Vibration control using nonlinear damped coupling

Maryam Ghandchi Tehrani[1*] and Vincenzo Gattulli[2]

[1]Institute of Sound and Vibration Research, University of Southampton, SO17 1EN, Southampton, UK
[2] University of L'Aquila, Italy

Abstract. In this paper, a dynamical system, which consists of two linear mechanical oscillators, coupled with a nonlinear damping device is considered. First, the dynamic equations are derived, then, an analytical method such as harmonic balance method, is applied to obtain the response to a harmonic base excitation. The response of the system depends on the excitation characteristics. A parametric study is carried out based on different base excitation amplitudes, frequencies, and different nonlinear damping values and the response of the system is fully described. For validation, time domain simulations are carried out to obtain the nonlinear response of the coupled system.

1. Introduction
Structural control deals with methodologies to design control systems for structural vibration attenuation even in the presence of strong external loadings such as the ones induced by seismic motion. Structural control methodologies can be grouped in three categories: active, semi-active and passive control [1].
The design solutions offered in the recent available literature aim to balance the opposite needs of synthesis and representativeness for either the damping system or the damped structure. A pair of simple oscillators or a pair of equivalent one-dimensional beams, coupled with a variety of damping devices, have often been employed as a synthetic but representative model to describe a wide class of structural realizations – for instance, adjacent tall buildings – or quasi-independent sub-systems that compose a single complex structure.
Several studies have been carried out for the optimization and design of the structures. For example, different strategies have been proposed for the optimal placement of viscous-type coupling devices into seismic joints to dissipate energy and to avoid hammering phenomena [2, 3]. A series of studies has also been devoted to dissipative interconnections realized through hysteretic dampers [4, 5], friction dampers [6] or semi-active devices [7]. In spite of the inherent complexity related to the description of the peculiar properties of each specific device, the simplest elasto-viscous constitutive laws are reproduced by the Kelvin–Voigt (KV) or Maxwell (Ma) model, fully described by two parameters, a stiffness and a viscous damping coefficient, whose assessment may play an important role in the preliminary design stage [8].
Nonlinear damping has been considered for the problem of energy harvesting and it has been demonstrated that it can extend the dynamic performance range of the harvester [9]. Quasi-linear models have been used to describe the dynamic response of the system with nonlinear damping. The quasi-linear model can provide a good representation since the response of the system with nonlinear damping does not include jump phenomena or bifurcation in contrast to that with nonlinear stiffness [10].
In this paper, a simple dynamic system composed of two linear oscillators is employed to analyze the control performance that can be achieved through a nonlinear damper connecting the two oscillators.

First the dynamic equations of the system are derived. Then, the response of the system subject to harmonic base excitation is obtained analytically using harmonic balance method and numerically using time-domain simulation. Then, the performance of the system with nonlinear damped coupling is compared with the system with linear damped coupling.

2. Theoretical development

Consider two simple linear oscillators with mass M_j and stiffness K_j, $(j=1,2)$, coupled by a linear stiffness *K*. U_1 and U_2 are the displacements of the two masses, F is the viscous force applied by a damper as depicted in Fig. 1.

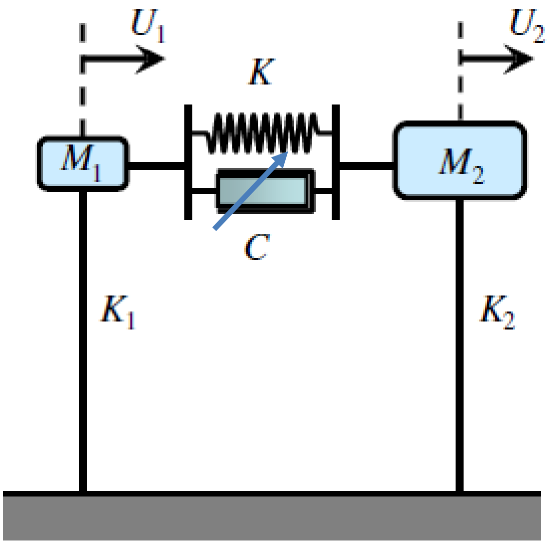

Figure 1. Vibration control of adjacent structures with nonlinear damping

The dynamic equations for the two dof system in Fig.1 can be written as,

$$M_1\ddot{U}_1 + F(\dot{U}_1,\dot{U}_2) + K_1(U_1 - U_g) + K(U_1 - U_2) = 0 \tag{1}$$

$$M_2\ddot{U}_2 - F(\dot{U}_1,\dot{U}_2) + K_2(U_2 - U_g) + K(U_2 - U_1) = 0 \tag{2}$$

where dot indicates the derivative with respect to time t. For nonlinear damping in the form of cubic damping, we have

$$F(\dot{U}_1,\dot{U}_2) = C(\dot{U}_1 - \dot{U}_2)^3 \tag{3}$$

For simplicity, we change the coordinate systems such that,

$$U_1 - U_g = Y \tag{4}$$

$$U_1 - U_2 = Z \tag{5}$$

which results in the following equations.

$$M_1(\ddot{Y} + \ddot{U}_g) + C\dot{Z}^3 + K_1 Y + KZ = 0 \tag{6}$$

$$M_2(\ddot{Y} - \ddot{Z} + \ddot{U}_g) - C\dot{Z}^3 + K_2(Y - Z) - KZ = 0 \tag{7}$$

Denoting, L as a reference length (for example 1 m), we can introduce the following dimensionless parameters and variables,

$$z=\frac{Z}{L};y=\frac{Y}{L};u_g=\frac{U_g}{L};\omega_j^2=\frac{K_j}{M_j};\beta=\frac{\omega_2}{\omega_1};\rho=\frac{M_2}{M_1};\kappa=\frac{K_2}{K_1};\eta=\frac{K}{\omega_1^2M_1};\gamma=\frac{C\omega_1L^2}{2m_1}\tau=\omega_1t \quad (8)$$

The equations of motion in the non-dimensional form become,

$$y''+2\gamma z'^3+\eta z+y=-u_g'' \quad (9)$$

$$y''-z''-\frac{2\gamma}{\rho}z'^3+\beta^2y-\left(\beta^2+\frac{\eta}{\rho}\right)z=-u_g'' \quad (10)$$

Writing the equations of motion (9,10) into the matrix form, yields,

$$\begin{bmatrix}0 & 1\\ -1 & 1\end{bmatrix}\begin{pmatrix}z''\\ y''\end{pmatrix}+\begin{bmatrix}2\gamma & 0\\ -\frac{2\gamma}{\rho} & 0\end{bmatrix}\begin{pmatrix}z'^3\\ y'^3\end{pmatrix}+\begin{bmatrix}\eta & 1\\ -\left(\beta^2+\frac{\eta}{\rho}\right) & \beta^2\end{bmatrix}\begin{pmatrix}z\\ y\end{pmatrix}=-\begin{pmatrix}1\\ 1\end{pmatrix}u_g'' \quad (11)$$

The system parameters are chosen such that, $\rho\geq 1$, $\kappa\geq 0$, and consequently $\beta\geq 0$.

3. Harmonic base excitation

Assuming tonal excitation for the base with an amplitude of U and the excitation frequency Ω,

$$u_g''=-\Omega^2U\cos\Omega\tau \quad (12)$$

using the harmonic balance method and taking the response only at the fundamental frequency, we have,

$$z(\tau)=A\cos\Omega\tau+B\sin\Omega\tau \quad (13)$$

and

$$y(\tau)=P\cos\Omega\tau+Q\sin\Omega\tau \quad (14)$$

The amplitudes of responses are therefore $Z=\sqrt{A^2+B^2}$ and $Y=\sqrt{P^2+Q^2}$. Substituting the two time responses and their derivatives into Eqs. (9) and (10), and ignoring higher order harmonics leads to,

$$\left(1-\Omega^2\right)z+\eta z+\frac{3}{2}\gamma\Omega^2Z^2z'=\Omega^2U\cos\Omega\tau \quad (15)$$

and,

$$\left(\beta^2-\Omega^2\right)y(\tau)+\left(\Omega^2-\frac{\eta}{\rho}-\beta^2\right)z(\tau)-\frac{3}{2}\frac{\gamma}{\rho}\Omega^2Z^2z'(\tau)=\Omega^2U\cos\Omega\tau \quad (16)$$

The equivalent linear damping for the nonlinear cubic damper in Eqs. (15) and (16) can be obtained from,

$$\xi_{1eqe}=\frac{3}{4}\gamma Z^2\Omega^2 \quad (17a)$$

$$\xi_{2eqe}=\frac{3}{4}\frac{\gamma}{\rho}Z^2\Omega^2 \quad (17b)$$

Please note that these are the non dimensional damping coefficient and their values can be greater than one.

Partitioning the coefficient of $\sin(\Omega\tau)$ and $\cos(\Omega\tau)$ for the Eq. (15) results in,

$$\left(1-\Omega^2\right)P+\eta A+\frac{3}{2}\gamma B\Omega^3\left(A^2+B^2\right)=\Omega^2U \quad (18a)$$

$$\left(1-\Omega^2\right)Q+\eta B-\frac{3}{2}\gamma A\Omega^3\left(A^2+B^2\right)=0 \tag{18b}$$

Similarly, partitioning the coefficients of $\sin(\Omega\tau)$ and $\cos(\Omega\tau)$ for the Eq (16), yields,

$$\left(\beta^2-\Omega^2\right)P+\left(\Omega^2-\frac{\eta}{\rho}-\beta^2\right)A-\frac{3}{2}\frac{\gamma}{\rho}\Omega^3 B\left(A^2+B^2\right)=\Omega^2 U \tag{19a}$$

$$\left(\beta^2-\Omega^2\right)Q+\left(\Omega^2-\frac{\eta}{\rho}-\beta^2\right)B+\frac{3}{2}\frac{\gamma}{\rho}\Omega^3 A\left(A^2+B^2\right)=0 \tag{19b}$$

The coefficients A, B, P and Q are the four unknowns that can be obtained from solving the four equations 18(a, b) and 19(a, b).

4. Numerical Example

For the numerical simulation, the following parameters are considered as given in [8].

$$\beta=2,\ \rho=3,\ \gamma=1,\ \eta=0.$$

The amplitude of the relative displacements Z and Y are obtained both from analytical solution by solving Eqs (17) and (18) as well as from time domain numerical simulation using Matlab ode45. The two undamped frequencies are 1 and 2.

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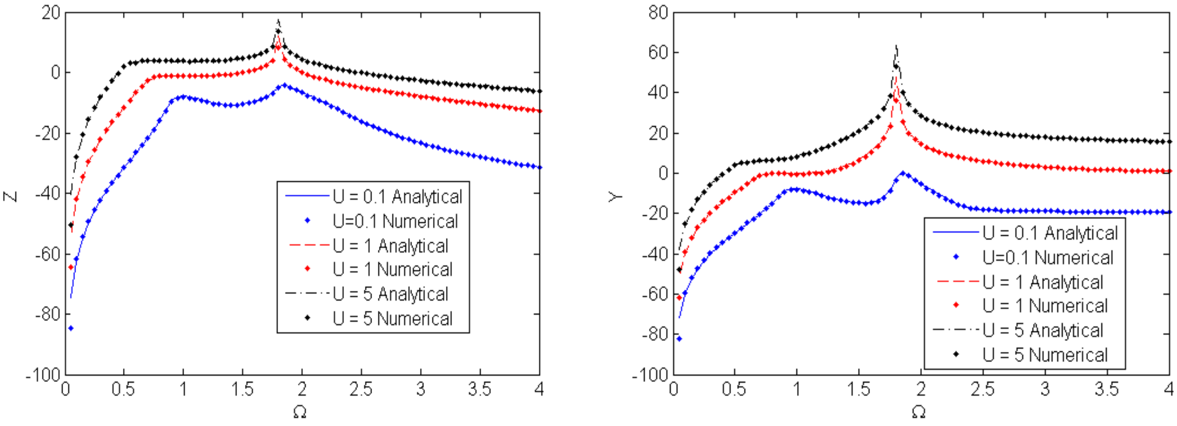
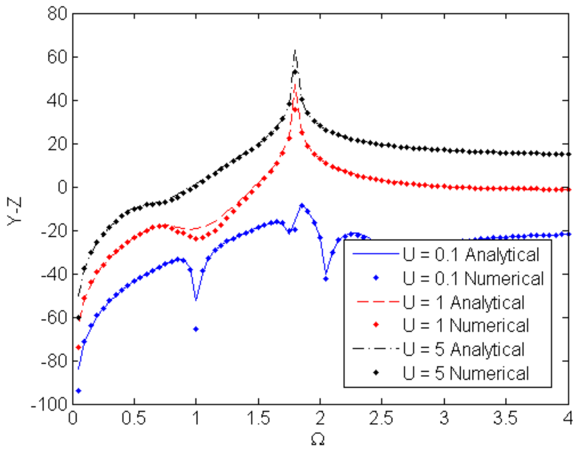
4.1. Effect of the base excitation amplitude U

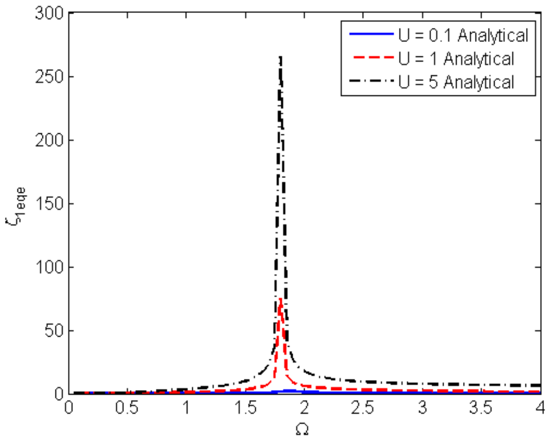
Figure 2(a-c) shows the amplitudes of the displacement responses as a function of normalised base excitation frequency for three different levels of base excitation amplitudes of 0.1 (blue solid line), 1 (red dashed line) and 5 (black dashed-dotted line). There is a good agreement between the harmonic balance approach and the numerical time domain simulation, confirming the validity of the analytical quasi-linear model. Several other numerical values for the system parameters have been considered and the harmonic balance method showed very good agreement with the results from time-domain simulations. The two peaks in Figure 2 correspond to the two modes of the structure. By increasing the excitation amplitudes, the amplitude of the responses increases. The equivalent linear damping ξ_{1eqe} is also plotted as a function of frequency, as shown in Figure 2(d). It can be seen that the equivalent damping increases particularly at the second mode when the excitation level increases.



(a) (b)



(c)



(d)

Figure 2. The effect of the base excitation amplitude U on the response of the nonlinear system as a function of the excitation frequency ,(b) relative displacement between the two masses Z,, (a) relative displacement between mass 1 and the base, Y (c) relative displacement between mass 2 and the base, Y-Z (d) equivalent damping ratio for mass 1

To compare the performance of the nonlinear damped coupling with the linear damped coupling, the relative displacement between the two masses is considered to be the same at a particular base excitation amplitude, for example, $U = 1$, at either resonance frequencies.

Figure 3 shows the plots of relative displacement between the two masses, Z, the relative displacement between the first mass and the base, Y the relative displacement between the second mass and the base, $Y - Z$, and the damping for nonlinear and linear systems, ζ_{eqe}, as a function of the base excitation amplitude, U. The results for the linear coupling are marked with dashed blue line and for the nonlinear coupling are marked with red solid line. The plot of the relative displacement, Z, in dB shows that the two lines cross when the base excitation amplitude is one. Above this level, the system with nonlinear coupling has relative amplitudes lower than the system with linear coupling. This is due to the fact that the slope of the line for the nonlinear coupling is 1dB/3dB, while the slope of the line for the linear coupling is 1dB/1dB. The nonlinear coupling achieves a better vibration control when the base excitation amplitude exceeds the amplitude of one.

The equivalent damping for the nonlinear system increases when the excitation amplitude increases. The linear damper is 0.5724Ns/m and it does not depend on the excitation level. The increase of the equivalent linear damping could justify the achievement of lower amplitudes of relative displacement by the nonlinear coupling.

The plot of Y also shows that the nonlinear coupling provides a better performance at excitation level above one compared to the linear coupling. However, the relative displacement between the second mass and the base, $Y - Z$ shows a slight increase in the amplitude, however this increase remains fixed at high excitation levels, since the slope of the nonlinear system becomes almost the same as the slope of the linear system.

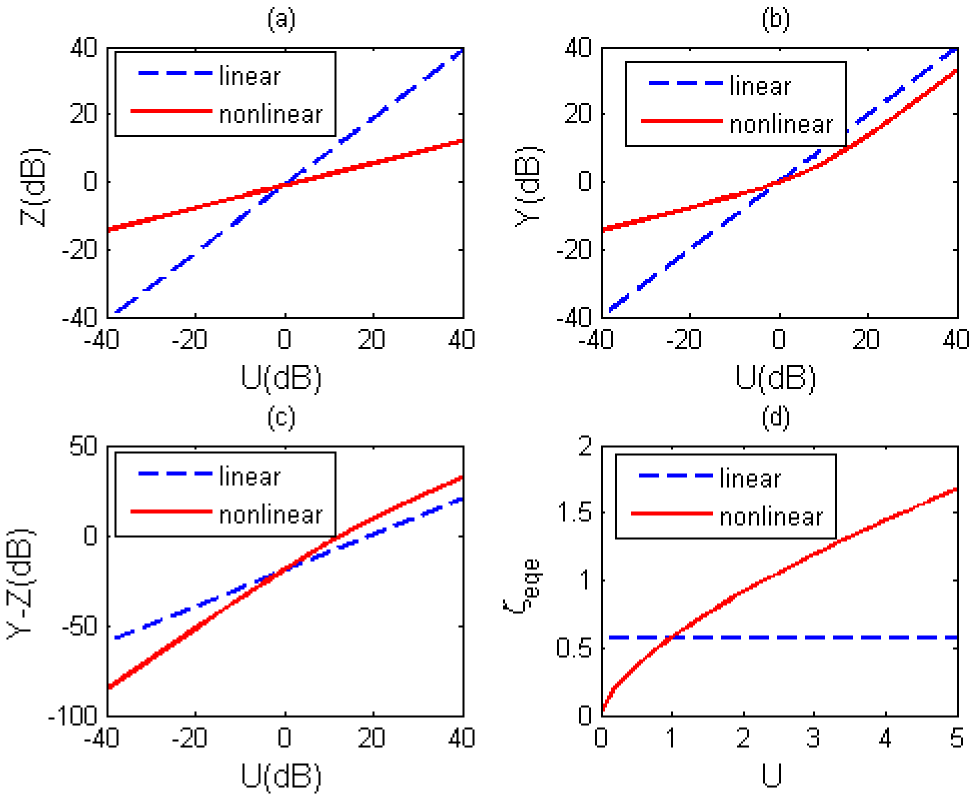


Figure 3. Comparison between the linear and nonlinear coupling when excited at the first undamped resonance frequency $\Omega = 1$, having the same relative displacement when $U = 1$. (a) relative displacement , Z, between the two masses in dB (b) displacement Y in dB, which is the relative motion between the first mass and the base (c) relative motion between the second mass and the base, Y-Z in dB (d) the equivalent damping for the nonlinear system

Similar plots are produced when the linear and nonlinear coupling have the same relative displacement at the second resonance and at an excitation amplitude of $U = 1$, as shown in Figure 4. The linear damping is chosen to be 3Ns/m for the linear coupling to provide the same relative displacement, Z , as the nonlinear system at the second natural frequency. The benefits of using nonlinear coupling can also be clearly seen in this case. The relative displacements between each mass and the base are almost the same as the ones for the linear system as shown in Figure 4(b) and (c).

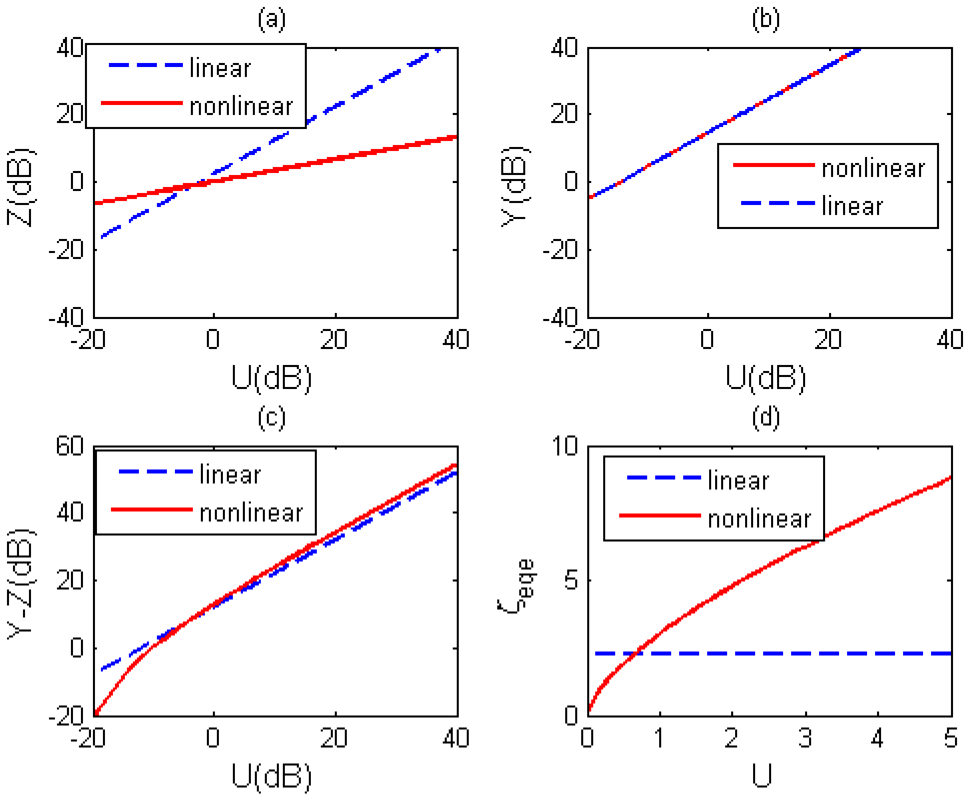


Figure 3. Comparison between the linear and nonlinear coupling when excited at the first undamped resonance frequency $\Omega = 2$, having the same relative displacement when $U = 1$. (a) relative displacement , Z, between the two masses in dB (b) displacement Y in dB, which is the relative motion between the first mass and the base (c) relative motion between the second mass and the base, Y-Z in dB (d) the equivalent damping for the nonlinear system

4.2. Effect of the Nonlinear damping γ

In this section three different values of nonlinear damping are considered and the amplitude of the responses are plotted as shown in Figure 5 when the base excitation amplitude $U = 1$. The increase of the nonlinear damping, reduces the amplitude of the relative displacement between the two masses, as shown in Figure 5(a). The equivalent linear damping increases as the nonlinear damping increases as expected from Eq.(17a). The equivalent damping at the frequency of the second mode is the highest due to the large amplitude of the relative displacement at that frequency, as can be seen in Figure 5(d). From Figures 5(b) and (c), not much effect can be observed on the relative amplitudes of the second mode, however the first mode is very well damped.

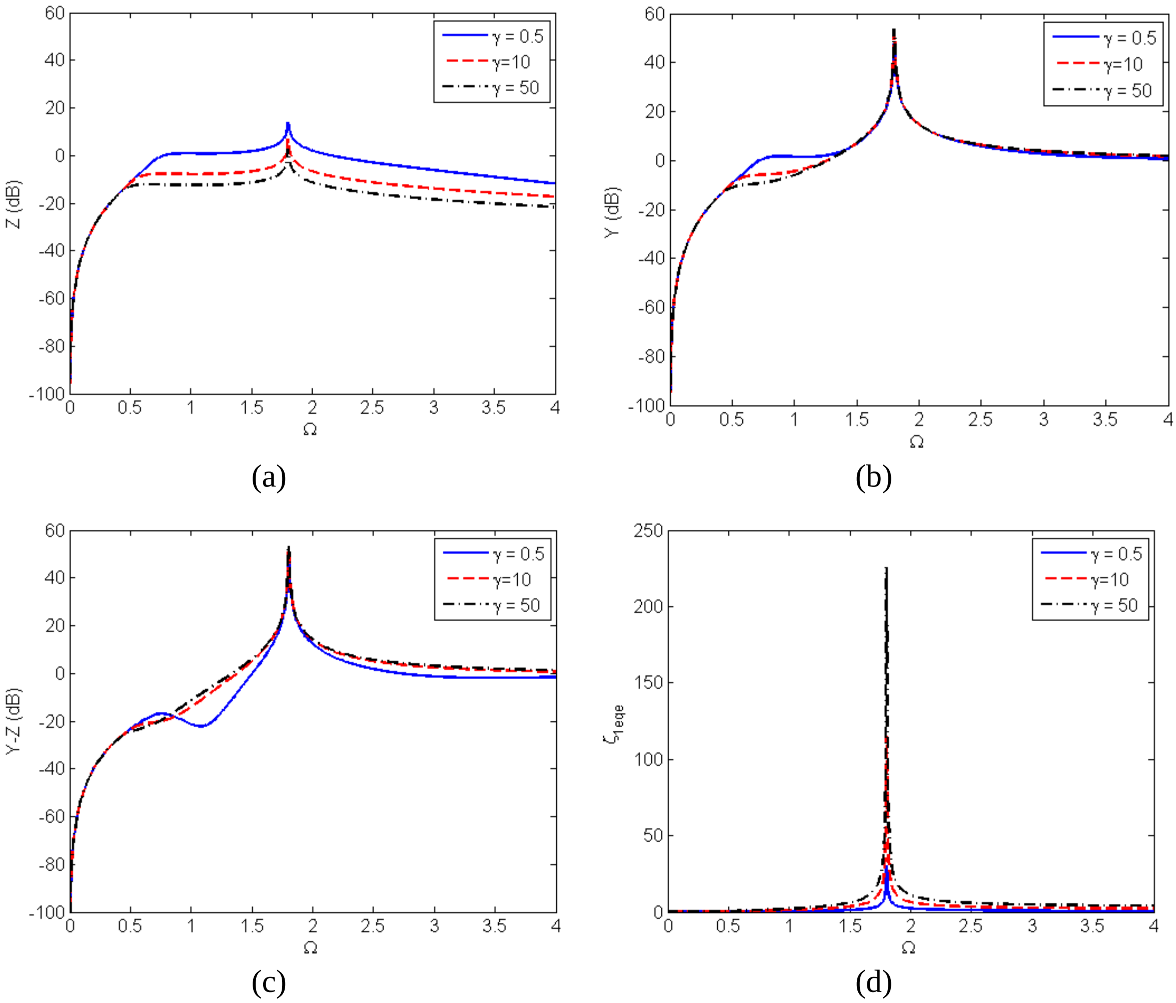


Figure 5. The effect of the nonlinear damping γ on the response of the system as a function of the excitation frequency ,(b) relative displacement between the two masses Z, (a) relative displacement between mass 1 and the base, Y (c) relative displacement between mass 2 and the base, Y-Z (d) equivalent damping ratio for mass 1

5. Conclusions

In this paper, the dynamic response of a two linear mechanical oscillators, coupled with a nonlinear damping subject to harmonic base excitation is presented. The model can represent a wide class of engineering structures such as adjacent tall buildings. The response of the system is investigated using different excitation amplitudes and frequencies. The performance of the nonlinear system is also compared with the linear system, when the two systems have the same amplitude at one of the two undamped natural frequencies of the system when the base is driven at an amplitude of one. For higher excitation levels than one, it is demonstrated that the nonlinear damper offers a better vibration control than the linear damper.

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