Experimental investigation of the variability in the dynamics of connected structures

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Abstract. Hydraulic pipes and cable bundles attached to host structures are widely found in engineering. This paper explores how variability in the connection points between structures affects the coupled dynamics. One at a time, two different one-dimensional waveguides are attached to a thin plate through a different set of point connections. Measurements considering randomly spaced connections were made and the experimental results are presented and compared to previously developed models. When multiples attachments are considered, the structure accommodates standing-like waves between the attachments, amplifying its response. It was possible to see the variability due the random spacing and, in a frequency-averaged sense, good agreement between the experimental data and the models were obtained. A comparison of the spatial response of the experiment and the infinite system is also presented.

1. Introduction

One-dimensional vibration waveguides, such as hydraulic pipes or cable bundles, connected to thin plates are widely found in engineering, but manufacturing processes lead to variability in the nominal properties of the materials and deviations from the design geometries [1]. This paper explores how deviations from the nominal position of the attachment points that connect the one-dimensional waveguide to the host-structure, namely a thin plate, affect the dynamic response of the system. Mobilities measurements were taken and are compared to analytical models. Previous experimental work [2] has measured the response of point-connected ribbed plates and compared the responses to results for infinite structures, but the effects of variability in the spacing of discrete attachments or for differences in the bending wavenumber ratio for the connected structures components were not previously investigated.

1.1. Mobility approach

Mobility is defined as the ratio of velocity to force as a function of frequency and it is usually a complex function in the frequency domain. It can be measured (or estimated) in both the translational or rotational sense. The point (or driving point) mobility can be found when the velocity and force are in the same direction and at the same point [3]. Mobility methods are extremely useful and a straightforward tool to analyse mechanical systems under periodic, transient or random loads. It can deal with simple lumped parameters and also with more elaborate coupled systems, in the form of a mobility matrix [4]. These methods can cope with the dynamic behaviour, in the frequency domain, of both a source and receiver

of vibration and then predict a coupled system performance in a manner analogous to what electrical engineers use for circuit analysis [3].

1.2. Using mobilities to couple an infinite beam to an infinite plate

Using mobility matrices, one can couple an infinite beam to an infinite plate through a set of point-connections. One can find the mobilities for an Euler-Bernoulli beam and a thin plate in [4] and they are given by:

$$Y_b^\infty(\omega, \alpha, \beta) = \frac{\dot{W}(x)}{F} = \frac{-\omega}{4EIk_b^3}(je^{-k_b r_{\alpha\beta}} - e^{-jk_b r_{\alpha\beta}}) \tag{1}$$

where Y_b^∞ is the mobility of the infinite beam as function of the frequency ω and the points on the beam at position α and β, $\dot{W}$ is the transverse velocity, F is the applied force, E is the Young's modulus I is the second moment of area, $k_b = \sqrt[4]{\frac{\rho S}{EI}}\sqrt{\omega}$ is the flexural wave number in the beam and $r_{\alpha\beta}$ is the distance between the points.

For an infinite plate:

$$Y_p^\infty(\omega, \alpha, \beta) = \begin{cases} \frac{\omega}{8Bk_p^2}\left[H_0^2(k_p r_{\alpha\beta}) - \frac{2j}{\pi}K_0(k_p r_{\alpha\beta})\right], & \alpha \neq \beta \\ \frac{1}{8\sqrt{Bm}}, & \alpha = \beta \end{cases} \tag{2}$$

where Y_p^∞ is the mobility of the infinite plate as function of the frequency ω and the points α and β, H_i^2 is an ith order Hankel function of the second kind, K_i is an ith order modified Bessel function of the second kind, B is the plate bending stiffness, $k_p = \sqrt[4]{\frac{m}{B}}\sqrt{\omega}$ is the flexural wavenumber in the plate and $r_{\alpha\beta}$ is the distance between the points.

A model algebraically formulated and then presented in the form of matrices, for coupling these structures through point connections was developed in [5] for scenarios where either the attachments are considered to be perfectly rigid links or when some flexibility, in a form of an elastic spring, is introduced. Equation (3) shows the velocity vector for points on the plate when rigid links are considered, whereas Eq. (4) shows the velocity vectors for the plate and the beam when elastic springs are used as connections. This analytical modelling will be used simulate numerical results for comparison with the experimental data.

$$\dot{w}_p = Y_p^\infty f - Y_p^\infty(Y_p^\infty + Y_b^\infty)^{-1}Y_p^\infty f \tag{3}$$

where $\dot{w}_p$ is the velocity vector of the connections points on the plate, Y_p^∞ is the mobility matrix of the infinite plate, f is the vector of the applied load applied to the plate and Y_b^∞ is the mobility matrix of the infinite beam.

$$\begin{Bmatrix} \dot{w}_p \\ \dot{w}_b \end{Bmatrix} = \begin{bmatrix} I & Y_p^\infty Y_b^{\infty-1} \\ \frac{1}{j\omega}\kappa & Y_b^{\infty-1} - \frac{1}{j\omega}\kappa \end{bmatrix}^{-1} \begin{Bmatrix} Y_p^\infty f \\ 0 \end{Bmatrix} \tag{4}$$

where $\dot{w}_p$ is the velocity vector of the connections points on the plate, $\dot{w}_b$ is the velocity vector of the connections points on the beam, I is the identity matrix, Y_p^∞ is the mobility matrix of the infinite plate,

Y_b^∞ is the mobility matrix of the infinite beam, $\boldsymbol{\kappa}$ is the spring stiffness matrix, ω is the frequency in rad/s and $\boldsymbol{f}$ is the vector of the applied load.

2. Description of the experiment

The experiment was designed to measure differences in the mobility of a coupled system, namely a finite beam attached to a finite plate, caused by variation in the spacing of the attachment points. Two different beams were used. The general properties of the plate and beams are given in Table 1. They were all made of mild steel.

Table 1. General properties of the plate and the beams.

Properties	Value
Density (kg/m³)	7850
Young's modulus (GPa)	200
Plate dimensions (l x b, mm)	750 x 350
Plate thickness (mm)	0.9
Beam 1 height (mm)	3
Beam 1 base (mm)	6
Beam 2 height (mm)	10
Beam 2 base (mm)	10
Beams 1 and 2 length (mm)	750
Beam 1 to plate bending wavenumber ratio	0.56
Beam 2 to plate bending wavenumber ratio	0.31
Regular spacing Δ (mm)	150

The experiment consisted of attaching one of the beams to the plate at five points using Neodymium magnets. Each magnet has a diameter of 4 mm and a height of 3 mm. Grade N52 magnets were used and, according to the manufacturer, each of the discs has a vertical pull of 9.3 N flush to a mild steel surface, which could have been verified using a standard pull test kit, but this was not part of the concerns of this paper. Also, when comparisons with the numerical models were made, it was assumed that there is no incorporation of the magnets mass or rotational coupling between beam and plate. For beam 1, the flexible beam, only one magnet was used at each attachment point, whereas in the case of beam 2, stiff beam, two magnets, side-by-side, were used at each attachment point.

The system was excited by a shaker at one of the attachment points at the beam and the input force was measured by an impedance head. It is possible to show algebraically that there is no difference in exciting the beam or the plate when rigid attachments are considered. The response was measured on the opposite side of the coupled system, on the plate, with a laser vibrometer, which measures the out of plane bending velocity at that point. Figure 1(a) shows schematically the experiment.
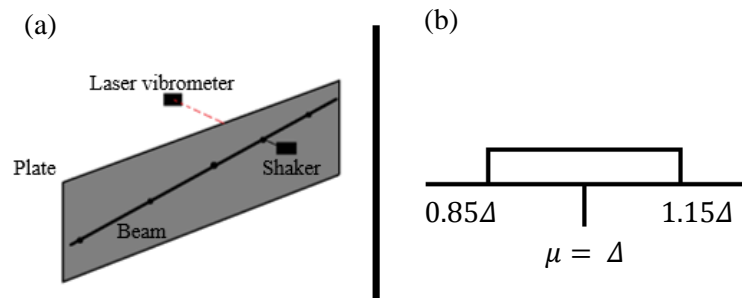

Figure 1. Experimental configuration (a) and pdf of the spacing (b).

For each of the beams, 15 repeatability tests were conducted by performing a disassemble and reassemble set of measurements were taken with the regular spacing Δ between the attachments. Also,

for each of the beams, 10 sets of random distributions of the spacing between the attachments were considered, with the pdf of the spacing being a uniform distribution, Figure 1(b). The spacing was allowed to vary up to 15% around its nominal equal spacing value of 150 mm. The driving attachment was kept the same and the other points moved relative to this one. Considering as the origin of the coordinate system the end of the beam closest to the shaker, the coordinates where the magnets were placed to connect the structures are given in Table 2, along with the spacing for each case.

Table 2. Coordinates and spacing between the attachments.

Set	Coordinates (mm) / Spacing Δ (mm)								
1	75		225		375		525		675
		150		**150**		**150**		**150**	
2	89		225		393		549		708
		136		**168**		**156**		**159**	
3	81		225		356		488		641
		144		**131**		**132**		**153**	
4	95		225		363		494		639
		130		**138**		**131**		**145**	
5	74		225		355		517		647
		151		**130**		**162**		**130**	
6	82		225		372		540		703
		143		**147**		**168**		**163**	
7	90		225		353		505		648
		135		**128**		**152**		**143**	
8	88		225		393		525		680
		137		**168**		**132**		**155**	
9	57		225		361		526		687
		168		**136**		**165**		**161**	
10	67		225		357		500		632
		158		**132**		**143**		**132**	
11	76		225		366		507		640
		149		**141**		**141**		**133**	

3. Results

This section presents the experimental data obtained. These results are compared to those that are predicted for infinite connected structures. When mentioned, the equivalent infinite system is defined as having the same number of attachments and the same spacing of the measured system between said attachments. Also, the infinite beams have the same cross-sectional area as the ones that were measured, whereas the infinite plate has the same thickness as the measured plate.

Figure 2 shows the difference that the random spacing produces on the measured mobility. This is obtained by referencing the randomly spaced cases to one of the evenly spaced in a dB scale. Therefore, the 0 dB is the selected evenly spaced scenario. It is also serves the purpose of showing the repeatability of the experiment and the inherent uncertainty in it. In both Figures 2(a) and (b), the black lines are the response of the 10 randomly spaced cases tested, the red line is the average of the 15 evenly spaced cases (when the system was disassembled and reassembled) and the yellow lines are the maxima and minima of these repeatability evenly spaced cases for each frequency. Figure 2(a) shows the results for beam 1, whereas Figure 2(b) shows the results for beam 2, the stiffer of the two beams.

In Figure 3, the mobility of one of the randomly spaced cases is compared to the equivalent infinite system. In both cases, Figures 3(a) and (b), the black lines are the measured mobility, the red lines are

the infinite system and the yellow lines are the frequency averaging of the experimental data. A bandwidth of 35 Hz was used for averaging.
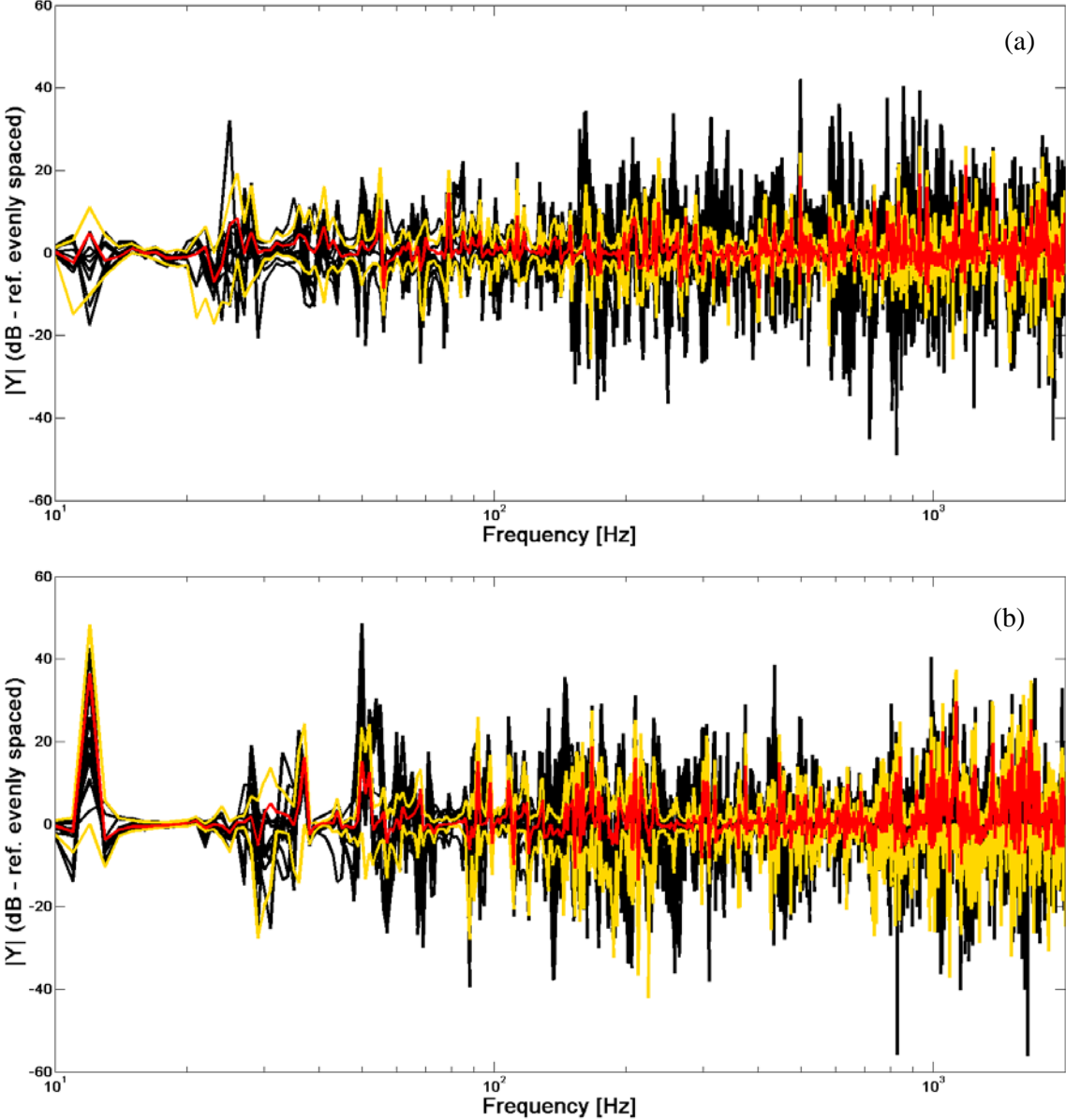

Figure 2. Differences due to random spacing versus repeatability. Beam 1 (less stiff beam) in (a) and beam 2 (stiffer beam) in (b). For both, ▬ are the randomly spaced cases, ▬ is the average of fifteen evenly spaced cases and the ▬ lines are the maxima and minima values for these evenly spaced cases at each frequency.

Figure 4 represents the same data as shown in Figure 3, but instead of referencing it to 1 m/s/N, the evenly spaced case is used once again as 0 dB. For the experimental data, the reference is the data of the selected evenly spaced case, whereas for the infinite system, the equivalent evenly spaced infinite system is the reference. As in Figure 3, for Figures 4(a) and 4(b), the black lines are the measured mobility, the red lines are the infinite system and the yellow lines are the frequency averaging of the experimental data.

Since all of the experimental data collected is referenced to the same value on a dB scale, 1 m/s/N, to change the reference for the evenly spaced case, one only needs to subtract the data from the new reference value, the evenly spaced case, as given by Equation (5).

$$\hat{Y}_{dB} = Y_{dB_r} - Y_{dB_e} \qquad (5)$$

where $\hat{Y}_{dB}$ is the experimental data now referenced to the evenly spaced case, Y_{dB_r} is the experimental data of the randomly spaced cases on a dB scale referenced to 1 m/s/N and Y_{dB_e} is the experimental data of the selected evenly spaced case, also on a dB scale referenced to 1 m/s/N.
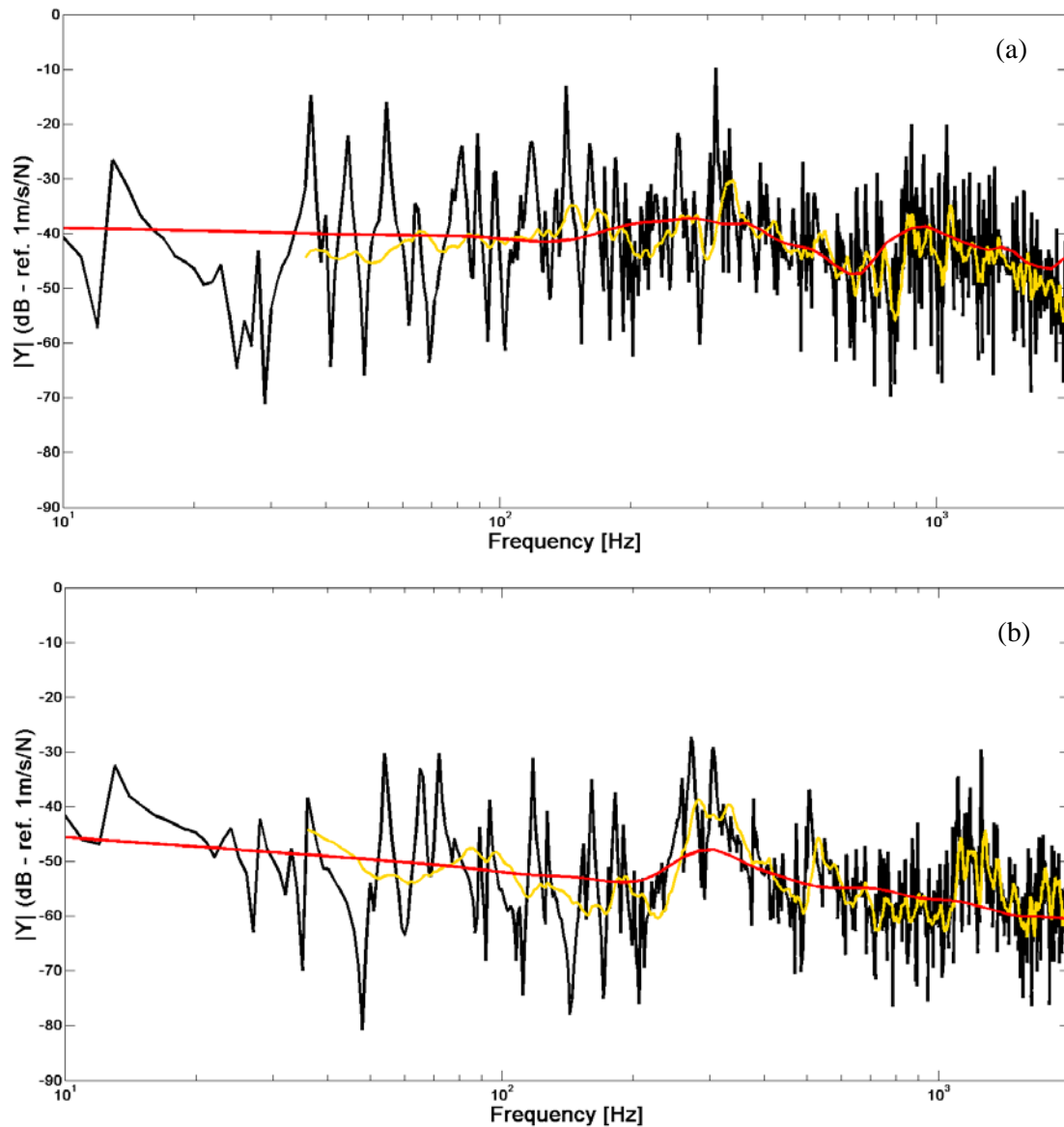

Figure 3. Randomly spaced cases, set number 3. Comparison with the infinite systems. Beam 1 in (a) and beam 2 in (b). For both, ▬ is the measured data, whereas ▬ is the equivalent infinite system and ▬ is the frequency averaged experimental data.

Around 300 Hz there is an amplification of the mobility response of the equivalent infinite system. Figure 5 shows spatial response of the experimental data and the equivalent infinite system at this frequency. In this case, a regular spacing of 150 mm between the attachments was considered. Although the data is shown for the evenly spaced case, the randomly spaced cases also exhibit the same behaviour in this region of the spectrum. The system was excited at the second connection point from right to left. The excitation was applied to the beam on the experiment and on the plate on the model. The results are shown for the combination of plate and beam 2, the stiff beam.
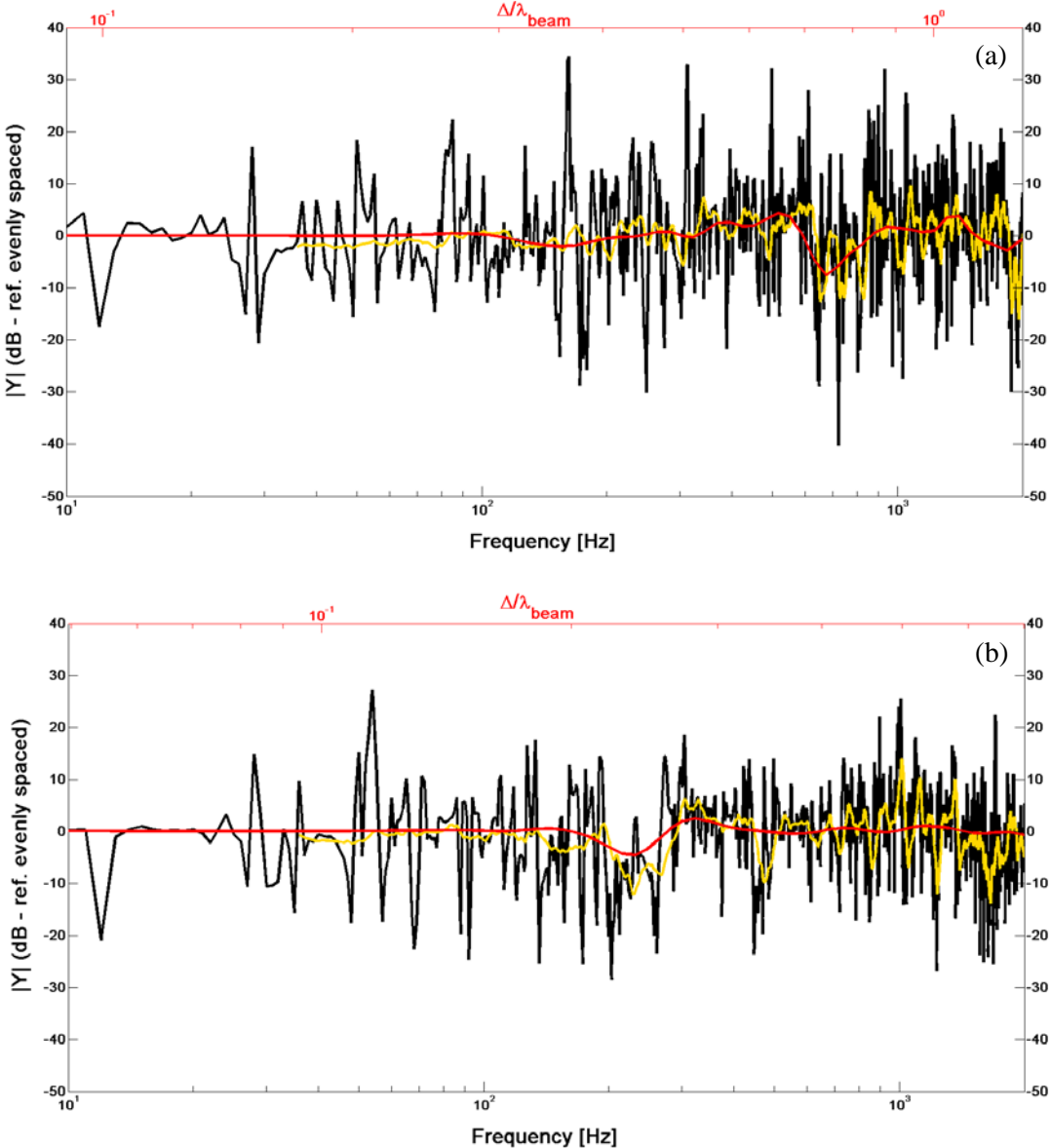

Figure 4. Comparison of the mobilities. Finite system versus the equivalent infinite system, both referenced to the adequate evenly spaced case. Beam 1 in (a) and beam 2 in (b). For both, ▬ is the measured data, whereas ▬ is the equivalent infinite system and ▬ is the frequency averaged experimental data.

Also, in Figure 4, an additional axis for x was added, in red at the top. Instead plotting the curves versus Frequency, it is possible to plot them versus the dimensionless parameter Δ/λ_{beam}, where Δ is the regular spacing, 150 mm, and λ_{beam} is the wavelength in the beam, $\lambda_{beam} = 2\pi \left(\frac{EI}{\rho A\omega^2}\right)^{\frac{1}{4}}$.
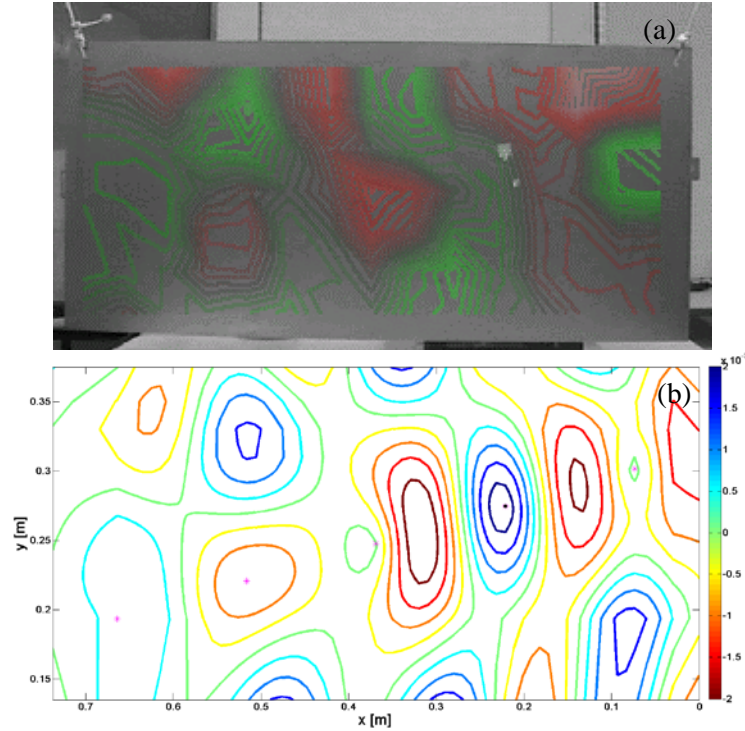

Figure 5. Plate and stiff beam. Comparison of the spatial response. In (a), the experimental data versus equivalent infinite system, in (b). The attachments are represented by *.

4. Discussion

In Figure 2, one can notice that when compared to the evenly spaced scenario, the dynamic response of both beams present variations that can be up to 40 dB, but most commonly reaching a difference up to 20 dB.

Although, in Figure 2(b), it is possible to see that the spread in the repeatability test is comparable to the variation from the random spacing at several points. Another way to look at this would be to compare the standard deviation values divided by the mean values, σ/μ, also known as coefficient variation, for both the randomly spaced cases and the evenly spaced cases at each frequency.

For beam 1, these coefficient of variation values at a frequency by frequency for the evenly spaced scenarios are oscillating around 0.25 at most frequency (with a maximum of 1.8 around 1.5 kHz), whereas the values for the randomly spaced cases are oscillating around 0.75. The coefficient of variation values, σ/μ, are also over 1.5 several times and could be up to 2.5, with a maximum value of a little over 2.5 at around 650 Hz for the randomly spaced cases.

For beam 2, the evenly spaced cases, the coefficient of variation also varies around 0.25 for most of the frequency range, but reaches up to 1.5 at several points, whereas the randomly spaced cases oscillate around 0.5, also reaching 1.5 multiple times with a maximum of 2.4 around 220 Hz. Even though the values for σ/μ are larger when randomly spaced attachments are considered, there are frequencies when

the magnitude of the evenly spaced cases are comparable to the randomly spaced cases, especially at frequencies above 1 kHz, where the data becomes less reliable with reduced coherence.

The literature reports that the response of the finite system, when frequency-averaged, should tend to that of the equivalent infinite system [6]. Both Figures 3(a) and 3(b) are consistent with that observation and comment. The frequency averaging of the experimental data was made by a moving averaging with a window of a constant size of 35 Hz.

As a matter of fact, even when the mobilities were referenced on a dB scale to the evenly spaced case, Figures 4(a) and 4(b), the frequency-averaged response of the system oscillates around that predicted by the equivalent infinite system with the same randomly spaced attachments. The reference value for plotting the red line is the magnitude of the mobility of the evenly spaced infinite system.

5. Conclusions and future work

The paper reports the findings of an experimental investigation into the effects that random variations of the position of the point connections have when simple but widely found structures are attached together. Neodymium magnets were used as the elements producing the connection. Two beams with different cross-sectional areas were considered.

From the results presented in [5], it is fair to say that the magnets behave like rigid links for the frequencies considered. Moreover, the standing-like wave predicted to happen between the attachments of the infinite system seems to match the response of the finite system, as shown in Figure (5).

The wavenumber ratio were not sufficiently different to notice any significant difference in the response amplitude due to the small variation of the position of the attachment points introduced.

The need for two magnets per connection point for beam 2, the stiffer beam, could have added some extra uncertainties in the position of the connecting points and rotational inertia of the connections, therefore producing results that are more scattered in the repeatability test. For this beam, it is possible to see the effects that varying the position of the attachments produce in the frequency range between 20 Hz and 500 Hz. Above the higher frequency, the inherent uncertainty of disassembling and reassembling the experiment gets comparable to the variation measured when random spacing was considered. Further study is required to analyse this.

Overall, it was also possible in a frequency average sense to show good agreement between the experimental data and the predicted equivalent coupled infinite systems.

Further steps in the study will be to include a more flexible one-dimensional waveguide, such as a cable bundle, preferable with a beam to plate bending wavenumber ratio larger than 1 and also attachment connections with flexibility. Future work will also develop a model that considers slowly varying properties of a beam, and try to use the work developed by Skudrzyk [7] to find the envelopes for the response of the coupled system.

References

[1]   Fabro A T 2014 *Ph.D. Thesis, University of Southampton* Propagation in waveguides with slowly changing variability

[2]   Mayr A R and Gibbs B M 2011 *Journal of Sound and Vibration* **330** 4798-4812

[3]   White R G 1986 *Noise and Vibration* White R G and Walker J G (Chichester: Ellis Horwood Limited) p 686

[4]   Gardonio P and Brennan M J 2004 *Advanced Applications in Acoustic Noise and Vibration* Fahy F and Walker J (London: Spon Press) p 389-447

[5]   Souza M R and Ferguson N S 2016 *Uncertainties 2016* Variability in the dynamic response of connected structures – A mobility apporach for point connections

[6]   Skudrzyk E 1968 *Simple and Complex Vibratory Systems* (London: The Pennsylvania State University Press)

[7]   Skudrzyk E 1980 *Journal of the Acoustical Society of America* **67(4)** 1105-1135