) above and [BK 3.34]. The results are:

$$\begin{aligned} P_c &\propto (1 + x)^2 \\ I_c &\propto (1 + x^{-1})^2 \end{aligned} \qquad (5)$$

These functions are shown in Fig.2.,where it is seen that neither curve possesses a minimum. This is in contrast to BK 'optimization', where $P(x)$ has a minimum, always at $x < 1$ (BK Fig.13). The intensity threshold approaches its smallest value for $\ell >> \ell_o$, the loss from α_o then dominating. In contrast the power threshold optimises for $\ell << \ell_o$ with the ε_o loss becoming the most important. For $\xi = \xi_c$ the two curves cross at $\ell = \ell_o$ with thresholds each four times larger than minimum. There is thus no clear compromise value for crystal length as there was for the focusing parameter; the actual length chosen will depend strongly on the laser pump power available, non-linear crystal parameters etc. Once again the BK 'optimum' length is inconveniently short for many infra-red materials (often less than 1mm), and it is therefore better to work with lengths substantially larger than ℓ_o.

In interpreting these results several points need to be considered. Firstly the intensity thresholds derived above refer to the focus of the Gaussian beam. Where surface damage is the limiting factor a slight correction may be required to find the surface intensity. Since, however, $\xi_c << 1$ for most materials, this correction is very small.

Secondly, the limits of $B > 1.68$, $\xi < 0.4$ must be observed for the results to be valid. Consideration of all cases in practical interest shows that the first condition is easily met and that ξ_c is then $<< 0.4$.

The third, and perhaps most important consideration, concerns the confocal parameters of the parametrically interacting beams. Assumed in BK, and therefore also in our own work, is the condition:

$$b_1 = b_2 = b_3 \ (\equiv b) \qquad [BK\ 3.12],$$

where b_1, b_2 and b_3 are the confocal parameters for the signal, idler and pump beams respectively. This limits, in general, application of our results to doubly-resonant oscillation (DRO).

Finally, in deriving (5), it has been assumed that $\alpha_o \ell << 1$ (as in BK). For some materials, (e.g. tellurium) this may not be valid. The effect will be to modify the exact forms of (5), but not the general conclusion that longer crystals should be used to minimize the intensity threshold. Further, in evaluating α_o, losses due to optical inhomogeneity which reduce the Q-factor of the resonator must be included; these losses are not normally detected by conventional spectro-photometer measurements.

Acknowledgements

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References

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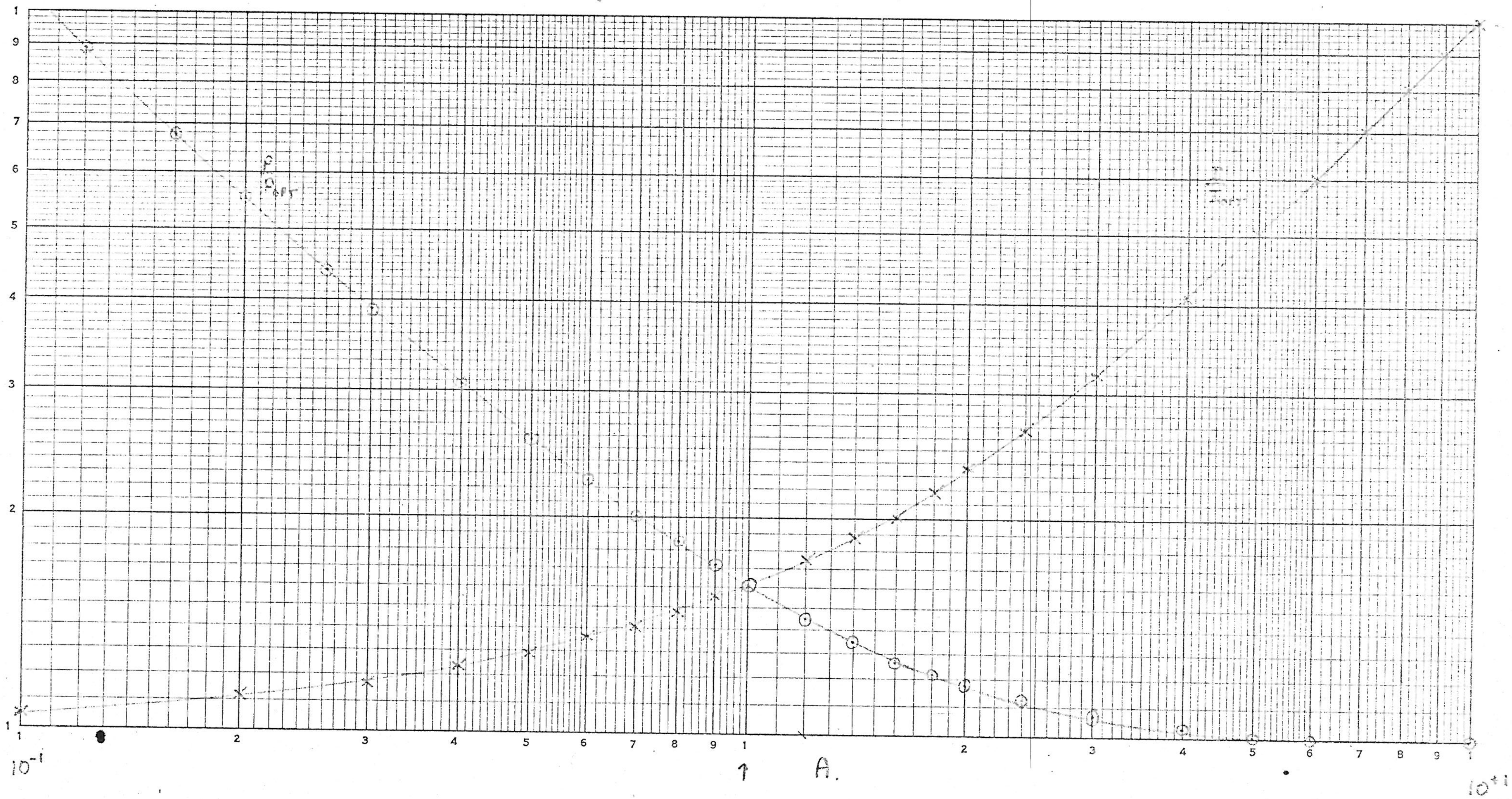
Figures

Figure 1. Plots of power threshold P/P_{opt} and intensity threshold I/I_{opt} ratios against normalized focusing parameter $A(= \xi/\xi_c)$.

Figure 2. Length dependence of power threshold, $(1 + x)^2$ and intensity threshold, $(1 + x^{-1})^2$ for compromise focusing, ξ_c. x is equal to ℓ/ℓ_o.

CHARTWELL Graph Data Ref. 5912 Log 1 Cycle x 2 Cycles



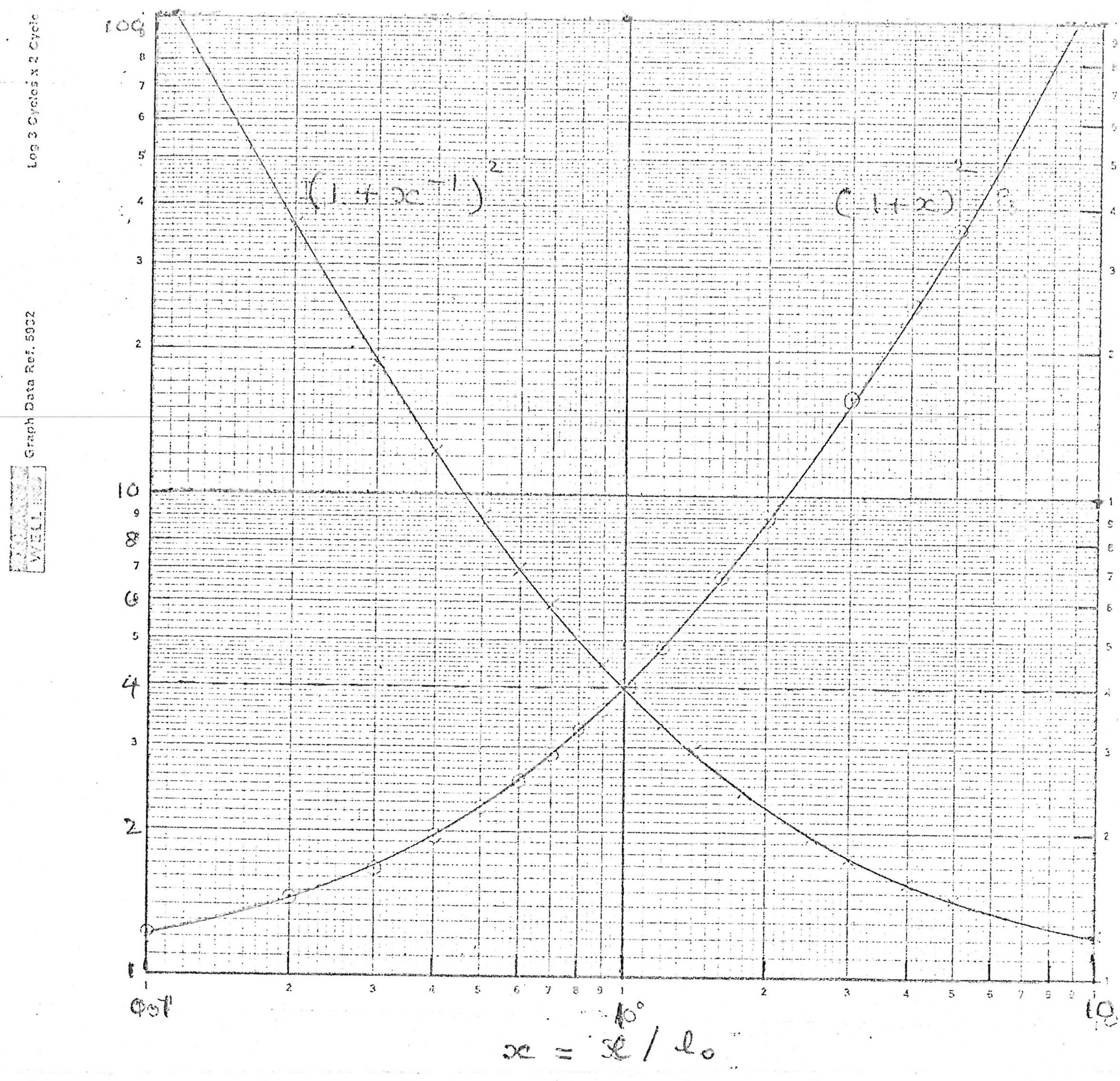


Fig. 2.