KRIGING BASED ROBUST OPTIMISATION ALGORITHM FOR MINIMAX PROBLEMS IN ELECTROMAGNETICS

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***Abstract* – The paper discusses some of the recent progress in kriging based worst-case design optimisation and proposes a new two-stage approach to solve practical problems. The efficiency of the infill points allocation is largely improved by adding an extra layer of optimisation enhanced by a validation process.**

I. PROBLEM SPECIFICATION

Robust optimisation is a relatively new term, its history can be dated back to 1989 when Taguchi first introduced the concept of design quality [1], and since then optimisation involving uncertainties has been increasingly drawing more attention. Due to the complexity of the optimisation problems in engineering design, the high level of non-linearity means these problems cannot be closely approximated by single linear or quadratic functions. Therefore, these problems are often solved by using direct search global optimisation algorithms. When evaluation of the underlying problem is expensive in terms of time or cost, surrogate modelling techniques are often implemented as an approximation and optimisation is applied to the surrogate model instead of the original problem.

The output f of a black-box function when the input variable x contains deterministic type of uncertainties can be expressed by a simplified equation (ignoring possible other sources of uncertainties and assuming the uncertainty ε is independent of the input variable x)

$$f \ = \ f(x+\varepsilon) \qquad (1)$$

where $\varepsilon[-\epsilon,\ \epsilon]$, the distribution of uncertainty ε, is unknown, but the magnitude is bounded to a given range ϵ.

II. A BRIEF REVIEW OF EXISTING APPROACHES

For a deterministic type of uncertainties, the basic approach is to transform the robust optimisation problem into a standard optimisation problem by optimising the worst-case of the original objective function, where multiple objective function evaluations are needed at each design stage. The number of objective function calls may be significantly increased and many unimportant and possibly nearly duplicated design points will be allocated during this process and thus making the optimisation extremely inefficient. This large number of function calls will be of particular concern to designers especially when the objective function is expensive to evaluate.

Recently, some more efficient kriging based approaches for solving worst-case optimisation problems have been proposed in literature. The authors of [3] use the mean and variants to assess the robustness, while their proposed strategy utilises the gradient information computed from the kriging model. In [4] the Expected Improvement (EI) infill sampling approach is combined with a relaxation procedure based on a kriging model. In [5] the EI infill sampling approach is applied to the worst-case response surface calculated based on the kriging model.

III. A TWO STAGE APPROACH

In this paper, we propose a two-stage approach for solving expensive worst-case optimisation problems. We focus on maximising the usage of available information while delaying the calculation of the worst-case value at sampling points to achieve a more efficient sampling scheme for the worst-case type of robust design optimisation.

The worst-case optimisation problem is often referred to as the minimax problem, with an extra ‘layer’ of optimisation, therefore the infill sampling criteria for global optimisation are often found ‘uncomfortable’ in the context of the worst-case optimisation problems. The worst-case value of the objective function at any given point does not depend on information given by that point alone (including the kriging prediction, MSE, gradient etc.), as information from its neighbouring points also needs to be taken into account.

The algorithm contains two stages; the first one is to update the kriging model by sequentially adding infill points at each iteration based on the worst-case expected improvement (WCEI) – this expected improvement measure is recalculated from standard EI, by taking the minimal EI value within the worst-case region of that design point (design site)

$$WCEI(x) = \max\{\min[EI(x+\varepsilon)]\,,0\} \qquad (2)$$
$$x+\varepsilon \in X$$

where X is a set of points located within the worst-case region of that unknown point x. A one-dimensional example is illustrated in Fig. 1, where the boundary ϵ of the worst-case design is ± 0.3.

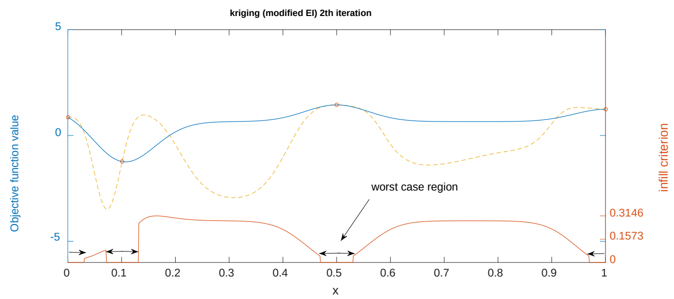

Fig. 1. The worst-case regions of existing design sites in a 1D example

The extra layer of the minimax problem is embedded within the WCEI; the new infill sampling point will be located where the minimal expected improvement around the target point is the largest. The WCEI is equal to zero at the locations within the worst-case region of existing design sites; consequently, these areas are banned as future infill locations at the model updating stage. During the process of model updating, the worst-case estimation of the objective function is computed simultaneously based on the kriging model constructed using the existing design sites at that iteration.

The second stage is triggered when the maximum WCEI within the design space becomes less than a predefined value or stage one has exceeded its allowance, if it has been imposed. The location of the worst-case estimated optimum is added as the next infill point and the associated objective function are evaluated. When the range of the underlying objective function surface is large, the actual worst-case optimum can differ from the estimated one; therefore, a validation process is necessary at stage two, after the worst-case optimum based on estimation is located. The worst-case region around the worst-case optimum is exploited and validated using a modified EI approach, where instead of calculating the improvement, the deterioration is computed to give an indication where the maximal worsening is located within the worst-case region of the worst-case optimum

$$E[D(x)] = \begin{cases} (\hat{y}(x) - y_{wc})\Phi(u(x)) + \hat{s}\phi(u), & s > 0 \\ 0, & s = 0 \end{cases}$$

$$u = \frac{\hat{y}(x) - y_{min}}{\hat{s}(x)}$$

This process can be repeated until the value of the expected deterioration is smaller than the predefined value.

IV. EXAMPLE

The worst-case optimisation process using the two-stage approach is illustrated by a one-dimensional test example. In Figs 2 and 3, the yellow dotted line depicts the original objective function, while the blue bold line shows the kriging model. The focus is on showing the mechanism of the proposed approach and how a position of the new infill point is decided.

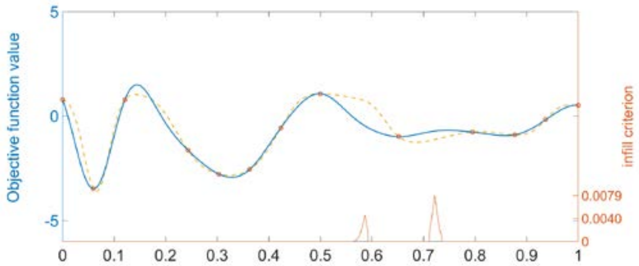

Fig. 2. The kriging model after 11th iteration

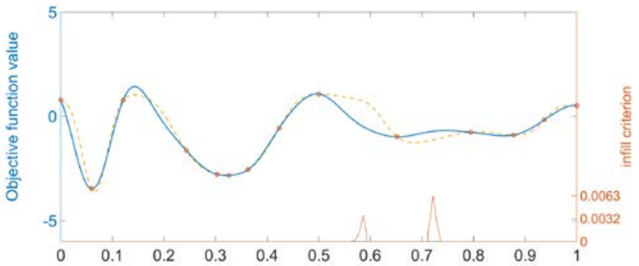

Fig. 3. The kriging model after 12th iteration

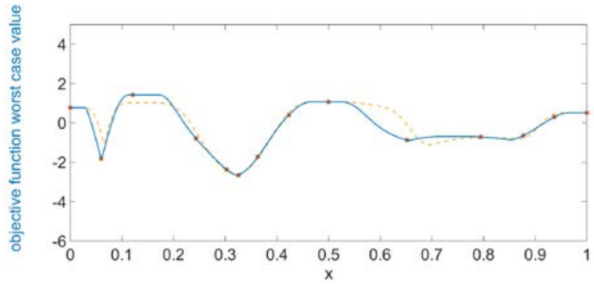

Fig. 4. The worst-case estimation based on the kriging model after 13th iteration

The maximum WCEI within the design space is less than the predefined value of 0.01, and the optimum at $x = 0.34$ of the worst-case estimation is taken as the next infill sampling point to be evaluated. The worst-case optimum is found in the 12th iteration in Fig. 3 at the location of the optimum point in the previous worst-case estimation, while Fig. 4 gives the final worst-case estimation of the objective function. The yellow dotted line in Fig. 4 is the worst-case response surface of the underlying objective function, and the blue bold line is the estimated worst-case response surface based on the kriging model. It will be noted that with the consideration of robustness the position of the final optimum is different to what has been suggested by Figs 2 and 3, in other words the robust optimum is different to the theoretical global one.

V. CONCLUSION AND DISSCUSION

A two-stage approach to worst-case optimisation problems has been proposed and details of the algorithm discussed. The suggested method does not compute the worst-case value, or the corresponding robustness measure, for any design site during the model updating stage, in order to avoid the objective function evaluation at a location that would contribute less to the overall model landscape, which would have taken place if the worst-case value was evaluated for the newly added infill point. Instead, the explicit search for the robust optimum takes place in the second stage after the model updating process has completed, with a validation process added to exploit the region around the estimated worst-case optimum. In the full-page version, the proposed optimisation method will be tested on a multi-dimensional practical electromagnetic design problem.

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