Successful N_2 leptogenesis with flavour coupling effects in realistic unified models

Pasquale Di Bari* and Stephen F. King†

School of Physics and Astronomy, University of Southampton,
Southampton, SO17 1BJ, United Kingdom

October 9, 2015

Abstract

In realistic unified models involving so-called $SO(10)$-inspired patterns of Dirac and heavy right-handed (RH) neutrino masses, the lightest right-handed neutrino N_1 is too light to yield successful thermal leptogenesis, barring highly fine tuned solutions, while the second heaviest right-handed neutrino N_2 is typically in the correct mass range. We show that flavour coupling effects in the Boltzmann equations may be crucial to the success of such N_2 dominated leptogenesis, by helping to ensure that the flavour asymmetries produced at the N_2 scale survive N_1 washout. To illustrate these effects we focus on N_2 dominated leptogenesis in an existing model, the A to Z of flavour with Pati-Salam, where the neutrino Dirac mass matrix may be equal to an up-type quark mass matrix and has a particular constrained structure. The numerical results, supported by analytical insight, show that in order to achieve successful N_2 leptogenesis, consistent with neutrino phenomenology, requires a "flavour swap scenario" together with a less hierarchical pattern of RH neutrino masses than naively expected, at the expense of some mild fine-tuning. In the considered A to Z model neutrino masses are predicted to be normal ordered, with an atmospheric neutrino mixing angle well into the second octant and the Dirac phase $\delta \simeq 20^\circ$, a set of predictions that will be tested in the next years in neutrino oscillation experiments. Flavour coupling effects may be relevant for other $SO(10)$-inspired unified models where N_2 leptogenesis is necessary.

*E-mail: P.Di-Bari@soton.ac.uk
†E-mail: king@soton.ac.uk

1 Introduction

It would be certainly desirable to extend the SM, realising a unified picture able to solve the flavour problem, explaining masses and mixing parameters of quarks and leptons, and at the same time to provide solution to the cosmological puzzles. In particular leptogenesis [1] is an attractive way to explain the matter-antimatter asymmetry of the Universe, that in terms of the baryon-to-photon number ratio is given by [2]

$$\eta_B^{\rm CMB} = (6.1 \pm 0.1) \times 10^{-10}\,. \tag{1}$$

Leptogenesis is a cosmological application of the see-saw mechanism, a minimal extension of the SM able to explain neutrino masses and mixing [3]. Despite an intense activity on various aspects of leptogenesis, there are not many definite realistic unified models that have been shown to lead to successful leptogenesis while explaining fermion masses, mixing and CP violation, although in the case of $SU(5)$ it is certainly possible to achieve successful N_1 leptogenesis (for a recent example see e.g. [4] which uses the sequential dominance results for N_1 leptogenesis discussed in [5]).

In this paper we are interested in N_2 leptogenesis in so-called $SO(10)$-inspired models with type I seesaw. For definiteness, we investigate the possibility that the A to Z Pati-Salam model proposed in [6] (see also [7]) can not only describe neutrino masses and mixing but also attain the correct value of the matter-antimatter asymmetry with leptogenesis. The right-handed (RH) neutrino mass spectrum in this model is very hierarchical, typical of $SO(10)$-inspired models (assuming type I seesaw). In this way the lightest RH neutrino N_1 is too light to generate a sizeable asymmetry [8] while on the other hand the next-to-lightest RH neutrino is heavy enough to be able potentially to generate the correct asymmetry realising the so called N_2 dominated scenario [9, 10, 11, 12] or simply N_2 leptogenesis. However in general it is non-trivial to be able to find a set of values of the parameters of the model for the lightest RH neutrino wash-out to be negligible, in such a way that the asymmetry generated by the next-to-lightest neutrinos survives, while at the same time producing values of the neutrino parameters compatible with the experimental results.

The non-trivial requirements of N_2 leptogenesis may be somewhat relaxed by taking into account additional flavour coupling effects in the Boltzmann equations, or more generally the kinetic mixing terms, that may transmit part of the initially produced flavour asymmetry from one particular flavour to other flavours. This increases the chances that the asymmetry generated in a particular flavour at the N_2 scale may survive the N_1 washout. The additional flavour coupling effects, which are not usually considered in the literature, arise mainly from the fact that lepton asymmetry produced at the N_2 scale into left-handed lepton doublets is also accompanied by hyper-charge asymmetry into Higgs asymmetry, giving the dominant effect, baryons asymmetries into quarks and lepton asymmetries into right-handed charged leptons, which are then transmitted to the other flavours which couple to these particles [13]. This results in new contributions to the total asymmetry that in a traditional N_1-dominated scenario would give just some small corrections ($\mathcal{O}(10\%)$) [14]. However, in a N_2-dominated scenario they can become dominant if, contrarily to the usual dominant terms produced in a flavour that is strongly washed by the lightest RH neutrinos, they are produced into a flavour that escapes the lightest RH neutrino wash-out. In this way the so called "flavour swap scenario" is realised [15].

In the A to Z of Flavour with Pati-Salam [6], all of these features are exemplified. In particular, the N_1 scale is too light to generate asymmetries thermally, while the flavour asymmetry produced at the N_2 scale, namely the τ flavour, is effectively washed out at the N_1 scale for all ranges of parameters consistent with the experimentally acceptable neutrino masses and mixings. In this example, flavour coupling effects come to the rescue, effectively transmitting part of the τ asymmetry into (a linear combination of) electron and muon asymmetries at the N_2 scale, completely (electron) or partly (muon) surviving washout at the N_1 scale. These features necessarily arise from the rigid structure of the Yukawa and Majorana matrices enforced by the model, leading to fairly precise predictions for PMNS parameters which can be compared to experiment. It has been already shown that the model can simultaneously fit both lepton and quark parameters. Here for simplicity we will focus on the leptonic sector and consider non-supersymmetric leptogenesis.

The layout of the remainder of the paper is as follows. In Section 2 we review neutrino masses in the A to Z model. In Section 3 we fit the neutrino parameters without imposing successful leptogenesis. In Section 4 we show how within a traditional calculation of the asymmetry ignoring flavour coupling effects one cannot find any good fit both to neutrino parameters and to the measured value of the asymmetry simultaneously. In Section 5 we show how flavour coupling rescues the model. In Section 6 we provide an analytical insight on the found solution within more general context of $SO(10)$-inspired leptogenesis highlighting different aspects both of the specific solution within the A to Z model and more generally of $SO(10)$-inspired models. In Section 7 we draw our conclusions.

2 Neutrino masses in the A to Z model

The lowest order lepton Yukawa matrices (in LR convention) and heavy Majorana mass matrix M_R are predicted by the model just below the high energy Pati-Salam breaking scale $\sim$ few $\times 10^{16}$ GeV. The charged lepton Yukawa matrix of the model is diagonal to excellent approximation, and the neutrino Lagrangian in this basis is given by,

$$-\mathcal{L} = \overline{N}_L Y^{'\nu} N'_R + N_R^{'T} M'_R N'_R + H.c. \tag{2}$$

with the neutrino Yukawa and Majorana matrices [6],

$$Y'^{\nu} = \begin{pmatrix} 0 & be^{-i3\pi/5} & 0 \\ ae^{-i3\pi/5} & 4be^{-i3\pi/5} & 0 \\ ae^{-i3\pi/5} & 2be^{-i3\pi/5} & ce^{i\phi} \end{pmatrix}, \quad M'_R = \begin{pmatrix} M'_{11}\, e^{2i\xi} & 0 & M'_{13}\, e^{i\xi} \\ 0 & M'_{22}\, e^{i\xi} & 0 \\ M'_{13}\, e^{i\xi} & 0 & M'_{33} \end{pmatrix}, \tag{3}$$

where $M'_{11}, M'_{13}, M'_{22}, M'_{33}$ are positive and real. Note that for simplicity it was also assumed in [6] that $M'_{13} = 0$ and $\phi = 0$, while here we shall allow these parameters (generally present in the model) to take non-zero values. Generally the model predicts $M'_{11} \ll M'_{22} \ll M'_{33}$ and $M'_{13} \sim M'_{22}$. Without any tuning of parameters, the model predicts the typical values,

$$M'_{11} \sim 10^5 \text{ GeV}, \;\; M'_{22} \sim M'_{13} \sim 10^{10} \text{ GeV}, \;\; M'_{33} \sim 10^{15} \text{ GeV}, \tag{4}$$

while

$$a : b : c \sim m_{\text{up}} : m_{\text{charm}} : m_{\text{top}} \sim 10^{-6} : 10^{-3} : 1\,. \tag{5}$$

However, successful leptogenesis requires some fine-tuning, leading to a more compressed spectrum of right-handed neutrino masses, as we shall see. Also ξ is chosen to be one of the complex fifth roots of unity: $\xi = 0, \pm 2\pi/5, \pm 4\pi/5$. We shall consider all cases.

We perform a unitary transformation U_R to the flavour basis where the right-handed Majorana mass matrix is diagonal with real, positive eigenvalues,

$$\begin{aligned} \mathcal{L} &= \overline{N}_L Y'^{\nu}\, U_R\, U_R^\dagger\, N'_R + N_R'^T\, U_R^*\, U_R^T\, M'_R\, U_R\, U_R^\dagger\, N'_R + H.c. & (6)\\ &= \overline{N}_L\, Y^{\nu}\, N_R + N_R^T\, M_R\, N_R + H.c. & (7) \end{aligned}$$

where

$$N_R = U_R^\dagger\, N'_R, \quad Y^{\nu} = Y'^{\nu}\, U_R, \quad M_R = U_R^T\, M'_R\, U_R = \text{diag}(M_1, M_2, M_3). \tag{8}$$

We parametrise the unitary matrix, assuming small angle rotations, approximately as

$$U_R = \begin{pmatrix} 1 & 0 & R_{13} e^{i\phi_{13}} \\ 0 & 1 & 0 \\ -R_{13} e^{-i\phi_{13}} & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\phi_{11}} & 0 & 0 \\ 0 & e^{i\phi_{22}} & 0 \\ 0 & 0 & e^{i\phi_{33}} \end{pmatrix}, \tag{9}$$

where the first matrix factor diagonalises the right-handed neutrino mass matrix and the second factor ensures that the right-handed neutrino masses M_i are real and positive. Thus the form of the neutrino Yukawa matrix in the flavour basis is $Y^{\nu} = Y'^{\nu}\, U_R$,

$$Y^{\nu} \approx \begin{pmatrix} 0 & be^{-i3\pi/5} e^{i\phi_{22}} & 0 \\ ae^{-i3\pi/5} e^{i\phi_{11}} & 4be^{-i3\pi/5} e^{i\phi_{22}} & a\, R_{13}\, e^{i(\phi_{33} - \xi - 3\pi/5)} \\ ae^{-i3\pi/5} e^{i\phi_{11}} - cR_{13} e^{i\phi} e^{i(\phi_{11} - \phi_{13})} & 2be^{-i3\pi/5} e^{i\phi_{22}} & ce^{i\phi} e^{i\phi_{33}} \end{pmatrix}. \tag{10}$$

The parameters of U_R are determined from the requirement that

$$M_R = U_R^T M_R' U_R = D_M \equiv \mathrm{diag}(M_1, M_2, M_3). \tag{11}$$

In particular, the matrix elements of the diagonal M_R must satisfy:

$$\begin{aligned}
(M_R)_{11} &= M_1 \approx e^{2i(\phi_{11}-\phi_{13})}(e^{2i(\xi+\phi_{13})}M_{11}' - 2e^{i(\xi+\phi_{13})}M_{13}'R_{13} + M_{33}'R_{13}^2)\,, \\
(M_R)_{13} &= 0 \approx e^{i(\phi_{11}-\phi_{13}+\phi_{33})}(e^{2i(\xi+\phi_{13})}M_{11}'R_{13} - e^{i(\xi+\phi_{13})}M_{13}'(R_{13}^2-1) - M_{33}'R_{13})\,, \\
(M_R)_{22} &= M_2 \approx e^{2i\phi_{22}}e^{i\xi}M_{22}'\,, \\
(M_R)_{33} &= M_3 \approx e^{2i\phi_{33}}(M_{33}' + e^{2i(\xi+\phi_{13})}M_{11}'R_{13}^2 + 2e^{i(\xi+\phi_{13})}M_{13}'R_{13})\,.
\end{aligned} \tag{12}$$

From these conditions (remembering that M_i are real and positive) we find:

$$R_{13} \approx \frac{M_{13}'}{M_{33}'},\ \phi_{13} = -\xi,\ \phi_{11} = -\xi + \frac{1}{2}\,\mathrm{Arg}\left[M_{11}' - \frac{M_{13}'^2}{M_{33}'}\right]\,,\ \phi_{22} \approx -\xi/2,\ \phi_{33} \approx 0\,, \tag{13}$$

leading to:

$$\begin{aligned}
M_1 &\approx \left|M_{11}' - 2M_{13}'R_{13} + M_{33}'R_{13}^2\right| \approx \left|M_{11}' - \frac{M_{13}'^2}{M_{33}'}\right|\,, \\
M_2 &\approx M_{22}'\,, \\
M_3 &\approx M_{33}' + M_{11}'R_{13}^2 + 2M_{13}'R_{13}\,.
\end{aligned} \tag{14}$$

Thus the neutrino Yukawa matrix in the flavour basis is given by,

$$Y^\nu \approx \begin{pmatrix} 0 & be^{-i\,(\xi/2+3\pi/5)} & 0 \\ a\,e^{-i(\xi+3\pi/5)} & 4be^{-i\,(\xi/2+3\pi/5)} & a\,R_{13}\,e^{-i(\xi+3\pi/5)} \\ a\,e^{-i\,(\xi+3\pi/5)}(1-\gamma) & 2be^{-i(\xi/2+3\pi/5)} & c\,e^{i\phi} \end{pmatrix}\,, \tag{15}$$

where

$$\gamma \approx (c/a)R_{13}e^{i(\phi+\xi+3\pi/5)}\,. \tag{16}$$

The Dirac neutrino masses, the eigenvalues of the neutrino Dirac mass matrix $m_\nu^D = v\,Y_{LR}^\nu$, are given by

$$m_{\nu 1}^D \sim a\,v/\sqrt{17},\quad m_{\nu 2}^D \sim \sqrt{17}\,b\,v\quad m_{\nu 3}^D \sim c\,v\,, \tag{17}$$

where the approximations are quite precise given the working assumptions leading to the typical values in Eq. (5).

For the up-type quark matrix we have two options,

$$Y^u = \begin{pmatrix} 0 & be^{-i3\pi/5} & 0 \\ ae^{-i3\pi/5} & 4be^{-i3\pi/5} & 0 \\ ae^{-i3\pi/5} & 2be^{-i3\pi/5} & c\,e^{i\phi} \end{pmatrix} \qquad \text{CASE A} \tag{18}$$

and

$$Y^u = \begin{pmatrix} 0 & \frac{1}{3}\,be^{-i3\pi/5} & 0 \\ ae^{-i3\pi/5} & \frac{4}{3}\,be^{-i3\pi/5} & 0 \\ ae^{-i3\pi/5} & \frac{2}{3}\,be^{-i3\pi/5} & 3\,ce^{i\phi} \end{pmatrix} \qquad \text{CASE B}\,. \tag{19}$$

These are simply related to the neutrino Yukawa matrix in the original basis, Y'^{ν}, by Clebsch relations for CASE B, while we have simply $Y^u = Y'^{\nu}$ for CASE A (in the basis of Eq.(3)) which is just the minimal $SO(10)$-like expectation that the Dirac neutrino mass matrix is identically equal to the up-type quark mass matrix. Notice that in the A to Z model there are "texture zeroes" in the (1,1), (1,3) and (2,3) entries of the Yukawa matrices above that will play a role in leptogenesis considerations.

The Dirac neutrino masses are simply related to the up-type quark masses, depending on the choice of model,

$$m^D_{\nu 1} = m_{\rm up}, \quad m^D_{\nu 2} = m_{\rm charm}, \quad m^D_{\nu 3} = m_{\rm top} \qquad \text{CASE A} \tag{20}$$

and

$$m^D_{\nu 1} \approx m_{\rm up}, \quad m^D_{\nu 2} \approx 3\,m_{\rm charm}, \quad m^D_{\nu 3} \approx \frac{1}{3}\,m_{\rm top} \qquad \text{CASE B}\,. \tag{21}$$

In this way the (real) parameters a, b and c are simply determined from the values of the up quark masses at the grand-unified scale [1] so that the (real) input parameters are: $M'_{11}, M'_{13}, M'_{22}, M'_{33}$ and phase ϕ. The phase ξ is restricted to be one of the complex fifth roots of unity: $\xi = 0, \pm 2\pi/5, \pm 4\pi/5$.

Seesaw mechanism in the flavour basis

Using the see-saw formula

$$m^\nu = -v^2\, Y^\nu\, M_R^{-1}\, Y^{\nu T} \tag{22}$$

in the flavour basis we find the neutrino mass matrix m^ν,

$$m^\nu \approx m_a\, e^{i\phi_a} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & (1-\gamma) \\ 0 & (1-\gamma) & (1-\gamma)^2 \end{pmatrix} + m_b\, e^{i\phi_b} \begin{pmatrix} 1 & 4 & 2 \\ 4 & 16 & 8 \\ 2 & 8 & 4 \end{pmatrix} + m_c\, e^{i\phi_c} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{23}$$

[1] We use the values $m_{\rm top} = 100\,{\rm GeV}$, $m_{\rm charm} = 400\,{\rm MeV}$ and $m_{\rm up} = 1\,{\rm MeV}$. [16]

$$m_a = \frac{a^2 v^2}{M_1},\ m_b = \frac{b^2 v^2}{M_2},\ m_c = \frac{c^2 v^2}{M_3}\,, \tag{24}$$

$$\phi_a = -2(\xi + 3\pi/5),\ \phi_b = -2(\xi/2 + 3\pi/5),\ \phi_c = 2\phi\,. \tag{25}$$

From Eqs.(17),(24) and (20) or (21) we find,

$$m_a \sim 17\frac{m_{\rm up}^2}{M_1},\ m_b \sim (9)\frac{m_{\rm charm}^2}{17M_2},\ m_c \sim \frac{m_{\rm top}^2}{(9)M_3}\,, \tag{26}$$

where the factors in brackets apply to CASE B, and these factors are simply unity for CASE A. Thus the three right-handed neutrino masses M_1, M_2, M_3 may be determined from m_a, m_b, m_c.

The Majorana neutrino mass matrix m^ν, defined by $\mathcal{L}_\nu = -\frac{1}{2}m^\nu \overline{\nu}_{\rm L}\nu^c_{\rm L}$ + h.c., is diagonalised by

$$U_{\nu_{\rm L}}\, m^\nu\, U_{\nu_{\rm L}}^T = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \tag{27}$$

The PMNS matrix is then given by

$$U_{\rm PMNS} = U_{e_{\rm L}} U_{\nu_{\rm L}}^\dagger\,. \tag{28}$$

We use a standard parameterization $U_{\rm PMNS} = R_{23}U_{13}R_{12}P$ in terms of $s_{ij} = \sin(\theta_{ij})$, $c_{ij} = \cos(\theta_{ij})$, the Dirac CP violating phase δ^l and further Majorana phases contained in $P = {\rm diag}(e^{i\frac{\beta_1}{2}}, e^{i\frac{\beta_2}{2}}, 1)$. The standard PDG parameterization [17] differs slightly due to the definition of Majorana phases which are by given by $P_{\rm PDG} = {\rm diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}})$. Evidently the PDG Majorana phases are related to those in our convention by $\alpha_{21} = \beta^l_2 - \beta^l_1$ and $\alpha_{31} = -\beta^l_1$, after an overall unphysical phase is absorbed by $U_{e_{\rm L}}$.

For example, using the input parameters $\xi = 4\pi/5$, $\phi = \gamma = 0$ and

$$m_a = 0.034 \text{ eV},\ m_b = 0.002 \text{ eV}, m_c = 0.002 \text{ eV}, \tag{29}$$

we find for the CASE A the following values for the physical neutrino masses,

$$m_1 = 0.00034 \text{ eV},\ m_2 = 0.0086 \text{ eV},\ m_3 = 0.050 \text{ eV}. \tag{30}$$

and the lepton mixing parameters,

$$\theta_{12} = 34.0^\circ,\ \theta_{13} = 9.1^\circ,\ \theta_{23} = 39.7^\circ,\ \delta = 260^\circ,\ \beta_1 = 321^\circ,\ \beta_2 = 75^\circ\,. \tag{31}$$

3 Fitting neutrino parameters

In this Section we perform a quantitative numerical analysis. We randomly generate a set of values of the five (continuous) parameters of the model ($\underline{y} \equiv M'_{11}, M'_{22}, M'_{33}, M'_{13}, \phi$) for each of the five choices of the discrete parameter ξ and from these we calculate the nine low energy neutrino parameters in m_ν. A recent global analysis [18] finds the values shown below (with 1σ errors).

The solar mass squared difference:

$$\Delta m^2_{12} \equiv m^2_2 - m^2_1 = 7.50^{+0.19}_{-0.17} \times 10^{-5}\,\mathrm{eV}^2. \tag{32}$$

The atmospheric mass squared difference, respectively for normal ordering (NO) and for inverted ordering (IO):

$$\begin{aligned} \Delta m^2_{31} &\equiv m^2_3 - m^2_1 = 2.457^{+0.047}_{-0.047} \times 10^{-3}\,\mathrm{eV}^2 \quad \text{and} \\ \Delta m^2_{32} &\equiv m^2_3 - m^2_2 = -2.449^{+0.048}_{-0.047} \times 10^{-3}\,\mathrm{eV}^2\,. \end{aligned} \tag{33}$$

The solar mixing angle:

$$\theta_{12} = 33.48^\circ\,{}^{+0.78^\circ}_{-0.75^\circ}\,. \tag{34}$$

The reactor mixing angle:

$$\theta_{13} = 8.50^\circ\,{}^{+0.20^\circ}_{-0.21^\circ}\,. \tag{35}$$

The atmospheric mixing angle, respectively for NO and for IO:

$$\theta_{23} = 42.3^\circ\,{}^{+3.0^\circ}_{-1.6^\circ} \quad \text{and} \quad \theta_{23} = 49.5^\circ\,{}^{+1.5^\circ}_{-2.2^\circ}\,. \tag{36}$$

The Dirac phase, respectively for NO and for IO:

$$\delta = 306^\circ\,{}^{+39^\circ}_{-70^\circ} \quad \text{and} \quad \delta = 254^\circ\,{}^{+63^\circ}_{-62^\circ}\,. \tag{37}$$

We see that, for the atmospheric mixing angle, within 1σ, there are currently two solutions, one in the first octant for NO and one in the second octant for IO, though the alternative choice of octant is in both cases only disfavoured at only $\sim 1.4\,\sigma$. Note also that, within 3σ, the Dirac phase δ can have any value.

The four parameters $\Delta m^2_{12}, \Delta m^2_{23}, \theta_{12}$ and θ_{13}, are measured quite accurately and precisely and their distributions are very well approximated by Gaussian distributions. On the other hand the atmospheric mixing angle is not only much less precisely measured and not Gaussianly distributed, but is also affected by larger systematic uncertainties and in particular different global analyses exclude maximal mixing with different statistical significance, in any case below 2σ, so that its determination, and in particular the deviation from maximality, should be still regarded as quite unstable [18, 19].

We have determined our best fit values minimising the quantity

$$\chi^2 \equiv \sum_{i=1}^{N} p_i^2 \,, \qquad p_i \equiv \frac{X_i^{\rm th}(\underline{y}) - \bar{X}_i}{\sigma_{X_i}} \,, \tag{38}$$

where the $X_i = \bar{X}_i \pm \sigma_{X_i}$'s are the $N = 5$ experimental parameters that we fit (in next Sections we will include the matter-antimatter asymmetry so that N rises to 6) and $X_i^{\rm th}(\underline{y})$ are their predicted values depending on the theoretical parameters of the model $\underline{y} = (M'_{11}, M'_{13}, M'_{22}, M'_{33}, \phi; \xi, {\rm J})$ ($\xi = 0, \pm 4\pi/5, \pm 2\pi/5$, J=CASE A, CASE B are discrete parameters). Since the Dirac phase δ can have any value within 3σ, and has certainly not a Gaussian distribution, we decided not to include it in the fit not to risk to prematurely exclude potential solutions but we will comment on how this would change including δ.

For the same reason for the atmospheric mixing angle we use very conservatively $\theta_{23} = 45.9^\circ \pm 3.5^\circ$ both for NO and IO (we have basically taken as a central value the average between the two minima and as error their halved separation). We could also have performed the fits distinguishing two different ranges for θ_{23}, one in the lower octant for NO and one in the higher octant for IO, but these are still too weak hints. Our choice cuts too low or too high values for θ_{23}, as established by all experiments even singularly taken, but it still treats the maximal mixing value as perfectly allowed and does not favour any of the two octants on the other, since, as already discussed, the hint coming from current experimental data is still too weak and unstable. In this way any emerging possible preference either for lower or for higher octant, as for maximal mixing, can be uniquely ascribed to the model itself and is not hidden by still unstable measurements.

Within the approximation that θ_{23} distribution is also Gaussianly distributed, the defined quantity χ^2 has a truly χ^2-distribution, being the sum of squares of independent Gaussian variables and the p_i's are the associated pulls.

Notice that the number of (continuous) theoretical parameters is equal to the number of experimental parameters (five) and, therefore the number of degrees of freedom vanishes. This implies that the minimisation of χ^2 can only provide best fit values (and prediction on the low energy phases) but cannot be regarded as a goodness of fitness of the model, since even vanishing χ^2 values could be potentially obtained with a fine tuned choice of the theoretical parameters $\underline{y}$ independently of the values of the experimental values. However, a preliminary indication of the goodness of the fitness is given by a comparison of the predicted value of the Dirac phase with the current best fit value Eq. (37), though, as we said, we certainly need a more precise measurement of δ to draw firmer conclusions.

In Table 1 and Table 2 we summarise the results of our analysis. We indicate the Majorana phases both in the convention ${\rm diag}(e^{i\,\frac{\beta_1}{2}}, e^{i\,\frac{\beta_2}{2}}, 1)$ and in the (PDG) convention

CASE	A		
ξ	$+4\pi/5$	0	$-4\pi/5$
$\chi^2_{\rm min}$	4.40	22.8	3.63
m_1/meV	0.20	0.021	0.022
m_2/meV $(p_{\Delta m^2_{12}})$	8.66 (+0.002)	8.62 (-0.38)	8.69 (+0.25)
m_3/meV $(p_{\Delta m^2_{13}})$	49.9 (+0.76)	48.9 (-1.34)	49.8 (+0.47)
$\theta_{12}/^\circ$ $(p_{\theta_{12}})$	33.5 (+0.04)	35.3 (2.37)	33.0 (-0.66)
$\theta_{13}/^\circ$ $(p_{\theta_{13}})$	8.37 (-0.64)	8.08 (-2.05)	8.42 (-0.37)
$\theta_{23}/^\circ$ $(p_{\theta_{23}})$	39.2 (-1.85)	34.3 (-3.3)	39.9 (-1.66)
$\delta/^\circ$	251	180	109
$\beta_1/^\circ$	225	338	173
$\beta_2/^\circ$	82	175	279
$\alpha_{21}/^\circ$	217	197	106
$\alpha_{31}/^\circ$	135	22	187
M'_{11}/GeV	6.5×10^5	6.6×10^5	2.2×10^6
M'_{22}/GeV	5.0×10^9	5.1×10^9	5.0×10^9
M'_{33}/GeV	7.2×10^{15}	8.6×10^{16}	6.8×10^{16}
M'_{13}/GeV	3.2×10^{10}	3.0×10^{11}	3.4×10^{11}
M'_{13}/M'_{22}	6.4	59.7	69
ϕ/π	1.27	1.80	1.64
M_1/GeV	5.1×10^5	4.2×10^5	4.9×10^5
M_2/GeV	5.0×10^9	5.1×10^9	5.0×10^9
M_3/GeV	7.2×10^{15}	8.6×10^{16}	6.8×10^{16}

Table 1: Results for the case A (NO and no leptogenesis).

$\text{diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}})$. The table refers to NO, since we found that the model cannot reproduce the neutrino mixing parameters for IO. In particular it badly fails in reproducing the neutrino mass spectrum.

Table 1 refers to CASE A (cf. eq. (18)) while Table 2 to CASE B (cf. eq. (19)). For both cases we show the results for $\xi = \pm 4\pi/5$ and for $\xi = 0$ since for $\xi = \pm 2\pi/5$ we could not find any fit with a value of $\chi^2 < 100$ so that they can be basically considered ruled out.

Let us list the main results postponing some comments to the conclusions.

- We do not find any solution for IO, the model is unable to reproduce IO neutrino masses.
- For $\xi = \pm 2\pi/5$ there are no solutions with $\chi^2_{\rm min} < 100$.
- Except for the case $\xi = 0$, the best fit values are obtained for the CASE A.

CASE	B		
ξ	$+4\pi/5$	0	$-4\pi/5$
$\chi^2_{\rm min}$	7.84	9.32	5.81
m_1/meV	0.008	3.0	0.004
m_2/meV ($p_{\Delta m^2_{12}}$)	8.58 (-0.81)	9.25 (+0.89)	8.72 (+0.62)
m_3/meV ($p_{\Delta m^2_{13}}$)	49.6 (+0.05)	48.8 (-1.85)	50.3 (+1.53)
$\theta_{12}/^\circ$ ($p_{\theta_{12}}$)	33.2 (-0.31)	32.3 (-1.49)	33.0 (-0.57)
$\theta_{13}/^\circ$ ($p_{\theta_{13}}$)	8.93 (+2.10)	8.84 (+1.66)	8.51 (+0.06)
$\theta_{23}/^\circ$ ($p_{\theta_{23}}$)	40.0 (-1.63)	44.55 (-0.33)	39.9 (-1.66)
$\delta/^\circ$	253	99	108
$\beta_1/^\circ$	335	66	170
$\beta_2/^\circ$	84	325	278
$\alpha_{21}/^\circ$	108	259	109
$\alpha_{31}/^\circ$	25	294	190
M'_{11}/GeV	7.2×10^5	2.9×10^5	6.6×10^6
M'_{22}/GeV	4.3×10^{10}	4.2×10^{10}	4.4×10^{10}
M'_{33}/GeV	2.2×10^{16}	9.7×10^{13}	4.3×10^{16}
M'_{13}/GeV	6.8×10^{10}	7.5×10^9	5.1×10^{11}
ϕ/π	1.62	1.23	1.62
M_1/GeV	5.1×10^5	2.9×10^5	4.9×10^5
M_2/GeV	4.3×10^{10}	4.2×10^{10}	4.4×10^{10}
M_3/GeV	2.2×10^{16}	9.7×10^{13}	4.3×10^{16}

Table 2: Results for the CASE B (NO and no leptogenesis).

- The best fit solutions that we obtained for NO, except for the case B with $\xi = 0$, seem to point to $\theta_{23} \sim 40^\circ$, certainly in the first octant and even below the current best fit value eq. (36) hinted by current global analyses.

- Taking into account the RH neutrino mixing parameter M'_{13} improves all fits and one finds values of $\chi^2_{\min}(M'_{13} \neq 0) \simeq \chi^2_{\min}(M'_{13} = 0) - 2$.

- Values of the Dirac phase close to the best fit value from global analyses (cf. eq. (37)) are attained for $\xi = +4\,\pi/5$ (both in CASE A and in CASE B). If further experimental data should further support these values ($\sim -90^\circ$), this would be the only surviving option.

4 Leptogenesis without flavour coupling

Let us now consider the calculation of the matter-antimatter asymmetry with leptogenesis. The strongly hierarchical RH neutrino mass spectrum in the A to Z model discussed in the previous section, with $M_1 \ll 10^9\,\mathrm{GeV}$ and $10^{12}\,\mathrm{GeV} \gg M_2 \gg 10^9\,\mathrm{GeV}$, necessarily points to a N_2-dominated leptogenesis scenario [9] since both the lighest and the heaviest RH neutrino decays produce a negligible asymmetry compared to the observed one. In this case the $B-L$ asymmetry, in a portion of co-moving volume containing one RH neutrino in ultra-relativistic thermal equilibrium, can be calculated as [9, 10]

$$\begin{aligned} N_{B-L}^{\mathrm{lep,f}} \;\simeq\; & \left\{ \left[\frac{K_{2e}}{K_{2\tau_2^\perp}}\, \varepsilon_{2\tau_2^\perp} \kappa(K_{2\tau_2^\perp}) + \left(\varepsilon_{2e} - \frac{K_{2e}}{K_{2\tau_2^\perp}}\, \varepsilon_{2\tau_2^\perp} \right) \kappa(K_{2\tau_2^\perp}/2) \right] \, e^{-\frac{3\pi}{8}\, K_{1e}} + \right. \\ & + \left[\frac{K_{2\mu}}{K_{2\tau_2^\perp}}\, \varepsilon_{2\tau_2^\perp}\, \kappa(K_{2\tau_2^\perp}) + \left(\varepsilon_{2\mu} - \frac{K_{2\mu}}{K_{2\tau_2^\perp}}\, \varepsilon_{2\tau_2^\perp} \right) \kappa(K_{2\tau_2^\perp}/2) \right] \, e^{-\frac{3\pi}{8}\, K_{1\mu}} + \\ & + \left. \varepsilon_{2\tau}\, \kappa(K_{2\tau})\, e^{-\frac{3\pi}{8}\, K_{1\tau}} \right\} \,, \end{aligned} \tag{39}$$

where $K_{2\tau_2^\perp} \equiv K_{2e} + K_{2\mu}$ and $\varepsilon_{2\tau_2^\perp} \equiv \varepsilon_{2e} + \varepsilon_{2\mu}$. This expression for the asymmetry neglects the flavour coupling effects studied in [15]. In the present model only the last term will survive, as we shall justify shortly, and we may drastically approximate the above expression to a much simpler one,

$$N_{B-L}^{\mathrm{lep,f}} \simeq \varepsilon_{2\tau}\, \kappa(K_{2\tau})\, e^{-\frac{3\pi}{8}\, K_{1\tau}} \,, \tag{40}$$

which says that the only relevant asymmetry is that one produced at the N_2 scale in the tauon flavour, simply given by $\varepsilon_{2\tau}\, \kappa(K_{2\tau})$ with our normalisation, followed by exponential washout $e^{-\frac{3\pi}{8}\, K_{1\tau}}$ the N_1 scale. [2]

[2] Of course in models with $M_1 \gtrsim 10^9\,\mathrm{GeV}$ or realising a crossing level solution [20] with M_1 and M_2 close enough to have resonant leptogenesis [21], this contribution must also be considered and the lightest RH neutrino washout is then not a crucial problem as in the N_2-dominated scenario. Examples of realistic $SO(10)$ models realising this case were discussed in [22].

The flavoured decay parameters $K_{i\alpha}$ are defined as

$$K_{i\alpha} \equiv \frac{\Gamma_{i\alpha} + \overline{\Gamma}_{i\alpha}}{H(T = M_i)} = \frac{|m_{D\alpha i}|^2}{M_i\, m_\star}\,, \tag{41}$$

where we introduced the neutrino Dirac mass matrix $m_D \equiv v\, Y^{\nu}_{LR}$. The $\Gamma_{i\alpha}$'s and the $\bar{\Gamma}_{i\alpha}$'s can be regarded as the zero temperature limit of the flavoured decay rates into α leptons, $\Gamma(N_i \to \phi^\dagger\, l_\alpha)$, and anti-leptons, $\Gamma(N_i \to \phi\, \bar{l}_\alpha)$, in a three-flavoured regime, where lepton quantum states can be treated as an incoherent admixture of the three flavour components. The efficiency factors at the production can be calculated using [23]

$$\kappa(K_{2\alpha}) = \frac{2}{z_B(K_{2\alpha})\, K_{2\alpha}} \left(1 - e^{-\frac{K_{2\alpha}\, z_B(K_{2\alpha})}{2}}\right)\,, \quad z_B(K_{2\alpha}) \simeq 2 + 4\, K_{2\alpha}^{0.13}\, e^{-\frac{2.5}{K_{2\alpha}}}\,. \tag{42}$$

This expression is valid for an initial thermal abundance but, as we will see in a moment, since the production will prove to occur in the strong wash-out regime, there is actually independence of the initial RH neutrino abundance. Moreover in this case the efficiency factor is well approximated by $\kappa(K_{2\alpha}) \simeq 0.5/K_{2\alpha}^{1.2}$.

The flavoured *CP* asymmetries, defined as $\varepsilon_{2\alpha} \equiv -(\overline{\Gamma}_{2\alpha} - \Gamma_{2\alpha})/(\Gamma_{2\alpha} + \overline{\Gamma}_{2\alpha})$, can be calculated in general as [24]

$$\varepsilon_{2\alpha} \;=\; \frac{3}{16\,\pi}\, \frac{M_2\, m_{\rm atm}}{v^2}\, \sum_{j\neq 2} \left(\mathcal{I}_{2j}^{\alpha}\, \xi(M_j^2/M_2^2) + \frac{2}{3}\, \mathcal{J}_{2j}^{\alpha}\, \frac{M_j/M_2}{M_j^2/M_2^2 - 1}\right)\,, \tag{43}$$

where we defined [25],

$$\mathcal{I}_{2j}^{\alpha} \equiv \frac{{\rm Im}\Big[\big(m_D^\dagger\big)_{i\alpha}\, \big(m_D\big)_{\alpha j} \big(m_D^\dagger m_D\big)_{ij}\Big]}{M_2\, M_j\, \widetilde{m}_2\, m_{\rm atm}}\,, \quad \mathcal{J}_{2j}^{\alpha} \equiv \frac{{\rm Im}\Big[\big(m_D^\dagger\big)_{i\alpha}\, \big(m_D\big)_{\alpha j} \big(m_D^\dagger m_D\big)_{ji}\Big]}{M_2\, M_j\, \widetilde{m}_2\, m_{\rm atm}}\,, \tag{44}$$

with $\widetilde{m}_2 \equiv (m_D^\dagger\, m_D)_{22}/M_2$, and

$$\xi(x) = \frac{2}{3} x \left[(1+x)\ln\left(\frac{1+x}{x}\right) - \frac{2-x}{1-x}\right]\,. \tag{45}$$

Terms $\propto \mathcal{I}_{21}^{\alpha}\, \xi(M_1^2/M_2^2), \mathcal{J}_{21}^{\alpha}, \mathcal{J}_{23}^{\alpha}$ are strongly suppressed in a way that in the N_2-dominated scenario the flavoured *CP* asymmetries $\varepsilon_{2\alpha}$'s can be approximated by

$$\varepsilon_{2\alpha} \simeq \frac{3}{16\pi} \frac{M_2\, m_{\rm atm}}{v^2}\, \mathcal{I}_{23}^{\alpha}\,. \tag{46}$$

This is because the two terms (for $j = 1$) from the interference with the lightest RH neutrinos are $\propto M_1/M_2$, while the second term from the interference with the heaviest RH neutrinos is $\propto M_2/M_3$ and, therefore, they are suppressed compared to $\mathcal{I}_{23}^{\alpha}$.

Let us now calculate $\mathcal{I}_{23}^{\alpha}$. First of all it is immediate to see that since $Y_{e3}^{\nu}=0$ then necessarily $\varepsilon_{2e}=0$. Even though the muon asymmetry does not exactly vanishes as the electronic one, however, it is suppressed as $\varepsilon_{2\mu}\propto a\,b^2\,M'_{13}/M'_3\sim 10^{-17}$ and can be neglected. This implies that, at least at lower order, the observed asymmetry can only be produced in the tauon flavour, as in Eq.(40). The expression for the final asymmetry, expressed in terms of the baryon-to-photon number ratio, then becomes extremely simple,

$$\eta_B \simeq a_{\rm sph}\,\frac{N_{B-L}^{\rm f}}{N_{\gamma}^{\rm rec}} \simeq 0.01\,\varepsilon_{2\tau}\,\kappa(K_{2\tau})\,e^{-\frac{3\pi}{8}\,K_{1\tau}}\,, \tag{47}$$

where $a_{\rm sph}=28/79$ is the fraction of $B-L$ asymmetry that is converted into a baryon asymmetry by sphaleron processes in equilibrium, $\kappa(K_{2\tau})$ is the efficiency factor for the tauon asymmetry at the end of the N_2-production and $N_{\gamma}^{\rm rec}$ is the number of photons at recombination in the given portion of co-moving volume. [3]

We have first of all to calculate $K_{1\tau}$ and check that it is possible to have $K_{1\tau}\lesssim 1$. From the general expression eq. (41) one has

$$K_{1\tau}=\frac{|m_{D\tau 1}|^2}{M_1\,m_\star}=\frac{v^2\,a^2}{m_\star\,M_1}\,|1-\gamma|^2\,, \tag{48}$$

showing that the quantity γ, defined in Eq. (16) and originating from the mixing parameter M'_{13}, plays a crucial role. Indeed since $v^2\,a^2/(m_\star\,M_1)\sim 20\gg 1$, the possibility to have $K_{1\tau}\lesssim 1$ necessarily relies on having $\mathrm{Arg}[\gamma]\simeq 0$ implying $\phi\sim-(\xi+3\pi/5)$.

Let us now calculate $\varepsilon_{2\tau}$ from the eq. (46). Considering that $\widetilde{m}_2=21\,v^2\,b^2/M_2$ and that $\mathcal{I}_{23}^{\tau}\simeq 4\,b^2\,c^2\,\sin(2\,\phi+\xi+6\,\pi/5)\,v^4/(M_2\,M_3\,\widetilde{m}_2\,m_{\rm atm})$, one finds

$$\varepsilon_{2\tau}\simeq\frac{c^2}{28\,\pi}\,\frac{M_2}{M_3}\,\sin(2\phi+\xi+6\pi/5)\,. \tag{49}$$

When the condition for $K_{1\tau}\lesssim 1$ on ϕ is imposed one has

$$\varepsilon_{2\tau}\simeq-\frac{c^2}{28\,\pi}\,\frac{M_2}{M_3}\,\sin\xi\,, \tag{50}$$

showing that out of the five possible values of ξ, only $\xi=-2\pi/5,-4\pi/5$ can lead to the correct sign of the asymmetry. An order-of-magnitude estimation gives then $|\varepsilon_{2\tau}|\sim 10^{-7}$–$10^{-6}$.

Finally from the eq. (41) we can calculate

$$K_{2\tau}=\frac{|m_{D\tau 2}|^2}{M_2\,m_\star}=\frac{4}{(9)}\,\frac{b^2\,v^2}{M_2\,m_\star}\sim 10\,, \tag{51}$$

[3] This expression is valid independently of the normalisation of the abundances N_X, since the normalization factor would cancel out in the ratio $N_{B-L}^{\rm f}/N_{\gamma}^{\rm rec}$.

that, plugged into the eq. (47) gives $\eta_B \sim 10^{-11}$–10^{-9}, showing that potentially the observed value of the asymmetry could be reproduced. However when one tries to fit simultaneously the asymmetry and the mixing parameters one finds that the condition $\gamma \simeq 1$, necessary to have $K_{1\tau} \lesssim 1$, is incompatible with the possibility to reproduce the correct values of the mixing parameters so that the asymmetry produced at the N_2 scale is afterwards completely washed out at the N_1 scale.

5 Leptogenesis with flavour coupling

As we have just seen the asymmetry is mainly produced by the next-to-lightest RH neutrinos in the tauon flavour but this asymmetry is fully washed-out by the lightest RH neutrinos since the condition $K_{1\tau} \lesssim 1$ is not compatible with the measured values of the mixing parameters.

However, one has also to consider that part of the asymmetry in the tauon flavour is transferred to the electron and muon flavours by flavour coupling effects due primarily to the fact that N_2-decays produce in addition to an asymmetry in the tauon lepton doublets also an (hyper charge) asymmetry in the Higgs bosons. This Higgs asymmetry unavoidably induces, through the inverse decays, also an asymmetry in the lepton doublets that at the production are a coherent admixture of electron and muon components. Therefore, in this case, inverse decays actually produce an asymmetry instead of wash it out as in a traditional picture. A somehow smaller effect is also due to the asymmetries stored into quarks and into right handed charged leptons.

The account of flavour coupling effects modifies the usually considered expression for the asymmetry eq. (47) resulting into [15]

$$\eta_B \simeq \sum_{\alpha=e,\mu,\tau} \eta_B^{(\alpha)} \, , \tag{52}$$

where in addition to the tauon contribution eq. (47) one also has an electron contribution

$$\eta_B^{(e)} \simeq -0.01\, \varepsilon_{2\tau}\, \kappa(K_{2\tau})\, \frac{K_{2e}}{K_{2e}+K_{2\mu}}\, C^{(2)}_{\tau^{\perp}\tau}\, e^{-\frac{3\pi}{8}\, K_{1e}} \, , \tag{53}$$

and a muon contribution given by

$$\eta_B^{(\mu)} \simeq -0.01\, \varepsilon_{2\tau}\, \kappa(K_{2\tau}) \left(\frac{K_{2\mu}}{K_{2e}+K_{2\mu}}\, C^{(2)}_{\tau^{\perp}\tau} - \frac{K_{1\mu}}{K_{1\tau}}\, C^{(3)}_{\mu\tau} \right) e^{-\frac{3\pi}{8}\, K_{1\mu}} \, . \tag{54}$$

It should be noticed how the source of the electron and muon asymmetries is in any case the tauon asymmetry, but part of this induces a muon and an electron asymmetry thanks to flavour coupling. The flavour coupling coefficients are given by $C^{(2)}_{\tau^{\perp}\tau} = 104/589$ and $C^{(3)}_{\mu\tau} = 142/537$. Notice that we are neglecting additional correcting terms containing products of the flavour coupling coefficients and we are also neglecting terms $\propto \varepsilon_{2\tau^\perp}$ since this is always too small to generate sizeable contributions. Also notice that from Eqs. (15) and (41) one can see immediately that $K_{1e} = 0$.

Notice moreover that the second term in the muon contribution comes from flavour coupling at the lightest RH neutrino wash-out that works in the same way as at the production: the lightest RH neutrino wash-out processes acting on muon lepton doublets, in the presence of a non-vanishing Higgs asymmetry, induce a muon asymmetry. However, this term is basically much smaller than the first term that gives the dominant contribution to the total asymmetry, indeed it dominates on the electronic term as well since $K_{2e} \ll K_{2\mu}$, and is in the end responsible for the two solutions that we found.

We have indeed performed again a numerical fit for all the different cases and this time we have found that, both for the case A and for the case B with $\xi = +4\pi/5$, there is indeed a solution, shown in Table 3, with an acceptably small χ^2 value. We found no other solution with a χ^2 lower than 100 for all the other cases ($\xi = 0, \pm 2\pi/5, -4\,\pi/5$) since these tend to give either a too small θ_{12} or a too small θ_{13} or both. In both cases one can see that $K_{1\mu} \lesssim 1$. In this way the flavour coupling induced asymmetry at the production in the muon flavour can survive the lightest RH neutrino wash out giving the dominant contribution to the final asymmetry. This is a nice example of the "flavour swap" scenario envisaged in [15].

It should be noticed that both solutions predict the atmospheric angle well in the second octant, a feature that will be relatively soon tested by new data from neutrino oscillation experiments. In fact the two solutions give quite similar predictions on δ and θ_{23}. In the Figure we have plotted, in the plane δ vs. θ_{23}, points corresponding to solutions with $\chi^2 < 10$. This gives an idea of the allowed region in this plane. It can be seen how for the CASE A the minimum values correspond to $(\delta, \theta_{23}) \sim (10^\circ, 51^\circ)$ while for the CASE B the whole region is slightly reduced (indeed $\chi^2_{\rm min} \simeq 5$ for the CASE A and $\chi^2_{\rm min} \simeq 6$ for the CASE B) and shifted to higher values both of δ and θ_{23} and the minimum values are $(\delta, \theta_{23}) \sim (14^\circ, 52^\circ)$. This shows that the CASE A is slightly more favoured compared to CASE B.

It should also be noticed that since now the number of degrees of freedom $\nu = 1$, the $\chi^2_{\rm min}$ can be regarded as an indication of the goodness of fit (g.o.f.), having in mind the previous discussion on the measurement of θ_{23}. Of course we also had the possibility to choose the value of the discrete parameter ξ and between CASE A and CASE B and this somehow made things a bit easier, but it is still intriguing that, despite its reduced number of parameters, the model can also account for the matter-antimatter of the Universe. It should also be said that if the current experimental information on δ is taken into account (cf. eq.(37)), one probably would have an additional contribution $\Delta\chi^2_\delta = p^2_\delta \gtrsim 3$ so that the allowed regions shown in the Figure are marginally compatible with current data on δ, especially in CASE B while in CASE A the portion around the

CASE	A	B
ξ	$+4\pi/5$	
$\chi^2_{\rm min}$	5.15	6.1
$M'_{11}/10^6$GeV	1.30	1.33
$M'_{22}/10^{10}$GeV	0.48	4.35
$M'_{33}/10^{12}$GeV	2.16	1.31
$M'_{13}/10^{10}$GeV	1.81	0.61
M'_{13}/M'_{22}	3.75	0.141
ϕ/π	0.795	0.788
$M_1/10^7$GeV	15	2.7
$M_2/10^{10}$GeV	0.483	4.35
$M_3/10^{12}$ GeV	2.16	1.31
$\vert\gamma\vert$	203	38
m_1/meV	2.3	2.3
m_2/meV ($p_{\Delta m^2_{12}}$)	8.93 (-0.22)	8.94 (-0.25)
m_3/meV ($p_{\Delta m^2_{13}}$)	49.7 (+0.17)	49.7 (+0.21)
$\sum_i m_i$/meV	61	61
m_{ee}/meV	1.95	1.95
$\theta_{12}/^\circ$ ($p_{\theta_{12}}$)	33.0 (-0.58)	33.0 (-0.66)
$\theta_{13}/^\circ$ ($p_{\theta_{13}}$)	8.40 (-0.47)	8.40 (-0.49)
$\theta_{23}/^\circ$ ($p_{\theta_{23}}$)	53.3 (+2.1)	54.0 (+2.3)
$\delta/^\circ$	20.8	23.5
$\beta_1/^\circ$	118	115
$\beta_2/^\circ$	281	278
$\alpha_{21}/^\circ$	163	162
$\alpha_{31}/^\circ$	242	245
$\rho/^\circ$	279	279
$\sigma/^\circ$	220	221
$\eta_B/10^{-10}$ (p_{η_B})	6.101 (+0.01)	6.101 (+0.01)
$\varepsilon_{2\tau}$	-8.1 $\times 10^{-6}$	-1.3×10^{-5}
$K_{1\mu}$	0.11	0.58
$K_{1\tau}$	4341	800
$K_{2\tau}$	7.3	7.3
$K_{2\mu}$	29.2	29.2
K_{2e}	1.8	1.8
$\vert(\widetilde{m}_\nu)_{11}\vert$/meV	6.6×10^{-6}	3.7×10^{-5}
$\vert(\widetilde{m}_\nu^{-1})_{33}\vert$/meV^{-1}	0.22	1.2
$\vert(\widetilde{m}_\nu)_{12}\vert$/meV	2.6×10^{-5}	1.5×10^{-4}

Table 3: Solutions found for flavour coupled leptogenesis with $\chi^2_{\rm min} < 100$ (neutrino masses are NO).

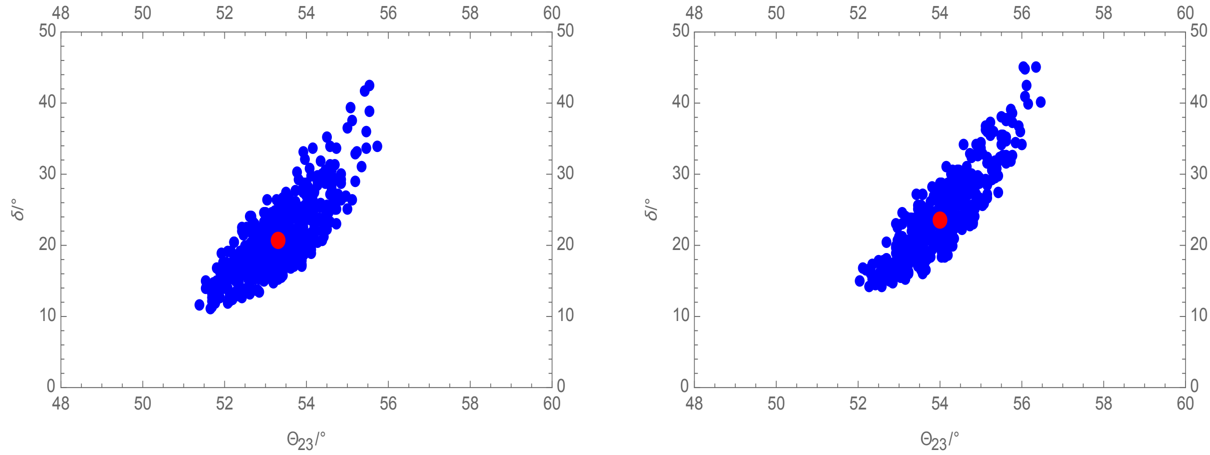

Figure 1: Scatter plots of points in the plane δ vs. θ_{23} with $\chi^2 < 10$ (leptogenesis included) for the CASE A (left) and for the CASE B (right) for NO and $\xi = +4\pi/5$. The red points correspond to the best fits solutions in Table 3. These predictions are subject to the theoretical uncertainties discussed in the text.

minimum values for (δ, θ_{23}) is still allowed at $\sim 2\sigma$. [4] However, one should also take into account that we are currently neglecting corrections from the charge lepton mass matrix that has been approximated to be diagonal and also uncertainties on the values of the up quark masses at high scale (see Footnote 1). This might help in shifting the allowed regions shown in the Figure toward more favoured experimental values. In any case the allowed regions shown in the Figure in the plane δ vs. θ_{23}, in combination with NO, are quite a strong prediction of the model that will be certainly tested during next years. Much more difficultly testable predictions, certainly not in a close future, are the very small value of the neutrinoless double beta decay effective neutrino mass ($m_{ee} \simeq 2\,\text{meV}$) and the small increase ($\simeq 3\,\text{meV}$) of the sum of the neutrino masses from its hierarchial value ($\sum_i m_i|_{m_1=0} \simeq 58\,\text{meV}$).

A significant feature of the best fit spectrum for both cases A and B in Table 3 is the compressed spectrum of right-handed neutrino masses as compared to the original estimates in Eq.(4). This stems from the requirement that successful N_2 leptogenesis is proportional to the ratio M_2/M_3 in Eq.(50) which must necessarily be larger than naively expected in Eq.(4). M_2 cannot change much, since according to sequential dominance and the charm quark mass relation in Eq.(26), it sets the solar neutrino mass scale. It follows that the only possibility is for M_3 to decrease which requires the parameter m_c to increase (see Eq.(26)) and hence the third seesaw matrix in Eq.(23), proportional to

[4]However, note that in Eq.(38) we would then have $N = 7$ so that the number of degrees of freedom would be 7-5=2 so that one could say that including δ would actually improve the fit, decreasing χ^2/d.o.f, for points with $\Delta\chi^2 \lesssim 5$. This point is right now quite indicative since the distribution of δ is highly non Gaussian.

diag(0, 0, 1), threatens to dominate the neutrino mass matrix and pull the atmospheric mixing away from its maximal value. This threat is averted by noting that, for large γ, the first matrix in Eq.(23) has a dominant (3,3) element of order γ^2 which may partially cancel the contribution from the third matrix leaving a resulting (3,3) element of order γ, of the same order as the (2,3) and (3,2) elements of the first matrix. This requires a fine tuning of one part in γ (tabulated in Table 3). In order to maintain the correct atmospheric neutrino mass, this increase in γ must be compensated by reducing the parameter m_a in proportion to γ which, from Eq.(26), requires M_1 to increase in proportion to γ. In the next section we obtain further analytic insight into our results from different perspectives.

6 Analytical insight from general $SO(10)$-inspired leptogenesis

It is interesting to get some understanding of the numerical results we obtained within the A to Z model from the more general context of $SO(10)$-inspired leptogenesis [26, 20, 11, 12]. In this case the asymmetry can be calculated analytically assuming that the spectrum of the neutrino Dirac mass matrix is hierarchical ($m^D_{\nu 1} \gg m^D_{\nu 2} \gg m^D_{\nu 3}$), an assumption certainly holding in our case. Here we present a simple generalisation of the results presented in [12] providing a very good explanation of the numerical results.

The starting point is to write the neutrino Dirac mass matrix in the bi-unitary parameterisation (mathematically equivalent to the singular value decomposition of m^D_ν),

$$m^D_\nu = \widetilde{V}^\dagger_L \, D_{m^D_\nu} \, \widetilde{U}_R \, , \tag{55}$$

where we defined $D_{m^D_\nu} \equiv \mathrm{diag}(m^D_{\nu 1}, m^D_{\nu 2}, m^D_{\nu 3})$. The unitary matrices $\widetilde{V}_L$ and $\widetilde{U}_R$ transform respectively the LH and RH neutrino fields from the flavour basis to the Yukawa basis. In our case, from the eq. (15) for the neutrino Yukawa matrix and parameterising $\widetilde{V}_L$ analogously to the leptonic mixing matrix as

$$\widetilde{V}_L = R_{23}(\theta^L_{23}) \, R_{13}(\theta^L_{13}) \, R_{12}(\theta^L_{12}) \, D(\Phi^L) \, , \tag{56}$$

one has tiny $\theta^L_{23}, \theta^L_{13} \ll 1^\circ$ so that $\widetilde{V}_L \simeq R_{12}(\theta^L_{12})$ with $\theta^L_{12} \simeq 14^\circ$. Notice that this angle would correspond to the Cabibbo angle θ_C in the quark sector, that is therefore overestimated by $\sim 1^\circ$. However, turning on down quark mass matrix (non-diagonal) terms, one can reproduce the correct value. Similar corrections are expected on the neutrino mixing angles from analogous charged lepton mass matrix correcting (non-diagonal) terms.

Inserting the eq. (55) into the see-saw formula eq. (22) written in the flavour basis, one obtains the following expression for the Majorana mass matrix in the Yukawa basis

$$\widetilde{M}_R \equiv \widetilde{U}^\star_R \, D_M \, \widetilde{U}^\dagger_R = -D_{m^D_\nu} \, \widetilde{V}^\star_L \, m^{-1}_\nu \, \widetilde{V}^\dagger_L \, D_{m^D_\nu} \, , \tag{57}$$

and for its inverse

$$\widetilde{M}_R^{-1} \equiv \widetilde{U}_R\, D_M^{-1}\, \widetilde{U}_R^{\dagger} = -D_{m_\nu^D}^{-1}\, \widetilde{V}_L\, m_\nu\, \widetilde{V}_L^T\, D_{m_\nu^D}^{-1} \,. \tag{58}$$

In the above we have transformed the heavy Majorana mass matrix from the diagonal basis $M_R = D_M$ in Eq.(11) to the basis in Eq.(55) in which the Dirac neutrino mass matrix is diagonal. The analytical expressions for the RH neutrino masses obtained in [12] in the approximation $\widetilde{V}_L \simeq I$ get extended for a general V_L simply replacing $m_\nu \to \widetilde{m}_\nu \equiv \widetilde{V}_L\, m_\nu\, \widetilde{V}_L^T$ [20] (the light neutrino mass matrix in the Yukawa basis) obtaining for the RH neutrino masses the following analytical expressions

$$M_1 \simeq \frac{(m_{\nu 1}^D)^2}{|(\widetilde{m}_\nu)_{11}|}\,, \quad M_2 \simeq \frac{(m_{\nu 2}^D)^2\,|(\widetilde{m}_\nu)_{11}|}{m_1\, m_2\, m_3\, |(\widetilde{m}_\nu^{-1})_{33}|}\,, \quad M_3 \simeq (m_{\nu 3}^D)^2\, |(\widetilde{m}_\nu^{-1})_{33}|\,. \tag{59}$$

In deriving the first and third equalities above we have used the strong up-type quark mass hierarchy (equal to the eigenvalues of the neutrino Dirac mass matrix in case A), and for the second equality we have taken the absolute value of the determinant of the seesaw formula. Plugging the expressions for M_2 and M_3 into the eq. (49) for $\epsilon_{2\tau}$ and in turn this into the eq. (54) for the dominant muonic contribution to η_B and taking into account that $K_{2\mu}/(K_{2\mu} + K_{2e}) = 16/17$, one finds

$$\eta_B \simeq -\frac{0.04\, c^2}{119\,\pi}\,\frac{M_2}{M_3}\,\kappa(K_{2\tau})\, C^{(2)}_{\tau^{\perp}\tau}\, e^{-\frac{3\pi}{8}\,K_{1\mu}}\, \sin(2\phi + \xi + 6\pi/5). \tag{60}$$

From the expressions for M_2 and M_3 in eq. (59) and, taking into account the relations (17), (20) and (21), one can also write for CASE A (CASE B)

$$\eta_B \simeq -\frac{0.04}{119\,\pi}\,\frac{(9)\, m^2_{\rm charm}\,|(\widetilde{m}_\nu)_{11}|}{v^2\, m_1\, m_2\, m_3\, |(\widetilde{m}_\nu^{-1})_{33}|^2}\,\kappa(K_{2\tau})\, C^{(2)}_{\tau^{\perp}\tau}\, e^{-\frac{3\pi}{8}\,K_{1\mu}}\, \sin(2\phi + \xi + 6\pi/5). \tag{61}$$

Finally one also has

$$K_{1\mu} = \frac{|m_{\nu 21}^D|^2}{m_\star\, M_1} \simeq \frac{(m_{\nu 2}^D)^2}{m_\star\, M_1}\,|\widetilde{U}_{R21}|^2 \simeq \frac{|(\widetilde{m}_\nu)_{12}|^2}{m_\star\,|(\widetilde{m}_\nu)_{11}|}\,, \tag{62}$$

where we used the approximations $\sin\theta_C \ll \cos\theta_C \simeq 1$ and the following analytical expression for $\widetilde{U}_R$ [20, 12]

$$\widetilde{U}_R \simeq \begin{pmatrix} 1 & -\frac{m_{\nu 1}^D}{m_{\nu 2}^D}\,\frac{\widetilde{m}^\star_{\nu 12}}{\widetilde{m}^\star_{\nu 11}} & \frac{m_{\nu 1}^D}{m_{\nu 3}^D}\,\frac{(\widetilde{m}_\nu^{-1})^\star_{13}}{(\widetilde{m}_\nu^{-1})^\star_{33}} \\ \frac{m_{\nu 1}^D}{m_{\nu 2}^D}\,\frac{\widetilde{m}_{\nu 12}}{\widetilde{m}_{\nu 11}} & 1 & \frac{m_{\nu 2}^D}{m_{\nu 3}^D}\,\frac{(\widetilde{m}_\nu^{-1})^\star_{23}}{(\widetilde{m}_\nu^{-1})^\star_{33}} \\ \frac{m_{\nu 1}^D}{m_{\nu 3}^D}\,\frac{\widetilde{m}_{\nu 13}}{\widetilde{m}_{\nu 11}} & -\frac{m_{\nu 2}^D}{m_{\nu 3}^D}\,\frac{(\widetilde{m}_\nu^{-1})_{23}}{(\widetilde{m}_\nu^{-1})_{33}} & 1 \end{pmatrix} \begin{pmatrix} e^{-i\,\frac{\Phi_1}{2}} & 0 & 0 \\ 0 & e^{-i\,\frac{\Phi_2}{2}} & 0 \\ 0 & 0 & e^{-i\,\frac{\Phi_3}{2}} \end{pmatrix}, \tag{63}$$

that we generalised here to the case when $\widetilde{V}_L \neq I$ and where the expressions for the phases Φ_i given in [12] can be also generalised in terms of $\widetilde{m}_\nu$ (we do not need them here). From these expressions for η_B we can now make some considerations that explain some of the features of the two found numerical solutions for $\xi = +4\pi/5$ (see Table 3) providing a useful analytical insight.

- Inserting the numerical values for M_2/M_3, $K_{1\mu}$ and ϕ given in Table 3 and 4 into the eq. (60), the observed value of η_B is indeed reproduced.
- For one of the possible choices of values for ξ, the effective leptogenesis phase is maximal for $\phi = 3\pi/20 - \xi/2 + n\,\pi$. For the case $\xi = 4\pi/5$ the phase is maximal for $\phi = 3\pi/4 + n\pi$ explaining quite well the best fit values found for ϕ (see Table 3).
- From the eqs. (59) we can derive an analytical expression for M_2/M_3, the other crucial parameter determining the value of the asymmetry (cf. eq. (60)), finding

$$\frac{M_2}{M_3} = \frac{(m_{\nu 2}^D)^2}{(m_{\nu 3}^D)^2}\,\frac{|(\widetilde{m}_\nu)_{11}|}{m_1\,m_2\,m_3\,|(\widetilde{m}_\nu^{-1})_{33}|^2}\,. \tag{64}$$

We have verified that indeed, inserting into this equation the measured values for the mixing angles and for the solar and atmospheric neutrino mass scales, one obtains the correct value of M_2/M_3 thanks to a reduction of $|(\widetilde{m}_\nu^{-1})_{33}|$ to values $\sim 0.1\,\mathrm{eV}^{-1}$ from phase cancellations (without cancellations one would have $|(\widetilde{m}_\nu^{-1})_{\tau\tau}| \sim 100\,\mathrm{eV}^{-1}$) and that this is necessarily accompanied by the reduction of $|(\widetilde{m}_\nu)_{11}|$ to values $\sim 10^{-6}\,\mathrm{eV}$ (without cancellations one would have $|(\widetilde{m}_\nu)_{11}| \sim 10\,\mathrm{meV}$) when a condition $K_{1\mu} \lesssim 24$ is also imposed (it is then much more general than $K_{1\mu} \lesssim 1$). One can see that this is indeed what happens from Table 3, where we also show the best fit values for $|(\widetilde{m}_\nu^{-1})_{33}|$, $|(\widetilde{m}_\nu)_{11}|$ and $|(\widetilde{m}_\nu)_{12}|$. It is interesting that this results into a stable value of the next-to-lightest RH neutrino mass ($M_2 \simeq 5 \times 10^9\,\mathrm{GeV}$ for the CASE A and $M_2 \simeq 2 \times 10^{10}\,\mathrm{GeV}$ for the case B) the same we obtained in Table 2 without imposing leptogenesis, while the heaviest RH neutrino mass reduces to values $M_3 \simeq 2 \times 10^{12}\,\mathrm{GeV}$. Correspondingly the lightest RH neutrino mass, though it does not play a direct role since the asymmetry is N_2-dominated, is necessarily forced to grow to values $M_1 \sim 10^{7\div 8}\,\mathrm{GeV}$. This is something we have already discussed at the end of Section 5 starting within the model parameterization and that we have now seen also from a bottom up perspective. Moreover the value of the lightest neutrino mass and of the neutrinoless double beta decay effective neutrino mass $m_1 \simeq m_{ee} \simeq 2\,\mathrm{meV}$, in agreement with what we obtained fully numerically in Table 3.

Therefore, the eqs. (59) do explain quite well the obtained results and they also seem to point to a more general result: given the measured values of the low energy neutrino parameters, the reduction of M_3 generally (the condition $K_{1\mu} \lesssim 24$ is quite general) implies an uplift of M_1 while M_2 remains stable at a scale $\sim 10^{10}\,\mathrm{GeV}$. In other words the current measurements are such that the condition for the realisation of the $M_2 - M_3$ crossing level (reduction of $|(\widetilde{m}_\nu^{-1})_{33}|$) necessarily implies also the occurrence of the $M_1 - M_2$ crossing level solution, that would lead to a quasi-degenerate RH neutrino mass spectrum in the close vicinity of the crossing level. We have checked this statement also verifying that if one let θ_{13} to be free, then this effect occurs only for $\theta_{13} \sim 7^\circ \div 15^\circ$. In other words, with current values of the neutrino oscillation parameters, a deviation from a strong hierarchy of the RH neutrino masses proportional to the squares of the up-quark masses, seems to point, for $K_{1\mu} \lesssim 24$, toward a RH neutrino spectrum where all three RH neutrino masses tend to get closer. This analytical insight seems to help understanding also recent numerical results where quark-lepton parameters have been fitted within $SO(10)$-models either excluding [27] or including [28] leptogenesis and either hierarchical or compact RH neutrino mass spectra have been found but never crossing level solutions with only two close RH neutrino masses.

- The best fit values of the flavoured CP asymmetry $\varepsilon_{2\tau}$ in Table 3, the source of the asymmetry, is well above the upper bound [8, 29, 25]

$$\varepsilon^{\mathcal{I}}_{1\alpha} \lesssim \frac{3}{16\,\pi}\,\frac{M_1\,m_{\rm atm}}{v^2}\,\sqrt{\frac{K_{1\alpha}}{K_1}} \simeq 10^{-6}\,\left(\frac{M_1}{10^{10}\,{\rm GeV}}\right)\,\sqrt{\frac{K_{1\alpha}}{K_1}}\,, \tag{65}$$

holding for the term in the lightest RH neutrino CP flavoured asymmetries analogous to the first term in eq. (43). Indeed one has $|\mathcal{I}^{\tau}_{23}| \simeq 17 \gg 1$ for the best fits of Table 3. This is possible if the absolute values of the entries of the orthogonal matrix are much larger than unity. This can be understood considering that the orthogonal matrix [30] in $SO(10)$-inspired models is given by ($U \equiv U_{PMNS}$)

$$\Omega \simeq \left(\begin{array}{ccc} -\frac{\sqrt{m_1\,|\widetilde{m}_{\nu 11}|}}{\widetilde{m}_{\nu 11}}\,U_{e1} & \sqrt{\frac{m_2\,m_3\,|(\widetilde{m}_\nu^{-1})_{33}|}{|\widetilde{m}_{\nu 11}|}}\,\left(U^\star_{\mu 1} - U^\star_{\tau 1}\,\frac{(\widetilde{m}_\nu^{-1})_{23}}{(\widetilde{m}_\nu^{-1})_{33}}\right) & \frac{U^\star_{31}}{\sqrt{m_1\,|(\widetilde{m}_\nu^{-1})_{33}|}} \\ -\frac{\sqrt{m_2\,|\widetilde{m}_{\nu 11}|}}{\widetilde{m}_{\nu 11}}\,U_{e2} & \sqrt{\frac{m_1\,m_3\,|(\widetilde{m}_\nu^{-1})_{33}|}{|\widetilde{m}_{\nu 11}|}}\,\left(U^\star_{\mu 2} - U^\star_{\tau 2}\,\frac{(\widetilde{m}_\nu^{-1})_{23}}{(\widetilde{m}_\nu^{-1})_{33}}\right) & \frac{U^\star_{32}}{\sqrt{m_2\,|(\widetilde{m}_\nu^{-1})_{33}|}} \\ -\frac{\sqrt{m_3\,|\widetilde{m}_{\nu 11}|}}{\widetilde{m}_{\nu 11}}\,U_{e3} & \sqrt{\frac{m_1\,m_2\,|(\widetilde{m}_\nu^{-1})_{33}|}{|\widetilde{m}_{\nu 11}|}}\,\left(U^\star_{\mu 3} - U^\star_{\tau 3}\,\frac{(\widetilde{m}_\nu^{-1})_{23}}{(\widetilde{m}_\nu^{-1})_{33}}\right) & \frac{U^\star_{33}}{\sqrt{m_3\,|(\widetilde{m}_\nu^{-1})_{33}|}} \end{array}\right)\,D_\Phi, \tag{66}$$

where $D_\Phi \equiv {\rm diag}(e^{-i\frac{\phi_1}{2}}, e^{-i\frac{\phi_2}{2}}, e^{-i\frac{\phi_3}{2}})$, an analytical expression that generalises that one given in [12] when $\widetilde{V}_L \neq I$. One can see that for the best fit values of $\widetilde{m}_{11}$ and $\widetilde{m}^{-1}_{33}$ one has that all the absolute values of the Ω entries in the first and third column are much higher than unity. This is confirmed by the numerical results we find for Ω, corresponding to the best fits in Table 3 for CASE A

$$\Omega^{\rm (CASEA)} \simeq \left(\begin{array}{ccc} -4.40016 - 15.9889\,i & 0.0930875 - 0.894045\,i & -16.0396 + 4.38107\,i \\ -15.9446 + 3.40333\,i & -1.15394 + 0.0537137\,i & 3.40494 + 15.9553\,i \\ -3.69174 + 4.35811\,i & 0.709793 + 0.204576\,i & 4.37787 + 3.64191\,i \end{array}\right), \tag{67}$$

and for CASE B

$$\Omega^{(CASEB)} \simeq \begin{pmatrix} -1.77835 - 6.85986\,i & 0.108413 - 0.897431\,i & -6.97828 + 1.73423\,i \\ -6.87598 + 1.34103\,i & -1.15331 + 0.0386159\,i & 1.34278 + 6.90018\,i \\ -1.64314 + 1.81259\,i & 0.710523 + 0.199612\,i & 1.85785 + 1.52677\,i \end{pmatrix} . \quad (68)$$

These specific expressions for Ω should give a clear idea of the involved fine tuned cancellations and explains why one can have $|\mathcal{I}_{23}^{\tau}| \gg 1$ (implying $\varepsilon_{2\tau} \gg (3M_2 m_{\rm atm}/(16\,\pi\,v^2))$ and more generally why the flavoured *CP* asymmetries can be enhanced in the vicinity of crossing level solutions [20] though RH neutrino masses are still hierarchical and leptogenesis is far from being resonant. However, our novel solution relying on flavour coupling, represents in this respect quite a large improvement compared to the commonly considered solutions based on $M_1 \gtrsim 10^9\,{\rm GeV}$, since this requires a further uplift of the lightest RH neutrino mass of at least two orders of magnitude and, therefore, even higher fine tuned cancellations.

- As we have seen the third important ingredient for the existence of the solution is to have $K_{1\mu} \lesssim 1$. This is crucial in the case A, while in the case B the electron contribution, sixteen times smaller than the muonic one, would be still sufficient (remember that $K_{1e} = 0$). The expression for $K_{1\mu}$ eq. (62) shows that this condition is realised for $|(\widetilde{m}_\nu)_{12}|^2 \lesssim m_\star\,|(\widetilde{m}_\nu)_{11}|$ that is indeed verified for the best fits in Table 3.

- We can compare our results to those presented in [11] for $\widetilde{V}_L \simeq V_{CKM}$ where flavour coupling was neglected. The values of the Majorana phases in Table 3, match with those in [11] though in a marginal region (for an easier comparison in Table 3 we also give the the Majorana phases in the convention ${\rm diag}(e^{i\,\rho}, 1, e^{i\,\sigma})$ as in [20, 11]). The same it is true for the values of $\theta_{13}, \theta_{23}, \delta$. In [11] it can be also seen how the ratio M_2/M_3 can get reduced around $m_1 \simeq 2.5\,{\rm meV}$. Moreover in [11] there is a lower bound on $M_2 \gtrsim 5 \times 10^{10}\,{\rm GeV}$ that corresponds to $\alpha_2 \equiv m^D_{\nu 2}/m_{\rm charm} \gtrsim 3$, while we find that successful leptogenesis is obtained for $M_2 \simeq 5 \times 10^9\,{\rm GeV}$ and we have $\alpha_2 = 1$ in the CASE A. This is because the *CP* asymmetry, thanks to the condition $K_{1\mu}$ that makes possible much lower values of M_2/M_3, is greatly enhanced compared to the upper bound eq. (65). Moreover in [11] there are no points with M_1 uplift. This is explained since there the solutions correspond to $K_{1\tau} \lesssim 1$ while here with flavour coupling the solutions correspond to $K_{1\mu} \lesssim 1$ and so there is an intrinsic difference. This shows that the account of flavour coupling indeed opens new solutions enlarging the allowed regions in the space of parameters, in particular making possible a reduction in the scale of leptogenesis set by M_2.

7 Conclusions

The A to Z model can not only provide a satisfactory fit to all parameters in the leptonic mixing matrix but can also reproduce the correct value of the matter-antimatter asymmetry with N_2-dominated leptogenesis. In this respect it is crucial to account for flavour coupling effects due to the redistribution of the asymmetry in particles that do not participate directly to the generation of the asymmetry, *in primis* the Higgs asymmetry. In particular a "flavour swap" scenario is realised whereby the asymmetry generated in the tauon flavour emerges as a surviving asymmetry dominantly in the muon flavour. The solution works even in the simplest case where the neutrino Dirac mass matrix is equal to the up quark mass matrix.

Neutrino masses are predicted to be NO, with an atmospheric neutrino mixing angle well into the second octant and the Dirac phase $\delta \simeq 20^\circ$, a set of predictions that will be tested in the next years in neutrino oscillation experiments. We expect these values to be slightly corrected by charged lepton mass matrix corrections and different theoretical uncertainties (for example in the values of the up quark masses at the high scale). In particular we note that charged lepton mixing corrections, although small in the A to Z model due to the (1,2) entry of charged lepton and down quark mass matrices being zero, may yield atmospheric mixing corrections of order one degree.

In conclusion, the novel solution that we presented involving indispensible flavour coupling, opens new possibilities for successful leptogenesis within realistic $SO(10)$-inspired models. Although there is fine tuning given by the parameter γ, the level of fine tuning is mild, certainly much smaller than in traditional quasi-degenerate solutions. The reason for the fine tuning is that the spectrum of RH neutrinos, must be compressed as compared to the "natural" $SO(10)$-inspired spectrum proportional to the squares of up-type quark masses. However, whereas models with $M_1 \gtrsim 10^9\,\mathrm{GeV}$, commonly considered in the literature, involve a very compact mass spectrum, approaching degeneracy, the compressed mass spectrum for the N_2-dominated case considered here remains very hierarchical.

Acknowledgements

PDB and SFK acknowledge partial support from the STFC Consolidated ST/J000396/1 and the European Union FP7 ITN-INVISIBLES (Marie Curie Actions, PITN- GA-2011-289442). PDB acknowledges financial support from the NExT/SEPnet Institute. We wish to thank for useful discussions K. Babu, B. Bajc, M. Re Fiorentin.

References

[1] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).

[2] P. A. R. Ade *et al.* [Planck Collaboration], Astron. Astrophys. **571** (2014) A16.

[3] P. Minkowski, Phys. Lett. B **67** (1977) 421; T. Yanagida, in Proceedings of the Workshop on Unified Theory and Baryon Number of the Universe, eds. O. Sawada and A. Sugamoto (KEK, 1979) p.95; P. Ramond, Invited talk given at Conference: C79-02-25 (Feb 1979) p.265-280, CALT-68-709, hep-ph/9809459; M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, eds. P. van Niewwenhuizen and D. Freedman (North Holland, Amsterdam, 1979) Conf.Proc. C790927 p.315, PRINT-80-0576; R. Barbieri, D. V. Nanopoulos, G. Morchio and F. Strocchi, Phys. Lett. B **90** (1980) 91; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44** (1980) 912.

[4] F. Bjorkeroth, F. J. de Anda, I. d. M. Varzielas and S. F. King, arXiv:1505.05504 [hep-ph]; F. Bjorkeroth, F. J. de Anda, I. de Medeiros Varzielas and S. F. King, JHEP **1506** (2015) 141 [arXiv:1503.03306 [hep-ph]].

[5] S. Antusch, S. F. King and A. Riotto, JCAP **0611** (2006) 011 [hep-ph/0609038]; S. F. King, Phys. Rev. D **67** (2003) 113010 [hep-ph/0211228].

[6] S. F. King, JHEP **1408** (2014) 130 [arXiv:1406.7005 [hep-ph]].

[7] F. Bjorkeroth and S. F. King, arXiv:1412.6996 [hep-ph]; S. F. King, JHEP **1401** (2014) 119 [arXiv:1311.3295 [hep-ph]]; S. F. King, Phys. Lett. B **724** (2013) 92 [arXiv:1305.4846 [hep-ph]]; S. F. King, JHEP **1307** (2013) 137 [arXiv:1304.6264 [hep-ph]].

[8] S. Davidson and A. Ibarra, Phys. Lett. B **535** (2002) 25 [hep-ph/0202239].

[9] P. Di Bari, Nucl. Phys. B **727** (2005) 318 [hep-ph/0502082].

[10] O. Vives, Phys. Rev. D **73** (2006) 073006 [hep-ph/0512160]; S. Blanchet and P. Di Bari, Nucl. Phys. B **807** (2009) 155 [arXiv:0807.0743 [hep-ph]]; X. G. He, S. S. C. Law and R. R. Volkas, Phys. Rev. D **78** (2008) 113001 [arXiv:0810.1104 [hep-ph]]; S. Antusch, P. Di Bari, D. A. Jones and S. F. King, Nucl. Phys. B **856** (2012) 180 [arXiv:1003.5132 [hep-ph]]; S. Blanchet, P. Di Bari, D. A. Jones and L. Marzola, JCAP **1301** (2013) 041 [arXiv:1112.4528 [hep-ph]]; P. Di Bari, S. King and M. Re Fiorentin, JCAP **1403** (2014) 050 [arXiv:1401.6185 [hep-ph]]; J. Zhang, Phys. Rev. D **91** (2015) 7, 073012 [arXiv:1502.04043 [hep-ph]].

[11] P. Di Bari and A. Riotto, Phys. Lett. B **671** (2009) 462 [arXiv:0809.2285 [hep-ph]]; P. Di Bari and A. Riotto, JCAP **1104** (2011) 037 [arXiv:1012.2343 [hep-ph]].

[12] P. Di Bari, L. Marzola and M. Re Fiorentin, Nucl. Phys. B **893** (2015) 122 [arXiv:1411.5478 [hep-ph]].

[13] R. Barbieri, P. Creminelli, A. Strumia and N. Tetradis, Nucl. Phys. B **575** (2000) 61 [hep-ph/9911315]. W. Buchmuller and M. Plumacher, Phys. Lett. B **511** (2001) 74 [hep-ph/0104189]; JHEP **0601** (2006) 068; E. Nardi, Y. Nir, E. Roulet and J. Racker, JHEP **0601** (2006) 164 [hep-ph/0601084]; S. Blanchet and P. Di Bari, Nucl. Phys. B **807** (2009) 155 [arXiv:0807.0743 [hep-ph]].

[14] F. X. Josse-Michaux and A. Abada, JCAP **0710** (2007) 009 [hep-ph/0703084].

[15] S. Antusch, P. Di Bari, D. A. Jones and S. F. King, Nucl. Phys. B **856** (2012) 180 [arXiv:1003.5132 [hep-ph]].

[16] H. Fusaoka and Y. Koide, Phys. Rev. D **57** (1998) 3986 [hep-ph/9712201].

[17] J. Beringer et al. (Particle Data Group), Phys. Rev. D86, 010001 (2012).

[18] M. C. Gonzalez-Garcia, M. Maltoni and T. Schwetz, JHEP **1411** (2014) 052 [arXiv:1409.5439 [hep-ph]].

[19] F. Capozzi, G. L. Fogli, E. Lisi, A. Marrone, D. Montanino and A. Palazzo, Phys. Rev. D **89** (2014) 093018 [arXiv:1312.2878 [hep-ph]]; D. V. Forero, M. Tortola and J. W. F. Valle, Phys. Rev. D **90** (2014) 9, 093006 [arXiv:1405.7540 [hep-ph]].

[20] E. K. Akhmedov, M. Frigerio and A. Y. Smirnov, JHEP **0309**, 021 (2003) [hep-ph/0305322].

[21] A. Pilaftsis, Phys. Rev. D **56** (1997) 5431.

[22] X. d. Ji, Y. c. Li, R. N. Mohapatra, S. Nasri and Y. Zhang, Phys. Lett. B **651** (2007) 195 [hep-ph/0605088].

[23] W. Buchmuller, P. Di Bari and M. Plumacher, Annals Phys. **315** (2005) 305 [hep-ph/0401240]; S. Blanchet and P. Di Bari, JCAP **0703** (2007) 018 [hep-ph/0607330].

[24] L. Covi, E. Roulet and F. Vissani, Phys. Lett. B **384** (1996) 169 [hep-ph/9605319].

[25] S. Antusch, P. Di Bari, D. A. Jones and S. F. King, Phys. Rev. D **86** (2012) 023516 [arXiv:1107.6002 [hep-ph]].

[26] W. Buchmuller and M. Plumacher, Phys. Lett. B **389** (1996) 73 [hep-ph/9608308]; E. Nezri and J. Orloff, JHEP **0304** (2003) 020 [hep-ph/0004227]; F. Buccella, D. Falcone and F. Tramontano, Phys. Lett. B **524** (2002) 241 [hep-ph/0108172]; G. C. Branco, R. Gonzalez Felipe, F. R. Joaquim and M. N. Rebelo, Nucl. Phys. B **640** (2002) 202 [hep-ph/0202030].

[27] A. Dueck and W. Rodejohann, JHEP **1309** (2013) 024 [arXiv:1306.4468 [hep-ph]].

[28] F. Buccella, D. Falcone, C. S. Fong, E. Nardi and G. Ricciardi, Phys. Rev. D **86** (2012) 035012 [arXiv:1203.0829 [hep-ph]]; G. Altarelli and D. Meloni, JHEP **1308** (2013) 021 [arXiv:1305.1001 [hep-ph]]; C. S. Fong, D. Meloni, A. Meroni and E. Nardi, JHEP **1501** (2015) 111 [arXiv:1412.4776 [hep-ph]].

[29] A. Abada, S. Davidson, A. Ibarra, F.-X. Josse-Michaux, M. Losada and A. Riotto, JHEP **0609** (2006) 010 [hep-ph/0605281].

[30] J. A. Casas and A. Ibarra, Nucl. Phys. B **618** (2001) 171 [hep-ph/0103065].