The effect of beam inclination on the performance of a passive vibration isolator using buckled beams

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Abstract. Passive vibration isolators are desired to have both high static stiffness to support large static load and low local stiffness to reduce the displacement transmissibility at frequencies greater than resonance. Utilization of a vertical buckled beam as a spring component is one way to realize such a stiffness characteristic since it exhibits a smaller ratio of local stiffness to static stiffness than that of a linear spring. This paper investigates the behaviour of a vibration isolator using inclined beams as well as vertical ones and examines the effect of beam inclination for the purpose of improving the isolation performance. The experimental system investigated has an isolated mass which is supported by a combination of two types of beams: buckled beams and constraining beams. The buckled beams can be inclined from the vertical by attachment devices, and the constraining beams are employed to prevent off-axis motion of the isolated mass. The results demonstrate that the inclination of the buckled beams reduces the resonance frequency and improves the displacement transmissibility at frequencies greater than resonance.

1. Introduction

Vertical vibration isolation using axially compressed buckled beam has the advantage that it has a low resonance frequency without large deflection when supporting a static load [1]. Winterflood et al. proposed a vibration isolator using such high static and low local stiffness characteristics of buckled beams [1,2]. Virgin and Davis analytically investigated the force-displacement relationship of a vertical buckled beam including the effects of initial geometric imperfections, and experimentally obtained the displacement transmissibility for a vibration isolator with such vertical buckled beams [3].

Some quasi-zero-stiffness systems using a combination of a positive and negative stiffness have also been proposed. Carrella et al. demonstrated that an oblique spring can have a negative stiffness characteristic in a certain range of vertical displacement and analytically investigated the static characteristics of a system consisting of one vertical and two oblique springs [4]. When this mechanism is used in a vibration isolator, parameters of the system are determined so that compressed oblique springs are in horizontal position when an isolated mass is at equilibrium. Force transmissibility and displacement transmissibility of this system were examined by the analytical and numerical approaches [5,6]. The vibration isolator with axially compressed beams instead of horizontal springs has also been proposed [7]. Liu et al. analyzed force and displacement transmissibility of the system which has such a quasi-zero-stiffness mechanism [8], and Fulcher et al. examined the vibration isolation performance experimentally as well as analytically [9]. Magnetic force has also been used to create negative stiffness characteristic [10,11], and both positive and negative stiffness characteristics [12].

The force-displacement relationship shown in reference [4] indicates that the vertical stiffness of an oblique spring decreases with an inclination angle due to geometric nonlinearity. This suggests that the resonance frequency of a vibration isolator with buckled beams may also be reduced by inclining the beams since a buckled beam can be regarded as a compressed spring.

In this paper, the effectiveness of inclining the buckled beams in a vibration isolator is examined. Stiffness characteristics of a vibration isolator with inclined as well as vertical beams are investigated and compared. Base excitation tests are also conducted to confirm the reduction in displacement transmissibility. The term "local stiffness" is referred as "dynamic stiffness" in some references. However, the latter is also used for the transfer function between force and displacement [13]. To prevent this confusion, the term "local stiffness" is used throughout in this paper.

2. Effect of beam inclination on stiffness

The arrangement of a vertical vibration isolation system considered in this paper is depicted in figure 1. An isolated mass is supported by two pairs of identical flat beams which are buckled in opposite directions, as shown in figure 1. Stiffness of the buckled beams in the vertical direction plays the role of a spring in conventional vibration isolation systems. The beams are set in the vertical position in figure 1(a) and at an angle θ with respect to the vertical in figure 1(b).

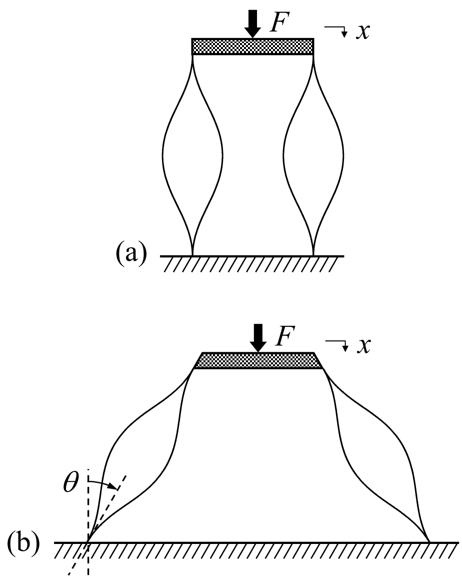

Figure 1. Vertical vibration isolation system consisting of an isolated mass and two pairs of buckled beams. The beams are set (a) in the vertical position and (b) at an angle θ with respect to the vertical.

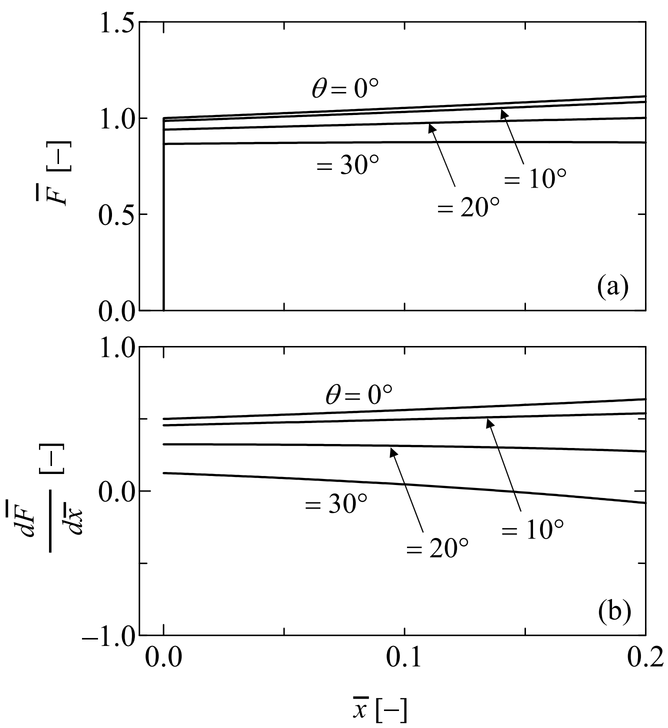

Figure 2. Theoretical (a) force-displacement and (b) stiffness-displacement characteristics of the set of buckled beams shown in figure 1.

The force-displacement relationship of a buckled beam with arbitrary clamp angles at both ends is formulated using elliptic integrals of the first and second kind [1]. The calculated results for the set of inclined beams in figure 1 are plotted in figure 2, where $\bar{x}$ is the vertical deflection of the beams which is taken from the unbuckled position and normalized by the beam length, and $\bar{F}$ is the vertical compressive force normalized by the buckling load for $\theta = 0°$. For supporting a large static load and reducing the natural frequency of the system, $\bar{F}$ should be large whereas $d\bar{F}/d\bar{x}$ should be small.

The figure demonstrates that both $\bar{F}$ and $d\bar{F}/d\bar{x}$ increase with displacement $\bar{x}$ in the case of $\theta = 0°$, and the force-displacement relationship becomes more flattened. This suggests that beam inclination is effective for reducing the local stiffness and consequently the resonance frequency of the vibration isolation system.

In the case of $\theta = 30°$, the value of $d\bar{F}/d\bar{x}$ is negative for $\bar{x} > 0.141$. If the intended equilibrium position of the isolated mass is in this range, an additional spring with appropriate positive stiffness needs to be attached in order to stabilize the system.

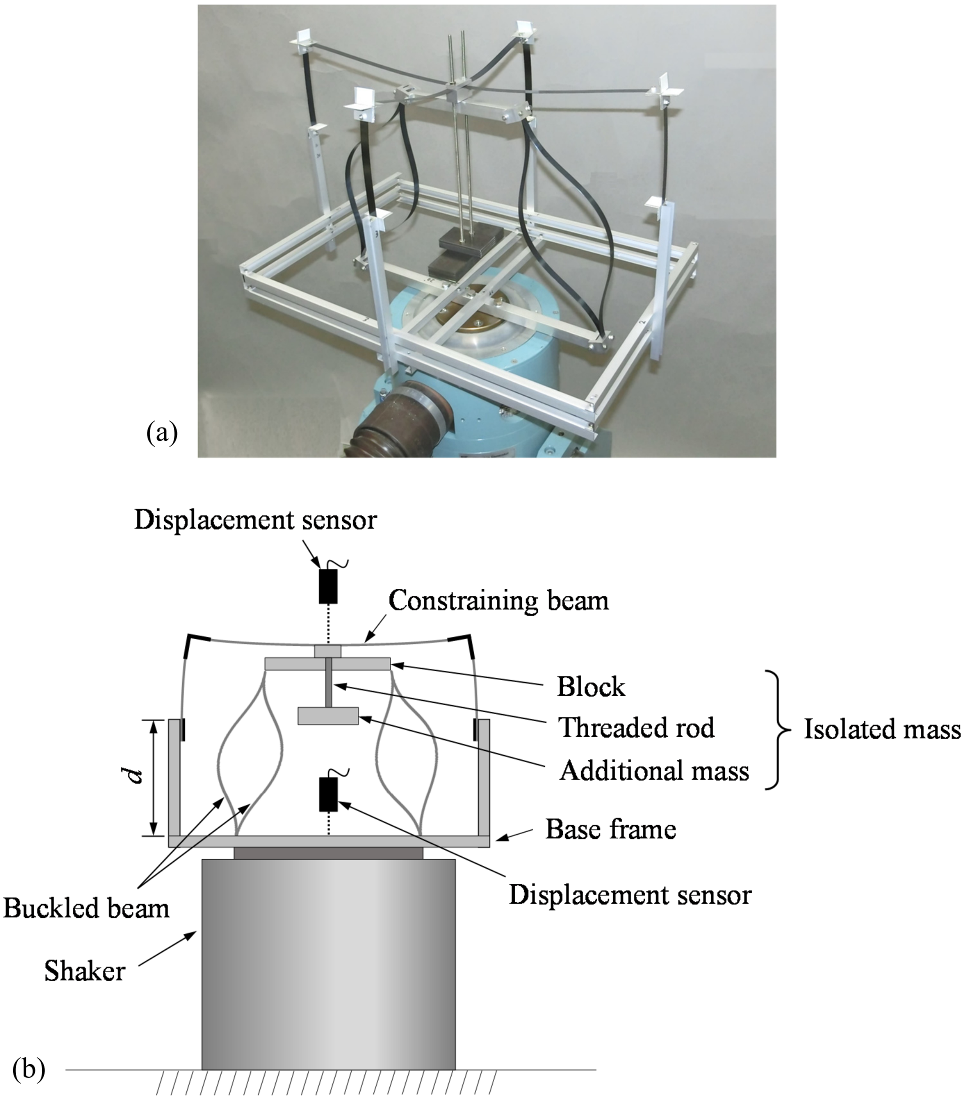

Figure 3. (a) Photograph and (b) schematic of the test rig.

3. Test rig

The test rig is shown in figure 3. The isolated mass consisting of a block, threaded rod and additional mass is supported by a combination of buckled beams and constraining beams. The isolated mass is 904.5 g $(=8.87\ \mathrm{N})$ in total and the size of the buckled beams is $300\ \mathrm{mm}\times 10\ \mathrm{mm}\times 0.3\ \mathrm{mm}$.

The buckled beams can be inclined to $\theta = 10°, 20°$ and $30°$ from the vertical by attachment devices. Note that θ represents the beam inclination before buckling. The constraining beams in the horizontal plane are employed to prevent off-axis motion of the isolated mass. Both types of beams are attached to the base frame which is mounted on a shaker table. To prevent initial deformation of the constraining beams, the height of the base frame, denoted by d in figure 3(b), is adjusted so that the constraining beams become flat when the vertical or inclined beams are not buckled.

To examine the isolation performance in the case when the system behaves in a linear fashion, the base frame was excited by a harmonic motion with small peak-to-peak amplitude of 1 mm. Displacements of the isolated mass and the base frame were measured by laser sensors, and then FFT analyses were conducted to obtain the displacement transmissibility.

4. Results

To confirm the effect of the beam inclination on the stiffness, the force-displacement relationship was investigated experimentally for the inclination angles $\theta = 0°, 10°, 20°$ and $30°$.
First, the relationship was measured in the vicinity of the equilibrium position of the isolated mass by using static loads of $F = 8.81\ \mathrm{N} \sim 8.91\ \mathrm{N}$. The results are shown in figure 4. Here, the displacement for $F = 8.81\ \mathrm{N}$ is set to zero in all cases of θ, and F_0 denotes the static load $(=8.87\ \mathrm{N})$ at the equilibrium position. As shown in the figure, the stiffness about the equilibrium position becomes lower as the beam inclination θ increases. The stiffness values calculated from the slopes of the fitted lines in figure 4 are listed in table 1. The effect of beam inclination is small between $\theta = 0°$ and $10°$, but stiffness decreases rapidly as θ increases to $20°$ and $30°$. Table 2 shows the natural frequency f_n measured from free vibrations for each value of θ. As predicted from table 1, the natural frequency decreases with θ.

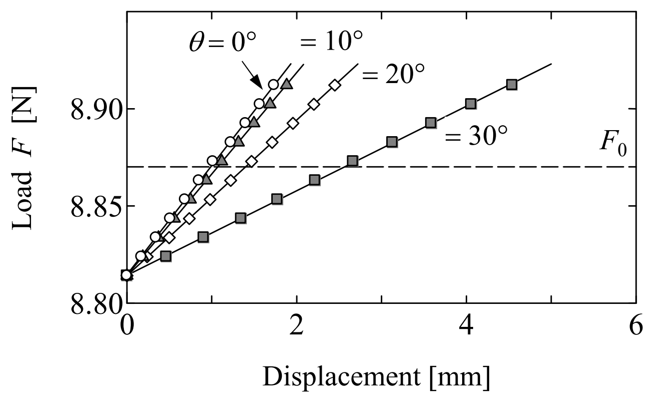

Figure 4. Force-displacement relationship in the vicinity of the equilibrium position of the isolated mass. The symbols ○, ▲, ◇, and ■ represent experimental results for $\theta = 0°, 10°, 20°$ and $30°$, respectively. F_0 denotes the static load $(=8.87\ \mathrm{N})$ at the equilibrium.

Table 1. Stiffness obtained from figure 4.

θ	Stiffness
0°	56.2 N/m
10°	52.2 N/m
20°	40.1 N/m
30°	21.7 N/m

Table 2. Natural frequency measured from free vibration.

θ	Natural frequency f_n
0°	1.26 Hz
10°	1.22 Hz
20°	1.07 Hz
30°	0.78 Hz

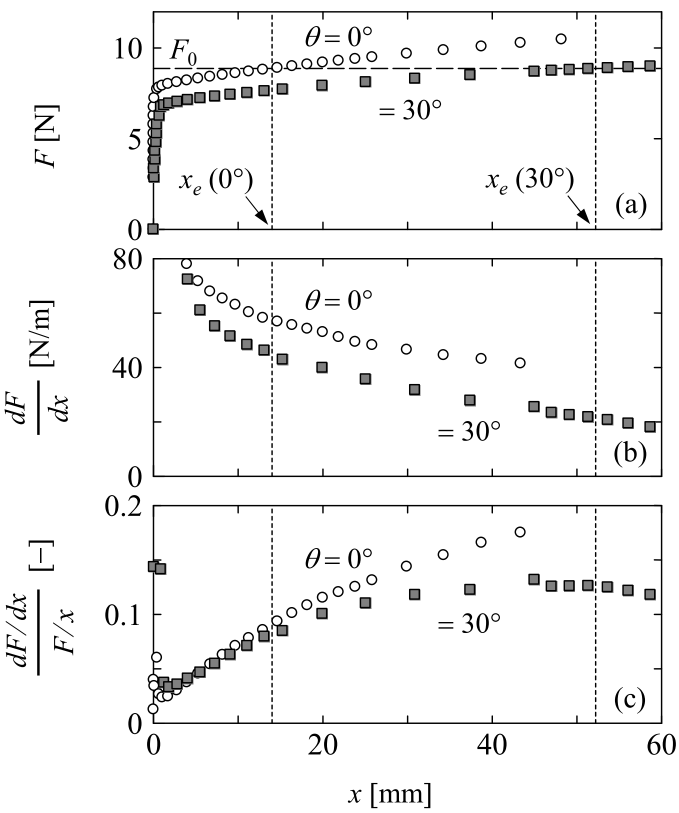

Figure 5. Force-displacement relationship in a wide range of displacement. $x_e(0°)$ and $x_e(30°)$ denote equilibrium positions of the isolated mass for $\theta = 0°$ and $30°$, respectively.

Figure 5 shows the force-displacement and stiffness-displacement relationships as well as the ratio of local stiffness to static stiffness which are measured in a wider range of displacement. Here, x is the displacement of the isolated mass from the unbuckled position. For clarity, only results for $\theta = 0°$ and $30°$ are plotted. Two vertical dotted lines indicate the equilibrium positions, $x_e(0°) = 14.0$ mm and $x_e(30°) = 52.2$ mm, of the isolated mass for each value of θ. Normalized values of these equilibrium positions are given as $\bar{x}_e(0°) = 0.047$ and $\bar{x}_e(30°) = 0.174$, respectively.

The results shown in figures 2 and 5 have the following different features which are due to the effects of the constraining beams in the test rig. Stiffness for $\theta = 30°$ is positive at the equilibrium position

$x = 52.2$ mm in figure 5 but it is negative at $\bar{x} = 0.174$ in figure 2. This means that the positive bending stiffness of the constraining beams counteracts the negative stiffness of the buckled beams at this position. In addition, the stiffness for $\theta = 0°$ decreases with displacement in figure 5 but increases in figure 2. This suggests that the stiffness of the constraining beams decreases with displacement, and therefore, reduction of the total stiffness results from the beam inclination itself and an accompanying reduction of stiffness contributed by the constraining beams.

A vibration isolator with a vertical linear spring has a ratio of local stiffness to static stiffness of 1.0. In contrast, the value of the ratio in figure 5 is of the order of 0.1 in the range of the experiment, which shows the advantage of using buckled beams as a spring component in a vibration isolator. In figure 5, moreover, the ratio is further reduced by beam inclination particularly in the range corresponding to larger static deflection. This suggests that beam inclination is effective for improving vibration isolation performance.

Finally, effectiveness of the beam inclination is examined by the base excitation test. Figure 6 shows the response of the isolated mass to the base frame at an excitation frequency of $f = 3$ Hz. It is seen that the response of the isolated mass for $\theta = 30°$ is much smaller than that for $\theta = 0°$. Table 3 shows displacement transmissibility values which are obtained from the amplitude ratio of the isolated mass to the base frame for the excitation frequency component. Values in parentheses are calculated values for an undamped vibration isolation system which are obtained from the theoretical formula $T = |1-(f/f_n)^2|^{-1}$ and the natural frequencies shown in table 2. It is confirmed that the isolation performance is improved by the beam inclination and the measured transmissibility agrees with the calculated results obtained from the theoretical formula for an undamped system.

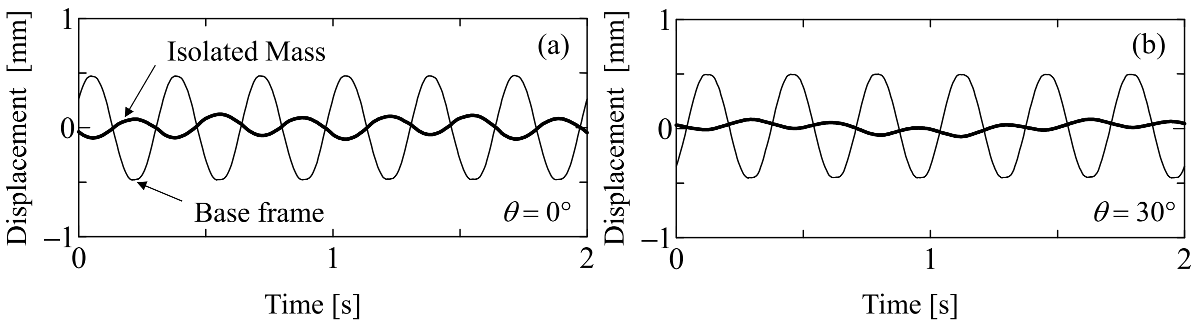

Figure 6. Response of the isolated mass to a base excitation at an excitation frequency $f = 3$ Hz in the case of (a) $\theta = 0°$ and (b) $\theta = 30°$.

Table 3. Measured displacement transmissibility. Values in parentheses are calculated values from the formula $T = |1-(f/f_n)^2|^{-1}$ and the natural frequency f_n shown in table 2.

f	$\theta = 0°$	$\theta = 30°$
1.4 Hz	12 dB (13 dB)	– 8 dB (– 7 dB)
2.0 Hz	– 4 dB (– 4 dB)	– 16 dB (– 15 dB)
3.0 Hz	– 14 dB (– 13 dB)	– 23 dB (– 23 dB)

5. Conclusion

The performance of a passive vibration isolator using buckled beams has been investigated. It has been found from theoretical calculations that inclination of the buckled beams reduces the local stiffness, which suggests that beam inclination is effective for reducing the resonance frequency of the system. An experimental study has also been done to verify the effectiveness. The results have demonstrated that beam inclination reduces the natural frequency and consequently improves vibration isolation performance at frequencies greater than resonance. It has also been found that constraining beams in the experimental rig have a positive but reducing stiffness with respect to the displacement. Improvement of the performance partly results from the stiffness reduction of the constraining beams which is accompanied by the beam inclination.

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