

1 Portfolio and Welfare Consequences  
2 of Debt Market Dominance

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4 **Abstract**

The ability to issue debt that pays in units of the domestic good leads a country to accumulate a large and negative net foreign asset position while maintaining a positive position in equity. This debt market advantage also helps to explain the weak relationship between the real exchange rate and relative consumption. Our stylized model matches the key facts about the U.S. international portfolio, the U.S. real exchange rate, and explains nearly 50% of the observed variation in the valuation effects. We find that taxing bond market transactions increases the volatility of the exchange rate, capital flows and allocations. In contrast, taxing equity positions stabilizes the exchange rate and capital flows while having little impact on the allocation. Lastly, the paper describes a global solution method for portfolio problems under incomplete markets.

5 *Keywords:* international portfolio choice, global imbalances, incomplete markets,  
6 bond market

7 *JEL codes:* F32, D52, C63

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8 **1. Introduction**

9 During the past several decades we have witnessed a growing financial integra-  
10 tion in the world economy. There has been an increase in both the volume of  
11 internationally-traded assets and the magnitude of cross-border gross capital flows.  
12 According to the dataset compiled by Lane and Milesi-Ferretti (2007b), the total  
13 gross foreign assets of the U.S. were 16% of GDP in 1970, stayed below 32% until  
14 1984 and increased to 131% of GDP in 2007. Total gross foreign liabilities were 12%  
15 of GDP in 1970, stayed below 30% until 1984 and subsequently soared to 148% of  
16 GDP in 2007.

17 The increase of net and gross international capital flows led to global imbalances  
18 and, in particular, to a significant deterioration of the U.S. net foreign asset (NFA)

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19 position. This weakening of the U.S. NFA position was driven by the accumulation  
20 of debt liabilities, while the NFA in equity improved. Gourinchas and Rey (2005) say  
21 that “as financial globalization accelerated its pace, the U.S. transformed itself from  
22 a world banker into a world venture capitalist, investing greater amounts into high  
23 yield assets such as equity and FDI”, while “its liabilities have remained dominated  
24 by bank loans, trade credit and debt, i.e. low yield safe assets”. Similar observations  
25 were made by Obstfeld and Rogoff (2000) and Mendoza et al. (2009) among others.  
26 Obstfeld (2004) states that for the U.S. “the striking change since the early 1980s is  
27 the sharp growth in foreign portfolio equity holdings”, while on the liabilities side,  
28 “the most dramatic percentage increase has been in the share of U.S. bonds held by  
29 foreigners.”

30 However, the U.S. is in a unique position: it is the only country that can borrow  
31 (and lend) almost exclusively in the domestic currency. The share of U.S. debt-like  
32 liabilities including government debt, corporate debt and bank loans denominated in  
33 U.S. dollars grew from 80.6% in 1990 to 88.3% in 2004, see Lane and Shambaugh  
34 (2009). For comparison, during the same period the share of U.K. debt-like liabilities  
35 denominated in British pounds averaged at 19.4%. Notably, most of the U.S. debt-  
36 like foreign assets were also denominated in the domestic currency amounting to  
37 87.4% in 2004. Therefore, international debt markets have been dominated by assets  
38 denominated in the U.S. dollar.<sup>3</sup> The U.S. currency has also dominated the trade in  
39 commodity markets, another significant source of short-term debt.<sup>4</sup>

40 In this paper, we ask if the U.S. dominance in the international debt market  
41 can account for the size and composition of the U.S. international portfolio and the  
42 dynamics of the real exchange rate. Did the observed debt market structure have

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<sup>3</sup>For a review see Eichengreen and Hausmann (2005). The introduction of the euro led to the increased share of euro-denominated internationally traded debt assets. However, the rise of the euro can be partially accounted for by increased financial flows within the Eurozone, while trade in euro assets with non-euro economies may be limited.

<sup>4</sup>See Goldberg and Tille (2009) for the analysis of the U.S. dollar’s role in international trade.

43 any implications for the international adjustment mechanism and what were the wel-  
44 fare implications? To address these questions, we build a model that allows us to  
45 match the size and the composition of the U.S. NFA position. The model must fea-  
46 ture incomplete financial markets, multiple internationally traded assets and many  
47 goods. Incomplete markets are necessary because with complete markets and time-  
48 separable preferences portfolios are typically constant and, therefore, capital flows  
49 are absent.<sup>5</sup> Multiple (at least three) assets are needed to distinguish debt and eq-  
50 uity positions and to model gross equity positions. Multiple goods are necessary to  
51 allow endogenous external adjustment via the real exchange rate as emphasized by  
52 Lane and Milesi-Ferretti (2001) and Obstfeld (2004). The model and the solution  
53 method must also apply to asymmetric economies. In a symmetric world, any im-  
54 balances are necessarily transitory unless there are multiple equilibria that naturally  
55 pose problems for the quantitative analysis. In order to generate a realistic external  
56 portfolio, we introduce two asymmetries into the model.

57 First, we model the privileged position of the U.S. by assuming that bonds traded  
58 in the world financial markets are denominated in the units of its domestic good.  
59 With this asymmetry our model predicts that the privileged country accumulates  
60 a significant debt while maintaining a sizable net position in equity. In the model,  
61 the domestic bond is a good hedge against fluctuation in consumption. But when  
62 domestic output increases, the domestic price level declines and consumption becomes  
63 more affordable. Yet, the domestic bond does not allow purchasing more consumption  
64 as its payoff declines. Hence, the domestic bond is sold. While this result relies on a  
65 low, yet reasonable, elasticity of substitution between goods, our numeric simulations  
66 show that the effect is very strong, allowing us to match the U.S. net position in debt  
67 equal to -38.0% of GDP.

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<sup>5</sup>For a proof of this statement see Judd et al. (2000).

68 Second, we assume that individuals in the U.S. are less risk-averse than elsewhere.<sup>6</sup>  
69 This is a reduced form way of modeling the fact, documented in Mendoza et al.  
70 (2009), that the U.S. has more developed financial markets than the rest of the  
71 world. With more developed financial markets, consumers are better able to insure  
72 away idiosyncratic risks and, hence, should be more inclined to invest in risky assets.  
73 In turn, Weil (1993) shows that increasing the risk-aversion coefficient or increasing  
74 the variance of individual income have the same effect on the equilibrium consumption  
75 function.

76 The small difference in risk aversion that we assume enables us to obtain a realistic  
77 portfolio of foreign assets. This asymmetry has little quantitative effect by itself, but  
78 it reinforces the bond-market advantage. Namely, a country with lower risk aversion is  
79 willing to hold a larger than the rest of the world fraction of wealth in equity. But the  
80 insurance service it provides is valued little given the size of observed macroeconomic  
81 risks. So, the increased investment in equity is nearly entirely financed by borrowing  
82 in the debt market. The effect on the overall NFA position is small. Adding the bond  
83 market advantage, above providing independent motives to borrow, lowers the cost  
84 of the existing debt and prompts the less-risk-averse country to increase investment  
85 in foreign equity. Similarly, a country with the bond market advantage is willing to  
86 borrow but this increases its exposure to fluctuations in domestic income. So, it will  
87 not invest in equity unless we increase its risk-tolerance.

88 The main contribution of this paper is to show that the U.S. external balance  
89 sheet can be matched using a relatively simple model. This model also matches two  
90 important facts in international finance: the home equity bias and the consumption-

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<sup>6</sup>An indirect evidence of an elevated risk appetite in the U.S. is provided in Rydqvist et al. (2009): 38.5% of U.S. individuals owned (directly) stocks in 2006. The second highest participation rate of 28.5% is in Canada; the participation in Japan, which is the largest holder of the U.S. debt, is only 18.1. Large European economies have even lower participation rates: 14.1 in the U.K., 12.5 in Germany and 6.9 in France. Moreover, the U.S. has maintained the lead in stock market participation since 1945.

91 real exchange rate disconnect. Equity home bias presents a direct restriction on the  
92 composition of the NFA that we target. The correlation between the real exchange  
93 rate and relative consumption (hence relative pricing kernels) determines which assets  
94 a country chooses to hold and which it decides to sell and the size and the stochastic  
95 properties of capital flows. Restricted by the above empirical facts, the model explains  
96 nearly 50% of the observed exchange rate related valuation effects emphasized by  
97 Lane and Milesi-Ferretti (2007b).<sup>7</sup>

98 On the methodological side, we explain how to solve international portfolio choice  
99 models by explicitly modeling the financial wealth distribution. We compute a global  
100 solution to a two-country two-good general equilibrium incomplete markets model by  
101 adapting the projection method developed in Judd et al. (2000) and Kubler and Schmedders  
102 (2003). We also show that there exists a wealth-recursive competitive equilibrium,  
103 providing a theoretical foundation for the solution methodology. The existence proof  
104 highlights the role of portfolio constraints (and debt limits in particular) that are  
105 necessary conditions for equilibrium existence but have, so far, been ignored in the  
106 previous body of work on international portfolio choice.

107 The rest of the paper is organized as follows. Section 2 gives a brief literature  
108 overview. Section 3 presents the model and the solution concept. Section 4 presents  
109 the numerical results and the transaction tax experiment. We conclude by stating the  
110 issues that remain to be solved. Details of the computational algorithm and proofs  
111 are relegated to the appendix.<sup>8</sup>

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<sup>7</sup>Our model requires additional elements to be consistent with the well-known asset pricing facts. So, we concentrate on the exchange rate channel, largely ignoring the valuation effects stemming from fluctuations in asset prices.

<sup>8</sup>Supplemental appendix is available online.

## 112 2. Related literature

113 We omit the vast literature on current account determination under complete  
114 markets that dates back to Razin and Helpman (1978). Among the research on in-  
115 complete markets, several papers are closely related to our work as they analyze  
116 the size and/or the composition of the U.S. NFA position. Mendoza et al. (2009)  
117 consider a model with heterogeneous agents, no aggregate risk and one good. Debt  
118 in their model is state contingent, which is an equity-like instrument. As a result,  
119 it is not clear how to separate debt from equity positions. More importantly, the  
120 debt instrument is contingent on an idiosyncratic employment shock and has no data  
121 counterpart. Caballero et al. (2008) show that differences in the countries' ability  
122 to supply financial assets may lead to substantial global imbalances. Our debt mar-  
123 ket advantage concept offers a new explanation while being complementary to the  
124 above two. Our model features aggregate uncertainty, exchange rate risk and a more  
125 realistic portfolio choice setting. So, it is more suitable for quantitative analysis.  
126 Gourinchas et al. (2010) also build a quantitative model addressing the privileged  
127 position of the U.S. They link the U.S. advantage to a lower risk aversion and the  
128 size of the U.S. economy as in Hassan (2012). But, among other short-comings, their  
129 model counter-factually predicts a zero NFA position for the U.S.; see table 5.

130 Coeurdacier et al. (2010), Heathcote and Perri (2013) and Berriel and Bhattarai  
131 (2013) provide competing explanations for the home equity bias. The first relies on  
132 the correlation between dividends and wages, the second emphasizes trade openness,  
133 and the third government spending shocks. All models are symmetric and, hence,  
134 have no predictions for the size and composition asymmetry of the observed NFA  
135 positions. Also none of the models analyzes valuation effects.

136 Devereux and Sutherland (2010) study valuation effects in a symmetric model. In  
137 their numerical example, countries buy domestic bonds and their mean NFA is always  
138 zero. The example also relies on preference shocks without which trade in equities col-

139 lapses. They report significant exchange rate driven valuation effects which, however,  
140 are a consequence of counterfactual predictions about the volatility of real exchange  
141 rates. In contrast, we analyze valuation effects in a calibrated model that closely  
142 matches the U.S. portfolio and the empirical properties of the U.S. real exchange  
143 rate.

144 All of the above models rely on the approximation technique that is criticized by  
145 Rabitsch et al. (2014). Pavlova and Rigobon (2010) study a continuous time model  
146 with a closed form solution. They do not present results from a calibrated model, but  
147 rather study simple examples. The model can generate substantial and volatile val-  
148 uation effects, but only by relying on demand shocks with volatility ranging between  
149 25 and 145% of output volatility. The latter implies an unreasonably volatile real  
150 exchange rate that is uncorrelated with the real variables. The direct implication is  
151 that the valuation effects are volatile and unpredictable. Our model has only output  
152 shocks and it predicts a stochastic process for the real exchange rate that is close  
153 to the data. So, it is more meaningful for the quantitative analysis of international  
154 financial adjustment.

155 Our paper also provides a new solution method for international portfolio models  
156 with incomplete financial markets. Below we briefly go over the inventory of existing  
157 solution methods and state the advantages of our method. For a more detailed  
158 discussion and formal numerical tests see Rabitsch et al. (2014).

159 Kollmann (2006) exploits the idea that an equilibrium allocation should be close to  
160 the efficient one. The author describes a portfolio that supports approximately the ef-  
161 ficient allocation given the associated price system. However, such a portfolio may not  
162 be optimal even in the class of considered portfolios, *e.g.* constant portfolios, as they  
163 are generally not welfare maximizing choices. The idea in Devereux and Sutherland  
164 (2009) and Tille and van Wincoop (2010) is to derive constant portfolios using the  
165 second-order approximation to the model with one asset and a guess for the NFA

166 position. One can then use the constant portfolios to derive a third-order approxima-  
167 tion to the “macro” portion of the model. The latter can, in turn, be used to derive  
168 a first-order approximation to the portfolio decisions. However, the solution depends  
169 on the NFA value assumed at the beginning. So, with an asymmetric model like  
170 ours, one needs to solve a model iteratively, continuously refining the approximation  
171 point. The authors provide no arguments as to why the iterative procedure should be  
172 convergent. Indeed, Rabitsch et al. (2014) show that this algorithm is unstable, and  
173 it can lead to nonsensical solutions. Pavlova and Rigobon (2007) develop a continu-  
174 ous time model that can be solved analytically. However, it is limited to logarithmic  
175 preferences and the Cobb-Douglas good aggregator. Evans and Hnatkovska (2012)  
176 use continuous time logarithmic utility approximation to the portfolio choice part of  
177 the problem within a discrete time framework.

178 Our method has several advantages over the existing work. First, it allows for  
179 more realistic financial market structures. Second, adding new assets in our model  
180 environment does not increase the dimension of the state space. Adding capital to  
181 it would make the model more difficult to compute because of the additional state  
182 variable(s). But, in our experience, policies are close to linear in capital and low  
183 cost approximation techniques could be used for the new states. The methodological  
184 advances, *e.g.* see Judd et al. (2011) and Maliar and Maliar (2014), also make the  
185 use of global solution methods for highly dimensional problems increasingly feasible.  
186 Third, a wide range of borrowing limits and portfolio restrictions can be analyzed.  
187 Without explicit restrictions on portfolios any incomplete markets model is ill-defined  
188 as it is not even clear if a competitive equilibrium can exist. Lastly, our solution  
189 method offers higher accuracy as we solve for a global, and not a local, solution. All  
190 in all, we believe it is a valuable addition to the existing stock of solution methods.



191 **3. The Model**

192 *3.1. Environment*

193 Time is discrete and indexed by  $t = 0, 1, 2, \dots$ . The exogenous state of the economy  
194  $z_t$  is a first-order Markov process with finitely many states,  $\mathcal{Z} = \{\bar{z}_1, \dots, \bar{z}_S\}$ , and a  
195 probability transition matrix  $\Pi$ . The initial state  $z_0$  is given. A partial history of the  
196 state realizations  $(z_0, \dots, z_t)$  is denoted by  $z^t$  and its probability by  $\pi(z^t|z_0)$ .

197 There are two countries, each populated by a representative household. Two  
198 perishable goods are traded every period. Good  $i$  produced in country  $i$  is traded at  
199 price  $p_i(z^t)$ .

200 Financial markets trade three financial assets: home and foreign equity and a  
201 bond. A claim to home equity pays  $d_1(z_t) > 0$  units of good 1 when the current state  
202 is  $z_t$  and a claim to foreign equity pays  $d_2(z_t) > 0$  units of good 2. The supply of each  
203 stock is fixed and normalized to 1. We assume further that there is no short-selling.  
204 The two equity claims are traded at the ex-dividend prices  $q_1(z^t)$  and  $q_2(z^t)$ . Bond's  
205 payoff consists of  $\alpha$  units of good 1 and  $1 - \alpha$  units of good 2. The market value of  
206 the payoff is:

$$p_b(z^t) \equiv \alpha p_1(z^t) + (1 - \alpha) p_2(z^t). \quad (1)$$

207 Negative positions in the bond are allowed subject to a borrowing limit described  
208 below. The bond is in net zero supply and it is traded at  $q_b(z^t)$ .

209 We interpret the case with  $\alpha = 1$  as the situation in which the world bond market  
210 only trades the bonds paying in goods of country 1. The case with  $\alpha = 0.5$  corresponds  
211 to the symmetric world bond market. Alternatively, we could have assumed that there  
212 is a menu of bonds that are subject to different trading costs. Our model situation is  
213 a special case of such a setting: we assume that all bonds but one have a prohibitively  
214 high trading cost.

215 The initial allocation of financial assets  $(\theta_{1,0}^i, \theta_{2,0}^i, b_0^i)_{i=1}^2$  is given.

216 The household in country  $i$  trades in financial and goods markets to maximize the  
 217 expected life-time utility given by:

$$U(c) = E \left[ \sum_{t=0}^{\infty} \beta^t u(g^i(c_1^i(z^t), c_2^i(z^t))) \middle| z_0 \right], \quad \beta \in [0, 1), u' > 0, u'' < 0. \quad (2)$$

218 Function  $g^i$  is a constant return to scale consumption aggregator. We assume that  
 219 the households' preferences display consumption home bias. Hence, the consumption  
 220 aggregate in country  $i$  is biased towards the domestically produced good  $i$ . In the case  
 221 of the CES aggregator, this assumption is isomorphic to assuming trade costs/taxes  
 222 if the latter are rebated back to households.

223 The household in country  $i$  receives non-financial income  $w_i(z_t)$  units of domestic  
 224 good  $i$  and  $\epsilon$  units of foreign good  $-i$ . This income represents wages and profits of  
 225 privately held companies. In our quantitative analysis we assign a negligible value  
 226 to  $\epsilon$ . So, for the clarity of the exposition we formulate the model as if households  
 227 received no endowment of foreign goods.

228 The budget constraint of the household living in country  $i$  after history  $z^t$  is:

$$p_1(z^t)c_1^i(z^t) + p_2(z^t)c_2^i(z^t) + q_1(z^t)\theta_1^i(z^t) + q_2(z^t)\theta_2^i(z^t) + q_b(z^t)b^i(z^t) = I^i(z^t), \quad (3)$$

229 where  $(\theta_1^i, \theta_2^i, b^i) \in R_+^2 \times [-B(z^t), \infty)$  is the portfolio of the consumer living in country  
 230  $i$  and consists of his positions in the two equity claims and the bond.  $I^i(z^t)$  is “cash-  
 231 in-hand” that consists of the market value of his non-financial income  $w_i$  and the  
 232 income that he receives from his financial portfolio (including dividends):

$$I^i(z^t) \equiv p_i(z^t)w_i(z^t) + \sum_{j=1}^2 (q_j(z^t) + p_j(z^t)d_j(z^t))\theta_j^i(z^{t-1}) + p_b(z^t)b^i(z^{t-1}). \quad (4)$$

233 In the infinite horizon model if no explicit borrowing limits are imposed the prob-  
 234 lem of the agent may not be well-defined. We impose the borrowing limit that is  
 235 close in spirit to Levine and Zame (1996), who require that it should be possible for  
 236 households to repay their obligations over a finite period of time. We require that the  
 237 households are able to do so in one period:

$$B^i(z^t) \equiv \min_{z_{t+1} \in \mathcal{Z}} \left[ \frac{p_i(z^{t+1})w_i(z_{t+1}) + \sum_{j=1}^2 \tilde{q}_j(z^{t+1})\theta_j^i(z^t)}{p_b(z^{t+1})} \right], \quad \forall z^t, \quad (5)$$

238 where  $\tilde{q}_j(z^t) \equiv q_j(z^t) + p_j(z^t)d_j(z_t)$  is the cum-dividend price of stock  $j$ . This con-  
 239 straint is sufficiently generous since households can borrow against their portfolio of  
 240 stocks.<sup>9</sup>

### 241 3.2. Wealth-recursive equilibrium

242 A competitive equilibrium is a price system  $\mathcal{P} = \{p_1(z^t), p_2(z^t), q_1(z^t), q_2(z^t), q_b(z^t) :$   
 243  $\forall z^t\}$ , an allocation  $\mathcal{C} = \{(c_1^i(z^t), c_2^i(z^t))_{i=1}^2 : \forall z^t\}$  and asset positions  $\mathcal{A} = \{(\theta_1^i(z^t), \theta_2^i(z^t), b^i(z^t))_{i=1}^2 :$   
 244  $\forall z^t\}$  such that:

- 245 1. given the price system  $\mathcal{P}$ , the allocation and asset positions solve each house-  
 246 hold's optimization problem;
- 247 2. financial and goods markets clear:  $\forall z^t, j = 1, 2,$

$$c_j^1(z^t) + c_j^2(z^t) = w_j(z^t) + d_j(z^t) \quad (6a)$$

$$\theta_j^1(z^t) + \theta_j^2(z^t) = 1 \quad (6b)$$

$$b^1(z^t) + b^2(z^t) = 0. \quad (6c)$$

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<sup>9</sup>Because equity short-selling is not allowed, this borrowing limit implies that debt can be at least as high as the lowest output realization. When wealth is distributed evenly, the borrowing capacity of a country is approximately  $p_j(z^t)e_j(z^t) + q_j(z^t)$ , that is, about 2.6 times that country's GDP,  $p_j(z^t)e_j(z^t)$ , in our calibration. In equilibrium this borrowing limit is never binding. But when a country's wealth share decreases, it reduces its portfolio of equities and the borrowing capacity shrinks to one GDP. So, the borrowing limit changes endogenously and pro-cyclically with economic conditions. Quantitatively, however, this fact has limited implications.

248 We normalize the price system so that:

$$p_1(z^t) + p_2(z^t) = 1, \quad \forall z^t.$$

249 In general, it is not feasible to compute the competitive equilibrium as defined  
 250 above. The usual reason is the curse of dimensionality: a natural state vector for this  
 251 model includes the portfolio holdings of each household. To sidestep this problem, we  
 252 restrict our attention to wealth-recursive equilibria. Duffie et al. (1994) call recursive  
 253 equilibria dynamically simple. Such equilibria may not exist if the state space is not  
 254 sufficiently rich. Duffie et al. (1994) show that the equilibrium will exist if the state  
 255 space includes all the equilibrium variables. A wealth-recursive Markov equilibrium  
 256 is a competitive equilibrium in which the distribution of wealth is a sufficient statistic  
 257 for all the equilibrium variables.

258 Let the financial wealth share of country 1 be denoted by  $\omega$ :

$$\omega(z^t) \equiv I^1(z^t)/[I^1(z^t) + I^2(z^t)] \in [0, 1], \quad \forall z^t. \quad (7)$$

259 The wealth share always lies in the unit interval under the borrowing limit (5). Im-  
 260 portantly, the total wealth depends only on the prices and the exogenous labor income  
 261 and dividend processes:

$$I^1(z^t) + I^2(z^t) = \sum_{j=1}^2 [p_j(z^t)(w_j(z_t) + d_j(z_t)) + q_j(z^t)]. \quad (8)$$

262 Notice also that the portfolio in the definition of  $\omega(z^t)$  was chosen after history  $z^{t-1}$ .  
 263 Then (7) implicitly defines a law of motion for the endogenous state  $\omega$ :

$$\omega(z^{t+1}) = \Omega(\omega(z^t), z^t, z_{t+1}), \quad \forall z^t, z_{t+1}. \quad (9)$$

264 Let  $x = ((c_1^i, c_2^i)_{i=1}^2, (\theta_1^i, \theta_2^i, b^i)_{i=1}^2, (\mu_1^i, \mu_2^i, \mu_b^i)_{i=1}^2, (p_1, p_2, q_1, q_2, q_b))$  be a vector of all  
 265 endogenous variables at any node of the history tree, where the Lagrange multipliers  
 266  $\mu_1^i, \mu_2^i, \mu_b^i$  correspond to short-selling constraints on stock 1, stock 2 and a borrowing  
 267 limit on the debt. Set  $\mathcal{X} = R_+^4 \times R_+^6 \times R_+^6 \times R_+^5$  contains all possible values of  
 268  $x$ . Define  $\rho : [0, 1] \times \mathcal{Z} \rightarrow \mathcal{X}$  to be a policy correspondence that maps the current  
 269 state, i.e. (wealth share, exogenous Markov state) pair, into the set of all equilibrium  
 270 variables.

271 A *wealth-recursive Markov equilibrium* is an equilibrium correspondence  $\rho : [0, 1] \times$   
 272  $\mathcal{Z} \rightrightarrows \mathcal{X}$  and a financial wealth transition map  $\Omega : [0, 1] \times \mathcal{Z} \rightarrow [0, 1]^{|\mathcal{Z}|}$  such that  
 273  $\forall (\omega, z) \in [0, 1] \times \mathcal{Z}$ :

$$0 = E \left[ \Phi \left( \rho(\omega, z), \omega, z, (\rho(\omega(z'), z'), \omega(z'), z')_{z' \in \mathcal{Z}} \right) \middle| z \right], \quad (10)$$

$$\omega(z') = \Omega(\omega, z, z'),$$

274 where  $\Phi$  is the system of equilibrium conditions. This system is written out ex-  
 275 plicitly in Appendix B.1. Theorem 1 below shows that a wealth-recursive Markov  
 276 equilibrium does exist. Assumptions (b) and (c) jointly guarantee that the autarkic  
 277 allocation provides at least some utility. Together with assumption (a) it implies that  
 278 equilibrium consumption is bounded away from zero.

279 **Theorem 1.** *Suppose the following conditions hold:*

- 280 a) *the utility is well-behaved and unbounded below:  $u^i(0) = -\infty, i \in \{1, 2\}$ ;*
- 281 b) *both goods are essential:  $g^i(0, x) = g^i(x, 0) = 0, \forall x \in \mathbb{R}_+, i \in \{1, 2\}$ ;*
- 282 c) *endowments are bounded below:  $\exists w_m > 0 : w_i(z) > w_m, \forall z \in \mathcal{Z}, i \in \{1, 2\}$ ;*
- 283 d) *households face short-selling constraints and borrowing limits (5).*

284 *Then there exists a wealth-recursive Markov equilibrium.*

285 Any numerical algorithm allows computing only approximate, not exact, equilibria  
 286 in which optimality conditions are met only approximately. Kubler and Polemarchakis

287 (2004) show that approximate equilibria may exist even if the exact equilibrium does  
 288 not. Theorem 1 assures that we are not computing an approximate solution for the  
 289 model that has no exact equilibrium.

290 Unfortunately, the policy correspondence  $\rho$  is not guaranteed to be single-valued.  
 291 Neither can it be established that the Markov equilibrium has an invariant ergodic  
 292 measure. Yet, we know of no work that establishes when a GE model with incomplete  
 293 markets and aggregate uncertainty possesses these properties.

### 294 3.3. Functional forms

295 For the rest of our analysis we specialize our setting. We assume a CRRA utility  
 296 function and a CES aggregator:

$$u^i(c, z) = \delta(z)c^{1-\gamma^i}/(1-\gamma^i), \quad \gamma^i > 0, \quad (11a)$$

$$g^1(c_1, c_2) = (sc_1^\phi + (1-s)c_2^\phi)^{1/\phi}, \quad s \in [0.5, 1], \phi \leq 1 \quad (11b)$$

$$g^2(c_1, c_2) = ((1-s)c_1^\phi + sc_2^\phi)^{1/\phi}. \quad (11c)$$

297 The elasticity of substitution (ES) between the two goods is  $\varepsilon \equiv \frac{1}{1-\phi}$ . The above  
 298 consumption aggregators imply the following aggregate price indices:

$$P^1 = (s^\varepsilon p_1^{1-\varepsilon} + (1-s)^\varepsilon p_2^{1-\varepsilon})^{1/(1-\varepsilon)}, \quad (12a)$$

$$P^2 = ((1-s)^\varepsilon p_1^{1-\varepsilon} + s^\varepsilon p_2^{1-\varepsilon})^{1/(1-\varepsilon)}. \quad (12b)$$

299 Let  $q \equiv p_1/p_2$  denote the terms of trade and let  $Q \equiv P^1/P^2$  denote the real exchange  
 300 rate. Finally, denote the fraction of income spent on domestic goods (in a symmetric  
 301 deterministic model) by  $\chi$ :

$$\chi = \frac{s^\varepsilon}{s^\varepsilon + (1-s)^\varepsilon}. \quad (13)$$

## 302 4. Numerical results

303 We solve the model numerically using the projection method. Namely, for each  
304  $z \in \mathcal{Z}$  we approximate the policy functions  $\rho(\omega, z)$  by cubic splines on  $[0, 1]$ . We  
305 use a uniform grid for  $\omega$ . The update for the equilibrium (price, policy and wealth  
306 transition) functions is obtained by solving the system (10). We iterate on the equi-  
307 librium functions until the change in the price system is less than  $10^{-4}$ , so that the  
308 change in the price system between two consecutive iterations is less than 1 basis  
309 point.<sup>10</sup> We also test the accuracy of our solution method on a model in which the  
310 only source of income is dividends. In this case, the optimal solution is a “linear  
311 sharing rule”. That is, trade in the two stocks is enough to achieve full consumption  
312 insurance and the markets are effectively complete, see Baxter and Jermann (1997)  
313 and Heathcote and Perri (2013). In this case our solution method performs extremely  
314 well with errors in the equilibrium conditions being close to machine precision.

315 Multiplicity is a plague for numerical analysis. We know of no work that would  
316 provide conditions under which a competitive equilibrium is unique. Kubler and Schmedders  
317 (2002) suggest that multiplicity of equilibria in models with incomplete financial mar-  
318 kets may be related to the multiplicity of efficient allocations. It is verified in Ap-  
319 pendix E.1 that the efficient allocation is indeed unique.

320 We now explain our calibration. We use annual data from 1984 until 2007 for  
321 the U.S. and for the OECD economies to construct nine moments described below,  
322 with details provided in Appendix F. The common discount factor  $\beta$  is chosen so that  
323 the average return on the bond is 4%, a common benchmark value. This moment  
324 is denoted by M1 in table 4. We choose  $(\gamma^1, \gamma^2) = (0.773, 1.680)$ , values that are  
325 close to the commonly-used logarithmic preferences, to match the U.S. net equity  
326 and debt positions: the NFA in equity is +20.8% of output (moment M2), and the

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<sup>10</sup>Off-grid errors in the equilibrium conditions are plotted in Appendix E.

327 NFA in debt is -38.0% (moment M3). We set  $(\phi, s) = (-0.385, 0.916)$  to match the  
328 volatility of the real exchange rate (moment M4) and the trade/GDP ratio (moment  
329 M5). The corresponding elasticity of substitution (ES) between goods is 0.722. Such  
330 a low ES would not be needed if the model featured non-tradeable goods or if we  
331 assumed a larger difference in risk-aversion. Parameters  $\sigma_e$  and  $\rho_e$  are set to match the  
332 standard deviation and the autocorrelation of log-output in the U.S. data (moments  
333 M6 and M7). The stochastic process for endowments is assumed to be a 9-state first-  
334 order Markov process, with the two endowment processes being independent. The  
335 stochastic processes for wages and dividends are constructed in the following way:

$$w_i = \bar{w} + (1 - s_d)(e_i - E(e_i)), \quad (14a)$$

$$d_i = \bar{d} + s_d(e_i - E(e_i)). \quad (14b)$$

336 Parameter  $\bar{d}$  is chosen so that the domestic stock market value to GDP ratio is  
337 1.611 as in the U.S. during 1988-2007 (moment M8). This implies  $\bar{d} = 0.0645$  and  
338  $\bar{w} = E(e^i) - \bar{d} = 0.9355$ . It can be shown that  $\sigma(\ln(d^i))/\sigma(\ln(w^i)) \approx (\bar{w}/\bar{d})s_d/(1 - s_d)$ .  
339 In the data this ratio equals 5.356 (moment M9) and it implies  $s_d = 0.270$ .

340 The persistence of output in the U.S. is statistically indistinguishable from the  
341 sample weighted average of the OECD economies (see table F.5). So, we assume that  
342 the persistence of output in the two model countries is the same as observed in the  
343 U.S. The correlation between the domestic and the foreign output is set to zero as  
344 the weighted average correlation in the data is only 0.137 and is deemed insignificant.

345 We consider several specifications. S0 is the benchmark model calibrated to the  
346 data: country 1 has an advantage in the bond market as debt is denominated in  
347 good 1 (the debt market advantage) and households in country 1 are less risk-averse:  
348  $\alpha = 1, \gamma_1 = 0.773 < \gamma_2 = 1.680$ . S1 $^\gamma$  allows for risk-aversion differences but imposes  
349 that the bond market is symmetric:  $\alpha = 0.5$ . S1 $^\alpha$  allows country 1 to enjoy the



		Value	Moment/Source
Discount factor	$\beta$	0.9615	Return on bond = 4% (M1)
Risk-aversion of country 1	$\gamma^1$	0.7730	$NFA_{debt}^{US} = -0.380, NFA_{equity}^{US} = +0.208$ (M2,M3)
Risk-aversion of country 2	$\gamma^2$	1.6800	Same as for $\gamma_1$
ES between goods	$\phi$	-0.3850	Volatility of the RER = 3.5% (M4)
Utility weight of dom. good	$s$	0.9158	Trade/GDP = $0.5(X+M)/(C+NX) = 15.5\%$ (M5)
Volatility of income	$\sigma_e^1$	0.0151	Volatility of log-income in the U.S. (M6)
Persistence of income	$\rho_e^1$	0.7520	Persistence of log-income in the U.S. (M7)
Dividends/income	$d$	0.0645	Stock market value/GDP ratio = 1.611 (M8)
$\sigma(\text{dividends})/\sigma(\text{wages})$	$s_d$	0.2697	$\sigma(\ln(d^i))/\sigma(\ln(w^i)) = 5.356$ (M9)

Table 1: Benchmark parameter values

350 bond market advantage, but it imposes that households in the two countries are  
351 equally risk-averse:  $\gamma_1 = \gamma_2 = 1.222$ , which is the average of the two values used in  
352 S0. S2 is the fully symmetric environment with  $\alpha = 0.5$  and  $\gamma_1 = \gamma_2 = 1.222$ . We  
353 adjust the elasticity of substitution between goods  $\phi$  and the weight on the domestic  
354 good  $s$  to match moments M4 and M5 in all the specifications. Thus we use  $(\phi, s) =$   
355  $(-0.385, 0.916)$  in the specifications S0,  $(-0.285, 0.900)$  in S1 $^\alpha$ ,  $(-0.176, 0.891)$  in S1 $^\gamma$ ,  
356 and  $(-0.215, 0.887)$  in S2. We simulate 500 series of length 200,000. Each simulation  
357 starts from  $\omega_0 = 0.5$ . The first half of each sample is deleted. Table 4 summarizes  
358 our simulation results. Sections 4.1-4.5 describe our results.

359 *4.1. The country that can issue domestic bonds accumulates a negative bond position*  
360 *and invests borrowed funds in foreign equity.*

361 First, we provide an intuitive explanation of why a country chooses to hold a neg-  
362 ative position in the domestic bond. To obtain analytic results, we restrict financial  
363 trade to the bond paying a unit of good 1 and assume that consumers in the two  
364 countries are equally risk-averse. We then follow the argument in Svensson (1988)  
365 and ask: “In a financial autarky, which country would be willing to pay more for the  
366 bond that pays one unit of the good produced in country 1?” We start with a stochas-  
367 tic discount factor of a consumer in country 1:  $M_{t+1}^1 = \beta(C_{t+1}^1/C_t^1)^{-\gamma} P_t^1/P_{t+1}^1$ , where

	Statistic	Data	S0	S1 $^\alpha$	S1 $^\gamma$	S2	S1 $^{\gamma'}$
<i>International investment</i>							
1	Net equity / output, M2	0.208	<i>0.208</i>	-0.001	0.131	0.000	0.072
2	Net debt / output, M3	-0.380	<i>-0.380</i>	-0.136	-0.185	0.000	-0.089
3	Net FA / output	-0.172	<i>-0.172</i>	-0.137	-0.054	0.000	-0.017
4	Home equity bias	0.934	<i>0.890</i>	0.974	0.926	0.981	-0.929
5	E( $\omega$ )	–	<i>0.453</i>	0.465	0.488	0.500	0.505
<i>Real exchange rate</i>							
6	std(RER), M4	0.035	<i>0.035</i>	0.035	0.035	0.035	0.086
7	cor(RER,C/C*)	0.265	<i>0.212</i>	0.340	0.442	0.430	0.865
8	cor(RER,Y/Y*)	-0.246	<i>-0.612</i>	-0.578	-0.554	-0.559	-0.268
9	E( $p_1$ )	–	<i>0.486</i>	0.491	0.497	0.500	0.502
<i>Capital flows</i>							
10	std(CA/Y)	0.014	<i>0.006</i>	0.006	0.007	0.007	0.014
11	cor(CA/Y,Y)	-0.059	<i>0.597</i>	0.889	0.840	0.875	0.960
12	E(  $\Delta CA^e$  / CA )	0.386	<i>0.182</i>	0.052	0.127	0.041	0.100
13	cor( $\Delta CA^e$ , CA)	-0.451	<i>-0.513</i>	-0.037	0.013	0.017	0.024

S0: debt market advantage and risk-aversion difference; S1 $^\gamma$ : risk-aversion difference; S1 $^\alpha$ : debt market advantage; S2: symmetric economies; S1 $^{\gamma'}$ : all parameters as in S0 except there is no debt advantage

Table 2: Moments in the data and in the model

368  $C_t^1$  denotes aggregate consumption of country  $i$ . The above expression implies that  
369 a consumer in country 1 would like to hold assets with a payoff that is a) negatively  
370 correlated with the domestic price inflation  $\pi_{t+1}^1 \equiv P_{t+1}^1/P_t^1$  and b) negatively corre-  
371 lated with the domestic consumption growth  $C_{t+1}^1/C_t^1$ . These are referred to as the  
372 CPI and the consumption hedging motives, respectively. The ES is important for the  
373 second motive as it directly impacts consumers' willingness to change their consump-  
374 tion bundle when the relative price of goods changes. The risk aversion coefficient  
375 governs the relative importance of the two motives. Using log-linear approximations  
376 and the financial autarky allocation as an approximation point, we obtain proposition  
377 1 that is proved in Appendix C.

378 *Proposition 1. If financial markets trade only a bond denominated in good 1 and*  
379 *the borrowing constraint is not binding, then country 1 will hold a negative position*  
380 *in the bond when  $\gamma(2\chi\varepsilon - 1) < 2\chi - 1$ .*

381 This condition brings to light two opposing forces. Consider the increase in coun-

382 try 1's output such that the price of good 1 declines by 1%. On the one hand, country  
 383 1's aggregate consumption increases by  $(2\chi\varepsilon - 1)\%$ . The increase in consumption is  
 384 larger when the ES is high (hence the income effect is small), and when the consump-  
 385 tion home bias is strong. At the same time the marginal utility of country 1 declines  
 386 by  $\gamma(2\chi\varepsilon - 1)\%$ . This prompts country 1 to purchase the bond, because its payoff  
 387 declines when country 1's valuation of extra consumption is low and vice versa. We  
 388 call this a consumption hedging effect. On the other hand, the aggregate price level in  
 389 country 1 declines by  $(2\chi - 1)\%$  and the country demands more consumption because  
 390 it is more affordable. The bond does not help satisfying this demand as its payoff de-  
 391 clines. We call this a CPI effect. If the CPI effect dominates the consumption hedging  
 392 effect, then country 1 should hold a negative position in the bond. Importantly, the  
 393 lower the ES, the stronger is the desire to sell domestic bonds. A decrease in the ES  
 394 dampens the response of the consumption relative to that of the price level.<sup>11</sup>

395 We now turn to the simulation results in table 4. With the asymmetric bond  
 396 market, country 1 accumulates debt and invests borrowed funds in the foreign equity  
 397 (refer to column S0). Wealthier country 2 drives the price of good 2 up,  $E(p_2) = 0.514$   
 398 (recall that  $p_1 + p_2 = 1$ ). Country 1, enjoying the bond market advantage, would  
 399 like to sell domestic debt and purchase more of the relatively cheap domestic equity.  
 400 The latter action is not possible due to the short-selling constraint. This tension is  
 401 removed if the price of good 1 decreases and country 1's equity becomes less attractive.

402 When the asymmetry in the bond market (column S1<sup>γ</sup>) is turned off, the debt NFA  
 403 position of country 1 is still a sizeable -18.5% of GDP. But the overall NFA position  
 404 is only -5.4%. This result is driven by the more risk-averse household in country 2  
 405 that wishes to accumulate relatively safe debt and sell relatively risky equity. If the

---

<sup>11</sup>In a special case with logarithmic preferences, equal utility weights on goods, and the ES between goods equal to one both the CPI and the consumption hedging motives are absent. This is a well-known result obtained in Cole and Obstfeld (1991) where trade in goods is sufficient to achieve perfect risk-sharing. For the risk aversion coefficient  $\gamma = 1.222$  country 1 sells the "domestic" bond when the ES is less than 0.92.

406 difference in risk aversion is turned off (column S1<sup>a</sup>), the position in debt is -13.6%  
 407 and the overall NFA position is -13.7%. In this specification, similarly to S0, the  
 408 household in country 1 sells bonds and invests in equity as this helps engineering the  
 409 income stream that better matches the desired consumption spending. Yet, without  
 410 the difference in risk-aversion, the net position in equity is negligible and negative,  
 411 unlike in the data. The reason is that the price of good 1 is low; therefore, the equity  
 412 in country 2 is more expensive and a larger fraction of the domestic equity must be  
 413 sold to afford the foreign equity. In a fully symmetric setting (column S2), the net  
 414 foreign positions are zero since both countries on average must hold the same portfolio  
 415 of assets.

416 We also point out the interaction between the bond market asymmetry and the  
 417 risk-aversion differences. Because the good prices are perfectly negatively correlated,  
 418 proposition 1 implies that increasing risk-aversion of country 2 should lead to it buying  
 419 more of the bond issued by country 1. At the same time, because country 2 is more  
 420 risk-averse, it has a stronger precautionary demand for the bond. But the larger  
 421 position in the bond forces country 2 to hedge it with a larger position in country 1's  
 422 equity. This additional exposure amplifies the precautionary demand.

#### 423 *4.2. Domestic stocks dominate the countries' portfolios*

424 Irrespectively of the specification, the share of domestic equity in the equity port-  
 425 folio is high and ranges from 0.890 under S0 to 0.981 under S2. To gain insight  
 426 into why home equity bias arises, consider the symmetric model with the common  
 427 risk-aversion parameter  $\gamma$ . We follow the approach described in Kollmann (2006) and  
 428 compute the constant portfolio that allows consumers in both countries to approxi-  
 429 mately finance the efficient consumption allocation.<sup>12</sup> Denote the optimal portfolio  
 430 of equities of country 1 by  $(\theta^*, 1 - \theta^*)$ . By symmetry, the share of home equity is:

---

<sup>12</sup>This analysis uses the price system that would prevail under complete financial markets. The optimal portfolio solves the approximate (first-order of accuracy) budget constraint.

431  $2\theta^* - 1$ . Proposition 2 in Appendix C shows that  $\theta^*$  must solve:

$$\underbrace{-(2\chi - 1)(1 - 1/\gamma)\hat{e}/\lambda}_{\text{relative consumption spending}} = \underbrace{(2\theta^* - 1)}_{\text{home equity bias}} \cdot \underbrace{\bar{d}(\hat{d} - \hat{e}/\lambda)}_{\text{rel. div. income}} + \underbrace{\bar{w}(\hat{w} - \hat{e}/\lambda)}_{\text{rel. labor income}}, \quad (15)$$

432 where  $\hat{d} = (s_d/\bar{d})\hat{e}$ ,  $\hat{w} = ((1 - s_d)/\bar{w})\hat{e}$  and  $\lambda = \phi - (2\chi - 1)^2(\phi - 1/\gamma)$  is the inverse  
 433 of the elasticity of the terms of trade  $p_1/p_2$  with respect to the relative output  $e_1/e_2$ .  
 434 This implies that changes in a country's relative dividend and labor income must  
 435 match fluctuations in the desired relative consumption spending.

436 First, when labor and dividend income are fixed proportions of output, as would  
 437 be the case with endogenous production and the Cobb-Douglas technology, the op-  
 438 timal portfolio would likely exhibit foreign equity bias.<sup>13</sup> This is so because the  
 439 non-tradeable labor income makes a country's total income too sensitive to domestic  
 440 output and this prompts the sale of domestic equity.

441 To understand home equity bias in the general case, consider a reaction to an  
 442 increase in country 1's output. Because dividends are positively correlated with the  
 443 output and are more volatile, the relative dividend paid by country 1's equity in-  
 444 creases. The relative labor income paid in country 1, on the other hand, decreases  
 445 as does the relative desired consumption spending. But because labor income consti-  
 446 tutes a large fraction of the produced output, the desired consumption spending net  
 447 of labor income increases. In other words, following the shock the desired consump-  
 448 tion spending declines, but labor income declines more. This prompts the economies  
 449 to hold a large share of domestic equity in their portfolios.

450 Finally, as  $\gamma$  increases, relative consumption spending becomes more sensitive to  
 451 changes in the relative income:  $(2\chi - 1)(1 - 1/\gamma)/\lambda$  is increasing in  $\gamma$ . So, relative

---

<sup>13</sup>When  $s_d = \bar{d}$  the equilibrium home equity bias is:

$$2\theta^* - 1 = [(2\chi - 1)(1 - 1/\gamma)/(1 - \lambda) - \bar{w}]/\bar{d}. \quad (16)$$

452 consumption spending net of labor income becomes negatively correlated with the  
 453 relative output as  $\gamma$  increases. In this case, it is optimal for countries to invest a  
 454 larger fraction of wealth in foreign equity. Indeed, when we increase the risk-aversion  
 455 parameter to  $\gamma = 5$  in the symmetric setting, the mean share of home equity decreases  
 456 to 0.494 consistent with our predictions. So, assuming a high value of risk aversion  
 457 to generate an equity premium in our model is not acceptable: this would negate the  
 458 home bias.<sup>14</sup>

459 *4.3. Correlation between the RER and relative consumption is low*

460 In a symmetric environment and under complete financial markets the relation  
 461 between the real exchange rate and the relative consumption of the two countries is:

$$\ln(Q_t) = -\gamma \ln(C_t^1/C_t^2). \quad (17)$$

462 The above relation implies that there should be a perfect negative correlation  
 463 between the logarithms of the two variables. However, we have incomplete markets,  
 464 and the extent to which our results deviate from this benchmark depends on the  
 465 degree of the market incompleteness. The short-selling constraint is more likely to  
 466 affect the results. The borrowing limit, as we mentioned above, binds with a very  
 467 low probability. To understand how short-selling constraints change the correlation  
 468 between the real exchange rate and relative consumption, consider an increase in  
 469 country 1's output. Then observe that if the markets were complete, the wealth as  
 470 we define it would be perfectly negatively correlated with output. But markets are  
 471 incomplete and when output in country 1 increases, its wealth and consumption also  
 472 increase. The real exchange rate may decrease because there is more of domestic  
 473 goods, or increase because wealthier country 1 demands more. But there is also

---

<sup>14</sup>Heathcote and Perri (2013) show that adding non-tradable goods or capital accumulation makes equity home bias a robust feature of this type of models.

474 a non-standard effect. Because country 1 wants to purchase more of the domestic  
 475 equity but cannot, the price of goods produced in country 1 must increase. This  
 476 pushes up the price of country 1's equity, curbs demand, and equilibrates the market  
 477 for country 1's equity. This is the strongest effect, and the overall result is that  
 478 there is a positive correlation between the exchange rate and relative consumption:  
 479 0.208 under S0. Consistent with our intuition, this correlation is higher under the  
 480 symmetric specification S2 because the home equity bias is more extreme and the  
 481 short-selling constraints are more active.

482 *4.4. Fluctuations in the real exchange rate account for a large fraction of current*  
 483 *account adjustments*

484 Suppose that the foreign good becomes more expensive and the value of foreign  
 485 equity increases. The relative wealth positions of the two countries change, even  
 486 though there is no explicit trade. Because the gross positions are large, fluctuations  
 487 in the real exchange rate add significantly to capital flows (the traditionally measured  
 488 current account, CA). Similar adjustments must be made to account for changes in  
 489 the market value of the long-term assets purchased in previous periods. The CA then  
 490 can be decomposed into three components (see Appendix A for details):

$$CA_t = \underbrace{q_{2t}(\theta_{2t+1}^1 - \theta_{2t}^1) - q_{1t}(\theta_{1t+1}^2 - \theta_{1t}^2)}_{\text{capital flow}} + q_{bt}b_{1t+1} - q_{bt-1}b_{1t}$$

*+ exchange rate adjustment + equity price adjustment.*

491 The two adjustment effects are commonly referred to as the valuation effects that  
 492 were first brought to everyone's attention by Lane and Milesi-Ferretti (2001). The  
 493 exchange rate adjustment is proportional to  $(Q_t - Q_{t-1})$  while the price adjustment  
 494 corresponding to equity claim  $j$  is proportional to  $(q_{jt} - q_{jt-1})$ . In the U.S. exchange  
 495 rate fluctuations account for 38.6% of the total variation in the U.S. current account

496 (see row 11 in table 4). Under the benchmark specification (S0) this number is 18.2%,  
497 while under the other specifications it does not exceed 12.7%.

498 In the symmetric setting S2 there is a strong home equity bias. The share of the  
499 foreign equity owned by domestic households is very small and this, in turn, limits  
500 the CA adjustment effect that stands at 4.1%. In contrast, in the asymmetric setting,  
501 especially S0 and S1<sup>7</sup>, the home bias is not as extreme allowing for more substantial  
502 adjustments.

503 Under S0 the correlation between the adjustment and the actual flow is -0.513 (row  
504 13), close to -0.451 in the data. This can be attributed to the asymmetric portfolios  
505 purchased by the countries. When country 1's output decreases, its consumption also  
506 decreases and the CA improves. However, country 1's investment in foreign equity  
507 decreases and so does the adjustment effect. As a result, the CA improves while the  
508 exchange rate adjustment decreases.

509 Notice that the exchange rate plays a stabilizing role. When there is capital  
510 outflow, namely a country accumulates net foreign wealth, the real exchange rate ap-  
511 preciates, and the value of foreign equity declines relative to domestic equity. Foreign  
512 equity becomes less attractive and this slows down or reverses the outflow. Finally,  
513 all of the above discussion also applies to current account adjustments stemming from  
514 changes in the equity prices.<sup>15</sup>

515 *4.5. The distribution of country 1's financial wealth share is skewed towards 0 and its*  
516 *mean is decreased significantly. But the debt market advantage has a negligible*  
517 *impact on country 1's welfare*

518 Figure 1 plots the stationary distribution of the financial income share. In the  
519 symmetric setting (S2) the countries' financial wealth share is 0.5 on average. In the  
520 main specification with the bond advantage and the risk-aversion difference (S0), the

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<sup>15</sup>If we combine price and exchange rate adjustments to the current account, then under S0 we obtain  $E(|\Delta CA^{adj}|/|CA|) = 0.333$  while in the data this number is 0.674.



521 mean financial wealth share is 0.453, 17.2% less than that of country 2. If the risk-  
522 aversion asymmetry is turned off ( $S1^\alpha$ ), the mean financial wealth share of country  
523 1 is 0.465, still a sizeable 13.1% difference when compared with country 2. So, the  
524 bond advantage alone significantly distorts the stationary distribution of financial  
525 wealth. Wealth distribution is positively skewed in the three asymmetric setups.  
526 Our calculation also show that under  $S0$ , country 1's wealth share does not decline  
527 below 0.37, while country 2's wealth share could be as low as 0.16. Namely, while  
528 both countries can experience disastrous declines in their wealth share, the worst-case  
529 scenario for country 2 is twice as severe as it is for country 1.

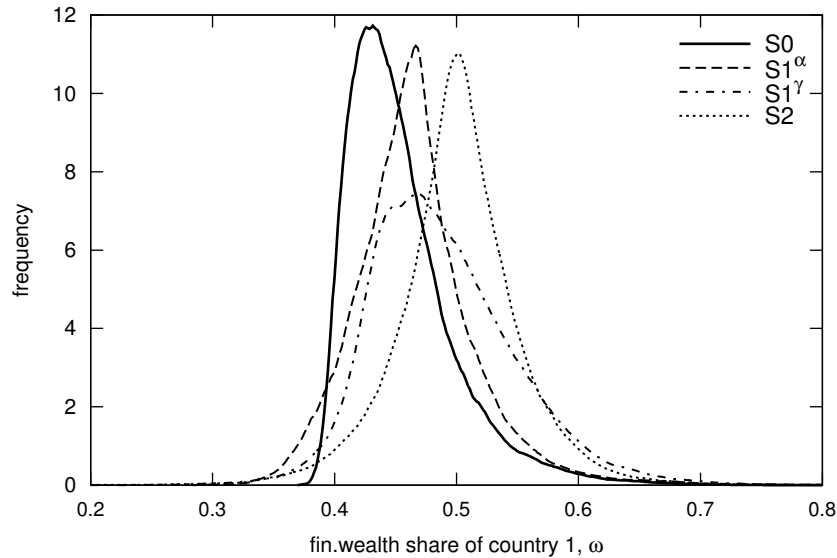


Figure 1: Stationary distribution of the financial income share.  
Note: Adding risk-aversion differences, compare  $S1^\gamma$  to  $S2$ , makes the distribution more dispersed. Adding debt market advantage, compare  $S1^\alpha$  to  $S2$ , skews the distribution to the right. Both frictions shift the distribution mean to the left.

530 Because financial markets are incomplete, the lower mean financial wealth share  
531 does not imply that the welfare of country 1 is affected adversely. Consider speci-  
532 fications  $S0$  and the new specification  $S1^{\gamma'}$ . The latter differs from  $S0$  only because  
533 neither country has a bond market advantage. So, all the preference and production  
534 parameters remain fixed across the two specifications and this facilitates our welfare  
535 comparison.  $S1^\gamma$  also adjusts  $(\phi, s)$  to match moments  $M4$  and  $M5$  unlike  $S1^{\gamma'}$ . We

536 report moments of the aggregate consumption and welfare for the two specifications  
537 in table 3. There are two effects. First, under S0, consumption in both countries is  
538 less volatile: 1.99% vs 3.76% for country 1 and 1.73% vs 3.69% for country 2. But,  
539 second, under S0 the average consumption is smaller in country 1, 0.685 vs 0.696,  
540 and higher in country 2, 0.706 vs 0.694. This means that country 2's welfare decid-  
541 edly improves and it should not object against country 1's dominance in the world  
542 debt markets. Importantly, the debt market advantage improves stability in both  
543 economies as evidenced by consumption volatility. Smoother consumption, in turn,  
544 implies less volatile real exchange rate and capital flows. So, county 1, designed to  
545 resemble the U.S., is a provider of global stability.

546 Above we compared ergodic properties of consumption processes in both countries.  
547 We now turn to welfare comparison using the following *ex ante* criterion:

$$W_{Sx}^i(\omega_0) = \sum_{z_0 \in \mathcal{S}} \bar{\pi}(z_0) V_{Sx}^i(\omega_0, z_0), \quad i = 1, 2,$$

548 where  $\bar{\pi}(z_0)$  is the ergodic distribution of the exogenous state and  $V_{Sx}^i$  is the optimal  
549 life-time utility of agent  $i$  under model  $Sx$ . We set  $\omega_0 = 0.5$ , that is, we start both  
550 economies with equal wealth. According to our criterion, welfare in each country is  
551 the same, save numerical errors, under S0 and S1 $\gamma'$ . We obtain the same results if we  
552 start the economies at other levels of  $\omega_0 \in [0.45, 0.55]$ . The reason is that it takes a  
553 long time for wealth distribution to deviate from its original position. Even after 100  
554 periods, country 1's expected wealth share is 0.498 and it commands a larger share  
555 of wealth with probability 0.366.

## 556 5. Transaction cost

557 Consider imposing a transaction cost on financial trades. Intuitively, this should  
558 stabilize financial flows, but is such a policy welfare improving? Which asset(s) should

	$E(C^1)$	$\sigma(C^1)$	welfare <sup>1</sup>	$E(C^2)$	$\sigma(C^2)$	welfare <sup>2</sup>
S0	0.685	0.020	105.106	0.706	0.017	-48.742
S1 <sup>γ'</sup>	0.696	0.038	105.106	0.694	0.037	-48.742

S0: debt market advantage and risk-aversion difference;

S1<sup>γ'</sup>: all parameters as in S0 except there is no debt advantage.

Table 3: The effect of bond market advantage: stochastic properties of aggregate cons. and welfare.  
Note: Bond market advantage substantially decreases consumption volatility in both countries while it has a negligible impact on welfare.

559 be taxed? We address these questions by imposing a cost on asset purchases. We  
560 consider two scenarios: in the first, only bond transactions are taxed, while in the  
561 second only equity is subject to a trading cost. Jeanne and Korinek (2010) propose  
562 to tax capital flows to undo the financial amplification stemming from the declining  
563 prices of collateral assets. The authors, however, do not make a distinction between  
564 equity and debt flows. In our setting taxing equity flows is beneficial, because it  
565 limits exposure to risky assets and reduces volatility of consumption and capital  
566 flows. Taxing debt financial flows may destabilize the world economy.

567 We make the following two modeling choices. First, we impose a quadratic trading  
568 cost to avoid issues with discontinuous policies and, hence, equilibrium existence.  
569 Second, the tax proceeds are not wasted but are rather rebated back to consumers.  
570 The rebate of country  $i$  is denoted by  $T^i(z^t)$ . Because consumers in the two countries  
571 pay the same cost, they also receive equal rebates. So, the budget constraint of  
572 country  $i$  is:

$$\begin{aligned}
I^i(z^t) + T^i(z^t) &= p_1(z^t)c_1^i(z^t) + p_2(z^t)c_2^i(z^t) \\
&+ \sum_{j=1}^2 [q_j(z^t)\theta_j^i(z^t) + 0.5\kappa_e[\theta_j^1(z^t) - \bar{\theta}_j^1]^2] \\
&+ q_b(z^t)b^i(z^t) + 0.5\kappa_b[b^i(z^t)]^2.
\end{aligned} \tag{18}$$

573 We use  $\bar{\theta}^1 = (1, 0)$ , so that any sale of equity claims abroad is taxed. However, we  
574 also obtain quantitatively similar results when instead we use the mean portfolio,

575  $\bar{\theta}^1 = (1.000, 0.127)$ , observed in the setting without the trading cost.

576 Consider first imposing a cost on the bond holdings. With  $\kappa_b > 0, \kappa_e = 0$ , triv-  
 577 ially, the magnitude of the countries' bond positions should decrease. This is indeed  
 578 true and the effect is extremely strong: with  $\kappa_b = 1bp$  bonds are not traded anymore.  
 579 With  $\kappa_b = 0.5bp$  the bond position of country 1 is -0.250, down from -0.380 as re-  
 580 ported in table 4. At the same time, country 1's NFA position in equity decreases  
 581 from 0.172 to 0.140, and the overall position improves from -0.172 to -0.110. Intu-  
 582 itively, since country 1 finds it more difficult to borrow, it must scale back its equity  
 583 positions. Insurance possibilities of the two countries are inhibited as the volatility of  
 584 consumption in country 1 and 2 increases by 10.7% and 16.0% respectively. Increased  
 585 consumption volatility leads to more volatile prices. Volatility of the RER and the  
 586 CA increases by 23.0% and 7.8% respectively. That is, taxing bond holdings makes  
 587 portfolios more rigid, while making consumption and exchange rate more volatile.  
 588 The mean consumption level of country 1 increases by 0.6%, while that of country 2  
 589 decreases by 0.6%. Despite the increased volatility, the importance of the exchange  
 590 rate adjustment to the CA decreases, now contributing 0.153 of the actual capital  
 591 flows.

	$(\kappa_b, \kappa_e)$	$NFA_{equity}$	$NFA_{debt}$	$std(C^1)$	$std(C^2)$	$std(RER)$	$std(\frac{CA^1}{Y^1})$
Bond tax	$(5bp, 0)$	0.140	-0.250	0.022	0.020	0.043	0.007
Equity tax	$(0, 5bp)$	0.138	-0.361	0.019	0.016	0.031	0.005

Table 4: Selected macroeconomic indicators when financial trade is taxed.

Note: Bond tax,  $\kappa_b > 0$ , has a larger impact on NFA composition. But equity tax,  $\kappa_e > 0$ , substantially decreases variance of all macroeconomic aggregates.

592 Next, consider imposing the same trading cost on equity claims:  $\kappa_e = 5bp, \kappa_b = 0$ .  
 593 Similarly to the case with a positive bond transaction tax, country 1's equity NFA  
 594 position decreases to 0.138. Country 1's net debt position improves marginally to  
 595 -0.361. As a result, the overall NFA position worsens to -0.222. Capital flows nearly  
 596 vanish because the external gross positions are small, and assets and liabilities are

597 close to being balanced. For the same reasons, the exchange rate adjustment to the  
598 CA contributes less than fraction 0.105 of the actual flows. Volatility of the real  
599 exchange rate and that of the current account in country 1 declines by 10.8% and  
600 11.1%, respectively. But the effect on the equilibrium allocation is smaller relative to  
601 the case with the bond transaction tax: mean consumption in country 1 decreases by  
602 0.4% while it increases by 0.4% in country 2. This suggests that the trade in bonds,  
603 rather than equity, is a more important contributor to international stability. When  
604 trade in the bond market is restricted, aggregate consumption levels must adjust  
605 more. When trade in equity claims is restricted, country 1's NFA becomes more  
606 extreme; yet, the effect on the allocation is smaller, the exchange rate and capital  
607 flows become less volatile.

608 After imposing a trading cost on any asset class, welfare changes are marginal, and  
609 this is expected in an endowment economy. For this reason, it may not be a useful tool  
610 for policy guidance. By examining separate properties of the equilibrium allocation,  
611 as we do above, it can be concluded that any policy inhibiting bond market operations  
612 could have substantial implications. Trading costs could also come in many disguises.  
613 For example, uncertainty about a country's fiscal budget may limit liquidity in the  
614 market for safe government debt. Our analysis suggests that the implications for the  
615 external position and the indirect effect on domestic demand cannot be ignored.

## 616 **6. Conclusions**

617 We study the implication of having a more developed market for debt on the ex-  
618 ternal balance sheet of a country. We use a general equilibrium model of international  
619 portfolio choice with a rich asset market structure and financial constraints. We allow  
620 countries to differ in two dimensions. First, we assume that only one country can  
621 issue debt that pays in units of its own good. Second, we assume that consumers in  
622 the same country are also less risk-averse which is a "reduced form" way of model-

623 ing a more developed domestic credit market. We show that having a bond market  
624 advantage prompts a country to accumulate debt. This effect is magnified signifi-  
625 cantly if the country is also less risk-averse. The latter asymmetry has little effect  
626 by itself, but it reinforces the bond market advantage. The asymmetry in the bond  
627 market also helps reducing the strong correlation between the real exchange rate and  
628 relative consumption. This brings our predictions about the real exchange rate closer  
629 to the data, and allows a meaningful description of the exchange rate channel of the  
630 international financial adjustment.

631 The short-selling constraints that we impose also play an important role. In the  
632 symmetric setting, this friction has a negligible effect on the allocation. But in the  
633 asymmetric setting it limits possible adjustments of the portfolio. For this reason it  
634 has a significant effect on the allocation and the pricing system, helping us match the  
635 observed U.S. international portfolio. While matching the net positions is relatively  
636 easy, the gross positions predicted by the model are largely understated. This can be  
637 partially explained by the fact that the U.S. equity market is only a small fraction of  
638 the world market. In our model it constitutes 50% and, to explain the gross positions,  
639 countries must hold only foreign equity in their portfolios, which is inconsistent with  
640 the home equity bias. This might be a fruitful direction to explore.

641 We use our setting to demonstrate how different markets contribute to “global  
642 stability.” Imposing a trading cost on bond purchases adversely affects the equilibrium  
643 allocations: there is high consumption inequality and it is more volatile. The real  
644 exchange rate and capital flows also become more volatile. In contrast, imposing a  
645 trading cost on equity purchases has a limited effect on the allocation and stabilizes  
646 the real exchange rate and capital flows. These results suggest that “slowing down”  
647 the trade in equity may be desirable. But one should also be wary of policies that  
648 may directly or indirectly affect the liquidity of the bond market.

649 Our work allows us to speculate about issuers of the potential dollar contenders.

650 The introduction of the Euro was an attempt to replace the U.S. dollar as the leading  
651 reserve currency. While it had been gaining the share in the world financial markets  
652 before the global financial crisis, the dollar has recovered its position since then as  
653 argued in Prasad (2014). According to our model the external position of the Euro  
654 area should worsen once its common currency resumes its trend. Similarly, once mar-  
655 kets trading yuan-denominated debt become transparent and the yuan is established  
656 as a reserve currency we expect China's external position to deteriorate.

657 Lastly, we do not address trends in the U.S. external portfolio positions. There  
658 could be several drivers of these trends. The most relevant in our view is the growth of  
659 the stock market value to GDP in the U.S. that increased from 83% to 204% between  
660 1988 and 2007. Similar changes occurred with the size of international trade in goods.  
661 We believe that financial and trade liberalizations have contributed significantly to  
662 the growth of the gross portfolio positions across the world. In our view, modeling  
663 these trends would be a valuable contribution.

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