Portfolio and Welfare Consequences of Debt Market Dominance

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Abstract
The ability to issue debt that pays in units of the domestic good leads a country to accumulate a large and negative net foreign asset position while maintaining a positive position in equity. This debt market advantage also helps to explain the weak relationship between the real exchange rate and relative consumption. Our stylized model matches the key facts about the U.S. international portfolio, the U.S. real exchange rate, and explains nearly 50% of the observed variation in the valuation effects. We find that taxing bond market transactions increases the volatility of the exchange rate, capital flows and allocations. In contrast, taxing equity positions stabilizes the exchange rate and capital flows while having little impact on the allocation. Lastly, the paper describes a global solution method for portfolio problems under incomplete markets.

Keywords: international portfolio choice, global imbalances, incomplete markets, bond market

JEL codes: F32, D52, C63

1. Introduction

During the past several decades we have witnessed a growing financial integration in the world economy. There has been an increase in both the volume of internationally-traded assets and the magnitude of cross-border gross capital flows. According to the dataset compiled by Lane and Milesi-Ferretti (2007b), the total gross foreign assets of the U.S. were 16% of GDP in 1970, stayed below 32% until 1984 and increased to 131% of GDP in 2007. Total gross foreign liabilities were 12% of GDP in 1970, stayed below 30% until 1984 and subsequently soared to 148% of GDP in 2007.

The increase of net and gross international capital flows led to global imbalances and, in particular, to a significant deterioration of the U.S. net foreign asset (NFA)

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position. This weakening of the U.S. NFA position was driven by the accumulation of debt liabilities, while the NFA in equity improved. Gourinchas and Rey (2005) say that “as financial globalization accelerated its pace, the U.S. transformed itself from a world banker into a world venture capitalist, investing greater amounts into high yield assets such as equity and FDI”, while “its liabilities have remained dominated by bank loans, trade credit and debt, i.e. low yield safe assets”. Similar observations were made by Obstfeld and Rogoff (2000) and Mendoza et al. (2009) among others. Obstfeld (2004) states that for the U.S. “the striking change since the early 1980s is the sharp growth in foreign portfolio equity holdings”, while on the liabilities side, “the most dramatic percentage increase has been in the share of U.S. bonds held by foreigners.”

However, the U.S. is in a unique position: it is the only country that can borrow (and lend) almost exclusively in the domestic currency. The share of U.S. debt-like liabilities including government debt, corporate debt and bank loans denominated in U.S. dollars grew from 80.6% in 1990 to 88.3% in 2004, see Lane and Shambaugh (2009). For comparison, during the same period the share of U.K. debt-like liabilities denominated in British pounds averaged at 19.4%. Notably, most of the U.S. debt-like foreign assets were also denominated in the domestic currency amounting to 87.4% in 2004. Therefore, international debt markets have been dominated by assets denominated in the U.S. dollar. The U.S. currency has also dominated the trade in commodity markets, another significant source of short-term debt.

In this paper, we ask if the U.S. dominance in the international debt market can account for the size and composition of the U.S. international portfolio and the dynamics of the real exchange rate. Did the observed debt market structure have

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3For a review see Eichengreen and Hausmann (2005). The introduction of the euro led to the increased share of euro-denominated internationally traded debt assets. However, the rise of the euro can be partially accounted for by increased financial flows within the Eurozone, while trade in euro assets with non-euro economies may be limited.

4See Goldberg and Tille (2009) for the analysis of the U.S. dollar’s role in international trade.
any implications for the international adjustment mechanism and what were the wel-
fare implications? To address these questions, we build a model that allows us to
match the size and the composition of the U.S. NFA position. The model must fea-
ture incomplete financial markets, multiple internationally traded assets and many
goods. Incomplete markets are necessary because with complete markets and time-
separable preferences portfolios are typically constant and, therefore, capital flows
are absent. Multiple (at least three) assets are needed to distinguish debt and eq-
uity positions and to model gross equity positions. Multiple goods are necessary to
allow endogenous external adjustment via the real exchange rate as emphasized by
Lane and Milesi-Ferretti (2001) and Obstfeld (2004). The model and the solution
method must also apply to asymmetric economies. In a symmetric world, any im-
balances are necessarily transitory unless there are multiple equilibria that naturally
pose problems for the quantitative analysis. In order to generate a realistic external
portfolio, we introduce two asymmetries into the model.

First, we model the privileged position of the U.S. by assuming that bonds traded
in the world financial markets are denominated in the units of its domestic good.
With this asymmetry our model predicts that the privileged country accumulates
a significant debt while maintaining a sizable net position in equity. In the model,
the domestic bond is a good hedge against fluctuation in consumption. But when
domestic output increases, the domestic price level declines and consumption becomes
more affordable. Yet, the domestic bond does not allow purchasing more consumption
as its payoff declines. Hence, the domestic bond is sold. While this result relies on a
low, yet reasonable, elasticity of substitution between goods, our numeric simulations
show that the effect is very strong, allowing us to match the U.S. net position in debt
equal to -38.0% of GDP.

\footnote{For a proof of this statement see Judd et al. (2000).}
Second, we assume that individuals in the U.S. are less risk-averse than elsewhere.\footnote{An indirect evidence of an elevated risk appetite in the U.S. is provided in Rydqvist et al. (2009): 38.5\% of U.S. individuals owned (directly) stocks in 2006. The second highest participation rate of 28.5\% is in Canada; the participation in Japan, which is the largest holder of the U.S. debt, is only 18.1. Large European economies have even lower participation rates: 14.1 in the U.K., 12.5 in Germany and 6.9 in France. Moreover, the U.S. has maintained the lead in stock market participation since 1945.} This is a reduced form way of modeling the fact, documented in Mendoza et al. (2009), that the U.S. has more developed financial markets than the rest of the world. With more developed financial markets, consumers are better able to insure away idiosyncratic risks and, hence, should be more inclined to invest in risky assets. In turn, Weil (1993) shows that increasing the risk-aversion coefficient or increasing the variance of individual income have the same effect on the equilibrium consumption function.

The small difference in risk aversion that we assume enables us to obtain a realistic portfolio of foreign assets. This asymmetry has little quantitative effect by itself, but it reinforces the bond-market advantage. Namely, a country with lower risk aversion is willing to hold a larger than the rest of the world fraction of wealth in equity. But the insurance service it provides is valued little given the size of observed macroeconomic risks. So, the increased investment in equity is nearly entirely financed by borrowing in the debt market. The effect on the overall NFA position is small. Adding the bond market advantage, above providing independent motives to borrow, lowers the cost of the existing debt and prompts the less-risk-averse country to increase investment in foreign equity. Similarly, a country with the bond market advantage is willing to borrow but this increases its exposure to fluctuations in domestic income. So, it will not invest in equity unless we increase its risk-tolerance.

The main contribution of this paper is to show that the U.S. external balance sheet can be matched using a relatively simple model. This model also matches two important facts in international finance: the home equity bias and the consumption-
real exchange rate disconnect. Equity home bias presents a direct restriction on the composition of the NFA that we target. The correlation between the real exchange rate and relative consumption (hence relative pricing kernels) determines which assets a country chooses to hold and which it decides to sell and the size and the stochastic properties of capital flows. Restricted by the above empirical facts, the model explains nearly 50% of the observed exchange rate related valuation effects emphasized by Lane and Milesi-Ferretti (2007b).

On the methodological side, we explain how to solve international portfolio choice models by explicitly modeling the financial wealth distribution. We compute a global solution to a two-country two-good general equilibrium incomplete markets model by adapting the projection method developed in Judd et al. (2000) and Kubler and Schmedders (2003). We also show that there exists a wealth-recursive competitive equilibrium, providing a theoretical foundation for the solution methodology. The existence proof highlights the role of portfolio constraints (and debt limits in particular) that are necessary conditions for equilibrium existence but have, so far, been ignored in the previous body of work on international portfolio choice.

The rest of the paper is organized as follows. Section 2 gives a brief literature overview. Section 3 presents the model and the solution concept. Section 4 presents the numerical results and the transaction tax experiment. We conclude by stating the issues that remain to be solved. Details of the computational algorithm and proofs are relegated to the appendix.

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7Our model requires additional elements to be consistent with the well-known asset pricing facts. So, we concentrate on the exchange rate channel, largely ignoring the valuation effects stemming from fluctuations in asset prices.

8Supplemental appendix is available online.
2. Related literature

We omit the vast literature on current account determination under complete markets that dates back to Razin and Helpman (1978). Among the research on incomplete markets, several papers are closely related to our work as they analyze the size and/or the composition of the U.S. NFA position. Mendoza et al. (2009) consider a model with heterogeneous agents, no aggregate risk and one good. Debt in their model is state contingent, which is an equity-like instrument. As a result, it is not clear how to separate debt from equity positions. More importantly, the debt instrument is contingent on an idiosyncratic employment shock and has no data counterpart. Caballero et al. (2008) show that differences in the countries’ ability to supply financial assets may lead to substantial global imbalances. Our debt market advantage concept offers a new explanation while being complementary to the above two. Our model features aggregate uncertainty, exchange rate risk and a more realistic portfolio choice setting. So, it is more suitable for quantitative analysis. Gourinchas et al. (2010) also build a quantitative model addressing the privileged position of the U.S. They link the U.S. advantage to a lower risk aversion and the size of the U.S. economy as in Hassan (2012). But, among other shortcomings, their model counter-factually predicts a zero NFA position for the U.S.; see table 5.

Coeurdacier et al. (2010), Heathcote and Perri (2013) and Berriel and Bhattarai (2013) provide competing explanations for the home equity bias. The first relies on the correlation between dividends and wages, the second emphasizes trade openness, and the third government spending shocks. All models are symmetric and, hence, have no predictions for the size and composition asymmetry of the observed NFA positions. Also none of the models analyzes valuation effects.

Devereux and Sutherland (2010) study valuation effects in a symmetric model. In their numerical example, countries buy domestic bonds and their mean NFA is always zero. The example also relies on preference shocks without which trade in equities col-
lapses. They report significant exchange rate driven valuation effects which, however, are a consequence of counterfactual predictions about the volatility of real exchange rates. In contrast, we analyze valuation effects in a calibrated model that closely matches the U.S. portfolio and the empirical properties of the U.S. real exchange rate.

All of the above models rely on the approximation technique that is criticized by Rabitsch et al. (2014). Pavlova and Rigobon (2010) study a continuous time model with a closed form solution. They do not present results from a calibrated model, but rather study simple examples. The model can generate substantial and volatile valuation effects, but only by relying on demand shocks with volatility ranging between 25 and 145% of output volatility. The latter implies an unreasonably volatile real exchange rate that is uncorrelated with the real variables. The direct implication is that the valuation effects are volatile and unpredictable. Our model has only output shocks and it predicts a stochastic process for the real exchange rate that is close to the data. So, it is more meaningful for the quantitative analysis of international financial adjustment.

Our paper also provides a new solution method for international portfolio models with incomplete financial markets. Below we briefly go over the inventory of existing solution methods and state the advantages of our method. For a more detailed discussion and formal numerical tests see Rabitsch et al. (2014).

Kollmann (2006) exploits the idea that an equilibrium allocation should be close to the efficient one. The author describes a portfolio that supports approximately the efficient allocation given the associated price system. However, such a portfolio may not be optimal even in the class of considered portfolios, e.g. constant portfolios, as they are generally not welfare maximizing choices. The idea in Devereux and Sutherland (2009) and Tille and van Wincoop (2010) is to derive constant portfolios using the second-order approximation to the model with one asset and a guess for the NFA
position. One can then use the constant portfolios to derive a third-order approxima-
tion to the “macro” portion of the model. The latter can, in turn, be used to derive a first-order approximation to the portfolio decisions. However, the solution depends on the NFA value assumed at the beginning. So, with an asymmetric model like ours, one needs to solve a model iteratively, continuously refining the approximation point. The authors provide no arguments as to why the iterative procedure should be convergent. Indeed, Rabitsch et al. (2014) show that this algorithm is unstable, and it can lead to nonsensical solutions. Pavlova and Rigobon (2007) develop a continuous time model that can be solved analytically. However, it is limited to logarithmic preferences and the Cobb-Douglas good aggregator. Evans and Hnatkovska (2012) use continuous time logarithmic utility approximation to the portfolio choice part of the problem within a discrete time framework.

Our method has several advantages over the existing work. First, it allows for more realistic financial market structures. Second, adding new assets in our model environment does not increase the dimension of the state space. Adding capital to it would make the model more difficult to compute because of the additional state variable(s). But, in our experience, policies are close to linear in capital and low cost approximation techniques could be used for the new states. The methodological advances, e.g. see Judd et al. (2011) and Maliar and Maliar (2014), also make the use of global solution methods for highly dimensional problems increasingly feasible. Third, a wide range of borrowing limits and portfolio restrictions can be analyzed. Without explicit restrictions on portfolios any incomplete markets model is ill-defined as it is not even clear if a competitive equilibrium can exist. Lastly, our solution method offers higher accuracy as we solve for a global, and not a local, solution. All in all, we believe it is a valuable addition to the existing stock of solution methods.
3. The Model

3.1. Environment

Time is discrete and indexed by \( t = 0, 1, 2, \ldots \). The exogenous state of the economy \( z_t \) is a first-order Markov process with finitely many states, \( Z = \{ \bar{z}_1, \ldots, \bar{z}_S \} \), and a probability transition matrix \( \Pi \). The initial state \( z_0 \) is given. A partial history of the state realizations \((z_0, \ldots, z_t)\) is denoted by \( z^t \) and its probability by \( \pi(z^t|z_0) \).

There are two countries, each populated by a representative household. Two perishable goods are traded every period. Good \( i \) produced in country \( i \) is traded at price \( p_i(z^t) \).

Financial markets trade three financial assets: home and foreign equity and a bond. A claim to home equity pays \( d_1(z_t) > 0 \) units of good 1 when the current state is \( z_t \) and a claim to foreign equity pays \( d_2(z_t) > 0 \) units of good 2. The supply of each stock is fixed and normalized to 1. We assume further that there is no short-selling. The two equity claims are traded at the ex-dividend prices \( q_1(z^t) \) and \( q_2(z^t) \). Bond’s payoff consists of \( \alpha \) units of good 1 and \( 1 - \alpha \) units of good 2. The market value of the payoff is:

\[
p_b(z^t) \equiv \alpha p_1(z^t) + (1 - \alpha)p_2(z^t). \tag{1}
\]

Negative positions in the bond are allowed subject to a borrowing limit described below. The bond is in net zero supply and it is traded at \( q_b(z^t) \).

We interpret the case with \( \alpha = 1 \) as the situation in which the world bond market only trades the bonds paying in goods of country 1. The case with \( \alpha = 0.5 \) corresponds to the symmetric world bond market. Alternatively, we could have assumed that there is a menu of bonds that are subject to different trading costs. Our model situation is a special case of such a setting: we assume that all bonds but one have a prohibitively high trading cost.
The initial allocation of financial assets \((\theta^i_1, 0, \theta^i_2, b^i_0)\) is given.

The household in country \(i\) trades in financial and goods markets to maximize the expected life-time utility given by:

\[
U(c) = E \left[ \sum_{t=0}^{\infty} \beta^t u(g^i(c^i_1(z^t), c^i_2(z^t))) \bigg| z_0 \right], \quad \beta \in [0, 1), u' > 0, u'' < 0. \tag{2}
\]

Function \(g^i\) is a constant return to scale consumption aggregator. We assume that the households’ preferences display consumption home bias. Hence, the consumption aggregate in country \(i\) is biased towards the domestically produced good \(i\). In the case of the CES aggregator, this assumption is isomorphic to assuming trade costs/taxes if the latter are rebated back to households.

The household in country \(i\) receives non-financial income \(w_i(z_t)\) units of domestic good \(i\) and \(\epsilon\) units of foreign good \(-i\). This income represents wages and profits of privately held companies. In our quantitative analysis we assign a negligible value to \(\epsilon\). So, for the clarity of the exposition we formulate the model as if households received no endowment of foreign goods.

The budget constraint of the household living in country \(i\) after history \(z^t\) is:

\[
p_1(z^t)c^i_1(z^t) + p_2(z^t)c^i_2(z^t)
+ q_1(z^t)\theta^i_1(z^t) + q_2(z^t)\theta^i_2(z^t) + q_b(z^t)b^i(z^t)
= I^i(z^t), \tag{3}
\]

where \((\theta^i_1, \theta^i_2, b^i)\) \(\in R^2_+ \times [-B(z^t), \infty)\) is the portfolio of the consumer living in country \(i\) and consists of his positions in the two equity claims and the bond. \(I^i(z^t)\) is “cash-in-hand” that consists of the market value of his non-financial income \(w_i\) and the income that he receives from his financial portfolio (including dividends):

\[
I^i(z^t) \equiv p_i(z^t)w_i(z^t) + \sum_{j=1}^{2} (q_j(z^t) + p_j(z^t)d_j(z^t))\theta^i_j(z^{t-1}) + p_b(z^t)b^i(z^{t-1}). \tag{4}
\]
In the infinite horizon model if no explicit borrowing limits are imposed the problem of the agent may not be well-defined. We impose the borrowing limit that is close in spirit to Levine and Zame (1996), who require that it should be possible for households to repay their obligations over a finite period of time. We require that the households are able to do so in one period:

\[
B^i(z^t) \equiv \min_{z_{t+1} \in \mathbb{Z}} \left[ \frac{p_i(z_{t+1})w_i(z_{t+1}) + \sum_{j=1}^2 \tilde{q}_j(z_{t+1})\theta_j^i(z^t)}{p_b(z_{t+1})} \right], \quad \forall z^t, (5)
\]

where \(\tilde{q}_j(z^t) \equiv q_j(z^t) + p_j(z^t)d_j(z_t)\) is the cum-dividend price of stock \(j\). This constraint is sufficiently generous since households can borrow against their portfolio of stocks.

3.2. Wealth-recursive equilibrium

A competitive equilibrium is a price system \(\mathcal{P} = \{p_1(z^t), p_2(z^t), q_1(z^t), q_2(z^t), q_b(z^t) : \forall z^t\}\), an allocation \(\mathcal{C} = \{(c^1_i(z^t), c^2_i(z^t))^2_{i=1} : \forall z^t\}\) and asset positions \(\mathcal{A} = \{(\theta_1^i(z^t), \theta_2^i(z^t), b^i(z^t))^2_{i=1} : \forall z^t\}\) such that:

1. given the price system \(\mathcal{P}\), the allocation and asset positions solve each household’s optimization problem;
2. financial and goods markets clear: \(\forall z^t, j = 1, 2,\)

\[
c^1_j(z^t) + c^2_j(z^t) = w_j(z^t) + d_j(z^t) \quad (6a)
\]

\[
\theta_j^1(z^t) + \theta_j^2(z^t) = 1 \quad (6b)
\]

\[
b^1(z^t) + b^2(z^t) = 0. \quad (6c)
\]

\(^9\)Because equity short-selling is not allowed, this borrowing limit implies that debt can be at least as high as the lowest output realization. When wealth is distributed evenly, the borrowing capacity of a country is approximately \(p_j(z^t)e_j(z^t) + q_j(z^t)\), that is, about 2.6 times that country’s GDP, \(p_j(z^t)e_j(z^t)\), in our calibration. In equilibrium this borrowing limit is never binding. But when a country’s wealth share decreases, it reduces its portfolio of equities and the borrowing capacity shrinks to one GDP. So, the borrowing limit changes endogenously and pro-cyclically with economic conditions. Quantitatively, however, this fact has limited implications.
We normalize the price system so that:

\[ p_1(z^t) + p_2(z^t) = 1, \quad \forall z^t. \]

In general, it is not feasible to compute the competitive equilibrium as defined above. The usual reason is the curse of dimensionality: a natural state vector for this model includes the portfolio holdings of each household. To sidestep this problem, we restrict our attention to wealth-recursive equilibria. Duffie et al. (1994) call recursive equilibria dynamically simple. Such equilibria may not exist if the state space is not sufficiently rich. Duffie et al. (1994) show that the equilibrium will exist if the state space includes all the equilibrium variables. A wealth-recursive Markov equilibrium is a competitive equilibrium in which the distribution of wealth is a sufficient statistic for all the equilibrium variables.

Let the financial wealth share of country 1 be denoted by \( \omega \):

\[ \omega(z^t) \equiv I_1(z^t)/[I_1(z^t) + I_2(z^t)] \in [0, 1], \quad \forall z^t. \quad (7) \]

The wealth share always lies in the unit interval under the borrowing limit (5). Importantly, the total wealth depends only on the prices and the exogenous labor income and dividend processes:

\[ I_1(z^t) + I_2(z^t) = \sum_{j=1}^{2} [p_j(z^t)(w_j(z_t) + d_j(z_t)) + q_j(z^t)]. \quad (8) \]

Notice also that the portfolio in the definition of \( \omega(z^t) \) was chosen after history \( z^{t-1} \). Then (7) implicitly defines a law of motion for the endogenous state \( \omega \):

\[ \omega(z^{t+1}) = \Omega(\omega(z^t), z^t, z_{t+1}), \quad \forall z^t, z_{t+1}. \quad (9) \]
Let \(x = ((c^i_1, c^i_2)^2_{i=1}, (\theta^i_1, \theta^i_2, b^i)^2_{i=1}, (\mu^i_1, \mu^i_2, \mu^i_b)^2_{i=1}, (p_1, p_2, q_1, q_2, q_b))\) be a vector of all endogenous variables at any node of the history tree, where the Lagrange multipliers \(\mu^i_1, \mu^i_2, \mu^i_b\) correspond to short-selling constraints on stock 1, stock 2 and a borrowing limit on the debt. Set \(X = \mathbb{R}^4_+ \times \mathbb{R}^6_+ \times \mathbb{R}^6_+ \times \mathbb{R}^5_+\) contains all possible values of \(x\). Define \(\rho : [0, 1] \times Z \to X\) to be a policy correspondence that maps the current state, i.e. (wealth share, exogenous Markov state) pair, into the set of all equilibrium variables.

A wealth-recursive Markov equilibrium is an equilibrium correspondence \(\rho : [0, 1] \times Z \Rightarrow X\) and a financial wealth transition map \(\Omega : [0, 1] \times Z \to [0, 1]^{|Z|}\) such that \(\forall (\omega, z) \in [0, 1] \times Z\):

\[
0 = E\left[\Phi\left(\rho(\omega, z), \omega, z, (\rho(\omega(z'), z'), \omega(z'), z')_{z' \in Z}\right)\right],
\]

\(\omega(z') = \Omega(\omega, z, z'),\)

where \(\Phi\) is the system of equilibrium conditions. This system is written out explicitly in Appendix B.1. Theorem 1 below shows that a wealth-recursive Markov equilibrium does exist. Assumptions (b) and (c) jointly guarantee that the autarkic allocation provides at least some utility. Together with assumption (a) it implies that equilibrium consumption is bounded away from zero.

**Theorem 1.** Suppose the following conditions hold:

a) the utility is well-behaved and unbounded below: \(u^i(0) = -\infty, i \in \{1, 2\}\);

b) both goods are essential: \(g^i(0, x) = g^i(x, 0) = 0, \forall x \in \mathbb{R}_+, i \in \{1, 2\}\);

c) endowments are bounded below: \(\exists w_m > 0 : w_1(z) > w_m, \forall z \in Z, i \in \{1, 2\}\);

d) households face short-selling constraints and borrowing limits (5).

Then there exists a wealth-recursive Markov equilibrium.

Any numerical algorithm allows computing only approximate, not exact, equilibria in which optimality conditions are met only approximately.
(2004) show that approximate equilibria may exist even if the exact equilibrium does not. Theorem 1 assures that we are not computing an approximate solution for the model that has no exact equilibrium.

Unfortunately, the policy correspondence $\rho$ is not guaranteed to be single-valued. Neither can it be established that the Markov equilibrium has an invariant ergodic measure. Yet, we know of no work that establishes when a GE model with incomplete markets and aggregate uncertainty possesses these properties.

3.3. Functional forms

For the rest of our analysis we specialize our setting. We assume a CRRA utility function and a CES aggregator:

$$u^i(c, z) = \delta(z)c^{1-\gamma^i}/(1 - \gamma^i), \quad \gamma^i > 0,$$

$$g^1(c_1, c_2) = (sc_1^\phi + (1 - s)c_2^\phi)^{1/\phi}, \quad s \in [0.5, 1], \phi \leq 1 \quad (11a)$$

$$g^2(c_1, c_2) = ((1 - s)c_1^\phi + sc_2^\phi)^{1/\phi}. \quad (11b)$$

The elasticity of substitution (ES) between the two goods is $\varepsilon \equiv \frac{1}{1 - \phi}$. The above consumption aggregators imply the following aggregate price indices:

$$P^1 = (s^\varepsilon p_1^{1-\varepsilon} + (1 - s)^\varepsilon p_2^{1-\varepsilon})^{1/(1-\varepsilon)}, \quad (12a)$$

$$P^2 = ((1 - s)^\varepsilon p_1^{1-\varepsilon} + s^\varepsilon p_2^{1-\varepsilon})^{1/(1-\varepsilon)}. \quad (12b)$$

Let $q \equiv p_1/p_2$ denote the terms of trade and let $Q \equiv P^1/P^2$ denote the real exchange rate. Finally, denote the fraction of income spent on domestic goods (in a symmetric deterministic model) by $\chi$:

$$\chi = \frac{s^\varepsilon}{s^\varepsilon + (1 - s)^\varepsilon}. \quad (13)$$
4. Numerical results

We solve the model numerically using the projection method. Namely, for each $z \in Z$ we approximate the policy functions $\rho(\omega, z)$ by cubic splines on $[0, 1]$. We use a uniform grid for $\omega$. The update for the equilibrium (price, policy and wealth transition) functions is obtained by solving the system (10). We iterate on the equilibrium functions until the change in the price system is less than $10^{-4}$, so that the change in the price system between two consecutive iterations is less than 1 basis point. We also test the accuracy of our solution method on a model in which the only source of income is dividends. In this case, the optimal solution is a “linear sharing rule”. That is, trade in the two stocks is enough to achieve full consumption insurance and the markets are effectively complete, see Baxter and Jermann (1997) and Heathcote and Perri (2013). In this case our solution method performs extremely well with errors in the equilibrium conditions being close to machine precision.

Multiplicity is a plague for numerical analysis. We know of no work that would provide conditions under which a competitive equilibrium is unique. Kubler and Schmedders (2002) suggest that multiplicity of equilibria in models with incomplete financial markets may be related to the multiplicity of efficient allocations. It is verified in Appendix E.1 that the efficient allocation is indeed unique.

We now explain our calibration. We use annual data from 1984 until 2007 for the U.S. and for the OECD economies to construct nine moments described below, with details provided in Appendix F. The common discount factor $\beta$ is chosen so that the average return on the bond is 4%, a common benchmark value. This moment is denoted by M1 in table 4. We choose $(\gamma^1, \gamma^2) = (0.773, 1.680)$, values that are close to the commonly-used logarithmic preferences, to match the U.S. net equity and debt positions: the NFA in equity is $+20.8\%$ of output (moment M2), and the

\[10\text{Off-grid errors in the equilibrium conditions are plotted in Appendix E.}\]
NFA in debt is -38.0% (moment M3). We set \((\phi, s) = (-0.385, 0.916)\) to match the volatility of the real exchange rate (moment M4) and the trade/GDP ratio (moment M5). The corresponding elasticity of substitution (ES) between goods is 0.722. Such a low ES would not be needed if the model featured non-tradeable goods or if we assumed a larger difference in risk-aversion. Parameters \(\sigma_e\) and \(\rho_e\) are set to match the standard deviation and the autocorrelation of log-output in the U.S. data (moments M6 and M7). The stochastic process for endowments is assumed to be a 9-state first-order Markov process, with the two endowment processes being independent. The stochastic processes for wages and dividends are constructed in the following way:

\[
\begin{align*}
    w_i &= \bar{w} + (1 - s_d)(e_i - E(e_i)), \\
    d_i &= \bar{d} + s_d(e_i - E(e_i)).
\end{align*}
\]

Parameter \(\bar{d}\) is chosen so that the domestic stock market value to GDP ratio is 1.611 as in the U.S. during 1988-2007 (moment M8). This implies \(\bar{d} = 0.0645\) and \(\bar{w} = E(e^i) - \bar{d} = 0.9355\). It can be shown that \(\sigma(\ln(d^i))/\sigma(\ln(w^i)) \approx (\bar{w}/\bar{d})s_d/(1-s_d)\). In the data this ratio equals 5.356 (moment M9) and it implies \(s_d = 0.270\).

The persistence of output in the U.S. is statistically indistinguishable from the sample weighted average of the OECD economies (see table F.5). So, we assume that the persistence of output in the two model countries is the same as observed in the U.S. The correlation between the domestic and the foreign output is set to zero as the weighted average correlation in the data is only 0.137 and is deemed insignificant.

We consider several specifications. S0 is the benchmark model calibrated to the data: country 1 has an advantage in the bond market as debt is denominated in good 1 (the debt market advantage) and households in country 1 are less risk-averse: \(\alpha = 1, \gamma_1 = 0.773 < \gamma_2 = 1.680\). S1 allows for risk-aversion differences but imposes that the bond market is symmetric: \(\alpha = 0.5\). S1\(a\) allows country 1 to enjoy the
Value | Moment/Source
--- | ---
Discount factor | $\beta$ | 0.9615 | Return on bond = 4% (M1)
Risk-aversion of country 1 | $\gamma^1$ | 0.7730 | NFA$^{US}_{debt} = -0.380$, NFA$^{US}_{equity} = +0.208$ (M2,M3)
Risk-aversion of country 2 | $\gamma^2$ | 1.6800 | Same as for $\gamma^1$
ES between goods | $\phi$ | -0.3850 | Volatility of the RER = 3.5% (M4)
Utility weight of dom. good | $s$ | 0.9158 | Trade/GDP = 0.5(X+M)/(C+NX) = 15.5% (M5)
Volatility of income | $\sigma^1_s$ | 0.0151 | Volatility of log-income in the U.S. (M6)
Persistence of income | $\rho^1_s$ | 0.7520 | Persistence of log-income in the U.S. (M7)
Dividends/income | $d$ | 0.0645 | Stock market value/GDP ratio = 1.611 (M8)
$\sigma$(dividends)/$\sigma$(wages) | $s_d$ | 0.2697 | $\sigma(ln(d^1))/\sigma(ln(w^1)) = 5.356$ (M9)

Table 1: Benchmark parameter values

bond market advantage, but it imposes that households in the two countries are equally risk-averse: $\gamma^1 = \gamma^2 = 1.222$, which is the average of the two values used in S0. S2 is the fully symmetric environment with $\alpha = 0.5$ and $\gamma^1 = \gamma^2 = 1.222$. We adjust the elasticity of substitution between goods $\phi$ and the weight on the domestic good $s$ to match moments M4 and M5 in all the specifications. Thus we use $(\phi, s) = (-0.385, 0.916)$ in the specifications S0, $(-0.285, 0.900)$ in S1$^\alpha$, $(-0.176, 0.891)$ in S1$^\gamma$, and $(-0.215, 0.887)$ in S2. We simulate 500 series of length 200,000. Each simulation starts from $\omega_0 = 0.5$. The first half of each sample is deleted. Table 4 summarizes our simulation results. Sections 4.1-4.5 describe our results.

4.1. The country that can issue domestic bonds accumulates a negative bond position and invests borrowed funds in foreign equity.

First, we provide an intuitive explanation of why a country chooses to hold a negative position in the domestic bond. To obtain analytic results, we restrict financial trade to the bond paying a unit of good 1 and assume that consumers in the two countries are equally risk-averse. We then follow the argument in Svensson (1988) and ask: “In a financial autarky, which country would be willing to pay more for the bond that pays one unit of the good produced in country 1?” We start with a stochastic discount factor of a consumer in country 1: $M^1_{t+1} = \beta(C^1_{t+1}/C^1_t)^{-\gamma}P^1_t/P^1_{t+1}$, where
<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>S0</th>
<th>S1α</th>
<th>S1γ</th>
<th>S2</th>
<th>S1γ′</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>International investment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Net equity / output, M2</td>
<td>0.208</td>
<td>0.208</td>
<td>-0.001</td>
<td>0.131</td>
<td>0.000</td>
<td>0.072</td>
</tr>
<tr>
<td>2 Net debt / output, M3</td>
<td>-0.380</td>
<td>-0.380</td>
<td>-0.136</td>
<td>-0.185</td>
<td>0.000</td>
<td>-0.089</td>
</tr>
<tr>
<td>3 Net FA / output</td>
<td>-0.172</td>
<td>-0.172</td>
<td>-0.137</td>
<td>-0.054</td>
<td>0.000</td>
<td>-0.017</td>
</tr>
<tr>
<td>4 Home equity bias</td>
<td>0.934</td>
<td>0.890</td>
<td>0.974</td>
<td>0.926</td>
<td>0.981</td>
<td>-0.929</td>
</tr>
<tr>
<td>5 E(ω)</td>
<td></td>
<td>-0.453</td>
<td>0.465</td>
<td>0.488</td>
<td>0.500</td>
<td>0.505</td>
</tr>
<tr>
<td><strong>Real exchange rate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 std(RER), M4</td>
<td>0.035</td>
<td>0.035</td>
<td>0.035</td>
<td>0.035</td>
<td>0.035</td>
<td>0.086</td>
</tr>
<tr>
<td>7 cor(RER,C/C*)</td>
<td>0.265</td>
<td>0.212</td>
<td>0.340</td>
<td>0.442</td>
<td>0.430</td>
<td>0.865</td>
</tr>
<tr>
<td>8 cor(RER,Y/Y*)</td>
<td>-0.246</td>
<td>-0.612</td>
<td>-0.578</td>
<td>-0.554</td>
<td>-0.559</td>
<td>-0.268</td>
</tr>
<tr>
<td>9 E(p1)</td>
<td>–</td>
<td>0.486</td>
<td>0.491</td>
<td>0.497</td>
<td>0.500</td>
<td>0.502</td>
</tr>
<tr>
<td><strong>Capital flows</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 std(CA/Y)</td>
<td>0.014</td>
<td>0.006</td>
<td>0.006</td>
<td>0.007</td>
<td>0.007</td>
<td>0.014</td>
</tr>
<tr>
<td>11 cor(CA/Y,Y)</td>
<td>-0.059</td>
<td>0.597</td>
<td>0.889</td>
<td>0.840</td>
<td>0.875</td>
<td>0.960</td>
</tr>
<tr>
<td>12 E(|\Delta CA^1|/</td>
<td>CA</td>
<td>)</td>
<td>0.386</td>
<td>0.182</td>
<td>0.052</td>
<td>0.127</td>
</tr>
<tr>
<td>13 cor(|\Delta CA^1|,CA)</td>
<td>-0.451</td>
<td>-0.513</td>
<td>-0.037</td>
<td>0.013</td>
<td>0.017</td>
<td>0.024</td>
</tr>
</tbody>
</table>

S0: debt market advantage and risk-aversion difference; S1γ: risk-aversion difference; S1α: debt market advantage; S2: symmetric economies; S1γ′: all parameters as in S0 except there is no debt advantage

Table 2: Moments in the data and in the model

\( C^1_t \) denotes aggregate consumption of country \( i \). The above expression implies that a consumer in country 1 would like to hold assets with a payoff that is a) negatively correlated with the domestic price inflation \( \pi^1_{t+1} \equiv P^1_{t+1}/P^1_t \) and b) negatively correlated with the domestic consumption growth \( C^1_{t+1}/C^1_t \). These are referred to as the CPI and the consumption hedging motives, respectively. The ES is important for the second motive as it directly impacts consumers’ willingness to change their consumption bundle when the relative price of goods changes. The risk aversion coefficient governs the relative importance of the two motives. Using log-linear approximations and the financial autarky allocation as an approximation point, we obtain proposition 1 that is proved in Appendix C.

**Proposition 1.** If financial markets trade only a bond denominated in good 1 and the borrowing constraint is not binding, then country 1 will hold a negative position in the bond when \( \gamma(2\chi \varepsilon - 1) < 2\chi - 1 \).

This condition brings to light two opposing forces. Consider the increase in coun-
try 1’s output such that the price of good 1 declines by 1%. On the one hand, country 1’s aggregate consumption increases by \((2\chi\varepsilon - 1)\%\). The increase in consumption is larger when the ES is high (hence the income effect is small), and when the consumption home bias is strong. At the same time the marginal utility of country 1 declines by \(\gamma(2\chi\varepsilon - 1)\%\). This prompts country 1 to purchase the bond, because its payoff declines when country 1’s valuation of extra consumption is low and vice versa. We call this a consumption hedging effect. On the other hand, the aggregate price level in country 1 declines by \((2\chi - 1)\%\) and the country demands more consumption because it is more affordable. The bond does not help satisfying this demand as its payoff declines. We call this a CPI effect. If the CPI effect dominates the consumption hedging effect, then country 1 should hold a negative position in the bond. Importantly, the lower the ES, the stronger is the desire to sell domestic bonds. A decrease in the ES dampens the response of the consumption relative to that of the price level.

We now turn to the simulation results in table 4. With the asymmetric bond market, country 1 accumulates debt and invests borrowed funds in the foreign equity (refer to column S0). Wealthier country 2 drives the price of good 2 up, \(E(p_2) = 0.514\) (recall that \(p_1 + p_2 = 1\)). Country 1, enjoying the bond market advantage, would like to sell domestic debt and purchase more of the relatively cheap domestic equity. The latter action is not possible due to the short-selling constraint. This tension is removed if the price of good 1 decreases and country 1’s equity becomes less attractive.

When the asymmetry in the bond market (column S1\(\gamma\)) is turned off, the debt NFA position of country 1 is still a sizeable -18.5% of GDP. But the overall NFA position is only -5.4%. This result is driven by the more risk-averse household in country 2 that wishes to accumulate relatively safe debt and sell relatively risky equity. If the

\[1\]

[1] In a special case with logarithmic preferences, equal utility weights on goods, and the ES between goods equal to one both the CPI and the consumption hedging motives are absent. This is a well-known result obtained in Cole and Obstfeld (1991) where trade in goods is sufficient to achieve perfect risk-sharing. For the risk aversion coefficient \(\gamma = 1.222\) country 1 sells the “domestic” bond when the ES is less than 0.92.
difference in risk aversion is turned off (column S1\(\alpha\)), the position in debt is -13.6% and the overall NFA position is -13.7%. In this specification, similarly to S0, the household in country 1 sells bonds and invests in equity as this helps engineering the income stream that better matches the desired consumption spending. Yet, without the difference in risk-aversion, the net position in equity is negligible and negative, unlike in the data. The reason is that the price of good 1 is low; therefore, the equity in country 2 is more expensive and a larger fraction of the domestic equity must be sold to afford the foreign equity. In a fully symmetric setting (column S2), the net foreign positions are zero since both countries on average must hold the same portfolio of assets.

We also point out the interaction between the bond market asymmetry and the risk-aversion differences. Because the good prices are perfectly negatively correlated, proposition 1 implies that increasing risk-aversion of country 2 should lead to it buying more of the bond issued by country 1. At the same time, because country 2 is more risk-averse, it has a stronger precautionary demand for the bond. But the larger position in the bond forces country 2 to hedge it with a larger position in country 1’s equity. This additional exposure amplifies the precautionary demand.

4.2. Domestic stocks dominate the countries’ portfolios

Irrespectively of the specification, the share of domestic equity in the equity portfolio is high and ranges from 0.890 under S0 to 0.981 under S2. To gain insight into why home equity bias arises, consider the symmetric model with the common risk-aversion parameter \(\gamma\). We follow the approach described in Kollmann (2006) and compute the constant portfolio that allows consumers in both countries to approximately finance the efficient consumption allocation.\(^{12}\) Denote the optimal portfolio of equities of country 1 by \((\theta^*, 1 - \theta^*)\). By symmetry, the share of home equity is:

\(^{12}\)This analysis uses the price system that would prevail under complete financial markets. The optimal portfolio solves the approximate (first-order of accuracy) budget constraint.
$2\theta^* - 1$. Proposition 2 in Appendix C shows that $\theta^*$ must solve:

$$-(2\chi - 1)(1 - 1/\gamma)\hat{c}/\lambda = \frac{(2\theta^* - 1)}{\text{relative consumption spending}} \cdot \bar{d}(\hat{d} - \hat{c}/\lambda) + \bar{w}(\hat{w} - \hat{c}/\lambda), \quad (15)$$

where $\hat{d} = (s_{d}/\bar{d})\hat{e}$, $\hat{w} = ((1 - s_{d})/\bar{w})\hat{e}$ and $\lambda = \phi - (2\chi - 1)^2(\phi - 1/\gamma)$ is the inverse of the elasticity of the terms of trade $p_1/p_2$ with respect to the relative output $e_1/e_2$.

This implies that changes in a country’s relative dividend and labor income must match fluctuations in the desired relative consumption spending.

First, when labor and dividend income are fixed proportions of output, as would be the case with endogenous production and the Cobb-Douglas technology, the optimal portfolio would likely exhibit foreign equity bias. This is so because the non-tradeable labor income makes a country’s total income too sensitive to domestic output and this prompts the sale of domestic equity.

To understand home equity bias in the general case, consider a reaction to an increase in country 1’s output. Because dividends are positively correlated with the output and are more volatile, the relative dividend paid by country 1’s equity increases. The relative labor income paid in country 1, on the other hand, decreases as does the relative desired consumption spending. But because labor income constitutes a large fraction of the produced output, the desired consumption spending net of labor income increases. In other words, following the shock the desired consumption spending declines, but labor income declines more. This prompts the economies to hold a large share of domestic equity in their portfolios.

Finally, as $\gamma$ increases, relative consumption spending becomes more sensitive to changes in the relative income: $(2\chi - 1)(1 - 1/\gamma)/\lambda$ is increasing in $\gamma$. So, relative

$\text{relative consumption spending} = (2\theta^* - 1) \cdot \bar{d}(\hat{d} - \hat{c}/\lambda) + \bar{w}(\hat{w} - \hat{c}/\lambda)$.

---

13 When $s_d = \bar{d}$ the equilibrium home equity bias is:

$$2\theta^* - 1 = [(2\chi - 1)(1 - 1/\gamma)/(1 - \lambda) - \bar{w}] / \bar{d}. \quad (16)$$
consumption spending net of labor income becomes negatively correlated with the relative output as \( \gamma \) increases. In this case, it is optimal for countries to invest a larger fraction of wealth in foreign equity. Indeed, when we increase the risk-aversion parameter to \( \gamma = 5 \) in the symmetric setting, the mean share of home equity decreases to 0.494 consistent with our predictions. So, assuming a high value of risk aversion to generate an equity premium in our model is not acceptable: this would negate the home bias.\(^1\)

4.3. *Correlation between the RER and relative consumption is low*

In a symmetric environment and under complete financial markets the relation between the real exchange rate and the relative consumption of the two countries is:

\[
\ln(Q_t) = -\gamma \ln(C^1_t/C^2_t). \tag{17}
\]

The above relation implies that there should be a perfect negative correlation between the logarithms of the two variables. However, we have incomplete markets, and the extent to which our results deviate from this benchmark depends on the degree of the market incompleteness. The short-selling constraint is more likely to affect the results. The borrowing limit, as we mentioned above, binds with a very low probability. To understand how short-selling constraints change the correlation between the real exchange rate and relative consumption, consider an increase in country 1’s output. Then observe that if the markets were complete, the wealth as we define it would be perfectly negatively correlated with output. But markets are incomplete and when output in country 1 increases, its wealth and consumption also increase. The real exchange rate may decrease because there is more of domestic goods, or increase because wealthier country 1 demands more. But there is also

\(^{14}\)Heathcote and Perri (2013) show that adding non-tradable goods or capital accumulation makes equity home bias a robust feature of this type of models.
a non-standard effect. Because country 1 wants to purchase more of the domestic
equity but cannot, the price of goods produced in country 1 must increase. This
pushes up the price of country 1’s equity, curbs demand, and equilibrates the market
for country 1’s equity. This is the strongest effect, and the overall result is that
there is a positive correlation between the exchange rate and relative consumption:
0.208 under S0. Consistent with our intuition, this correlation is higher under the
symmetric specification S2 because the home equity bias is more extreme and the
short-selling constraints are more active.

4.4. Fluctuations in the real exchange rate account for a large fraction of current
account adjustments

Suppose that the foreign good becomes more expensive and the value of foreign
equity increases. The relative wealth positions of the two countries change, even
though there is no explicit trade. Because the gross positions are large, fluctuations
in the real exchange rate add significantly to capital flows (the traditionally measured
current account, CA). Similar adjustments must be made to account for changes in
the market value of the long-term assets purchased in previous periods. The CA then
can be decomposed into three components (see Appendix A for details):

\[
CA_t = q_{2t}(\theta^2_{2t+1, t} - \theta^2_{2t}) - q_{1t}(\theta^2_{1t+1, t} - \theta^2_{1t}) + q_{bt}b_{1t+1} - q_{bt-1}b_{1t} \\
+ \text{exchange rate adjustment} + \text{equity price adjustment.}
\]

The two adjustment effects are commonly referred to as the valuation effects that
were first brought to everyone’s attention by [Lane and Milesi-Ferretti (2001)]. The
exchange rate adjustment is proportional to \(Q_t - Q_{t-1}\) while the price adjustment
corresponding to equity claim \(j\) is proportional to \(q_{jt} - q_{jt-1}\). In the U.S. exchange
rate fluctuations account for 38.6% of the total variation in the U.S. current account
(see row 11 in table[1]). Under the benchmark specification (S0) this number is 18.2%, while under the other specifications it does not exceed 12.7%.

In the symmetric setting S2 there is a strong home equity bias. The share of the foreign equity owned by domestic households is very small and this, in turn, limits the CA adjustment effect that stands at 4.1%. In contrast, in the asymmetric setting, especially S0 and S1', the home bias is not as extreme allowing for more substantial adjustments.

Under S0 the correlation between the adjustment and the actual flow is -0.513 (row 13), close to -0.451 in the data. This can be attributed to the asymmetric portfolios purchased by the countries. When country 1’s output decreases, its consumption also decreases and the CA improves. However, country 1’s investment in foreign equity decreases and so does the adjustment effect. As a result, the CA improves while the exchange rate adjustment decreases.

Notice that the exchange rate plays a stabilizing role. When there is capital outflow, namely a country accumulates net foreign wealth, the real exchange rate appreciates, and the value of foreign equity declines relative to domestic equity. Foreign equity becomes less attractive and this slows down or reverses the outflow. Finally, all of the above discussion also applies to current account adjustments stemming from changes in the equity prices.\[15\]

4.5. The distribution of country 1’s financial wealth share is skewed towards 0 and its mean is decreased significantly. But the debt market advantage has a negligible impact on country 1’s welfare

Figure[1] plots the stationary distribution of the financial income share. In the symmetric setting (S2) the countries’ financial wealth share is 0.5 on average. In the main specification with the bond advantage and the risk-aversion difference (S0), the

\[15\] If we combine price and exchange rate adjustments to the current account, then under S0 we obtain \( E(\frac{|\Delta CA^{adj}|}{|CA|}) = 0.333 \) while in the data this number is 0.674.
mean financial wealth share is 0.453, 17.2% less than that of country 2. If the risk-aversion asymmetry is turned off (S1\(^{\alpha}\)), the mean financial wealth share of country 1 is 0.465, still a sizeable 13.1% difference when compared with country 2. So, the bond advantage alone significantly distorts the stationary distribution of financial wealth. Wealth distribution is positively skewed in the three asymmetric setups. Our calculation also show that under S0, country 1’s wealth share does not decline below 0.37, while country 2’s wealth share could be as low as 0.16. Namely, while both countries can experience disastrous declines in their wealth share, the worst-case scenario for country 2 is twice as severe as it is for country 1.

![Figure 1: Stationary distribution of the financial income share.](image)

Note: Adding risk-aversion differences, compare S1\(^{\gamma}\) to S2, makes the distribution more dispersed. Adding debt market advantage, compare S1\(^{\alpha}\) to S2, skews the distribution to the right. Both frictions shift the distribution mean to the left.

Because financial markets are incomplete, the lower mean financial wealth share does not imply that the welfare of country 1 is affected adversely. Consider specifications S0 and the new specification S1\(^{\gamma'}\). The latter differs from S0 only because neither country has a bond market advantage. So, all the preference and production parameters remain fixed across the two specifications and this facilitates our welfare comparison. S1\(^{\gamma}\) also adjusts (\(\phi, s\)) to match moments M4 and M5 unlike S1\(^{\gamma'}\). We
report moments of the aggregate consumption and welfare for the two specifications in table\textsuperscript{3}. There are two effects. First, under S0, consumption in both countries is less volatile: 1.99\% vs 3.76\% for country 1 and 1.73\% vs 3.69\% for country 2. But, second, under S0 the average consumption is smaller in country 1, 0.685 vs 0.696, and higher in country 2, 0.706 vs 0.694. This means that country 2’s welfare decidedly improves and it should not object against country 1’s dominance in the world debt markets. Importantly, the debt market advantage improves stability in both economies as evidenced by consumption volatility. Smoother consumption, in turn, implies less volatile real exchange rate and capital flows. So, country 1, designed to resemble the U.S., is a provider of global stability.

Above we compared ergodic properties of consumption processes in both countries. We now turn to welfare comparison using the following \textit{ex ante} criterion:

\[
W_{Sx}^i(\omega_0) = \sum_{z_0 \in S} \bar{\pi}(z_0)V_{Sx}^i(\omega_0, z_0), \quad i = 1, 2,
\]

where $\bar{\pi}(z_0)$ is the ergodic distribution of the exogenous state and $V_{Sx}^i$ is the optimal life-time utility of agent $i$ under model $Sx$. We set $\omega_0 = 0.5$, that is, we start both economies with equal wealth. According to our criterion, welfare in each country is the same, save numerical errors, under S0 and S1*\gamma. We obtain the same results if we start the economies at other levels of $\omega_0 \in [0.45, 0.55]$. The reason is that it takes a long time for wealth distribution to deviate from its original position. Even after 100 periods, country 1’s expected wealth share is 0.498 and it commands a larger share of wealth with probability 0.366.

5. \textbf{Transaction cost}

Consider imposing a transaction cost on financial trades. Intuitively, this should stabilize financial flows, but is such a policy welfare improving? Which asset(s) should
be taxed? We address these questions by imposing a cost on asset purchases. We consider two scenarios: in the first, only bond transactions are taxed, while in the second only equity is subject to a trading cost. Jeanne and Korinek (2010) propose to tax capital flows to undo the financial amplification stemming from the declining prices of collateral assets. The authors, however, do not make a distinction between equity and debt flows. In our setting taxing equity flows is beneficial, because it limits exposure to risky assets and reduces volatility of consumption and capital flows. Taxing debt financial flows may destabilize the world economy.

We make the following two modeling choices. First, we impose a quadratic trading cost to avoid issues with discontinuous policies and, hence, equilibrium existence. Second, the tax proceeds are not wasted but are rather rebated back to consumers. The rebate of country $i$ is denoted by $T^i(z^t)$. Because consumers in the two countries pay the same cost, they also receive equal rebates. So, the budget constraint of country $i$ is:

$$I^i(z^t) + T^i(z^t) = p_1(z^t)c^i_1(z^t) + p_2(z^t)c^i_2(z^t) + \sum_{j=1}^{2}[q_j(z^t)\theta_j^t(z^t) + 0.5\kappa_c[\theta_j^t(z^t) - \bar{\theta}_j^t]^2] + q_b(z^t)b^i(z^t) + 0.5\kappa_b[b^i(z^t)]^2.$$

We use $\bar{\theta}_1 = (1, 0)$, so that any sale of equity claims abroad is taxed. However, we also obtain quantitatively similar results when instead we use the mean portfolio,

<table>
<thead>
<tr>
<th></th>
<th>$E(C^1)$</th>
<th>$\sigma(C^1)$</th>
<th>welfare$^1$</th>
<th>$E(C^2)$</th>
<th>$\sigma(C^2)$</th>
<th>welfare$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S0</td>
<td>0.685</td>
<td>0.020</td>
<td>105.106</td>
<td>0.706</td>
<td>0.017</td>
<td>-48.742</td>
</tr>
<tr>
<td>S1'</td>
<td>0.696</td>
<td>0.038</td>
<td>105.106</td>
<td>0.694</td>
<td>0.037</td>
<td>-48.742</td>
</tr>
</tbody>
</table>

S0: debt market advantage and risk-aversion difference;
S1': all parameters as in S0 except there is no debt advantage.

Table 3: The effect of bond market advantage: stochastic properties of aggregate cons. and welfare.

Note: Bond market advantage substantially decreases consumption volatility in both countries while it has a negligible impact on welfare.
Consider first imposing a cost on the bond holdings. With $\kappa_b > 0, \kappa_e = 0$, trivially, the magnitude of the countries’ bond positions should decrease. This is indeed true and the effect is extremely strong: with $\kappa_b = 1bp$ bonds are not traded anymore. With $\kappa_b = 0.5bp$ the bond position of country 1 is -0.250, down from -0.380 as reported in table 4. At the same time, country 1’s NFA position in equity decreases from 0.172 to 0.140, and the overall position improves from -0.172 to -0.110. Intuitively, since country 1 finds it more difficult to borrow, it must scale back its equity positions. Insurance possibilities of the two countries are inhibited as the volatility of consumption in country 1 and 2 increases by 10.7% and 16.0% respectively. Increased consumption volatility leads to more volatile prices. Volatility of the RER and the CA increases by 23.0% and 7.8% respectively. That is, taxing bond holdings makes portfolios more rigid, while making consumption and exchange rate more volatile. The mean consumption level of country 1 increases by 0.6%, while that of country 2 decreases by 0.6%. Despite the increased volatility, the importance of the exchange rate adjustment to the CA decreases, now contributing 0.153 of the actual capital flows.

<table>
<thead>
<tr>
<th>$(\kappa_b, \kappa_e)$</th>
<th>NFA$_{equity}$</th>
<th>NFA$_{debt}$</th>
<th>std$(C^1)$</th>
<th>std$(C^2)$</th>
<th>std(RER)</th>
<th>std$(\Delta(\text{CA}))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond tax $(5bp, 0)$</td>
<td>0.140</td>
<td>-0.250</td>
<td>0.022</td>
<td>0.020</td>
<td>0.043</td>
<td>0.007</td>
</tr>
<tr>
<td>Equity tax $(0, 5bp)$</td>
<td>0.138</td>
<td>-0.361</td>
<td>0.019</td>
<td>0.016</td>
<td>0.031</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Table 4: Selected macroeconomic indicators when financial trade is taxed.
Note: Bond tax, $\kappa_0 > 0$, has a larger impact on NFA composition. But equity tax, $\kappa_e > 0$, substantially decreases variance of all macroeconomic aggregates.

Next, consider imposing the same trading cost on equity claims: $\kappa_e = 5bp, \kappa_b = 0$. Similarly to the case with a positive bond transaction tax, country 1’s equity NFA position decreases to 0.138. Country 1’s net debt position improves marginally to -0.361. As a result, the overall NFA position worsens to -0.222. Capital flows nearly vanish because the external gross positions are small, and assets and liabilities are
close to being balanced. For the same reasons, the exchange rate adjustment to the
CA contributes less than fraction 0.105 of the actual flows. Volatility of the real
exchange rate and that of the current account in country 1 declines by 10.8% and
11.1%, respectively. But the effect on the equilibrium allocation is smaller relative to
the case with the bond transaction tax: mean consumption in country 1 decreases by
0.4% while it increases by 0.4% in country 2. This suggests that the trade in bonds,
rather than equity, is a more important contributor to international stability. When
trade in the bond market is restricted, aggregate consumption levels must adjust
more. When trade in equity claims is restricted, country 1’s NFA becomes more
extreme; yet, the effect on the allocation is smaller, the exchange rate and capital
flows become less volatile.

After imposing a trading cost on any asset class, welfare changes are marginal, and
this is expected in an endowment economy. For this reason, it may not be a useful tool
for policy guidance. By examining separate properties of the equilibrium allocation,
as we do above, it can be concluded that any policy inhibiting bond market operations
could have substantial implications. Trading costs could also come in many disguises.
For example, uncertainty about a country’s fiscal budget may limit liquidity in the
market for safe government debt. Our analysis suggests that the implications for the
external position and the indirect effect on domestic demand cannot be ignored.

6. Conclusions

We study the implication of having a more developed market for debt on the ex-
ternal balance sheet of a country. We use a general equilibrium model of international
portfolio choice with a rich asset market structure and financial constraints. We allow
countries to differ in two dimensions. First, we assume that only one country can
issue debt that pays in units of its own good. Second, we assume that consumers in
the same country are also less risk-averse which is a “reduced form” way of model-
ing a more developed domestic credit market. We show that having a bond market advantage prompts a country to accumulate debt. This effect is magnified significantly if the country is also less risk-averse. The latter asymmetry has little effect by itself, but it reinforces the bond market advantage. The asymmetry in the bond market also helps reducing the strong correlation between the real exchange rate and relative consumption. This brings our predictions about the real exchange rate closer to the data, and allows a meaningful description of the exchange rate channel of the international financial adjustment.

The short-selling constraints that we impose also play an important role. In the symmetric setting, this friction has a negligible effect on the allocation. But in the asymmetric setting it limits possible adjustments of the portfolio. For this reason it has a significant effect on the allocation and the pricing system, helping us match the observed U.S. international portfolio. While matching the net positions is relatively easy, the gross positions predicted by the model are largely understated. This can be partially explained by the fact that the U.S. equity market is only a small fraction of the world market. In our model it constitutes 50% and, to explain the gross positions, countries must hold only foreign equity in their portfolios, which is inconsistent with the home equity bias. This might be a fruitful direction to explore.

We use our setting to demonstrate how different markets contribute to “global stability.” Imposing a trading cost on bond purchases adversely affects the equilibrium allocations: there is high consumption inequality and it is more volatile. The real exchange rate and capital flows also become more volatile. In contrast, imposing a trading cost on equity purchases has a limited effect on the allocation and stabilizes the real exchange rate and capital flows. These results suggest that “slowing down” the trade in equity may be desirable. But one should also be wary of policies that may directly or indirectly affect the liquidity of the bond market.

Our work allows us to speculate about issuers of the potential dollar contenders.
The introduction of the Euro was an attempt to replace the U.S. dollar as the leading reserve currency. While it had been gaining the share in the world financial markets before the global financial crisis, the dollar has recovered its position since then as argued in Prasad (2014). According to our model the external position of the Euro area should worsen once its common currency resumes its trend. Similarly, once markets trading yuan-denominated debt become transparent and the yuan is established as a reserve currency we expect China’s external position to deteriorate.

Lastly, we do not address trends in the U.S. external portfolio positions. There could be several drivers of these trends. The most relevant in our view is the growth of the stock market value to GDP in the U.S. that increased from 83% to 204% between 1988 and 2007. Similar changes occurred with the size of international trade in goods. We believe that financial and trade liberalizations have contributed significantly to the growth of the gross portfolio positions across the world. In our view, modeling these trends would be a valuable contribution.

**Bibliography**


