On the equivalence of
Finite Element, Finite Difference and Finite Integral Methods in Computational Electromagnetics

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Abstract

In this keynote address, the numerical 3D formulations using scalar Ω, *V* and vector ***A***, ***T*** potentials will be examined for electromagnetic fields at low frequency. The focus will be on the Finite Difference Method (FEM), Finite Integration Technique (FIT) and Finite Element Method (FEM) using nodal and edge elements, while the cell and equivalent network approaches will also be considered. It has been shown that for cuboidal elements the FEM equations may be presented in a form analogous to the FDM and FIT equations. The coefficients defining by volume integrals in FEM need to be expressed in an approximate manner to accomplish this similarity. Moreover, the nodes in FDM should be properly distributed and associated with middle points of relevant edges, facets and volumes.

It will be shown that the equations usually obtained via a variational approach may be more conveniently derived using integral methods employing a geometrical description of the interpolating functions of edge and facet finite elements. The multi-branch electric and magnetic circuit models will be discussed to explain FEM, FIT and FDM equations in a language of circuit theory. For the vector potential ***A***, ***T*** formulations the equations of FEM, FIT and FDM represent the loop (mesh) equations of equivalent reluctance/resistance networks for loops around the element edges. However, for the scalar potentials Ω, *V* these equations are analogous to the nodal equations of equivalent permeance/conductance network for element nodes.

The equivalence between a description of multiply connected windings in the finite element space using edge values of the vector potential $\boldsymbol{T}_0$ and the approach arising from the classical *mmf* distribution formulation will be explained. A method of simulating movement and calculating electromagnetic forces/torque using FEM, FIT, FDM will also be considered. A unified form of the stress tensor for all discussed methods will be presented.

For specially selected examples, comparisons will be made between the results obtained using the different methods for both scalar and vector potential formulations. Both the attractive and repulsive forces in a ‘test’ system containing cuboidal permanent magnets will be analyzed. Under the assumption of homogeneity the forces acting in the system may be found analytically so that the accuracy of numerical calculations can be assessed.

Finally, the analogies between FEM, FIT, FDM and magnetic or electric networks have been found very helpful in teaching, especially if students are already familiar with one of the methods; this will be particularly true when field methods are introduced to students already conversant with circuit theory.