A study on calculation method for mechanical impedance of air spring

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Abstract. This paper proposes an approximate analytic method of obtaining the mechanical impedance of air spring. The sound pressure distribution in cylindrical air spring is calculated based on the linear air wave theory. The influences of different boundary conditions on the acoustic pressure field distribution in cylindrical air spring are analysed. A 1-order ordinary differential matrix equation for the state vector of revolutionary shells under internal pressure is derived based on the non-moment theory of elastic thin shell. Referring to the transfer matrix method, a kind of expanded homogeneous capacity high precision integration method is introduced to solve the non-homogeneous matrix differential equation. Combined the solved stress field of shell with the calculated sound pressure field in air spring under the displacement harmonic excitation, the approximate analytical expression of the input and transfer mechanical impedance for the air spring can be achieved. The numerical simulation with the Comsol Multiphysics software verifies the correctness of theoretical analysis result.

1. Notation

ω	Angular frequency	c	Sound velocity
k	Wave number	ρ	Shell density of air spring
ρ_0	Air density inside the air spring	β_{0m}	0 point sequence of first order Bessel function
E	Elasticity modulus of shell material	μ	Poisson's ratio of shell material
δ	Thickness of shell	N_1、N_2	Tensions of unit length in the direction of longitude and latitude
p	Internal pressure	L	Height of air spring
R	Radius of air spring	U、W	Displacements in the direction of longitude and latitude
ζ_0	Amplitude of displacement harmonic excitation	M	Mass of upper cover plate

2. Introduction

Air spring is a kind of vibration isolation element utilizing compressibility of gases to isolate vibration after gases are filled into flexible closed container. Mechanical impedance is an important parameter describing performance of elastic element, which reflects the relationship between excitation and response of elastic element, and is a main index of vibration isolation design and performance optimization of elastic element[1-2]. The mechanical impedance can be acquired in two ways, i.e.

analytic calculation and experiment test. For the elastic element with homogeneity and regular shape, the approximate expression of mechanical impedance can be acquired through analytic method. The advantage of analytic method is that the corresponding impedance of elastic element can be obtained and optimized through adjusting corresponding parameters qualitatively and quantitatively[3]. However, for air spring, the acquisition of mechanical impedance mainly refers to experiment and analysis on the base of gas state equation generally neglecting the influence of flexible shell[4]. In the theoretical analysis, it is believed that the gas inside air spring is of uniform change and the stiffness of air spring will not change with frequency. Obviously, it is unreasonable. This paper proposes an approximate analytic method of obtaining the mechanical impedance of air spring through solving the sound pressure distribution inside air spring and the shell stress field of air spring. It is of particular importance in the optimization design of air spring vibration isolation.

3. Assumptions and parameters

Before modelling, the assumptions are made as below:

(1) Air inside air spring is viscous gas.

(2) Vibration amplitude is small, that is, the whole system is linear.

(3) Cover plates and other accessories are rigid approximately and have no resonance during the vibration.

(4) Shell is made of homogeneous linear elastic material.

(5) Do not consider the influence of shell deformation on its Lame's coefficients.

(6) Shell of air spring is elastic thin shell, and the bending moment and twisting moment on its cross section are neglected.

Cylindrical air spring model is shown as in Figure 1. The initial pressure inside the air spring is assumed to be 0.3Mpa. Other parameters required by calculation and simulation are shown in Table 1.

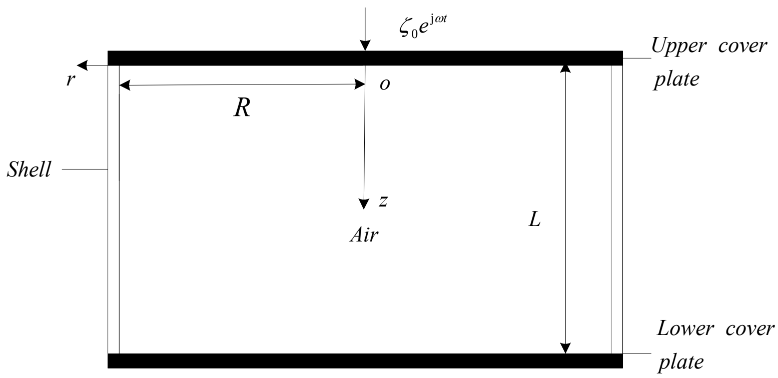

Figure 1. Cylindrical air spring model.

Table 1. Parameters of air spring required by calculation and simulation.

M (kg)	R (m)	L (m)	ρ (kg·m^{-3})	ρ_0 (kg·m^{-3})	c (m·s^{-1})	E (Mpa)	δ (m)	μ	ζ_0 (m)
100	0.165	0.138	1200	3.78	340	30	0.002	0.48	0.001

4. Sound pressure field inside cylindrical air spring

The upper cover plate of air spring suffers from displacement harmonic excitation, as shown in Figure 1, which can lead to the disturbance of the air volume element nearby the upper cover plate. According to the linear air fluctuation theory, the solution of sound pressure equation refers to the fixed value solution of Helmholtz under fixed boundary condition[5]. The expression of Helmholtz equation is shown as below

$$\nabla^2 p + k^2 p = 0 \qquad (k = \omega / c) \tag{1}$$

The shell in Figure 1 is assumed to be flexible approximately, and the boundary conditions for solution of sound pressure equation are shown as below

$$\begin{cases} z=0 & \dot{\zeta}_z=\zeta_0 \\ z=L & \dot{\zeta}_z=0 \\ r=R & p=0 \end{cases} \tag{2}$$

According to equation (1) and (2), sound pressure in cylindrical air spring under the excitation of harmonic displacement can be solved and expressed as

$$p_1=\sum_{m=1}^{\infty}\frac{-2\mathrm{j}\rho_0\omega\dot{\zeta}_0\,\mathrm{J}_0(\alpha_m r)\cosh[Q_m(L-z)]e^{\mathrm{j}\omega t}}{Q_m\beta_{0m}\sinh(LQ_m)\mathrm{J}_1(\beta_{0m})} \tag{3}$$

in which $Q_m=\left[\left(\frac{\beta_{0m}}{R}\right)^2-k^2\right]^{1/2}$, $\alpha_m=\frac{\beta_{0m}}{R}$.

Equation (3) shows that the distribution of sound pressure field in the cylindrical air spring with flexible shell will be effected by boundary conditions, and is a function about its height and radius. For the cylindrical air spring with rigid shell, the vertical velocity at the boundary where $r=R$ is zero[5], that is

$$\dot{V}_r(r,z,t)_{r=R}=0 \tag{4}$$

And then the sound pressure equation (3) can be simplified as

$$p_2=\mathrm{j}\rho_0 c\frac{\dot{\zeta}_0}{\sinh(kL)}\cosh\left[k(L-z)\right]e^{\mathrm{j}\omega t} \tag{5}$$

According to equation (5), it can be seen that the sound pressure is a function only related to height. Applying the Comsol Multiphysics software, the numerical simulations of sound pressure in the cylindrical air spring with rigid and flexible shell are shown in Figure 2 and Figure 3 respectively, which are consistent with the conclusions of theoretical analysis.

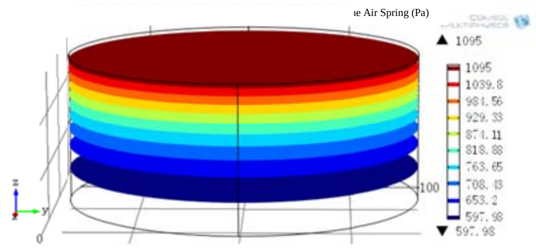

Figure 2. Sound pressure field distribution in cylindrical air spring with rigid shell.

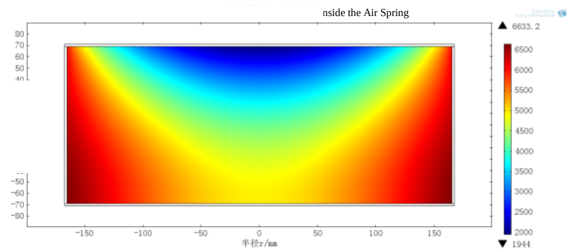

Figure 3. Sound pressure field distribution in cylindrical air spring with flexible shell.

5. Shell stress field of cylindrical air spring

Figure 4 demonstrates the orthogonal curvilinear coordinate system for cylindrical shell element $HH_1H_2H_3$ and its stress analysis, where N_1 and N_2 refer to forces of unit width along α and β directions; k_1 and k_2 are curvatures of cylindrical shell along α and β directions, and equal 0 and $1/R$ respectively; A and B are Lame's coefficients and equal to 1 and R respectively. According to non-moment theory of elastic thin shell, dynamic balance equation of cylindrical shell element $HH_1H_2H_3$ can be obtained as below[6]

$$\begin{cases} \dfrac{\partial N_1}{\partial \alpha} + \rho\delta\omega^2 U = 0 \\ -\dfrac{N_2}{R} + \rho\delta\omega^2 W + p = 0 \end{cases} \tag{6}$$

and its elastic equation can be expressed as[6]

$$\begin{cases} \dfrac{\partial U}{\partial \alpha} = \dfrac{N_1 - \mu N_2}{E\delta} \\ \dfrac{W}{R} = \dfrac{N_2 - \mu N_1}{E\delta} \end{cases} \tag{7}$$

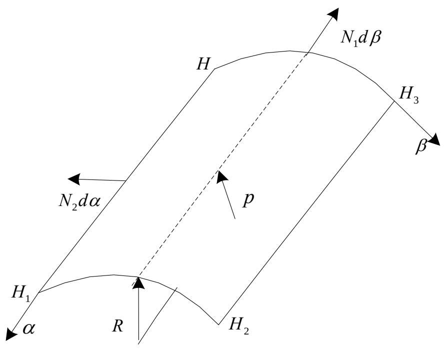

Figure 4. Infinitesimal stress analysis of cylindrical shell.

The simultaneous solution for equation (6) with equation (7) can be expressed as the following matrix form

$$\frac{\partial}{\partial \alpha}\begin{bmatrix} N_1 \\ U \end{bmatrix} = \begin{bmatrix} 0 & -A_0 \\ -B_0 & 0 \end{bmatrix}\begin{bmatrix} N_1 \\ U \end{bmatrix} + \begin{bmatrix} 0 \\ C \end{bmatrix} \tag{8}$$

where $A_0 = \rho\delta\omega^2$, $B_0 = \dfrac{(R\omega\mu)^2\rho - (R\omega)^2\rho + E}{E\delta[(R\omega)^2\rho - E]}$, $C = \dfrac{\mu q_2 R}{\delta(\rho R^2\omega^2 - E)}$.

Set $\{Z\} = \begin{bmatrix} N_1 \\ U \end{bmatrix}$, $[H] = \begin{bmatrix} 0 & -A_0 \\ -B_0 & 0 \end{bmatrix}$, $[Q] = \begin{bmatrix} 0 \\ C \end{bmatrix}$, equation (8) can also be represented as

$$\frac{\partial}{\partial \alpha}\{Z\} = [H]\{Z\} + [Q] \tag{9}$$

Expanding the equation (9), a homogeneous expansion matrix differential equation can be obtained as below

$$\frac{\partial}{\partial \alpha}\begin{bmatrix} N_1 \\ U \\ 1 \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 & -A_0 & 0 & 0 \\ -B_0 & 0 & C & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} N_1 \\ U \\ 1 \\ \alpha \end{bmatrix} \tag{10}$$

Set $\{\tilde{Z}\} = \begin{bmatrix} N_1 \\ U \\ 1 \\ \alpha \end{bmatrix}$, which is the expansion status vector, $[G] = \begin{bmatrix} 0 & -A_0 & 0 & 0 \\ -B_0 & 0 & C & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, which is a constant expansion matrix. Equation (10) can also be expressed as

$$\frac{\partial}{\partial \alpha}\{\tilde{Z}\} = [G]\{\tilde{Z}\} \tag{11}$$

Introducing the transfer matrix $\left[T(\alpha_2 \to \alpha_1)\right]$ along α direction[9], the expansion status vectors of both ends satisfy

$$\left\{\tilde{Z}(\alpha_2)\right\} = [T(\alpha_2 \to \alpha_1)]\left\{\tilde{Z}(\alpha_1)\right\} = [T]\left\{\tilde{Z}(\alpha_1)\right\} \tag{12}$$

where

$$\left[T\right] = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ T_{31} & T_{32} & T_{33} & T_{34} \\ T_{41} & T_{42} & T_{43} & T_{44} \end{bmatrix} \tag{13}$$

According to equation (11) and (12), the transfer matrix $\left[T\right]$ can be solved as the following expression

$$\left[T\right] = \exp\left\{\left[G\right]L\right\} \tag{14}$$

It can be obtained from precise integration method [7] that

$$\exp\left\{[G]L\right\} = I + [G]L + \frac{\left\{[G]L\right\}^2}{2!} \cdots + \frac{\left\{[G]L\right\}^n}{n!} \tag{15}$$

where I is unit matrix.

The relation between the expansion status vectors of both ends along α direction can be expressed as below

$$\begin{bmatrix} N_{12} \\ U_2 \\ 1 \\ \alpha_2 \end{bmatrix} = [T]\begin{bmatrix} N_{11} \\ U_1 \\ 1 \\ \alpha_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & L & 1 \end{bmatrix}\begin{bmatrix} N_{11} \\ U_1 \\ 1 \\ \alpha_1 \end{bmatrix} \tag{16}$$

Equation (16) can also be written as

$$\begin{bmatrix} N_{12} - A_2 \\ N_{11} - A_1 \end{bmatrix} = \begin{bmatrix} \dfrac{T_{11}}{T_{21}} & \dfrac{T_{12}T_{21} - T_{11}T_{22}}{T_{21}} \\ \dfrac{1}{T_{21}} & -\dfrac{T_{22}}{T_{21}} \end{bmatrix}\begin{bmatrix} U_2 \\ U_1 \end{bmatrix} \tag{17}$$

where $A_1 = -\dfrac{T_{23}}{T_{21}} - \dfrac{T_{24}}{T_{21}}\alpha_1$, $A_2 = \dfrac{T_{13}T_{21} - T_{11}T_{23}}{T_{21}} + \dfrac{T_{14}T_{21} - T_{11}T_{24}}{T_{21}}\alpha_1$.

According to equation (17) and the boundary conditions in equation (2), the dynamical force acted on both ends of cylindrical air spring can be derived as below

$$\begin{cases} N_{12} = T_{11}U_2 / T_{21} + A_2 \\ N_{11} = U_2 / T_{21} + A_1 \end{cases} \tag{18}$$

6. Expression of mechanical impedance

According to the definition of mechanical impedance[10], equation (3) and (18), the input and transfer mechanical impedance of cylindrical air spring with one fixed end can be expressed as below

$$\begin{cases} Z_{11} = \mathrm{j}\omega M + \displaystyle\sum_{m=1}^{\infty} \frac{-\mathrm{j}4\pi\rho\omega R^2}{Q_m \beta_{0m}^2 \sinh(LQ_m)} + \frac{2\pi R(T_{11}\zeta / T_{21} + A_2)}{\mathrm{j}\omega\zeta} \\ Z_{12} = \displaystyle\sum_{m=1}^{\infty} \frac{-\mathrm{j}4\pi\rho\omega R^2 \cosh(LQ_m)}{Q_m \beta_{0m}^2 \sinh(LQ_m)} + \frac{2\pi R(\zeta / T_{21} + A_1)}{\mathrm{j}\omega\zeta} \end{cases} \tag{19}$$

7. Simulation Verification

The acoustic structure coupling module in the Comsol Multiphysics software is applied to simulate the internal sound pressure field and shell stress field of cylindrical air spring. Figure 5 shows the theoretical analysis and simulation results which are each other consistent.

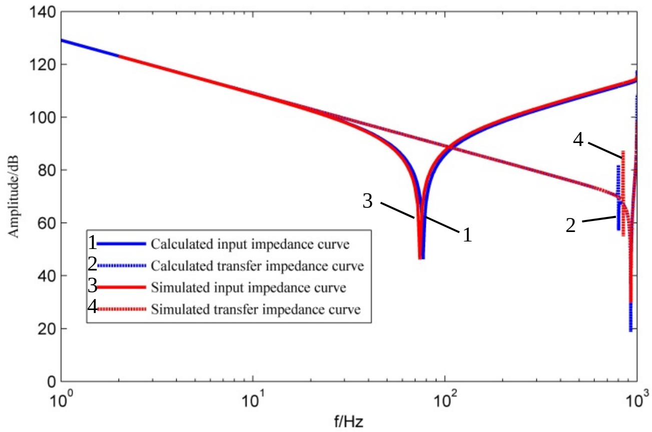

Figure 5. Calculation and simulation impedance curve of cylindrical air spring.

8. Conclusion

This paper proposes an approximate analytic method of obtaining the mechanical impedance of air spring through solving the sound pressure distribution inside air spring and the transfer matrix of shell. Based on theoretical analysis and simulation results, the available conclusions are as follows:

(1) The distribution of sound pressure field in the cylindrical air spring with flexible shell is effected by boundary conditions, which is a function about its height and radius. For the cylindrical air spring with rigid shell, the distribution of inner sound pressure field is only related to height. The simulation and theoretical results are consistent.

(2) A homogeneous expansion matrix differential equation for the state vector of revolutionary shells under internal pressure is derived based on the non-moment theory of elastic thin shell. Referring to the transfer matrix method and the expanded homogeneous capacity high precision integration method, the equation can be solved and the shell stress field of cylindrical air spring is obtained. Combined the stress field of shell with the sound pressure field in air spring under the displacement harmonic excitation, the approximate analytical expression of the input and transfer mechanical impedance for the air spring can be achieved. Although neglecting the air-structure and acoustic-structure interaction, the error between the theoretical analysis and simulation results is small.

Next, the method applied to calculating the mechanical impedance of other types of air spring such as bellow type or diaphragm type will be further developed and verified.

Acknowledgement

This work was funded by China Scholarship Council (CSC), and supported by Program for New Century Excellent Talents in University (NCET) from Chinese Ministry of Education and the National Natural Science Foundation of China within Grant No. 51303209. The supports are greatly appreciated. The authors would like to acknowledge the support of Institute of Sound and Vibration Research, University of Southampton, UK.

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