Novel Subcarrier-Allocation Schemes for Downlink MC DS-CDMA Systems

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***Abstract*—This paper addresses the subcarrier allocation in downlink multicarrier direct-sequence code-division multiple access (MC DS-CDMA) systems, where one subcarrier may be assigned to several users who are then distinguished from each other by their unique direct-sequence spreading codes. We first analyze the advantages and shortcomings of some existing subcarrier-allocation algorithms in the context of the MC DS-CDMA. Then, we generalize the worst subcarrier avoiding (WSA) algorithm to a so-called worst case avoiding (WCA) algorithm, which achieves better performance than the WSA algorithm. Then, the WCA algorithm is further improved to a proposed worst case first (WCF) algorithm. Furthermore, we propose an iterative worst excluding (IWE) algorithm, which can be employed in conjunction with the WSA, WCA, and the WCF algorithms, forming the IWE-WSA, IWE-WCA, and the IWE-WCF subcarrier-allocation algorithms. The complexities of these algorithms are analyzed, showing that they are all low-complexity subcarrier-allocation algorithms. The error performance is investigated and compared, demonstrating that we can now be very close to the optimum performance attained by the high-complexity Hungarian algorithm.**

***Index Terms*—Multicarrier, DS-CDMA, MC DS-CDMA, OFDMA, LTE/LTE-A, resource-allocation, subcarrier-allocation, greedy, complexity.**

I. INTRODUCTION

IN wireless communications, multicarrier signalings have attracted wide attention as one of the promising candidates for high speed broadband wireless communications. In multicarrier systems, multicarrier modulation/demodulation can be implemented with the aid of low-complexity fast Fourier transform (FFT) techniques. When appropriately configurated, some multicarrier schemes, such as orthogonal frequency division multiple access (OFDMA) and orthogonal multicarrier DS-CDMA, employ the capability to suppress inter-symbol interference (ISI) [1], [2]. Furthermore, the multicarrier DS-CDMA (MC DS-CDMA), in which each subcarrier uses direct-sequence (DS) spreading, employs a high number of degrees-of-freedom for high-flexibility design and reconfiguration [2].

It is now well-known that exploiting the time-varying characteristics of wireless channels is capable of significantly enhancing the quality-of-service (QoS) of wireless communication systems. Specifically, with the aid of dynamic subcarrier-allocation to users, promising energy- and spectral-efficiency can be attained by making use of the embedded multiuser diversity [3]. Owing to its above-mentioned metrics, subcarrier-allocation in broadband multicarrier systems, such as in LTE/LTE-A OFDMA, now becomes highly important. In literature, such as in [3]–[10], various subcarrier-allocation algorithms have been proposed and studied for downlink OFDMA systems and other multicarrier systems. Specifically, the (unfair) greedy algorithm has been investigated in [4] without considering the fairness, which aims at maximizing the total sum rate of downlinks. By contrast, in [5], [6], the (fair) greedy algorithm has been studied, when fairness is taken into account, making each user select the best subcarrier(s) from the available subcarriers. However, in terms of reliability, the users allocated the subcarriers at the late stages of the fair greedy algorithm often have poor performance. In order to circumvent the shortcomings of the fair greedy algorithm, in [7], a worst subcarrier avoiding (WSA) algorithm has been proposed for subcarrier-allocation in the downlink OFDMA and frequency division multiple access (FDMA) systems. The studies in [7] demonstrate that the WSA algorithm can effectively avoid assigning users the subcarriers of the poorest channel qualities, and can hence attain higher reliability than the fair greedy algorithm. In subcarrier-allocation, the Hungarian algorithm [11] is recognized the optimum algorithm in the sense of maximum reliability, which has been investigated, for example, in [7], [10]. However, the Hungarian algorithm is of high complexity for implementation in the OFDMA systems with a high number of subcarriers supporting a high number of users.

In LTE/LTE-A downlink OFDMA systems, the number of subcarriers is usually very high, which is up to 2048, and the number of users supported may also be very high. These characteristics generate some problems, such as, the PAPR problem, and may prevent schedulers from employing the optimum or even some promising sub-optimum subcarrier-allocation schemes, due to their complexity constraint. As the complexity of the optimum or sub-optimum subcarrier-allocation algorithms is mainly dependent on the number of subcarriers, reducing the number of subcarriers may effectively decrease the operation complexity of these algorithms. It is well-known that, owing to the employment of DS spreading, the MC DS-CDMA can use a significantly lower number of subcarriers than the multicarrier schemes, such as the OFDMA, which do not employ DS spreading. Furthermore, MC DS-CDMA employs the flexibility to configure its number of subcarriers according to the frequency-selectivity of wireless channels, so that each subcarrier experiences independent fading. In this case, the number of subcarriers of MC DS-CDMA will be at the order of the number of time domain resolvable paths of wireless

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Manuscript received October 10, 2013; revised March 28, 2014; accepted June 24, 2014. The associate editor coordinating the review of this paper and approving it for publication was M. C. Gursoy.

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Digital Object Identifier 10.1109/TWC.2014.2338853

channels and, hence, will usually be low [1]. Therefore, in MC DS-CDMA, the relatively high-complexity optimum or near-optimum subcarrier-allocation algorithms may be employed in order to achieve the best possible performance.

A range of researches [12]–[18] have been dedicated to the field of resource allocation in the MC CDMA and MC DS-CDMA systems. The allocations of transmission rate, subcarrier and power have been considered in MC-CDMA system in [14] for minimizing the total transmission power when given certain bit error rate (BER) requirements. The authors of [16], [17] have compared the capacity performance of the MIMO-OFDMA and MIMO-MC-CDMA systems, when adaptive power allocation is employed. In [13], adaptive allocations of subchannel, power and alphabet size have been addressed in a distributed MC DS-CDMA system, in order to minimize the transmit power under the constraint of packet rate.

Against the background, in this contribution, we study the subcarrier-allocation in MC DS-CDMA systems. First, some representative algorithms, including the greedy-family algorithms, WSA algorithm, etc., are introduced to and studied in association with the MC DS-CDMA systems. Then, a range of subcarrier-allocation algorithms aiming at maximizing the reliability of downlink MC DS-CDMA systems are proposed. Furthermore, we propose a scheme, namely iterative worst excluding (IWE) scheme, which allows the proposed subcarrier-allocation algorithms to achieve even better performance. In this paper, the BER performance of the MC DS-CDMA systems employing various subcarrier-allocation algorithms is investigated, when assuming that subcarrier channels experience independent fading. Our simulation results reveal that the proposed algorithms may significantly outperform the existing suboptimal algorithms. Furthermore, the IWE scheme is effective for further improving the BER performance of some subcarrier-allocation algorithms.

The rest of the paper is organized as follows. Section II introduces the system model and gives the main assumptions. Section III states the principles of the proposed subcarrier-allocation algorithms. Section IV discusses some existing subcarrier-allocation algorithms and details the proposed algorithms. Section V introduces the IWE scheme. Section VI analyzes and compares the complexity of the considered subcarrier-allocation algorithms. Section VII provides the BER results and, at last, conclusions are summarized in Section VIII.

II. SYSTEM MODELS

We consider a downlink MC DS-CDMA system which consists of one base station (BS) communicating with K mobile users. We assume that each of the communicating terminals, including BS and K mobile users, employs one antenna for signal receiving and transmission. Signals transmitted from BS to mobile users are MC DS-CDMA signals using time (T)-domain DS spreading [1] and the spreading factor is expressed as N. For clarity, the variables and notations used in this paper are summarized as follows:

K	Number of mobile users;
$\mathcal{K}$	Set of user indexes, defined as $\mathcal{K} = \{0, 1, \ldots, K-1\}$;
N	Spreading factor of DS spreading;
M	Number of subcarriers of MC DS-CDMA systems;
$\mathcal{M}$	Set of subcarrier indexes, defined as $\mathcal{M} = \{0, 1, \ldots, M-1\}$;
$h_{k,j}$	Channel gain of subcarrier j of user k;
$\boldsymbol{C}$	$(N \times K)$-dimensional spreading matrix with columns consisting of the spreading sequences taken from a $(N \times N)$ orthogonal matrix. Note that, some columns of $\boldsymbol{C}$ may be the same in the case of $K > N$. In this case, the corresponding users are operated on different subcarriers;
$\mathcal{F}_j$	Set of indexes for up to N users assigned to subcarrier j;
$\lvert\mathcal{F}\rvert$	Cardinality of the set $\mathcal{F}$, representing the number of elements in set $\mathcal{F}$;
P_k	Transmission power for user k;
P	Total transmission power of BS, $P = \sum_{k\in\mathcal{K}} P_k$;
$A_{k,j}$	Channel quality of subcarrier j of user k, $A_{k,j} = \lvert h_{k,j}\rvert^2/(2\sigma^2)$, where $\sigma^2 = 1/(2\bar{\gamma}_s)$ denotes the single-dimensional noise power at a mobile user and $\bar{\gamma}_s$ denotes the average signal-to-noise ratio (SNR) per symbol.

In this paper, we assume that each user is allocated one spreading code of one subcarrier. Consequently, we have $\bigcup_{j\in\mathcal{M}} \mathcal{F}_j = \mathcal{K}$, $\mathcal{F}_j \bigcap \mathcal{F}_i = \emptyset$ for $i \neq j$, and there are possibly N users sharing one subcarrier. Let us assume that the data symbols to be transmitted by the BS to the K mobile users are expressed as $\boldsymbol{x} = [x_0, x_1, \ldots, x_{K-1}]^T$, where x_k is the data symbol to user k, which is assumed to satisfy $E[x_k] = 0$ and $E[|x_k|^2] = 1$. Furthermore, let us assume that the j'th subcarrier is assigned to user k. Then, considering that the M subcarriers are orthogonal, the signal received by user k from the j'th subcarrier can be written as

$$\boldsymbol{y}_k = h_{k,j'} \boldsymbol{C}_k \boldsymbol{P} \boldsymbol{W} \boldsymbol{x} + \boldsymbol{n}_k \tag{1}$$

where, in addition to the notations mentioned previously, $\boldsymbol{y}_k$ is a length-N observation vector, $\boldsymbol{n}_k = [n_{k,0}, \ldots, n_{k,N-1}]^T$ is a length-N noise vector at user k, while $\boldsymbol{C}_k$ is a $(N \times K)$ matrix formed from $\boldsymbol{C}$ by setting those columns corresponding to the subcarriers different from the kth user's subcarrier to zero vectors, as the result of using orthogonal subcarriers. In this paper, we assume that uplinks and downlinks are operated in the time-division duplex (TDD) mode. Hence, an uplink channel and its corresponding downlink channel can be assumed to be reciprocal. In this way, the BS is capable of obtaining the knowledge of all the KM downlink channels and, hence, it can preprocess the signals to be transmitted by setting $\boldsymbol{W} = \mathrm{diag}\{w_0, w_1, \ldots, w_{K-1}\}$, where $w_k = h^*_{k,j'}/\sqrt{|h_{k,j'}|^2}$ and $(\cdot)^*$ denotes the conjugate operation. We assume that the channel-inverse power-allocation scheme is employed and, in (1), the power assigned to each user can be expressed in matrix form as $\boldsymbol{P} = \mathrm{diag}\{P_0, P_1, \ldots, P_{K-1}\}$. Consequently, after the despreading for user k using its spreading code $\boldsymbol{c}_k$, the kth column of $\boldsymbol{C}$, it can be shown that the decision variable generated by user k is

$$z_k = P_k \sqrt{|h_{k,j'}|^2} x_k + n_k \tag{2}$$

which yields the SNR $\gamma_k = P_k |h_{k,j'}|^2 \bar{\gamma}_s = P_k A_{k,j'}$. Explicitly, when allocating user k a subcarrier with higher channel quality $A_{k,j'}$, it attains a higher SNR and hence a lower error rate.

Note that the above considered MC DS-CDMA scheme can be straightforwardly extended to the scenarios where each of the users demands multiple data streams depending on the data rate required by the user. In this case, let q_k represent the number of data streams of user k $(k \in \mathcal{K})$. Then, we have the constraint of $\sum_{k\in\mathcal{K}} q_k \leq MN$ on the resource allocation, meaning that the total number of data streams does not exceed MN in order to avoid interference. In this extended MC DS-CDMA system, if $q_k \leq N$, user k can be assigned one subcarrier and its q_k data streams can be supported by assigning the user q_k different spreading codes. By contrast, if $q_k > N$, then, user k may be assigned multiple spreading codes and multiple subcarriers, in order to support the q_k data streams.

Note furthermore that our MC DS-CDMA scheme represents a generalized multicarrier scheme for studying resource allocation. First, when $N = 1$, i.e., when there is no DS spreading, the MC DS-CDMA scheme is reduced to the conventional OFDMA. Correspondingly, we only require subcarrier-allocation, but no code-allocation. Second, when given the total bandwidth of a MC DS-CDMA system, there exists a trade-off between the number of subcarriers M and the spreading factor N, which determines the bandwidth of subcarriers. Hence, in a MC DS-CDMA system, the number of subcarriers can be reconfigured according to the communication environments, so that each of the subcarriers experiences flat fading, while different subcarriers experience relatively independent fading. Specifically, when operated in an environment where fading is highly frequency-selective, the system may be configured with a relatively high number of subcarriers but a relatively low spreading factor, in order to guarantee that all subcarriers experience flat fading. By contrast, when the communication environment becomes less frequency-selective, the system may be reconfigured to use a smaller number of subcarriers but a bigger spreading factor. Owing to the reduced number of subcarriers and the increased bandwidth per subcarrier channel, different subcarriers will experience less correlated fading, the complexity of subcarrier-allocation can be reduced and, furthermore, the PAPR problem can be mitigated.

III. GENERAL THEORY OF RESOURCE ALLOCATION

In the MC DS-CDMA system, where M subcarriers are employed to support K users, when the power- and subcarrier-allocation are aimed to maximize the system reliability, the optimization problem can be described as

$$\begin{aligned}\cup\{\mathcal{F}_j, P_k\}^* &= \arg\min_{\cup\{\mathcal{F}_j,P_k\}}\{\bar{P}_e\}\\ &= \arg\min_{\cup\{\mathcal{F}_j,P_k\}}\left\{\frac{1}{K}\sum_{k\in\mathcal{K}}\bar{P}_e^{(k)}\right\},\\ \text{s.t.}\quad & \cup_{j\in\mathcal{M}}\mathcal{F}_j = \mathcal{K},\ \mathcal{F}_j\cap\mathcal{F}_l=\emptyset \text{ for } j\neq l,\\ & \sum_{k\in\mathcal{K}} P_k = P \end{aligned} \tag{3}$$

where "s.t." stands for "subject to", $\bar{P}_e$ denotes the system's average BER and $\bar{P}_e^{(k)}$ denotes the average BER of user k. In (3), $\cup\{\mathcal{F}_k, P_k\}$ stands for searching all the possible candidates for all users, while $\cup\{\mathcal{F}_k, P_k\}^*$ contain the final results for power- and subcarrier-allocation of all the users. In practice, however, it is often very hard to solve the optimization problem of (3). Since the average BER $\bar{P}_e$ in various of multicarrier communications is usually dominated by the subcarrier with the lowest SNR [8]. Consequently, in some references, such as in [7], [19], [20], power- and subcarrier-allocation algorithms are designed to maximize the minimum SNR of users.

According to [7], [8], power- and subcarrier-allocation can be carried out separately without loss of much performance but having much lower implementation complexity. Therefore, in this contribution, we assume that power- and subcarrier-allocation are executed separately in two steps. Specifically, after subcarrier-allocation, power-allocation is carried out according to the channels of the subcarriers allocated to different users. In this paper, the channel-inverse assisted power-allocation is employed, which has been proved to be optimum in the sense of maximizing the reliability. Under this power-allocation strategy, user k is allocated the power [8]

$$P_k = P\left(\sum_{l=1}^{K} A_l^{-1}\right)^{-1} A_k^{-1}, \quad k\in\mathcal{K} \tag{4}$$

where A_k denotes the channel quality of the subcarrier assigned to user k. After the power-allocation, it can be shown that the SNR of user k is

$$\gamma_k = \gamma_c = P\left(\sum_{l\in\mathcal{K}} A_l^{-1}\right)^{-1}, \quad k\in\mathcal{K} \tag{5}$$

which is independent of the index k, implying that all the users attain the same SNR γ_c and, hence, they also have the same error probability.

From (5) we can know that, in order to maximize the SNR, the subcarrier-allocation algorithms should be designed aiming to maximize $\left(\sum_{l\in\mathcal{K}} A_l^{-1}\right)^{-1}$, yielding the optimization problem

$$\begin{aligned}\cup\{\mathcal{F}_j\}^* &= \arg\max_{\cup\{\mathcal{F}_j\}}\left\{\left(\sum_{l\in\mathcal{K}} A_l^{-1}\right)^{-1}\right\},\\ \text{s.t.}\quad & \cup_{j\in\mathcal{M}}\mathcal{F}_j=\mathcal{K},\ \mathcal{F}_j\cap\mathcal{F}_l=\emptyset \text{ for } j\neq l. \end{aligned} \tag{6}$$

To solve the above optimization problem, exhaustive search may be carried out, which however has extremely high complexity and prevents the algorithm from practical implementation, when the number of subcarriers and the number of users are relatively high. In literature, the Hungarian algorithm [11] is aimed to solve the optimization problem of (6) with lower complexity than the exhaustive search. However, its complexity is still too high for practical implementation, especially, when there are a large number of subcarriers supporting many users, which is usually the case in LTE/LTE-A systems.

In order to minimize the complexity, in this contribution, we focus on the sub-optimum algorithms, which motivate to

TABLE I
CHANNEL QUALITY MATRIX FOR $K = 8$ USERS OF $M = 4$ SUBCARRIERS

S \ U	0	1	2	3	4	5	6	7
0	3.73	4.95	5.06	0.34	2.37	5.04	1.59	3.42
1	1.39	2.01	0.52	4.71	5.02	8.32	10.60	2.12
2	0.41	1.63	4.52	0.87	0.91	3.50	2.49	0.65
3	2.13	5.07	4.57	2.55	3.22	0.49	1.20	0.02

maximize the SNR by maximizing the worst channel quality of the subcarriers allocated to the users, as suggested by the study in [7]. This is because, according to (6), the value of $\left(\sum_{l\in\mathcal{K}} A_l^{-1}\right)^{-1}$ is mainly determined by the minimum of $\{A_0, A_1, \ldots, A_{K-1}\}$. Correspondingly, the optimization problem can be stated as

$$\begin{aligned} \cup\{\mathcal{F}_j\}^* &= \arg\max_{\cup\{\mathcal{F}_j\}} \left\{\min_{l\in\mathcal{K}}\{A_l\}\right\}, \\ \text{s.t.} \quad & \cup_{j\in\mathcal{M}} \mathcal{F}_j = \mathcal{K},\ \mathcal{F}_j \cap \mathcal{F}_l = \emptyset \text{ for } j \neq l. \end{aligned} \tag{7}$$

Note that, the WSA algorithm in [7] has been designed to solve the optimization problem of (7) for the downlink OFDMA system. As our studies and performance results show, our proposed subcarrier-allocation algorithms, including the WCA, WCF, IWE-WCA as well as the IWE-WCF algorithms, are capable of finding better solutions for subcarrier-allocation and achieving better error performance than the WSA algorithm.

Note additionally that, in principle, the subcarrier-allocation algorithms proposed in this paper as well as the WSA algorithm [7] all belong to the greedy family, which motivate to attain high throughput. Our algorithms can maintain all the merits of the conventional greedy algorithm [5], while circumventing its disadvantage of low reliability. This is because our algorithms aim to maximize the reliability via maximizing the achievable SNR. Therefore, they do not generate a trade-off on the throughput, since throughput is an increasing function of SNR.

IV. SUBCARRIER-ALLOCATION ALGORITHMS

In this section, we first review the principles of two representative low-complexity subcarrier-allocation algorithms, namely the greedy algorithm and the WSA algorithm. Their advantages and drawbacks are analyzed, against which a range of subcarrier-allocation algorithms are proposed and investigated. Along with our analysis, an example is introduced, which employs $M = 4$ subcarriers to support $K = 8$ mobile users. Therefore, each subcarrier can be assigned to two users, which are distinguished by their DS spreading codes of length $N = 2$. In this example, the channel qualities corresponding to the four subcarriers of the eight users are illustrated in Table I, where the first row and first column denote the user indexes and subcarrier indexes, respectively. Furthermore, the total transmission power $P = 1$ is assumed for the example considered. From the above discussion, we can realize that the main difference between the subcarrier-allocation in OFDMA systems and that in MC DS-CDMA systems is that one subcarrier is only assigned to one user in the OFDMA systems, while one subcarrier may be assigned to multiple users in the MC DS-CDMA systems. Let us first consider the greedy algorithm.

A. Greedy Algorithm

In the context of the greedy algorithm [5], a subcarrier is always allocated to the two users (in contrast to one in OFDMA) having the best channel qualities among the users still requiring subcarriers. For the example considered, the subcarrier-allocation is carried out one by one from the first subcarrier to the last. Specifically, subcarrier 0 is allocated to users 2 and 5, as they have the two highest channel qualities on subcarrier 0 among the eight users. Hence, the allocation set for subcarrier 0 is updated to $\mathcal{F}_0 = \{2, 5\}$. Similarly, subcarrier 1 is allocated to users 4 and 6, as they have the best channel qualities among the remaining users for this subcarrier, yielding $\mathcal{F}_1 = \{4, 6\}$. Similarly, we can obtain $\mathcal{F}_2 = \{1, 3\}$ and $\mathcal{F}_3 = \{0, 7\}$. According to the allocation results and (5), it can be shown that the attainable SNR is given by $\gamma_c = \left(\sum_{k\in\mathcal{F}_j} A_k^{-1}\right)^{-1} = 0.019$, while the worst (minimum) channel quality of the allocated subcarriers is $\min_{k\in\{\mathcal{F}_j\}}\{A_{k,j}\} = 0.02$, which dominates the attainable SNR and hence the achievable error performance.

Explicitly, the greedy algorithm has the advantage of low-complexity. However, at the later stages of allocation, the algorithm may have to assign users the subcarriers with very poor channel qualities, as there are no other options. As the above example shows, at the last stage, subcarrier 3 has to be allocated to user 7, which results in the poorest channel quality of $A_{7,3} = 0.02$.

B. Worst Subcarrier Avoiding Algorithm

The WSA algorithm is designed to avoid assigning users the subcarriers having the worst channel qualities [7]. With the aid of the example of Table I, the principles of the WSA algorithm can be illustrated as follows.

Firstly, for each of the subcarriers, the worst channel quality is identified, denoted by bold font in (8). It can be readily known that the worst channel qualities corresponding to the four subcarriers are $A_0^{(\min)} = 0.34$ for subcarrier 0, $A_1^{(\min)} = 0.52$ for subcarrier 1, $A_2^{(\min)} = 0.41$ for subcarrier 2 and $A_3^{(\min)} = 0.02$ for subcarrier 3. Secondly, the subcarriers are arranged in the ascending order as {3,0,2,1} according to their worst channel qualities, forming a matrix shown as

$$\left[\begin{array}{ccccccccc} & \text{U0} & \text{U1} & \text{U2} & \text{U3} & \text{U4} & \text{U5} & \text{U6} & \text{U7} \\ \text{S3} & 2.13 & \underline{5.07} & \underline{4.57} & 2.55 & 3.22 & 0.49 & 1.20 & \mathbf{0.02} \\ \text{S0} & \underline{3.73} & 4.95 & 5.06 & \mathbf{0.34} & 2.37 & \underline{5.04} & 1.59 & 3.42 \\ \text{S2} & \mathbf{0.41} & 1.63 & 4.52 & 0.87 & \underline{0.91} & 3.50 & \underline{2.49} & 0.65 \\ \text{S1} & 1.39 & 2.01 & \mathbf{0.52} & \underline{4.71} & 5.02 & 8.32 & 10.60 & \underline{2.12} \end{array}\right] \tag{8}$$

where, again, the worst channel qualities are represented by boldface values. Finally, based on the above-derived matrix, the subcarriers are allocated to the eight users in the principles of the greedy algorithm, as discussed in Section IV-A, from the first row to the last row, yielding the allocation results $\mathcal{F}_0 = \{0, 5\}$, $\mathcal{F}_1 = \{3, 7\}$, $\mathcal{F}_2 = \{4, 6\}$, and $\mathcal{F}_3 = \{1, 2\}$, corresponding to the underlined numbers in (8). With the aid of (5), the attainable SNR is evaluated to be $\gamma_c = \left(\sum_{k\in\mathcal{F}_j} A_k\right)^{-1} = 0.29$, when assuming the total transmission power $P = 1$. Furthermore, from (8) we can know that the worst channel quality of the allocated subcarriers is $\min_{k\in\{\mathcal{F}_j\}}\{A_{k,j}\} = 0.91$. Explicitly, the WSA algorithm significantly improves both the worst channel quality and the attainable SNR per subcarrier, in comparison with that obtained by the greedy algorithm. Owing to the above, the WSA algorithm is expected to achieve better error performance than the greedy algorithm [7].

C. *Worst Case Avoiding Algorithm*

From the analysis in Section IV-B, we may classify the WSA algorithm as a subcarrier-oriented WSA algorithm, which is capable of avoiding assigning the $(M-1)$ worst channels when there are in total M subcarriers [7]. Specifically, for the considered example, the WSA algorithm can guarantee not to assign the three worst channels and, in most cases, the four worst can be avoided. In the MC DS-CDMA systems where the number of users is more than the number of subcarriers, in order to achieve better error performance, the subcarrier-allocation may be operated in the user-oriented mode, which may avoid assigning more of the worst channels. Inspired by the observation, in this subsection, we generalize the WSA algorithm to a so-called worst case avoiding (WCA) algorithm, the principles of which is first illustrated below.

When the WCA algorithm is employed, it always tries to avoid as many as possible the worst channels. The WCA algorithm is operated either in the subcarrier-oriented mode, i.e., WSA, or in the user-oriented mode. Specifically, for the example considered, as the number of users is higher than the number of subcarriers, the user-oriented mode will avoid a higher number of worst channels than the subcarrier-oriented WSA algorithm. In this case, the WCA algorithm first arranges the users in an ascending order of {7,3,0,5,2,4,6,1} according to their worst channel qualities of four subcarriers, yielding

$$\begin{bmatrix} & \mathrm{U7} & \mathrm{U3} & \mathrm{U0} & \mathrm{U5} & \mathrm{U2} & \mathrm{U4} & \mathrm{U6} & \mathrm{U1} \\ \mathrm{S0} & \underline{3.42} & \mathbf{0.34} & \underline{3.73} & 5.04 & 5.06 & 2.37 & 1.59 & 4.95 \\ \mathrm{S1} & 2.12 & \underline{4.71} & 1.39 & \underline{8.32} & \mathbf{0.52} & 5.02 & 10.60 & 2.01 \\ \mathrm{S2} & 0.65 & 0.87 & \mathbf{0.41} & 3.50 & 4.52 & \mathbf{0.91} & \underline{2.49} & \underline{\mathbf{1.63}} \\ \mathrm{S3} & \mathbf{0.02} & 2.55 & 2.13 & \mathbf{0.49} & \underline{4.57} & \underline{3.22} & \mathbf{1.20} & 5.07 \end{bmatrix} \quad (9)$$

In (9) the channel qualities in boldface are the worst channel qualities of the users. Then, based on the ordered matrix (9), the subcarrier-allocation is carried out based on the greedy algorithm, one user at a stage, from the first to the last column. Consequently, the allocation results are $\mathcal{F}_0 = \{0, 7\}$, $\mathcal{F}_1 = \{3, 5\}$, $\mathcal{F}_2 = \{1, 6\}$, and $\mathcal{F}_3 = \{2, 4\}$. It can be shown that the SNR achieved by the WCA algorithm is $\gamma_c = 0.41$, and the worst channel quality of the allocated subcarriers is $\min_{k\in\{\mathcal{F}_j\}}\{A_{k,j}\} = 1.63$.

Straightforwardly, the proposed WCA algorithm is capable of achieving better allocation results than the WSA algorithm, as the WSA is a special case of the WCA. For the considered example, both the worst channel quality and the achievable SNR are improved in comparison with that obtained by the WSA algorithm. Furthermore, it can be shown that the WCA algorithm is capable of preventing allocating at least $\max\{K - N, M - 1\}$ worst channels, instead of at least $(M-1)$ of the WSA algorithm.

In summary, the WCA algorithm can be stated as follows.

Algorithm 1: (**Worst Case Avoiding Algorithm**)

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Initialization
Subcarrier-oriented mode is chosen when $M \geq K$, otherwise, user-oriented mode is selected when $M < K$. Set $\tilde{\mathcal{M}} = \mathcal{M}$, $\tilde{\mathcal{K}} = \mathcal{K}$.
1) Worst channel quality identification
    User-oriented mode—Find each user's worst channel quality: $A_k^{(\min)} = \min_{j\in\mathcal{M}}\{A_{k,j}\}$.
    Subcarrier-oriented mode—Find each subcarrier's worst channel quality: $A_j^{(\min)} = \min_{k\in\mathcal{K}}\{A_{k,j}\}$.
2) User (or Subcarrier) ordering
    User-oriented mode—Arrange users in ascending order according to the worst channel qualities as $\{i_0, i_1, \ldots, i_{K-1}\}$, if $A_{i_0}^{(\min)} \leq A_{i_1}^{(\min)} \leq \cdots \leq A_{i_{K-1}}^{(\min)}$.
    Subcarrier-oriented mode—Arrange subcarriers in ascending order according to the worst channel qualities as $\{q_0, q_1, \ldots, q_{M-1}\}$, if $A_{q_0}^{(\min)} \leq A_{q_1}^{(\min)} \leq \cdots \leq A_{q_{M-1}}^{(\min)}$.
3) Allocation
    Based on the above-derived order, subcarrier-allocation is carried out one-by-one:
    User-oriented mode—First, at the $i_k$th stage, subcarrier $j^*$ is allocated to user $i_k$: $j^* = \arg\max_{j\in\tilde{\mathcal{M}}}\{A_{i_k,j}\}$, $i_k \in \mathcal{K}$. Then, if subcarrier $j^*$ has been assigned to $N = K/M$ users, it is removed from $\tilde{\mathcal{M}}$: $\tilde{\mathcal{M}} \leftarrow \tilde{\mathcal{M}} - \{j^*\}$.
    Subcarrier-oriented mode—First, at the $q_m$th stage, user $k^*$ is allocated to subcarrier $q_m$: $k^* = \arg\max_{k\in\tilde{\mathcal{K}}}\{A_{k,q_m}\}$, $q_m \in \mathcal{M}$. Then, if user $k^*$ has been assigned the required number of subcarriers, it is deleted from $\tilde{\mathcal{K}}$: $\tilde{\mathcal{K}} \leftarrow \tilde{\mathcal{K}} - \{k^*\}$.
```

D. *Worst Case First Algorithm*

According to the WCA algorithm described in Section IV-C, as the example shows, user 2 is allocated the subcarrier at the fifth stage, as its worst channel quality is $A_{2,1} = 0.52$, which is the fifth worst of the users. However, from (9) we observe that subcarriers 0 and 1 cannot be the options for user 2, as each of these two subcarriers has been assigned to two users. In this case, the worst channel quality of user 2's available subcarriers becomes $A_{2,2} = 4.52$, which is much larger than that of users 4, 6, and 1's available subcarriers (which are 0.91, 1.2, and 1.63, respectively). Therefore, in order to maximize the system's reliability, it would be beneficial to allocate the subcarriers to users 4, 6, and 1 before assigning the subcarrier to user 2.

Based on the above observation, we propose the WCF algorithm, which re-order the users (or subcarriers) according to the worst channel qualities of the available subcarriers (users). Specifically for the MC DS-CDMA with $K > M$, during each stage, the algorithm first finds the worst channel quality of the unassigned users among only the subcarriers available for allocation, rather than finding the worst channel quality of the unsigned users among all the subcarriers, as done by the WCA algorithm. In detail, for the example considered, the WCF algorithm completes the allocation user by user in 8 stages, which can be demonstrated as

$$\left[\begin{array}{ccccccccc} & \text{U7} & \text{U3} & \text{U0} & \text{U5} & \text{U4} & \text{U6} & \text{U1} & \text{U2} \\ \text{S0} & \underline{3.42} & \mathbf{0.34} & \underline{3.73} & 5.04 & 2.37 & 1.59 & 4.95 & 5.06 \\ \text{S1} & 2.12 & \underline{4.71} & 1.39 & \underline{8.32} & 5.02 & 10.60 & 2.01 & 0.52 \\ \text{S2} & 0.65 & 0.87 & \mathbf{0.41} & 3.50 & \mathbf{0.91} & \underline{2.49} & \mathbf{1.63} & \underline{\mathbf{4.52}} \\ \text{S3} & \mathbf{0.02} & 2.55 & 2.13 & \mathbf{0.49} & \underline{3.22} & \mathbf{1.20} & \underline{5.07} & 4.57 \end{array}\right] \tag{10}$$

where the eight columns stand for the eight stages of allocation, the channel qualities in boldface are the minimum of the users' channel qualities of the available subcarriers at the eight stages. As shown in (10), at the first stage, the eight users' worst channel qualities of the subcarriers are the same as those in boldface in (9). In this case, user 7 ($A_{7,3} = 0.02$) is the worst and it is first assigned subcarrier 0 with the best channel quality of 3.42 among the four subcarriers. Similarly, as seen in (10), users 3, 0, and 5 are assigned subcarriers 1, 0, and 1, respectively, during the second, third and fourth stages. At this moment, we can see from (10) that the worst channel qualities of the available subcarriers for the four remaining users are $A_{1,2} = 1.63$ for user 1, $A_{2,2} = 4.52$ for user 2, $A_{4,2} = 0.91$ for user 4 and $A_{6,3} = 1.20$ for user 6, respectively. As we can see, the worst channel quality of the subcarriers available to user 2 becomes $A_{2,2} = 4.52$ instead of $A_{2,1} = 0.52$, as subcarrier 1 (also subcarrier 0) has already been assigned to two users in the previous four stages and cannot be assigned to other users. Therefore, at the fifth stage, a subcarrier is assigned to user 4, which is subcarrier 3. Similarly, subcarriers can be assigned to users 6, 1, and 2. From (10) we can know that the final allocation results are $\mathcal{F}_0 = \{0, 7\}$, $\mathcal{F}_1 = \{3, 5\}$, $\mathcal{F}_2 = \{2, 6\}$, and $\mathcal{F}_3 = \{1, 4\}$. The achievable SNR of the system is $\gamma_c = 0.49$ and the worst channel quality of the assigned subcarriers is $\min_{k\in\{\mathcal{F}_j\}}\{A_{k,j}\} = 2.49$.

In comparison with the WCA algorithm, as shown in Section IV-C, user 1 is forced to select subcarrier 2 at the last stage, which results in the poorest channel quality of $A_{1,2} = 1.63$. By contrast, under the WCF algorithm, user 1 has two options to choose either subcarrier 2 or subcarrier 3 at the seventh stage, and is then assigned the better subcarrier 3, which results in a channel quality of $A_{1,3} = 5.07$, which is significantly higher than $A_{1,2} = 1.63$ obtained by the WCA algorithm.

When comparing the WCF the WCA, it is not hard to know that the WCF algorithm is capable of yielding the highest achievable SNR as well as the highest worst channel quality, as demonstrated by the above example. As the above example shows, the WCF algorithm successfully avoids assigning the worst channel quality by preventing the unreasonable allocation for user 2 at the fifth stage by the WCA algorithm. Therefore, the proposed WCF algorithm provides a more reliable and efficient way of subcarrier-allocation, while simultaneously captures all the advantages of the WCA algorithm. In summary, the WCF algorithm is stated as:

```
Algorithm 2: (Worst Case First Algorithm)
Initialization
    User-oriented mode is chosen when $M < K$, subcarrier-oriented mode is used when $M \geq K$. Set $\tilde{\mathcal{K}} = \mathcal{K}$, $\tilde{\mathcal{M}} = \mathcal{M}$. Set $\mathcal{F}_j = \emptyset$ for all $j \in \mathcal{M}$.
Repeat
    1) User-oriented mode—Identify the worst channel quality of each user: $A_k^{(\min)} = \min_{j\in\tilde{\mathcal{M}}}\{A_{k,j}\}$, for all $k \in \tilde{\mathcal{K}}$.
       Subcarrier-oriented mode—Identify the worst channel quality of each subcarrier: $A_j^{(\min)} = \min_{k\in\tilde{\mathcal{K}}}\{A_{k,j}\}$, for all $j \in \tilde{\mathcal{M}}$.
    2) User-oriented mode—Find the user with the minimum of the worst channel qualities: $k^* = \arg\min_{k\in\tilde{\mathcal{K}}}\{A_k^{(\min)}\}$.
       Subcarrier-oriented mode—Find the subcarrier with the minimum of the worst channel qualities: $j^* = \arg\min_{j\in\tilde{\mathcal{M}}}\{A_j^{(\min)}\}$.
    3) User-oriented mode—Assign user $k^*$ the subcarrier with the best channel quality: $q' = \arg\max_{q\in\tilde{\mathcal{M}}}\{A_{k^*,q}\}$, then $\mathcal{F}_{q'} \leftarrow \mathcal{F}_{q'} \bigcup\{k^*\}$.
       Subcarrier-oriented mode—Allocate subcarrier $j^*$ to the user with the best channel quality: $i' = \arg\max_{i\in\tilde{\mathcal{K}}}\{A_{i,j^*}\}$, then $\mathcal{F}_{j^*} \leftarrow \mathcal{F}_{j^*} \bigcup\{i'\}$.
    4) User-oriented mode—Remove user $k^*$ from $\tilde{\mathcal{K}}$: $\tilde{\mathcal{K}} \leftarrow \tilde{\mathcal{K}} - \{k^*\}$. Remove subcarrier $q'$ from $\tilde{\mathcal{M}}$ if $|\mathcal{F}_{q'}| = N$: $\tilde{\mathcal{M}} \leftarrow \tilde{\mathcal{M}} - \{q'\}$.
       Subcarrier-oriented mode—Remove subcarrier $j^*$ from $\tilde{\mathcal{M}}$: $\tilde{\mathcal{M}} \leftarrow \tilde{\mathcal{M}} - \{j^*\}$. Remove user $i'$ from $\tilde{\mathcal{K}}$ if it has been assigned the required number of subcarriers: $\tilde{\mathcal{K}} \leftarrow \tilde{\mathcal{K}} - \{i'\}$.
    5) Stop if $\tilde{\mathcal{K}} = \emptyset$, or $\tilde{\mathcal{M}} = \emptyset$.
```

V. ITERATIVE WORST EXCLUDING ALGORITHMS

In this section, we propose a general algorithm called as the iterative worst excluding (IWE), which can be employed in associated with various of subcarrier-allocation algorithms, such as those considered in the previous sections. With the aid of the IWE algorithm, the error rate performance of subcarrier-allocation algorithms may achieve further improvement. Let us first illustrate the principles of the IWE algorithm.

A. *Iterative Worst Excluding Algorithm*

As the name suggests, the proposed IWE algorithm aims to achieve an improved BER performance by iteratively updating the associated channel quality matrix. During each iteration, the IWE algorithm removes the worst channel qualities of the candidate subcarriers or the candidate users, before carrying out the subcarrier-allocation. After the subcarrier-allocation at an iteration, the allocation results obtained are compared with those obtained from the last iteration, in order to observe whether any performance improvement is gained. If there is performance gain, the algorithm continues to the next iteration. Finally, the algorithm stops, when there is no further performance improvement. In the followings, we demonstrate the principles of the IWE algorithm in conjunction with the WCF subcarrier-allocation algorithm, which can be referred to as the IWE-WCF algorithm. Furthermore, we compare the IWE-WCF algorithm with the other algorithms proposed in the previous sections.

In the context of the IWE-WCF algorithm, the WCF algorithm is first carried out based on the channel quality matrix given in Table I during the first (initial) iteration.

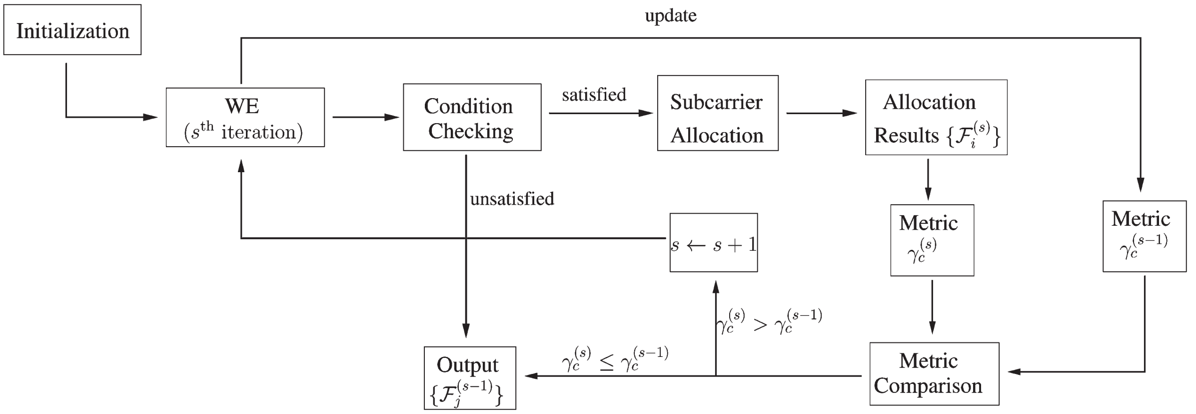


Fig. 1. Flowchart showing the steps of the IWE algorithm.

Correspondingly, the allocation results are given in the Section IV-D and the attainable SNR is $\gamma_c^{(1)} = 0.49$, where the superscript of (1) indicates the first iteration. At the second iteration, the worst channel qualities of the eight users are eliminated before operating again the WCF algorithm, in order to avoid assigning them to users. More specifically, the process of the second iteration can be shown with the aid of (11)

$$\left[\begin{array}{ccccccccc} & \text{U7} & \text{U3} & \text{U0} & \text{U6} & \text{U4} & \text{U5} & \text{U2} & \text{U1} \\ \text{S0} & \underline{3.42} & \times & \underline{3.73} & \mathbf{1.59} & 2.37 & 5.04 & 5.06 & 4.95 \\ \text{S1} & 2.12 & \underline{4.71} & \mathbf{1.39} & \underline{10.60} & 5.02 & 8.32 & \times & 2.01 \\ \text{S2} & \mathbf{0.65} & \mathbf{0.87} & \times & 2.49 & \times & \underline{\mathbf{3.50}} & \underline{\mathbf{4.52}} & \times \\ \text{S3} & \times & 2.55 & 2.13 & \times & \underline{\mathbf{3.22}} & \times & 4.57 & \underline{\mathbf{5.07}} \end{array}\right] \qquad 2^{\text{nd}}\ \text{iteration} \tag{11}$$

where "×" stands for the worst channel quality of an user which is removed before the subcarrier-allocation, referred to as worst excluding (WE). After the WE, we can see in (11) that subcarrier 0 can be allocated to any of the remaining 7 users. We define these 7 users as the candidate users of subcarrier 0, expressed as $\tilde{\mathcal{F}}_0 = \{0, 1, 2, 4, 5, 6, 7\}$. Simultaneously, we can see that subcarrier 1 also has 7 candidate users. However, both subcarrier 2 and 3 have only five candidate users.

Following the WE process, the algorithm carries out the condition checking, in order to know whether the subcarrier-allocation can be completed based on the updated channel quality matrix. In order to fulfill the allocation, two conditions have to be met. Otherwise, the following subcarrier-allocation will not be carried out and the algorithm stops. In detail, the two conditions are as follows.

Condition (a): The number of candidate users of each subcarrier exceeds, K/M, of the number of users to be assigned to one subcarrier. This condition can be expressed as

$$|\tilde{\mathcal{F}}_j| \ge K/M, \quad \forall\, j \in \mathcal{M}. \tag{12}$$

Condition (b): Each subcarrier can only be assigned to K/M different users and each user is only assigned one subcarrier, which can be expressed as

$$|\tilde{\mathcal{F}}_j \cup \tilde{\mathcal{F}}_q| \ge 2K/M, \quad j \ne q, \quad \forall\, j,\, q \in \mathcal{M}. \tag{13}$$

Specifically, for the example considered, we can observe from the updated matrix in (11) that the above two conditions can be met. Thus, it guarantees that each subcarrier can be allocated to two different users and each user attains one subcarrier. Therefore, we can proceed the WCF algorithm based on the updated matrix of (11). This process can also be shown with the aid of (11), where the boldface value under each user is the worst channel quality among the remaining users. Upon following the principles of the WCF algorithm, the new allocation results can be obtained, which are shown by the underlined values in (11). The results are $\mathcal{F}_0^{(2)} = \{0, 7\}$, $\mathcal{F}_1^{(2)} = \{3, 6\}$, $\mathcal{F}_2^{(2)} = \{2, 5\}$, and $\mathcal{F}_3^{(2)} = \{1, 4\}$. It can be shown that the achievable SNR of the system is $\gamma_c^{(2)} = 0.53$, while the worst channel quality of the allocated subcarriers is $\min_{k \in \{\mathcal{F}_j^{(2)}\}}\{A_{k,j}\} = 3.42$.

From the results of the second iteration, we can see that both the SNR and the worst channel quality are improved in comparison with those obtained from the first iteration. Therefore, the IWE-WCF algorithm continues to the third iteration, and the WE process is again first carried out, yielding

$$\left[\begin{array}{ccccccccc} & \text{U0} & \text{U1} & \text{U2} & \text{U3} & \text{U4} & \text{U5} & \text{U6} & \text{U7} \\ \text{S0} & 3.73 & 4.95 & 5.06 & \times & \times & 5.04 & \times & 3.42 \\ \text{S1} & \times & \times & \times & 4.71 & 5.02 & 8.32 & 10.60 & 2.12 \\ \text{S2} & \times & \times & \times & \times & \times & \times & 2.49 & \times \\ \text{S3} & 2.13 & 5.07 & 4.57 & 2.55 & 3.22 & \times & \times & \times \end{array}\right] \qquad 3^{\text{rd}}\ \text{iteration} \tag{14}$$

Then, the two required conditions are checked. Explicitly, the candidate user set of subcarrier 2 contains only one user and becomes $\tilde{\mathcal{F}}_2 = \{6\}$. However, for the example considered, each subcarrier is required to be allocated to $N = 2$ users. Hence, condition (a) described in (12) is not satisfied, and the algorithm hence stops. Consequently, the results obtained from the second iteration are taken as the final allocation results.

For convenience, the main steps of the IWE assisted subcarrier-allocation algorithms can be described by the flow chart in Fig. 1. In detail, during the initialization of the IWE algorithm, with the specific subcarrier-allocation algorithm is chosen, and the initial (first) iteration of subcarrier-allocation is carried out. After the initialization, the IWE scheme proceeds to the second iteration, and sets $s = 2$. During each iteration with $s \geq 2$, the WE process is first carried out, as shown in the figure. Note that, the WE can be operated either in user direction or in subcarrier direction, which is dependent on the subcarrier-allocation algorithm employed, the number of subcarriers as well as the number of users involved. For example, when the IWE-WCF algorithm is employed, the WE is carried out in user direction. By contrast, when the IWE-WSA algorithm is used, the WE process is operated in subcarrier direction, i.e., the worst channel quality of each of the subcarriers is removed. As shown in Fig. 1, following the WE block, the algorithm checks the conditions for assignment. When the two conditions as mentioned in this section are satisfied, it proceeds to the subcarrier-allocation. Otherwise, the IWE algorithm stops and takes the results obtained in the $(s-1)$th (previous) iteration as the final subcarrier-allocation. If the sth iteration of subcarrier-allocation is carried out, the allocation results of the sth (current) iteration are compared with those of the previous iteration against the performance metric. If performance is improved, the algorithm continues to the next iteration. Otherwise, the IWE algorithm stops and the allocation results from the previous iteration are taken as the final allocation results.

B. Characteristics of Iterative Worst Excluding Algorithm

The IWE algorithm employs a range of advantages in the sense of improving the error performance in comparison with the various subcarrier-allocation algorithms found in references. First, the IWE algorithm can be easily implemented in conjunction with an existing subcarrier-allocation algorithm, in order to enhance its performance, as discussed in Section V-A. The core of the IWE algorithm is the WE process, which meliorates the channel quality matrix prior to operating subcarrier-allocation. Based on the improved channel quality matrix, the subcarrier-allocation followed can hence improve the error performance. Second, the subcarrier-allocation algorithm assisted by the IWE algorithm can always guarantee error performance improvement in comparison with that without using the IWE. In Section V-A, we only described the operation procedure of the IWE-WCF algorithm. Similarly, we can also form the IWE aided WSA (IWE-WSA) algorithm, the IWE aided WCA (IWE-WCA) algorithm, etc., the performance of which will be evaluated in Section VII. It should be noted that, the greedy algorithm was designed not to maximize the minimum of channel qualities as the optimization problem given in (7). Hence, the IWE algorithm may not assist the greedy algorithm and its extensions in improving the error performance. Finally, from our studies, we find that the IWE algorithm is usually operated with a low number of iterations, which guarantees the IWE aided algorithms low complexity.

As the number of iterations required by the IWE algorithm is an important factor, which affects the performance and complexity of the associated subcarrier-allocation algorithms, in Table II, we summarize the average number of iterations required by the various IWE aided subcarrier-allocation algorithms for some cases. For this table, we assumed for the considered downlink MC DS-CDMA system that all subcarriers of all users experience independent Rayleigh fading and the Gaussian noise of the same variance. Furthermore, we assumed that the number of users supported by the system is $K = MN$. Each of the results in the table was obtained by averaging over the outcomes of 10^5 simulations. From the results, we can observe that the three IWE aided subcarrier-allocation algorithms always require a low average number of iterations, which is $\bar{S} < 3$ for all the considered cases. Moreover, from the table, a few other observations can be identified. First, given a constant N value, it can be shown that the average number of iterations normalized by the number of subcarriers M, i.e., $\bar{S}/M$, decreases explicitly as M increases, even though, for most cases, the average number of iterations $\bar{S}$ slightly increases as M becomes larger. Second, for most cases, $\bar{S}$ in general becomes smaller as the spreading factor increases for a constant M. Furthermore, the IWE-WSA algorithm requires in average a slightly bigger number of iterations than the other two algorithms considered. This is mainly because the IWE-WSA algorithm carries out the WE operations in subcarrier direction, while the other two algorithms run the WE operations in user direction.

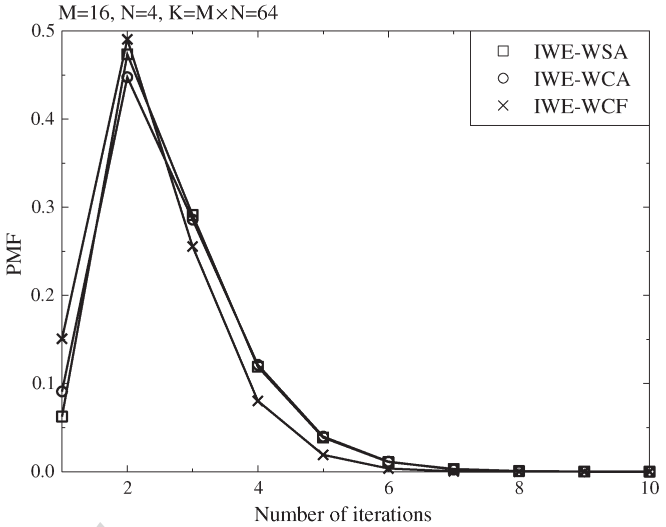


Fig. 2. Distribution of the number of iterations required by the IWE aided subcarrier-allocation algorithms.

Furthermore, in Fig. 2, we illustrate the probability mass function (PMF) of the number of iterations required by the three IWE aided subcarrier-allocation algorithms, where the results are obtained from 10^5 realizations. Associated with the studies, we assumed $M = 16$, $K = 64$, and $N = 4$. It can be observed that the number of iterations is a variable and, for most cases, the allocation requires 2 iterations. However, the allocation process sometimes requires up to 6 iterations. Furthermore, the probability of requiring 8 iterations is nearly zero, which is still much smaller than the number of users $K = 64$. From Table II and Fig. 2, we therefore can conclude that the IWE aided algorithms usually demand a low number of iterations, which ensures a low complexity for implementation. Note that, in practice, we may set the maximum number of iterations to three or four, which guarantees the most of the available gain, while limit the complexity.

TABLE II
AVERAGE NUMBER OF ITERATIONS FOR THE IWE AIDED SUBCARRIER-ALLOCATION ALGORITHMS

Algorithm	IWE-WCF				IWE-WCA				IWE-WSA			
N \ M	4	8	16	32	4	8	16	32	4	8	16	32
1	1.76	2.14	2.44	2.58	2.20	2.75	2.74	2.73	2.20	2.75	2.74	2.73
2	1.70	2.12	2.40	2.55	1.96	2.45	2.65	2.72	2.42	2.64	2.67	2.71
4	1.60	2.03	2.34	2.52	1.78	2.30	2.61	2.71	2.43	2.55	2.65	2.70
8	1.49	1.89	2.25	2.46	1.63	2.18	2.56	2.70	2.42	2.51	2.64	2.69

VI. COMPLEXITY ANALYSIS

In this section, we analyze the complexity of the proposed subcarrier-allocation algorithms and that of the other related algorithms. In our analysis, we assume that the same power-allocation scheme is used for all the subcarrier-allocation algorithms. Furthermore, the complexity reflects the number of comparisons required by the subcarrier-allocation algorithms.

First, the complexity of the greedy algorithm and that of the WSA algorithm can be found, for example, in [7], which are both $\mathcal{O}(K^2)$ for the MC DS-CDMA systems with $K \geq M$. Specifically, the number of comparisons required by the WSA algorithm can be expressed as

$$\mathcal{C}^{(\mathrm{WSA})} = M(K-1) + 2M \ln M + \frac{1}{2}K(K-1). \tag{15}$$

The complexity of the WCA algorithm depends on the specific operations. First, the K users are ordered from the worst to the best according to their worst channel qualities. This process requires $K(M-1) + 2K \ln K$ comparisons. Then, for the subcarrier-allocation, the upper-bound happens when each subcarrier is assigned to $(N-1)$ users during the first $(K-M)$ stages. In this case, $(K-M)(M-1) + M(M-1)/2$ comparisons are required. When considering the above analysis, the number of comparisons required by the WCA algorithm satisfies

$$\begin{aligned}\mathcal{C}^{(\mathrm{WCA})} &\leq K(M-1) + 2K \ln K + (K-M)(M-1) \\ &\quad + \frac{1}{2}M(M-1) \\ &\leq \left(2K - \frac{M}{2}\right)(M-1) + 2K \ln K. \end{aligned} \tag{16}$$

From (16), we can be implied that the WCA algorithm has a complexity of $\mathcal{O}(KM)$.

Similarly, the complexity of the WCF algorithm has an upper-bound, which happens when each of the M subcarriers is assigned to $(N-1)$ users during the first $(K-M)$ allocation stages. In this case, $K(M-1)$ comparisons are needed for the K users to find their worst channel qualities during the first $(K-M+1)$ stages. Then, $\sum_{m=2}^{M-1}(M-m) = (M-1)(M-2)/2$ comparisons are required for re-identifying the worst channel quality during the last $(M-1)$ stages. Moreover, during each stage, the WCF algorithm finds the minimum of the channel qualities of the $k\,(k = K, K-1, \ldots, 1)$ available users, which requires $K(K-1)/2$ comparisons. Except user ordering, the allocation process of the WCF algorithm is the same as that of the WCA algorithm, which requires $(K-M)(M-1) + M(M-1)/2$ comparisons. Consequently, the upper-bound for the number of comparisons required by the WCF algorithm can be expressed as

$$\begin{aligned}\mathcal{C}^{(\mathrm{WCF})} &\leq K(M-1) + \frac{1}{2}(M-1)(M-2) + \frac{1}{2}K(K-1) \\ &\quad + (K-M)(M-1) + \frac{1}{2}M(M-1) \\ &\leq (2K-1)(M-1) + \frac{1}{2}K(K-1) \end{aligned} \tag{17}$$

According to (17), we can readily know that the WCF algorithm has a complexity of $\mathcal{O}(K^2)$, as $K > M$ is assumed.

Let us now consider the complexity of the IWE-WSA algorithm. First, during the sth iteration, the WE process searches for the worst channel qualities of the M subcarriers, which have already been identified by the WSA operations during the $(s-1)$th iteration. Therefore, there is no complexity contribution by the WE process during the sth iteration. Second, we can easily find that the condition checking requires $\mathcal{C}^{(\mathrm{checking})} = M + M(M-1)/2$ operations during the sth $(s \geq 2)$ iteration. Note that, at the sth iteration, the number of comparisons required by the WSA-assisted subcarrier-allocation is $\mathcal{C}^{(\mathrm{allocation})}(s) = \mathcal{C}^{(\mathrm{WSA})} - \mathcal{C}^{(\mathrm{reduce})}(s)$, where $\mathcal{C}^{(\mathrm{reduce})}(s) = 2M(s-1)$ denotes the number of comparisons reduced as a result that some of the worst channels are removed by the WE process. When considering all the above, the number of comparisons required by the IWE-WSA algorithm can be expressed as

$$\begin{aligned}\mathcal{C}^{(\mathrm{IWE\text{-}WSA})} &= (S-1)\mathcal{C}^{(\mathrm{checking})} + \sum_{s=1}^{S} \mathcal{C}^{(\mathrm{allocation})}(s) \\ &= \left(\frac{1}{2}SK + SM\right)(K-1) \\ &\quad + \left(\frac{1}{2}M^2 + \frac{1}{2}M - SM\right)(S-1) + 2SM \ln M \end{aligned} \tag{18}$$

when assuming that S iterations are used. Equation (18) shows a complexity of $\mathcal{O}(SK^2)$ for the IWE-WSA algorithm.

In the context of the IWE-WCA and IWE-WCF algorithms, their complexity can be analyzed in the similar way as that for the IWE-WSA algorithm, in conjunction with WCA and WCF algorithms, respectively. It can be shown that the number of comparisons required by these two algorithms can be expressed as

$$\begin{aligned}\mathcal{C}^{(\mathrm{IWE\text{-}WCA})} &\leq \left(2SK - \frac{1}{2}SM\right)(M-1) \\ &\quad + \left(\frac{1}{2}M^2 + \frac{1}{2}M - SK\right)(S-1) + 2SK \ln K, \end{aligned} \tag{19}$$

TABLE III
COMPLEXITY OF VARIOUS SUBCARRIER-ALLOCATION ALGORITHMS

Algorithm	Complexity
Hungarian	$\mathcal{O}(K^3)$ [11]
Greedy	$\mathcal{O}(K^2)$
WUF Greedy	$\mathcal{O}(K^2)$
Maximal Greedy	$\mathcal{O}(\alpha K^2)$
WSA	$\mathcal{O}(K^2)$
WCA	$\mathcal{O}(KM)$
WCF	$\mathcal{O}(K^2)$
IWE-WSA	$\mathcal{O}(SK^2)$
IWE-WCA	$\mathcal{O}(SK^2)$
IWE-WCF	$\mathcal{O}(SK^2)$

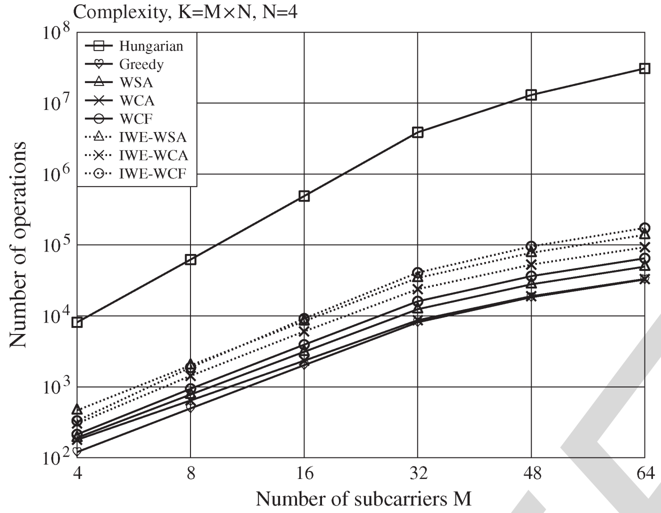


Fig. 3. Number of comparisons required by various subcarrier-allocation algorithms when $N = 4$.

$$\mathcal{C}^{(\text{IWE-WCF})} \leq \frac{1}{2}SK(K-1) + (2SK-S)(M-1) + \left(\frac{1}{2}M^2 + \frac{1}{2}M - SK\right)(S-1), \tag{20}$$

respectively. Therefore, the complexity of both the IWE-WCA and the IWE-WCF algorithms are $\mathcal{O}(SK^2)$.

In Table III, we summarize the complexity of the various subcarrier-allocation algorithms. Note that, the maximal greedy algorithm [10] requires a complexity of $\mathcal{O}(\alpha K^2)$, where $\alpha\,(\geq M)$ is the size of the search space. In Section VII, we assume that the maximal greedy algorithm uses a random search space having the size $\alpha = M$. Furthermore, in Figs. 3 and 4, we compare the number of operations required by the various subcarrier-allocation algorithms with respect to the number of subcarriers employed by the MC DS-CDMA systems. Note that, in both figures, the number of operations are either the exact values or the upper-bound of the algorithms. The number of comparisons of the IWE algorithms were obtained from (18)–(20). From both figures, we can see that the greedy and WCA algorithms always require the least number of comparisons, while the Hungarian algorithm [11], [21] needs the highest number of comparisons. When $N = 4$ in Fig. 3, the greedy algorithm demands the lowest number of comparisons when $M \leq 32$. However, when $N = 8$ in Fig. 4, the WCA algorithm always has the lowest operations. Observing from the two figures, we can know that the complexity of the proposed WCA and WCF algorithms are at the same level as that of the WSA and greedy algorithm. Moreover, for the considered examples, we find that the number of comparison required by the IWE-aided subcarrier-allocation algorithms is slightly less than twice of the number of comparisons required by the original corresponding algorithms without invoking the IWE algorithm.

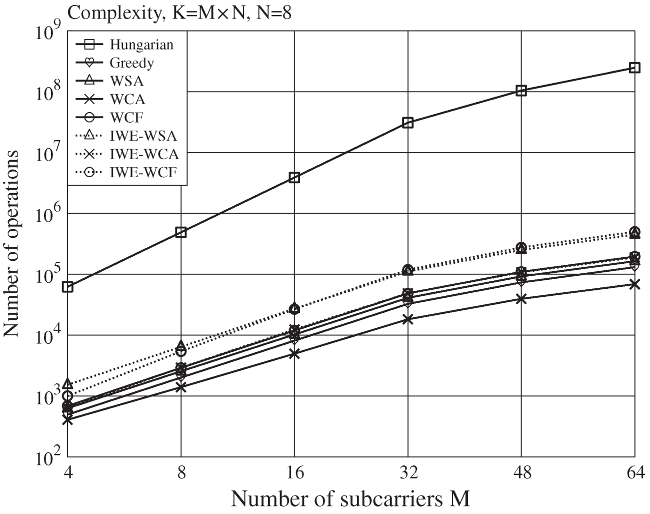


Fig. 4. Number of comparisons required by various subcarrier-allocation algorithms when $N = 8$.

VII. PERFORMANCE RESULTS

In this section, we provide a range of simulation results, in order to demonstrate and compare the achievable error performance of the downlink MC DS-CDMA systems employing the proposed and the other subcarrier-allocation algorithms considered. In our studies, we assume the Quadrature Phase-Shift Keying (QPSK) baseband modulation and that all the subcarriers experience independent flat Rayleigh fading. The number of users supported by the MC DS-CDMA is $K = MN$, with M being the number of subcarriers and N the length of the orthogonal DS spreading codes. Furthermore, for all the subcarrier-allocation algorithms considered, we assume that the channel-inverse assisted power-allocation is employed, under the constraint that the total transmission power is $P = K$.

Fig. 5 demonstrates the BER performance of the MC DS-CDMA system employing various of subcarrier-allocation algorithms, when $K = 64$ users are supported by $M = 16$ subcarriers. Hence, each subcarrier supports 4 users. From the figure, we can obtain the following observations. First, the Hungarian algorithm gives the best BER performance, while the greedy algorithm yields the worst performance. Both the WUF greedy algorithm [9] and the maximal greedy algorithm [10], which assumes a random search space of size $\alpha = M$, slightly outperform the greedy algorithm. As the greedy-class algorithms aim to maximize the sum of the channel qualities, rather than maximizing the reliability, such as the SNR in (5), the greedy-class algorithms in general achieve poorer BER performance than the other reliability motivated algorithm. Second, as seen in Fig. 5, the proposed WCA, WCF, especially

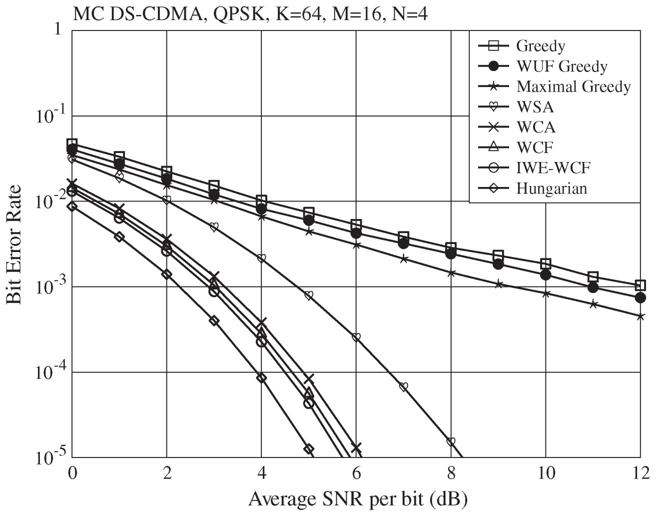


Fig. 5. BER comparison of the downlink MC DS-CDMA systems employing various subcarrier-allocation algorithms, when subcarriers experience independent Rayleigh fading.

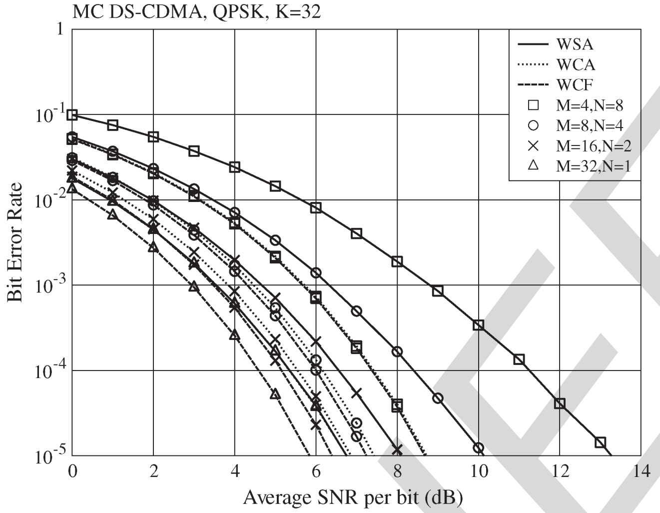


Fig. 6. BER comparison of the downlink MC DS-CDMA systems employing the WSA, WCA, and WCF algorithms, when subcarriers experience independent Rayleigh fading.

the IWE-WCF algorithms are capable of significantly outperforming the greedy-class algorithms as well as the WSA algorithm. Third, for the specific system parameters considered, the WCF algorithm has better BER performance than the WCA algorithm. This is because the WCF algorithm can avoid assignment of more number of worst subcarriers than the WCA algorithm. Finally, by invoking the IWE scheme, further error performance improvement can be attained with a penalty of double complexity. The achievable BER of the IWE-WCF algorithm is close to that achieved by the Hungarian algorithm, and the difference is only 0.7 dB.

Fig. 6 compares the BER performance of the MC DS-CDMA systems employing the WSA, WCA and the WCF algorithms for $K = 32$ users. In general, the proposed WCA and WCF algorithms always yield better BER performance than the WSA algorithm. As discussed in Section IV, the WSA algorithm implements the assignment by avoiding the worst channel qualities in a subcarrier-oriented mode. Hence, its performance

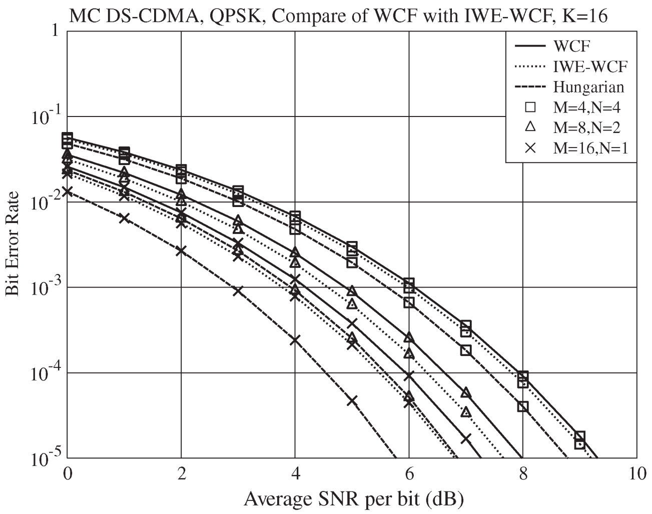


Fig. 7. BER comparison of the downlink MC DS-CDMA systems employing the WCF and the IWE-WCF algorithms, when subcarriers experience independent Rayleigh fading.

depends on the frequency-selective diversity. By contrast, for the MC DS-CDMA systems employing DS spreading, the number of users supported is usually higher than the number of subcarriers, as considered in Fig. 6. In this case, the WCA and WCF algorithms avoid the worst channel qualities in a user-oriented mode and achieve much higher diversity than the WSA scheme. Furthermore, from Fig. 6 we observe that, when given $K = MN$ a constant, the BER performance of the three algorithms improves as M becomes larger. The reason behind the observation is that we assumed that all subcarriers experience independent fading regardless of the number of subcarriers. This assumption implies that more subcarriers results in higher diversity. In this case, the advantage of the WCA algorithm over the WSA algorithm becomes smaller as the ratio of K/M becomes bigger. Furthermore, when $M = K = 32$ and $N = 1$, both the WCA and WSA achieve the same BER, as, in this case, the MC DS-CDMA is reduced to an OFDMA system without T-domain spreading. Consequently, the user-oriented diversity is the same as the subcarrier-oriented diversity. By contrast, as shown in Fig. 6, the advantage of the WCF algorithm over the WCA algorithm is enhanced as M increases, when given $K = MN$ a constant. Specifically, when $M = 32$ and $N = 1$, the WCF algorithm has 0.6 dB SNR gain over the WCA algorithm at the BER of 10^{-5}. From the above, we can know that, when all subcarriers experience independent fading, the number of subcarriers has a significant impact on the performance of the considered subcarrier-allocation algorithms.

Figs. 7–9 show the BER gain of employing the IWE algorithm for the WCF, WCA and WSA algorithms, respectively. Under the various cases, the BER improvement can be obtained by introducing the IWE algorithm. Thus, this observation confirms the benefit of using the IWE algorithm in association with subcarrier-allocation algorithms. By comparing the three figures, we observe that the IWE-WCF algorithm always has the best BER performance, while the IWE-WSA has the worst performance among the three IWE aided algorithms. This observation maintains the same for the three algorithms without using the IWE algorithm in Fig. 6. From Figs. 7 and 8, we

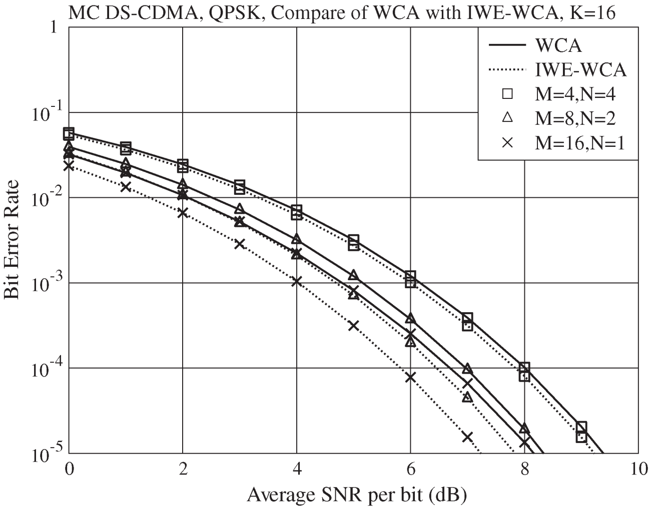


Fig. 8. BER comparison of the downlink MC DS-CDMA systems employing the WCA and the IWE-WCA algorithms, when subcarriers experience independent Rayleigh fading.

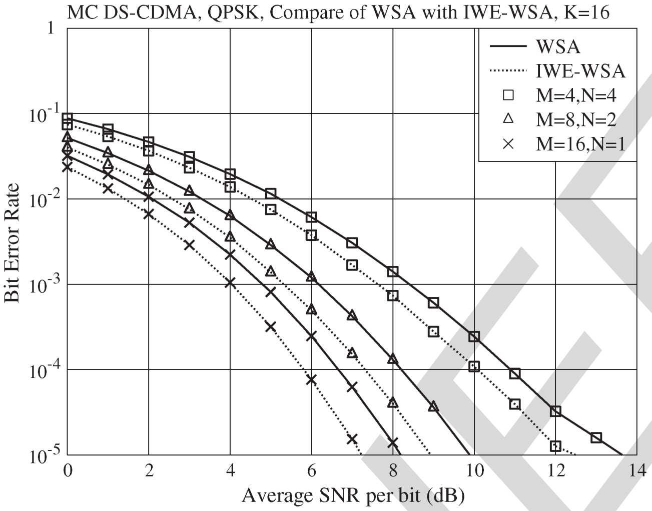


Fig. 9. BER comparison of the downlink MC DS-CDMA systems employing the WSA and the IWE-WSA algorithms, when subcarriers experience independent Rayleigh fading.

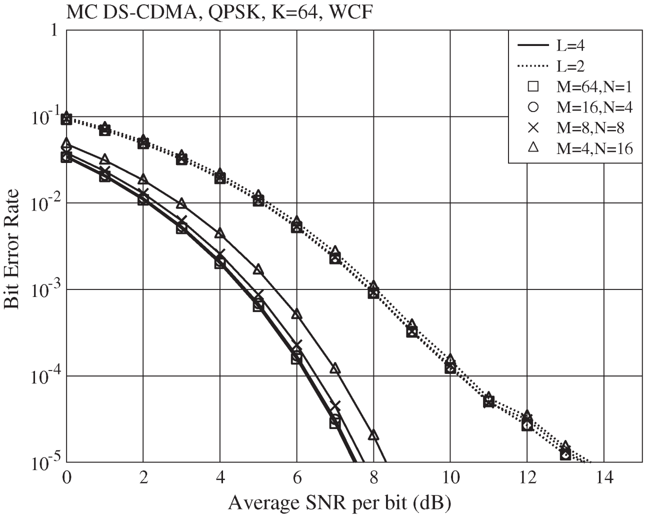


Fig. 10. BER of the downlink MC DS-CDMA systems employing the WCF algorithm, when subcarriers experience frequency selective Rayleigh fading with L number of time domain resolvable paths.

observe that the improvement of using the IWE scheme for the WCF and the WCA algorithms gets larger as the number of subcarriers M becomes bigger. By contrast, in Fig. 9, the BER advantage of using the IWE remains the same, which is about 1 dB, as the number of subcarriers M becomes bigger. As discussed in Section V, the WE process of the IWE-WCA and IWE-WCF algorithms excludes the worst subcarrier for each user during an iteration, but the worst user of each subcarrier is eliminated during every iteration for the IWE-WSA algorithm. Therefore, the BER performance of the IWE-WCF and IWE-WCA algorithms is highly affected by the subcarrier diversity, whereas that of the IWE-WSA algorithm is dominated by the user diversity. In Fig. 9, the number of users is $K = 16$ for all cases, thus they obtain a similar BER gain when employing the IWE algorithm.

So far, we have assumed that all subcarriers of a MC DS-CDMA system experience independent fading, regardless of the number of subcarriers. When given the frequency selectivity of a wireless channel, this assumption may not be true. In this case, the fading experienced by different subcarriers in fact becomes more correlated, as the number of subcarriers increases. Therefore, in Fig. 10, we study the BER performance of the MC DS-CDMA employing the WCF algorithm, when the number of time-domain resolvable paths is fixed to $L = 2$ or 4, i.e., when given the frequency selectivity of wireless channels. Explicitly, when $L = 2$, using $M = 4$ subcarriers is sufficient for attaining all the frequency diversity. By contrast, when $L = 4$, $M = 16$ subcarriers are required to achieve all the frequency diversity.

VIII. CONCLUSION

We have proposed a range of fair subcarrier-allocation algorithms and investigated them in the context of the MC DS-CDMA, where the number of users supported may be higher than the number of subcarriers. By analyzing the characteristics of the WSA algorithm that is beneficial to the systems with subcarriers more than users, we have generalized the WSA algorithm to the WCA algorithm, which is suitable for any multicarrier systems. Following our detailed analysis of these algorithms, we have proposed the WCF algorithm, which is capable of further improving the reliability of MC DS-CDMA systems. Moreover, an IWE algorithm has been proposed for application in conjunction with the WSA, WCA or the WCF, resulting in the IWE-WSA, IWE-WCA or the IWE-WCF algorithm. Our studies show that an IWE-assisted algorithm always improves the reliability of the original algorithm. The IWE-WCA algorithm outperforms the IWE-WSA algorithm, while the IWE-WCF algorithm achieves the highest reliability among these three. Furthermore, our results demonstrate that the reliability attained by these IWE-WCF algorithms is close to that achieved by the high-complexity optimum Hungarian algorithm. Additionally, the complexity of the proposed subcarrier-allocation algorithms has been analyzed and compared with that of the low-complexity greedy algorithm. We can argue that all our proposed subcarrier-allocation algorithms have the merit of low-complexity.

Note that, the observations obtained from this paper are in general suitable for the MC DS-CDMA systems, where different users are allocated with different numbers of subcarriers or/and spreading codes. This is because the relative advantages and disadvantages of the considered subcarrier-allocation algorithms are only determined by the diversity available from the channel quality matrix, i.e., by the values of K and M, but not by the numbers of data streams of the users.

REFERENCES

[1] L.-L. Yang, *Multicarrier Communications*. Chichester, U.K.: Wiley, 2009.

[2] L.-L. Yang and L. Hanzo, "Multicarrier DS-CDMA: A multiple access scheme for ubiquitous broadband wireless communications," *IEEE Commun. Mag.*, vol. 41, no. 10, pp. 116–124, Oct. 2003.

[3] K. Letaief and Y. J. Zhang, "Dynamic multiuser resource allocation and adaptation for wireless systems," *IEEE Trans. Wireless Commun.*, vol. 13, no. 4, pp. 38–47, Aug. 2006.

[4] J. Jang and K. B. Lee, "Transmit power adaptation for multiuser OFDM systems," *IEEE J. Sel. Areas Commun.*, vol. 21, no. 2, pp. 171–178, Feb. 2003.

[5] K. Kim and J. Kim, "A 2-D subcarrier allocation scheme for capacity enhancement in a clustered OFDM system," *IEICE Trans. Commun.*, vol. E90-B, no. 7, pp. 1880–1883, Jul. 2007.

[6] K. A. D. Teo, Y. Otani, and S. Ohno, "Adaptive subcarrier allocation for multi-user OFDM system," *IEICE Trans. Commun.*, vol. E89-A, no. 11, pp. 3131–3137, Nov. 2006.

[7] T. Liu, C. Yang, and L.-L. Yang, "A low-complexity subcarrier-power allocation scheme for frequency-division multiple-access systems," *IEEE Trans. Wireless Commun.*, vol. 9, no. 5, pp. 1564–1570, May 2010.

[8] D. Palomar, J. Cioffi, and M. Lagunas, "Joint Tx–Rx beamforming design for multicarrier MIMO channels: A unified framework for convex optimization," *IEEE Trans. Signal Process.*, vol. 51, no. 9, pp. 2381–2401, Sep. 2003.

[9] N. Y. Ermolova and B. Makarevitch, "Performance of practical subcarrier allocation schemes for OFDM," in *Proc. IEEE 18th Int. Symp. PIMRC*, Sep. 2007, pp. 1–4.

[10] O. Nwamadi, X. Zhu, and A. Nandi, "Dynamic subcarrier allocation for single carrier-FDMA systmes," in *Proc. EUSIPCO*, Aug. 2008, pp. 1–5.

[11] H. W. Kuhn, "The Hungarian method for the assignment problem," *Naval Res. Logistics Quart.*, vol. 2, no. 1-2, pp. 83–97, Mar. 1995.

[12] T. Jia and A. Duel-Hallen, "Improved channel allocation for multicarrier CDMA with adaptive frequency hopping and multiuser detection," *IEEE Trans. Commun.*, vol. 57, no. 11, pp. 3389–3396, Nov. 2009.

[13] Z. Wang, D. Yang, and L. Milstein, "Multi-user resource allocation for a distributed multi-carrier DS-CDMA network," *IEEE Trans. Commun.*, vol. 60, no. 1, pp. 143–152, Jan. 2012.

[14] P.-W. Fu and K.-C. Chen, "Rate, sub-carrier, power allocations for multicarrier CDMA with LMMSE multiuser detection," *IEEE Trans. Wireless Commun.*, vol. 6, no. 5, pp. 1574–1580, May 2007.

[15] Y. Wang, G. Xie, Y. Liu, and N. Li, "Feasible resource allocation with partial channel information for grouped MC-CDMA system," in *Proc. ISCIT*, Oct. 2007, pp. 683–688.

[16] E. Lo *et al.*, "Adaptive resource allocation and capacity comparison of downlink multiuser MIMO-MC-CDMA and MIMO-OFDMA," *IEEE Trans. Wireless Commun.*, vol. 6, no. 3, pp. 1083–1093, Mar. 2007.

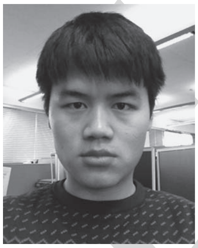
[17] P. Chan *et al.*, "Performance comparison of downlink multiuser MIMO-OFDMA and MIMO-MC-CDMA with transmit side information—Multi-cell analysis," *IEEE Trans. Wireless Commun.*, vol. 6, no. 6, pp. 2193–2203, Jun. 2007.

[18] J. Zhu and Y. Bar-Ness, "Power allocation algorithm in MC-CDMA," in *Proc. IEEE ICC*, 2002, vol. 2, pp. 931–935.

[19] Y. Liang, V. Veeravalli, and H. Poor, "Resource allocation for wireless fading relay channels: Max–Min solution," *IEEE Trans. Inf. Theory*, vol. 53, no. 10, pp. 3432–3453, Oct. 2007.

[20] C. Y. Ng and C. W. Sung, "Low complexity subcarrier and power allocation for utility maximization in uplink OFDMA systems," *IEEE Trans. Wireless Commun.*, vol. 7, no. 5, pp. 1667–1675, May 2008.

[21] J. Munkres, "Algorithms for the assignment and transportation problems," *J. Soc. Ind. Appl. Math.*, vol. 5, no. 1, pp. 32–38, Mar. 1957.



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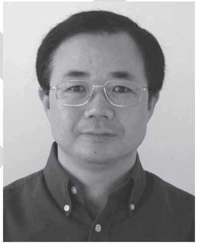

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