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Long-range collision avoidance for shared space simulation based on social forces

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Abstract

Shared space is an innovative approach to improve environments where both pedestrians and vehicles are present, with integrated layouts to balance priority. The Social Force Model (SFM) was used to visualise pedestrian and car trajectories so that peaks of density and pressure at critical locations are avoided. This paper extends the SFM to consider a long-range collision detection and collision resolution strategy. The determination of potential conflicts is enhanced using principle component analysis for a set of agent's prior speeds and directions. This long-range collision avoidance strategy results in more realistic SFM-based trajectories for pedestrians and cars in shared spaces.

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Keywords: social force model; long-range collision detection; collision resolution; shadow; principle component analysis; shared space

1. Introduction

Since the appearance of motorised transport, there has been debates on the extent to which standardised and vehicle-dominated streetscape cause negative effect on the public neighbourhoods. Hamilton-Baillie (2008) explains that individuals tend to spend less time in public areas if they perceive streets to be less attractive for their social interaction activities or mobility. As a result, the quality of these spaces declines and human activities transfer from public to private spaces. Hence, urban design is moving towards shared space as an alternative to traditional designs (Jones and Boujenko (2011); Schonauer et al. (2012)). The shared space concept focuses on changing the way streets function by reducing the dominance of vehicles. This is often obtained through the use of a single surface, with the goal of integrated space use (Department for Transport (2011)). Shared space allocates more degrees of freedom to its traffic participants. As a result of this increased freedom, traffic engineers have become concerned about the consequences of the new designs for pedestrians, as well as the delay for vehicular traffic. Computer modelling allows an examination of the trade-offs between travel times and vehicular and pedestrian traffic (Ishaque and Noland (2007)). Modelling and simulation, in combination with visualisation of future shared space designs, allow designers to test different

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mixed traffic situations. These should help to improve the basic concept and achieve solutions for issues such as optimal traffic capacities or delays. A combination of two traffic systems requires a way of finding paths to respective destinations for different road users. A mixed-traffic system should also be able to reproduce the most common behaviour patterns and handle interactions within heterogeneous traffic. Many theoretical models have been proposed in the literature (Zhang and Kim (2005); Franca et al. (2009); Chraibi and Seyfried (2010)) to uncover the laws which govern vehicle and pedestrian traffic dynamics separately. However, there is limited research on interactions within heterogeneous traffic. Hence, there is a need to investigate the interaction between different transportation modes in shared areas.

Anvari et al. (2012) discusses behavioural patterns of pedestrians and cars in shared space areas using the Social Force Model (SFM) and argues that short range repulsive forces in the SFM leads to excessively frequent urgent detours to avoid collisions. Hence, this paper explores a long-range collision avoidance method to handle the potential car-pedestrian road conflicts that might occur by following the SFM exclusively.

The collision avoidance issues are explored in different transportation systems such as crowd dynamics (Llorca et al. (2011); Ahn et al. (2011); Park et al. (2012); Golas et al. (2013)), air traffic management (Pallottino and Feron (2002); Sislak et al. (2011); Luongo et al. (2011)) and mobile ground robot applications (Mejias et al. (2010); Albaker and Rahim (2010); Knepper and Rus (2012)). The common idea is to detect a potential collision by a specific threat prediction model (e.g. straight or probabilistic or precise prediction method). Then, a collision resolution strategy (e.g. predefined or protocol based or optimised or reactive path planning) is applied based on estimations about how other users behave during a certain prediction horizon (e.g. constant acceleration or constant turn radius). The time and distance to a potential collision is then evaluated based on their relative distance, velocity and acceleration. For instance, Coelingh et al. (2010) describe an Automatic Emergency Braking system with a pedestrian detection and collision avoidance model. This system is capable of avoiding accidents for vehicle speeds up to 35 $\frac{\text{km}}{\text{h}}$. Their prediction model is dependant on the time to the collision and the time required to avoid the collision considering a safe collision avoidance distance and vehicle dynamics. The time to avoid a collision using factors such as the perception-reaction time and braking time is studied in vehicle-to-vehicle interactions (Tang and Yip (2010)). Furthermore, Ahn et al. (2011) extracted general principles of human behaviours in an outdoor pedestrian area from multiple video recordings and used real trajectories of pedestrians to estimate long term collisions. A collision avoidance algorithm was formulated based on a colliding area and a shift influencing area around pedestrians. Golas et al. (2013) proposed a probabilistic long-range collision avoidance strategy for crowd simulations by smoothening the effect of distance. In this approach, the velocity of pedestrians is updated based on their desired velocity and the estimation of future behaviour patterns of other pedestrians. In their prediction model, agents' current position and velocity is linearly extrapolated and considered as future behaviour of pedestrians. The prognosis of this model increases as agents get closer to each other.

In this paper, the SFM is extended to consider a long-range Collision Detection and Collision Resolution (CDCR) strategy. In Section 2, the SFM for shared space users is briefly recalled. The Principle Component Analysis (PCA) is introduced in Section 3 for a set of agent's prior speeds and directions in order to enhance the determination of an agent's shadow. The proposed solution returns the minimum speed change for each agent while deviating as little as possible from their desired direction of movement.

2. Social Force Model for shared space users

2.1. Geometrical agent modelling for pedestrians and cars

In the SFM by Helbing et al. (2000), each simulated pedestrian has a certain body size which can be expressed by circles (symmetrical configuration) of a radius r_α. Since the definition of shared space is the integration of pedestrians and vehicles, a car has been introduced by an ellipse with the radius $r_\gamma(\varphi_{\gamma U})$ (Anvari (2012); Anvari et al. (2012)). As shown in Fig. 1, the radius $r_\gamma(\varphi_{\gamma U})$ depends on the angle between the desired direction of a car γ and the centre of a close-by pedestrian $U = \alpha$ or car $U = \delta$. The radius of the ellipse $r_\gamma(\varphi_{\gamma U})$ in polar coordinates is described by Equation 1.

$$r_\gamma(\varphi_{\gamma U}) = \frac{w}{\sqrt{1 - \epsilon^2 cos^2(\varphi_{\gamma U})}}, \text{ where } \epsilon = \frac{\sqrt{l^2 - w^2}}{l} \tag{1}$$

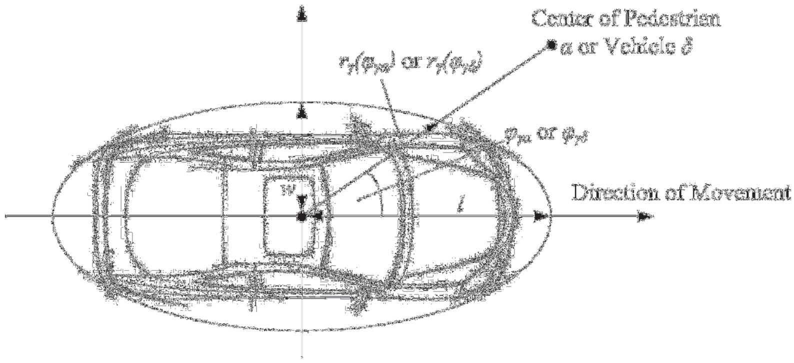


Fig. 1. Vehicle modelling using a geometrical approximation of an ellipse.

where $2l$ and $2w$ are the average length and width of a modelled car.

2.2. Force-based modelling

Since vehicles and pedestrians move within shared space environments, the original SFM for pedestrians has been considered and applied to a model for cars (Anvari (2012); Anvari et al. (2012)). The sum of the force terms exerted on a car γ from pedestrian α, boundary b and car δ can be seen in Equation 2. Each summand is explained briefly in the following paragraphs.

$$\frac{d\vec{v}_\gamma(t)}{dt} = \vec{f}_\gamma^0 + \sum_{\delta(\delta\neq\gamma)} \vec{f}_{\gamma\delta}^{\text{soc}} + \sum_\alpha \vec{f}_{\gamma\alpha}^{\text{soc}} + \sum_b \vec{f}_{\gamma b} + \vec{\xi} \tag{2}$$

2.2.1. Driving force for cars

The driving force of a car is similar to the one applied for pedestrians in the original SFM. This force term describes the motivation of a driver to move towards a certain (intermediate) destination. Driver γ is assumed to move in a desired direction $\vec{e}_\gamma$ with a desired speed v_γ^0 adapted to the actual velocity $\vec{v}_\gamma(t)$ within a constant relaxation time τ_γ.

$$\vec{f}_\gamma^0 = \frac{v_\gamma^0 \cdot \vec{e}_\gamma(t) - \vec{v}_\gamma(t)}{\tau_\gamma}, \text{ where } \vec{e}_\gamma(t) = \frac{\vec{r}_\gamma^k - \vec{r}_\gamma}{|\vec{r}_\gamma^k - \vec{r}_\gamma|} \tag{3}$$

The shortest path to the final destination for car γ is defined as a sequence of intermediate destinations (Anvari et al. (2012)). Therefore, the desired direction $\vec{e}_\gamma$ points in the direction of the next intermediate destination $\vec{r}_\gamma^k$ on the shortest path to the final destination.

2.2.2. Interaction between cars and shared space users

Shared space layouts aim to achieve a constant traffic flow by reducing stop-and-go behaviours (Shearer (2011)). Drivers try to adapt to the behaviour of other shared space users. Any deviation from their path to their destination is mainly due to local social interactions. The interaction force $\vec{f}_{\gamma U}^{\text{soc}}$ between a car γ with either another car ($U = \delta$) or a pedestrian ($U = \alpha$) is presented by Equation 4. This socio-psychological force $\vec{f}_{\gamma U}^{\text{soc}}$ is to keep a certain distance from nearby agents. An exponential function is applied to reflect the role of distance. The repulsive force increases when agents get closer and almost vanishes when they move far away from each other.

$$\vec{f}_{\gamma U}^{\text{soc}} = A_{\gamma U} e^{\frac{r_{\gamma U} - d_{\gamma U}}{B_{\gamma U}}} \vec{n}_{\gamma U} F_{\gamma U} \tag{4}$$

$\vec{n}_{\gamma U}$ is the normalized vector pointing from another user (car or pedestrian) to car γ. $A_{\gamma U}$ and $B_{\gamma U}$ are constant parameters that represent the interaction strength and interaction range of the repulsive force $\vec{f}_{\gamma U}^{\text{soc}}$ which require calibration. $d_{\gamma U}$ is the distance between the centre of agents and $r_{\gamma U}$ is the sum of their radii. The sum of the radii is therefore, $r_{\gamma U} = r_\gamma + r_U$. The radius r_γ depends on the angle $\varphi_{\gamma U}$ between the desired direction of a car and the direction of a neighbouring pedestrian or car.

Similar to Helbing et al. (2002), the anisotropic character of interactions is included to provide a more realistic form of forces. Considering that car movements are restricted to change of direction and lateral movement is not possible, an effective field of view is included in the form factor $F_{\gamma U}$ (see Equation 5). In addition, the cases for a pedestrian predicting the presence of a car or a car following another car are distinguished because in contrast to pedestrians a car driver does not only react to cars in front but also to those behind it.

$$F_{\gamma U} = \left(\lambda_\gamma + (1 - \lambda_\gamma)\frac{1 + cos(\varphi_{\gamma U})}{2}\right) \cdot q \tag{5}$$

q is the effective factor that distinguishes between car-pedestrian or car-car interactions. The effective field of view for car drivers is considered 60^o based on the area that is overlooked by drivers with easy head movements (Morgan and Blanco (2010)). Regarding car-pedestrian interaction, q is:

$$\begin{aligned} &q = 1, \text{ if } -30^o \leq \varphi_{\gamma\alpha} \leq 30^o \\ &q = 0, \text{ otherwise} \end{aligned} \tag{6}$$

Considering car-car interaction, the following can be summarised for the effective factor q:

$$\begin{aligned} &q = 1, \text{ if} \begin{cases} -30^o \leq \varphi_{\gamma\alpha} \leq 30^o \ \& \\ (180^o - 30^o) \leq \varphi_{\gamma\alpha} \leq (180^o + 30^o) \end{cases} \\ &q = 0, \text{ otherwise} \end{aligned} \tag{7}$$

By varying λ_γ, the influence of the exerted forces of the cars behind the leading car changes.

2.2.3. Social Force Model extension to pedestrians

Since the original SFM (Helbing (1991)) only considers forces exerted by pedestrians and obstacles onto other pedestrians, forces exerted by vehicles onto pedestrians need to be included. The existence of cars in a shared space environment is expressed by a new socio-repulsive force $\vec{f}^{soc}_{\alpha\gamma}$ from cars to pedestrians (Equation 8). This new force explains the most important interaction behaviour of a pedestrian keeping a certain distance to the nearby car since no physical interaction should occur.

$$\frac{d\vec{v}_\alpha(t)}{dt} = \vec{f}^0_\alpha + \sum_{\beta(\beta\neq\alpha)} \vec{f}_{\alpha\beta} + \sum_b \vec{f}_{\alpha b} + \sum_\gamma \vec{f}^{soc}_{\alpha\gamma} + \vec{\xi} \tag{8}$$

Similar to the interaction force between pedestrians in the SFM, an exponential function is applied to pedestrian α to represent the influence of distance between pedestrians and the close-by car γ as

$$\vec{f}^{soc}_{\alpha\gamma} = A_{\alpha\gamma} e^{\frac{r_{\alpha\gamma} - d_{\alpha\gamma}}{B_{\alpha\gamma}}} \vec{n}_{\alpha\gamma} F_{\alpha\gamma} \tag{9}$$

where $r_{\alpha\gamma} = r_\alpha + r_\gamma$; $d_{\alpha\gamma}$ is the distance between the centre of pedestrian α and car γ, $\vec{n}_{\alpha\gamma}$ is the normalized vector from car γ to pedestrian α. The form factor $F_{\alpha\gamma}$ is also set similar to Equation 5 to explain the anisotropic behaviour of pedestrian α when facing car γ.

3. Direction and speed prediction using Principle Component Analysis

The model in Section 2 can be defined as a continuous model in time as the sum of forces are determined for a time interval which is considered to be small (e.g. 0.1 s). Hence, the current velocities (the speed and direction of movements) are updated and continuously change over time. When predicting future locations of agents and potential collisions between agents, uncertainties are introduced if one solely relies on the current movements of an agent. In this section, we propose a velocity predictor considering a priori knowledge from a set of past velocities based on the PCA and, therefore, reduce uncertainties.

The PCA was initially introduced by Hotelling (1933). However, a mathematical ansatz (Abdi and Williams (2010)) can be found earlier by Pearson (1901), Cauchy (1829) and Jorden (1874). The PCA is formalised as a multivariant statistical technique that analyses a list of data arrays and describes them as a set of orthogonal principal components. APCA is applied to both find the direction of variation and reduce data. The latter idea is of interest within this paper.

Moeslund (2001) and Jackson (2001)report that the PCA technique is now applied using a priori knowledge about agents' speed and direction. The result of the PCA is then provided to the conflict avoidance strategy in order to detect and resolve potential collisions (see Section 4). In this paper, $N = 50$ previous velocities $\vec{v}_U$ of agents are considered and summarised in the following matrix:

$$\vec{V}_U = \begin{pmatrix} v_{U,x_1} & \cdots & v_{U,x_N} \\ v_{U,y_1} & \cdots & v_{U,y_N} \end{pmatrix} \tag{10}$$

Using the PCA, the data will be re-structured. The purpose is to reduce the amount of components, which is equivalent to a linear transformation and a dimensional reduction respectively. Principal components are extracted which are perpendicular to each other. These are then linear combinations of the original data. Thus, the covariance matrix and its eigenvectors need to be determined.

The arithmetic mean for the x- and y-components of the matrix $\vec{V}_U$ are calculated as:

$$\vec{\bar{v}}_U = \begin{pmatrix} \bar{v}_x \\ \bar{v}_y \end{pmatrix} = \begin{pmatrix} \frac{1}{N} \sum\limits_{i=1}^{N} \vec{v}_{U,x_i} \\ \frac{1}{N} \sum\limits_{i=1}^{N} \vec{v}_{U,y_i} \end{pmatrix} \tag{11}$$

Equation 11 is used to identify the covariance matrix:

$$\Sigma = \frac{1}{N} \sum_{i=1}^{N} (\vec{\bar{v}}_U - \vec{v}_{U,i})(\vec{v}_{U,i} - \vec{\bar{v}}_U)^{\mathrm{T}} \tag{12}$$

As mentioned, Equation 12 is a symmetric matrix, so $\vec{\Sigma} = \vec{\Sigma}^{\mathrm{T}}$. The Schur decomposition separates Equation 12 into an orthogonal matrix $\vec{M}$ and a diagonal matrix $\vec{\Lambda}$, so that

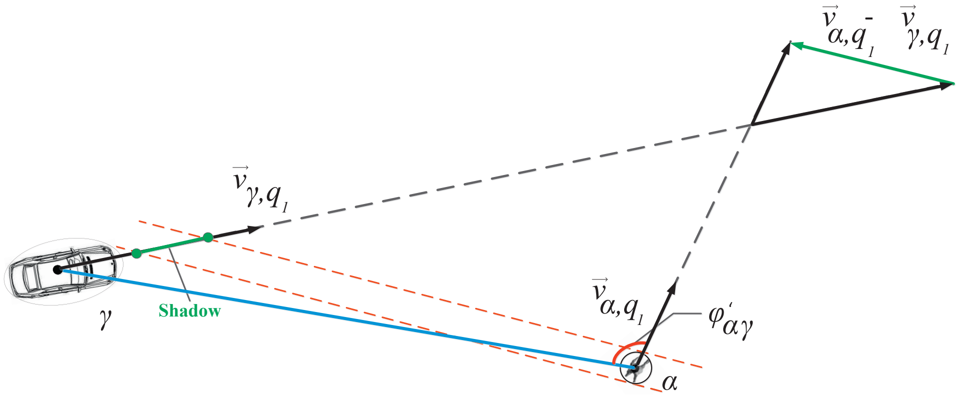
$$\vec{M}^{\mathrm{T}} \vec{\Sigma} \vec{M} = \vec{\Lambda} = \mathrm{diag}\left(\lambda_1^{\mathrm{Eigenvalue}}, ..., \lambda_r^{\mathrm{Eigenvalue}}\right) \tag{13}$$

Here, $\lambda_j^{\mathrm{Eigenvalue}}$ for $j = 1, ..., r$ are the eigenvalues of the covariance matrix $\vec{\Sigma}$ and $\vec{M}$ contains the equivalent eigenvectors $\vec{v}_{U,q_j}$. It is assumed that the eigenvectors $\vec{v}_{U,q_j}$ are ordered by the size of the corresponding eigenvalues, so that $\vec{v}_{U,q_1}$ relates to largest eigenvalue which is used to predict potential conflicts in Section 4.

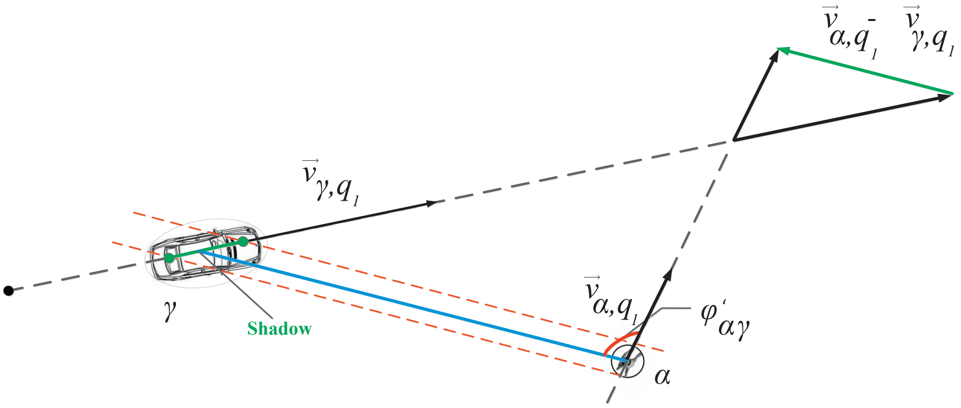
4. Long-range collision detection and collision resolution (CDCR) strategy for the SFM

The strategy for potential conflict avoidance is implemented for car-pedestrian and car-car interactions. A prediction of an interaction between two agents which results in a potential conflict is determined within a certain distance to each other. The conflict occurs if physical contact is estimated to be at a future time interval between these two agents. The term 'shadow' is introduced to detect potential conflicts.

Reaction strategies can be classified into speed change, steering change or a combination of both. Here, agents prevent potential conflicts using a combination of speed and direction change based on their relative position. The aim is then to find the minimum velocity change $\Delta\vec{v}^{\min}$ for each agent while deviating as little as possible from their desired direction of movement.



(a) Car γ does not intersect the shadow generated by pedestrian α



(b) Car γ intersects the shadow generated by pedestrian α

Fig. 2. Geometric construction for conflict detection.

4.1. Prediction of potential conflicts

If two agents are within a distance larger than their interaction range B_U, the possibility of a potential conflict is analysed. The conflict avoidance constraints are explained based on the geometrical considerations of two agents. Fig. 2 illustrates the predicted intersecting trajectories of a pedestrian ($U_1 = \alpha$) and a car ($U_2 = \gamma$). The position, direction of movement and initial velocity of pedestrian α and car γ determined by the PCA are shown in Fig. 2 (a). $\varphi_{\alpha\gamma}$ is the angle between the direction of movement of pedestrian α and the force $\vec{f}_{\alpha\gamma}$ exerted from car γ to pedestrian α. Two lines are indicated parallel to the velocity difference from the PCA that are tangential to pedestrian α in order to assign a section on the desired direction of car γ given by the PCA. This section is defined as the shadow of pedestrian α along the direction of car γ similar to air traffic management systems (Pallottino and Feron (2002)). A potential conflict is detected as soon as car γ intersects the shadow as shown in Fig. 2 (b). This is explained mathematically in Equation 14.

$$\frac{(v_{y_\alpha,q_1} - v_{y_\gamma,q_1})}{(v_{x_\alpha,q_1} - v_{x_\gamma,q_1})} < tan(\varphi'_{\alpha\gamma}) \tag{14}$$

Here, $\varphi'_{\alpha\gamma}$ is the angle between the largest eigenvalue and the location of the conflicting agent. The time t^{CPA} indicating the period to reach the location of minimum distance d^{CPA} between the agents at their Closest Point of Approach (CPA) needs to be determined. The minimum distance at the CPA is $d^{\mathrm{CPA}} = \left| r_\alpha(t^{\mathrm{CPA}}) - r_\gamma(t^{\mathrm{CPA}}) \right|$ and should not be less than the sum of their radii ($r_\alpha + r_\gamma(\varphi_{\gamma\alpha})$) as shown in Fig. 3. The position of agent α and γ is given by $(x_\alpha(t), y_\alpha(t))$ and $(x_\gamma(t), y_\gamma(t))$. Their time-dependent velocity vectors are $(v_{x_\alpha,q_1}(t), v_{y_\alpha,q_1}(t))$ and $(v_{x_\gamma,q_1}(t), v_{y_\gamma,q_1}(t))$.

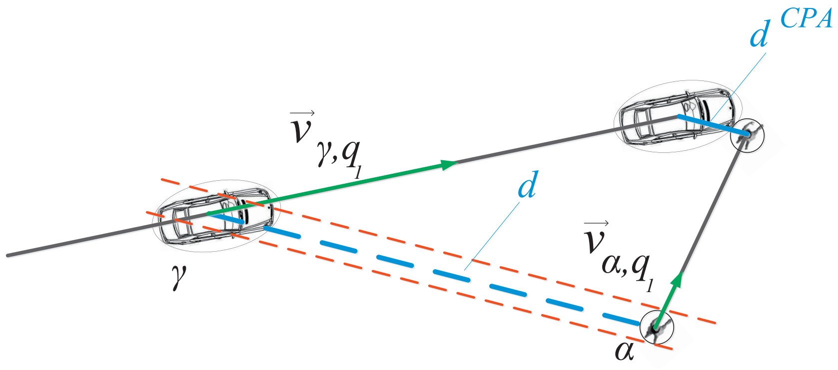


Fig. 3. Closest Point Approach (CPA) illustration between car γ and pedestrian α.

At any time instance t, the distance between the two agents is given by $d(t) = \sqrt{\Delta x(t)^2 + \Delta y(t)^2}$. The time to the minimum distance is now calculated as $t^{\text{CPA}} = -\frac{\Delta x \Delta v_{x,q_1} + \Delta y \Delta v_{y,q_1}}{\Delta v_{x,q_1}^2 + \Delta v_{y,q_1}^2}$. The time t^{CPA} to reach the CPA should be positive (in other words, in future) and less than a defined higher bound of $t^{\text{CPA,max}}$ (in other words, in close future). If the time $0 < t_{\gamma\alpha}^{\text{CPA}} < t_{\gamma\alpha}^{\text{CPA,max}}$ and distance to reach the CPA is less than a certain value (in other words, in close future), the conflict avoidance strategy is activated. In the following section, an optimisation is applied in order to avoid conflict between agents.

4.2. Optimisation of minimum speed and direction change

The agent with a higher speed starts to accelerate and deviate whereas the other agent decelerates and deviates accordingly. An optimisation is applied to calculate a minimum velocity change $\Delta\vec{v}^{\text{min}} = \vec{v}^{\text{opt}}(t) - \vec{v}(t)$ in order to avoid conflicts. The cost function $c(v_x^{\text{opt}}(t), v_y^{\text{opt}}(t))$ in Equation 15 is optimised.

$$c\,(v_x^{\text{opt}}(t), v_y^{\text{opt}}(t)) = (v_x^{\text{opt}}(t) - v_{x,q_1}(t))^2 + (v_y^{\text{opt}}(t) - v_{y,q_1}(t))^2 \tag{15}$$

Equation 15 is to be minimised subject to constraints which are linear inequalities of variables used in the cost function. Firstly, the optimal speed should be within a defined speed interval $v_U^{\text{min}} < v_U^{\text{opt}} < v_U^{\text{max}}$. Secondly, the minimum distance between the agents at the CPA should be more than the sum of their radii. Thirdly, the distance to

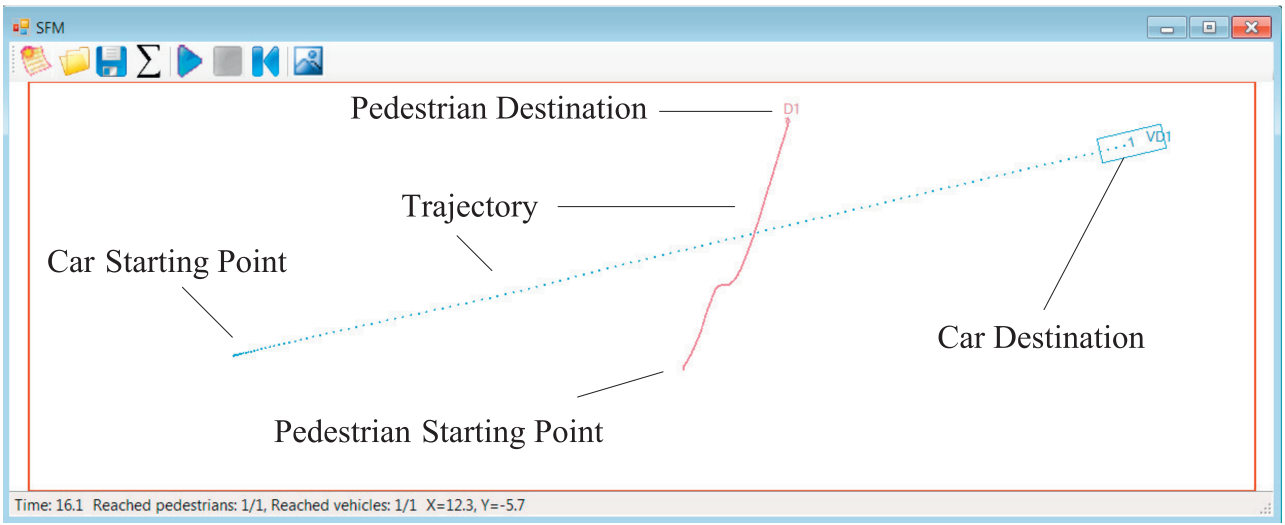


Fig. 4. Simulation of the interaction between a car and pedestrian without conflict avoidance force.

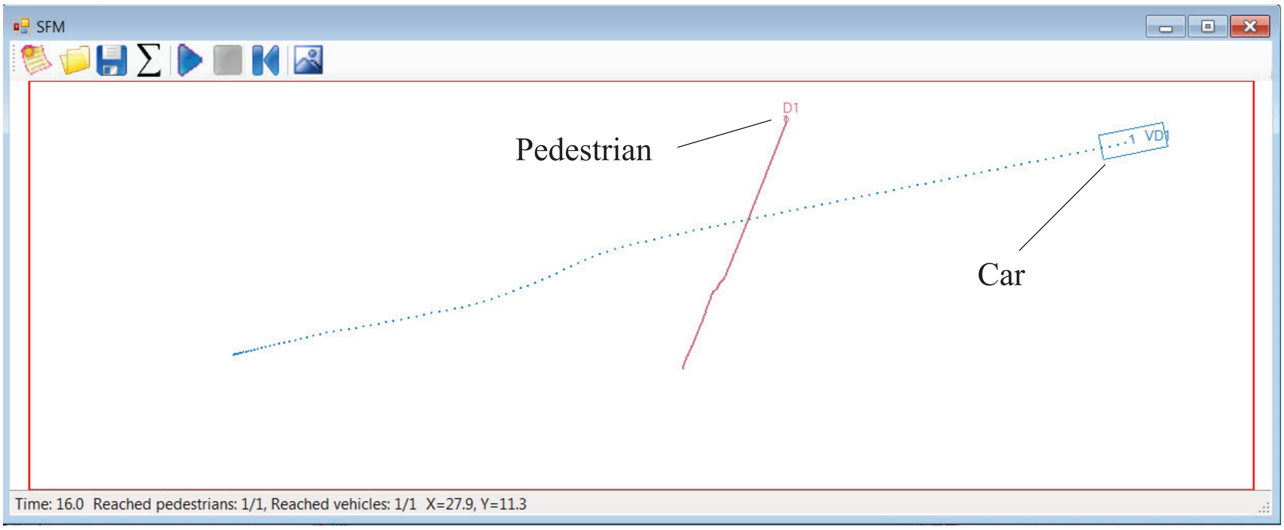


Fig. 5. Simulation of the interaction between a car and pedestrian with conflict avoidance force.

reach the CPA should be less than a certain value in order to be considered as a potential conflict. The optimisation problem incorporating all these constraints can be formulated as follows:

$$\begin{aligned} &\textbf{Minimize} && c\,(v_x^{\text{opt}}(t), v_y^{\text{opt}}(t)) \\ &\textbf{Subject to} && v_U^{\text{min}} < v_U^{\text{opt}} < v_U^{\text{max}} \\ & && d_{\gamma\alpha}^{\text{CPA}} > r_\alpha + r_\gamma(\varphi_{\gamma\alpha}) \\ & && \text{Distance to reach the CPA} > \text{Minimum acceptable distance} \end{aligned}$$

A conflict avoidance force $\vec{f}_U^{\text{conflict}} = \frac{\Delta\vec{v}^{\text{min}}}{\tau_U}$ is calculated and added to the sum of forces. A simulation of intersecting trajectories of a pedestrian and a car is presented before and after including the conflict avoidance force in Fig. 4 and 5. According to social forces, pedestrian α starts decelerating and deviating from the desired direction of movement when within the interaction range B_α to a car (see Fig. 4), without prior evaluation of the potential conflict. This is while pedestrian α and car γ start deviating from their desired direction of movement much earlier, as a result of conflict avoidance constraints in Fig. 5.

5. Conclusions and future work

This paper extends the work conducted by Anvari et al. (2012) describing the key characteristics of shared space users based on the Social Force Model (SFM). Potential road conflicts that might occur by following the SFM exclusively have been investigated and an approach presented to predict and handle these potential conflicts between pedestrians and cars based on their current positions and estimated direction and speed of movements. A potential conflict is detected as soon as agents enter each other's shadow. Using the Principle Component Analysis, the estimation of an agents shadow is enhanced for a set of agents prior velocities. A resolution strategy is chosen by a combination of speed and heading direction change. Conflict avoidance constraints are assigned to each agent to provide realistic conflict free interactions between pedestrians and cars while deviating as little as possible from the desired direction of movement. An optimisation process has been proposed to calculate the minimum velocity change for both agents in order to avoid conflicts.

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