

Experimental Realisation of a Thermal Squeezed State of Levitated Optomechanics

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We experimentally squeeze the thermal motional state of an optically levitated nanosphere, by fast switching between two trapping frequencies. The measured phase space distribution of our particle shows the typical shape of a squeezed thermal state, from which we infer up to 2.7dB of squeezing along one motional direction. The experiment features a large number of thermal excitations, therefore remaining in the classical regime. Nevertheless, we argue that the manipulation scheme described here could be used to achieve squeezing below the zero-point level, if preceded by ground state cooling of the levitated mechanical oscillator. Additionally, a higher degree of squeezing could in principle be achieved by repeating the frequency-switching protocol multiple times.

While squeezing a quantum state of light has a long history of experiments, the squeezing of a massive mechanical harmonic oscillator has so far not seen many experimental realisations. The first demonstration of squeezing in a classical mechanical oscillator was by Rugar et al. [1]. Squeezing of classical motional states in electromechanical devices by parametric amplification and weak measurement has been studied subsequently [2] as well as for an optomechanical system [3]. Squeezing below the zero-point level of a high-frequency mechanical oscillator has only been experimentally demonstrated very recently, in a microwave optomechanical device [4]. Also only very recently a hybrid photonic-phononic waveguide device has shown the shared properties of optomechanical two-mode squeezing [5]. Here we demonstrate squeezing via non-adiabatic frequency shifts as a tool for manipulating the classical state of a levitated optomechanical system.

Theory— We consider a nanosphere of mass m trapped in a harmonic potential. Along the z axis, we can manipulate the system by switching between two Hamiltonians \hat{H}_1, \hat{H}_2 , where $\hat{H}_j = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega_j^2\hat{z}^2$, \hat{z}, \hat{p} denote the z -components of the canonical position and momentum operators, and the trapping frequency may assume two distinct values: ω_1 or $\omega_2 < \omega_1$. As we shall see shortly, our squeezing protocol relies on the rapid (i.e. *non-adiabatic*) switching between the two Hamiltonians [6, 7]. It is instructive to write down the annihilation operators, say \hat{a} and \hat{b} , corresponding to the two trap frequencies ($\hbar = 1$):

$$\hat{a} = \sqrt{\frac{m\omega_1}{2}}\left(\hat{z} + \frac{i\hat{p}}{m\omega_1}\right), \quad \hat{b} = \sqrt{\frac{m\omega_2}{2}}\left(\hat{z} + \frac{i\hat{p}}{m\omega_2}\right). \quad (1)$$

Through simple algebra one may notice that \hat{a} and \hat{b} are related by a squeezing transformation of the form $\hat{b} = \cosh(r)\hat{a} - \sinh(r)\hat{a}^\dagger$, with $r \equiv \frac{1}{2}\log\left(\frac{\omega_2}{\omega_1}\right)$ the squeezing parameter. We may exploit the mathematical relationship between modes \hat{a} and \hat{b} to generate mechanical squeezing, as follows. Let the particle be initially pre-

pared in an arbitrary state (in our experiment, this will be a thermal state of \hat{H}_1). At time $t = 0$ we suddenly change the trapping frequency from ω_1 to ω_2 , such that the Hamiltonian becomes \hat{H}_2 . We then let the system evolve until a time $t = \tau$, before rapidly switching back to Hamiltonian \hat{H}_1 . [In the following we also refer to τ as the squeezing pulse duration.] In the Heisenberg picture, this amounts to a simple harmonic evolution $\hat{b} \rightarrow \hat{b}e^{-i\omega_2\tau}$ for the operator \hat{b} . In terms of the quadratures $\hat{X} = (\hat{a} + \hat{a}^\dagger)/\sqrt{2}$, $\hat{P} = -i(\hat{a} - \hat{a}^\dagger)/\sqrt{2}$, however, the transformation is nontrivial:

$$\begin{pmatrix} \hat{X} \\ \hat{P} \end{pmatrix} \rightarrow M \begin{pmatrix} \hat{X} \\ \hat{P} \end{pmatrix} \quad (2)$$

$$M = \begin{pmatrix} \cos(\omega_2\tau) & e^{2r}\sin(\omega_2\tau) \\ -e^{-2r}\sin(\omega_2\tau) & \cos(\omega_2\tau) \end{pmatrix}, \quad (3)$$

where the matrix M embodies a combination of rotation and squeezing in the phase space of mode \hat{a} . Note that, in general, the squeezed quadrature will be a linear combination of \hat{X} and \hat{P} . The associated squeezing parameter $\lambda(\tau)$ is encoded in the singular values of M , and can be found as follows. From the (easily verified) property $\det(MM^\top) = 1$, it follows that we can parametrize the eigenvalues of MM^\top as $(\mu, 1/\mu)$ for some parameter $\mu > 0$. Note that μ quantifies the deformation of the *variances* of the rotated quadratures. It then follows that, in dB units, the mechanical squeezing parameter reads

$$\lambda(\tau) = 10|\log_{10}(\sqrt{\mu})|. \quad (4)$$

The analytical expression for $\lambda(\tau)$ is unwieldy if τ is left generic. It is however readily verified that maximum squeezing can be obtained by setting $\omega_2\tau = \frac{\pi}{2}$, in which case $\lambda_{\max} = 10\log_{10}(\omega_1/\omega_2)$.

A related technique for squeezing generation, based on the continuous sinusoidal modulation of the trapping potential has been proposed for the squeezing of trapped ions [8], but has not yet been realised experimentally.

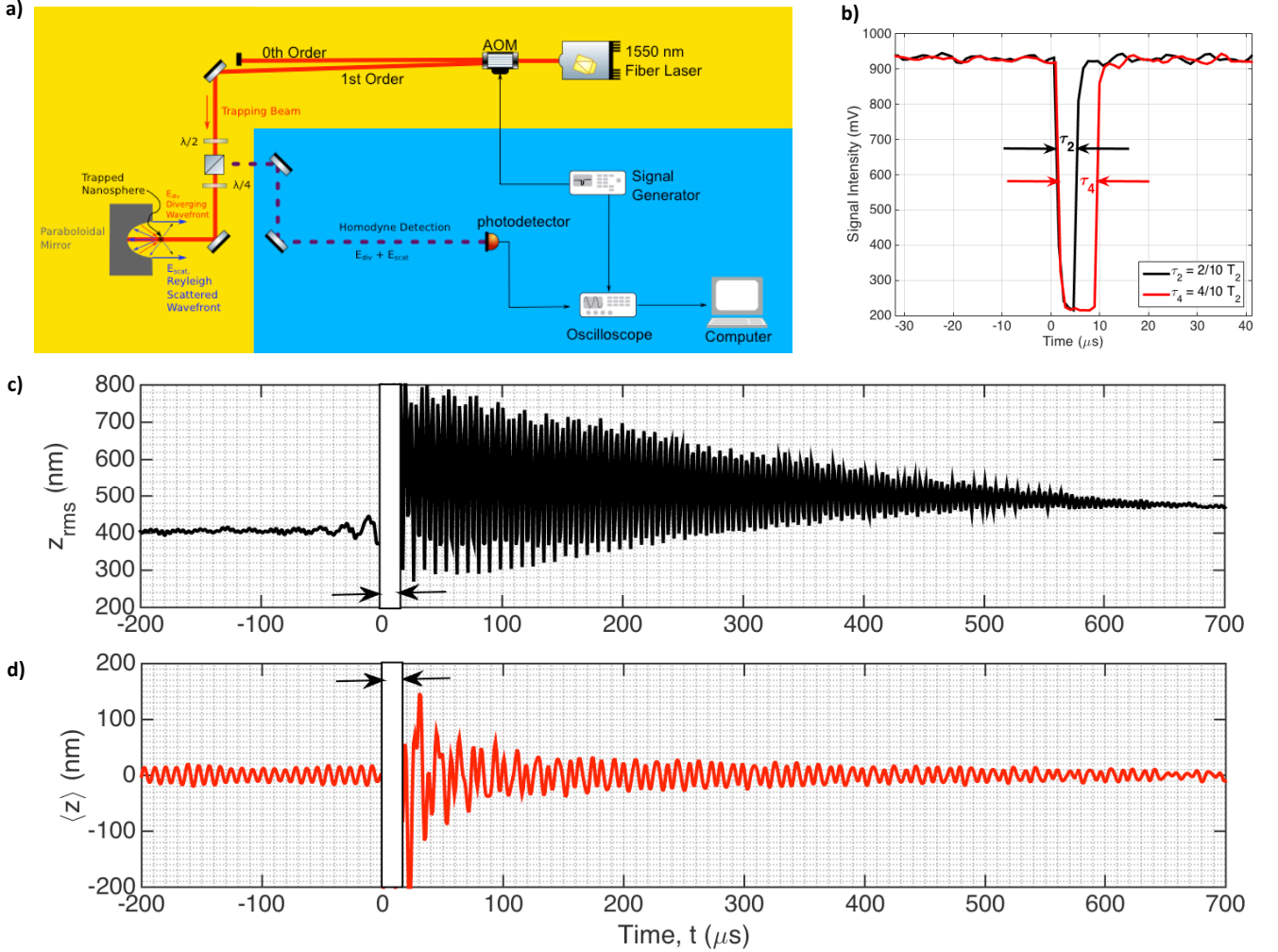


FIG. 1. **Experimental implementation of squeezing levitated optomechanics.** In a) we show the schematic for the squeezing setup. The yellow region is the trapping system and the blue region is for the detection as well as pulse application. b) is the negative square pulse applied as seen by the photodetector. Two different pulse widths are shown, τ_2 and τ_4 . c) The curve shows the root mean square position change with time, $z_{\text{rms}}(t) = \sqrt{\langle (z - \langle z \rangle)^2 \rangle}$ of 1500 pulse sequences applied to the same particle and therefore the mean position change of the particle relative to the centre of the distribution. The oscillation for t less than zero are consequences of band pass filtering. d) The curve is the time-dependence of the mean position $\langle z \rangle$ of the particle - the motion of the center of the thermal distribution. It also shows oscillations after the squeezing pulsed was applied.

Our technique is similar to the so-called “bang-bang control”, which has been recently exploited to generate coherent states of trapped ions by a sudden displacement of the trap minimum [9].

Experiments– We trap a silica nanosphere of radius 32nm (± 5 nm) and mass of 3.1×10^{-19} kg ($\pm 1 \times 10^{-19}$ kg) in an optical dipole trap. The size of the particle is evaluated from fitting a Lorentzian to the power spectral density of the signal, as described in [10] and shown in Fig. 3b. We use a 1550nm laser, directed into a parabolic mirror which focusses the light to a diffraction limited spot, where the particle is trapped. Experiments are performed in a vacuum chamber at pressure of 1×10^{-1} mbar. In this pressure regime the damping of the motion of the particle by random collisions with background gas is lin-

ear in pressure p_{gas} and the related damping coefficient can be approximated by

$$\Gamma \approx 15.8 \frac{r^2 p_{\text{gas}}}{m v_{\text{gas}}}, \quad (5)$$

with m and r are the radius and the mass of the nanosphere, and $v_{\text{gas}} = \sqrt{3k_B T / m_{\text{gas}}}$ is the mean thermal velocity of the background gas of mass m_{gas} [11]. We evaluate $\Gamma = 2\pi \times 227$ Hz ($\pm 2\pi \times 9$ Hz) for this experiment while the main uncertainty in mass comes from the pressure measurement ($\sim 15\%$).

As shown in Fig.1a) the position of the single nanosphere is measured using an optical homodyne method. More details about the particle trapping and detection can be found elsewhere [10].

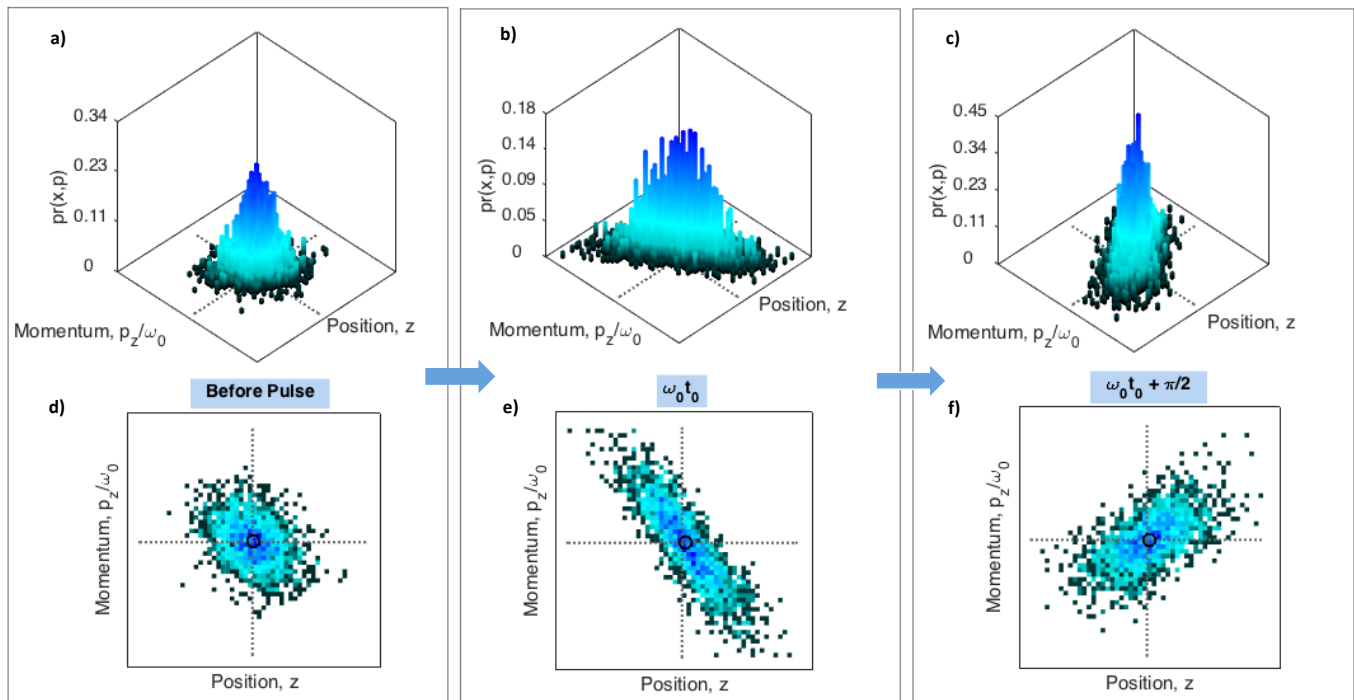


FIG. 2. **Phase-space plots of experimental results for squeezing a thermal state of motion of a trapped nanoparticle.** (a-c) show 3D phase space distribution whilst (d-f) are their 2D projections. (a), (d) show the state of the particle motion before the pulse is applied, which can be described by a Gaussian distribution as typical for a thermal state. (b), (e) shows the distribution shortly after the pulse has been applied and the clear signature of squeezing is observed. (c) and (f) show the distribution at time ($\frac{1}{4}T_2$) after the squeezing pulse. The squeezed state rotates in phase space while squeezing degrades with time. The latter is explained by a dephasing effect of the motion by background gas collisions. The observed oscillation of the center of the distribution is discussed with the help of mean position $\langle z \rangle$ in Fig.1d.

A squeezing pulse of duration τ is applied by switching between two different trapping laser powers P_1 and P_2 (see Fig.1b) using a free space acousto-optical modulator (AOM). The trapping frequency is given by $\omega = \sqrt{k_0/m}$, where $k_0 = 8\alpha P / (c\pi\epsilon_0 w_f^4)$ for motion in z -direction, with α the polarizability of the particle, c the speed of light, ϵ_0 the electric field constant and w_f the waist of the laser beam at the focal point. Therefore switching the laser power by changing the voltage applied to the AOM, we switch between trap frequencies $\omega_1 = 2\pi \times 112\text{kHz}$ and $\omega_2 = 2\pi \times 50\text{kHz}$. The timescale of the switch is limited by AOM bandwidth, which is more than 1MHz and therefore much faster than the trap frequencies. Therefore the switch is regarded as instantaneous compared to the motion of the particle in the trap.

The same signal generator which is used to generate the squeezing pulse triggers an oscilloscope to record a time trace of duration of one second. The same single pulse sequence is repeated 1500 times for the same particle trapped, while allowing for sufficient time between the pulses for the oscillating particle to return to a thermal state. The recorded time traces initially include signals from x , y and z motional degrees of freedom. However, the pulse scheme is only optimised for a single motional

frequency, namely, the one in z -direction which is perpendicular to the mirror surface. We filter the signal around the ω_z frequency peak to extract the impact of the pulses on the z motion alone. The root mean square (rms) of the position of the particle z_{rms} is used to analyse the state of the motion, see Fig.1c).

Results– The Fourier transform of the oscillation in the mean position $\langle z \rangle$ and - therefore the motion of the centre of the thermal distribution - after the pulse shows components of three different frequencies: ω_1 , ω_2 and $(\omega_1 + \omega_2)$, see Fig.1d). This suggests that: (i) together with the desired squeezing operation, our pulse imparts a small phase-space displacement to the particle; (ii) our dynamics features small nonlinearities and memory effects. To correct for this we subtract the average displacement from the data, while we try to account the remaining effects and experimental imperfections through an effective dephasing model (see supplement [12]). While this is not expected to accurately describe the fine details of our dynamics, it appears to predict the correct trends.

Initially z_{rms} is constant as the phase of the oscillation is random between the 1500 individual pulse experiments. After the squeezing pulse the motion shows phase-coherent oscillations, which decay within hundreds

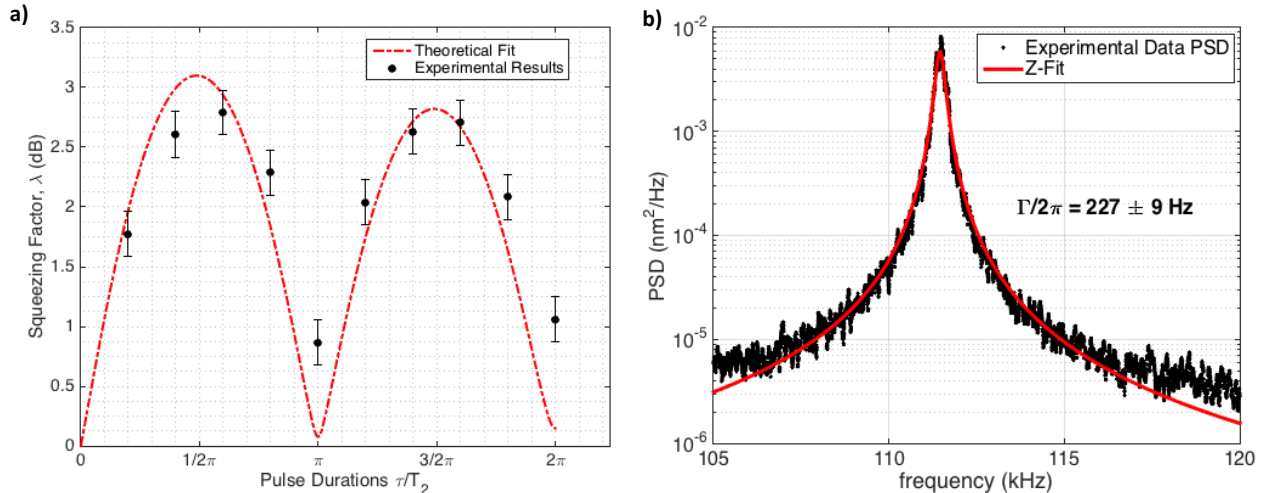


FIG. 3. **Quantitative analysis of the squeezing effect.** a) shows the squeezing factor λ , which is extracted experimentally from comparing the ratio of the minor-major axis of the elliptic state after the pulse to the initial thermal state before the pulse, depending on the duration of the squeezing pulse length τ in units of the oscillation period T_2 . The fit to the data has been done with the model according to Eqn. (10, in the supplement) including phase noise generated by random collisions with background gas. The noise model is described in the supplement [12]. b) shows the Lorentzian model fit to the power spectral density of the z -motion. The fit is used to extract the mass/size of the particle as well as the collisional damping rate Γ according to Eqn.(5).

of microseconds. The rms oscillation decays within about $680\mu\text{s}$ to $690\mu\text{s}$, which gives a rate of thermalisation to the temperature of the bath of background gas molecules by collisions between 230Hz to 234Hz. This is in perfect agreement with the value for Γ as extracted by the Lorentzian fit.

We are operating in the classical regime, in that we observe quadrature variances that are several orders of magnitude larger than the zero-point fluctuations of the mechanical oscillator. Therefore, we may estimate the particle’s momentum by simply taking the time differential of the position measurement. This approach neglects the so-called “collapse of the wavefunction” induced by the measurement, and would require revision in the quantum domain. Applying this strategy to the data set of 1500 repetitive samples we generate the phase space distribution of the trapped particle’s motion as affected by the squeezing pulse. Figs.2a),d) show the distribution of the system before the pulse is applied. This initial distribution we find is nearly Gaussian, while the small asymmetry can be explained with a non-linear response of the measurement to the position for large oscillation amplitudes [13].

After the pulse we observe the typical features associated with squeezing. The applied pulse deforms the phase-space distribution of the particle, which then displays the typical asymmetric shape of a squeezed state where one direction is much more elongated than the other. See Figs.2b),e) for the state after the application of the pulse. This distribution then rotates in phase

space according to the harmonic oscillator evolution, see for instance Figs.2c),f). During this evolution the initial squeezing effect is reduced towards a Gaussian distribution of a thermal state; we attribute this to thermalization via collisions with the background gas.

The squeezing parameter λ , (see Eqn.(4)), depends on the duration τ for which the pulse is applied. The squeeze of the distribution can be completely removed if the duration of the pulse is $\tau = \frac{1}{2}T_2$, with ($T_2 = 2\pi/\omega_2$), while the maximum squeezing is achieved when the pulse lasts for a quarter of one full oscillation, $\tau = \frac{1}{4}T_2$. The experimental results for variable pulse durations is shown in Fig.3a). While the theoretical variation of λ depending on τ agrees qualitatively with the experimental result, the largest squeezing factor we achieve experimentally is only 2.7dB, which is less than the theoretical expected. As anticipated, we can obtain a reasonable fit of the data through an effective pure dephasing model, as shown in the supplement. For the best fit, as shown in Fig.3a) we obtain $\omega_2 = 2\pi \times 53\text{kHz}$ and a dephasing rate $\gamma = 2\pi \times 778\text{Hz}$, which were kept as free parameters. This indicates that the motion of the particle during the squeezing operation is affected by further noises, the closer analysis of which is the focus of ongoing research.

Conclusion– The demonstrated squeezing technique could be used for enhanced sensing and metrology based on levitated optomechanics such as for force sensing applications [14] and non-equilibrium dynamics studies [13]. The quantum regime—and therefore truly quantum squeezing—may be approached if the motional state

can be initially prepared by cooling [15–17] which has recently resulted in centre of mass motion temperatures of trapped nanoparticles at around 1mK [10, 11, 18] or by state preparation using quantum measurement techniques [19, 20] which have been demonstrated for membrane and cantilever optomechanical devices [21, 22]. Future work will include the investigation of an appropriate measure to quantify the non-classical nature of the squeezed state achieved.

Finally, we would like to comment our measurement scheme, which relies on a continuous observation of the particle’s position. At first, this might appear to be undesirable as the quantum regime is approached. Yet it has recently been shown that, if correctly accounted for, this type of measurement may in fact improve the achievable mechanical squeezing [20].

Acknowledgements– We would like to thank for funding the EPSRC [EP/J014664/1], The Leverhulme Trust [RPG-2016-046], the John F Templeton Foundation [39530] and the Foundational Questions Institute [FQXi]. MSK thanks Danny Segal for discussions and a financial support by the EPSRC (EP/K034480/1) and the Royal Society.

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SUPPLEMENT: NOISE MODEL

In this section we refine our theoretical description to take into account phase diffusion of the mechanical oscillator. During the squeezing pulse, we will assume that the dynamics of mode \hat{b} is dictated by a *pure dephasing* [23] master equation of the form ($\hbar = 1$)

$$\dot{\rho} = -i[H_2, \rho] - \frac{\gamma}{2}[\hat{b}^\dagger \hat{b}, [\hat{b}^\dagger \hat{b}, \rho]], \quad (6)$$

where $H_2 = \omega_2 \hat{b}^\dagger \hat{b}$, ρ is the density matrix of the oscillator, γ is the phase diffusion rate and $[\hat{A}, \hat{B}] \equiv \hat{A}\hat{B} - \hat{B}\hat{A}$ the commutator. It is convenient to formulate the problem in terms of covariance matrices [24]. Since our state is initially thermal, it is easy to check that $\langle \hat{a} \rangle = \langle \hat{b} \rangle = 0$ throughout the dynamics. We can thus define the covariance matrix of mode \hat{a} simply as

$$\sigma_a = \begin{pmatrix} 2\langle \hat{X}^2 \rangle & \langle \hat{X}\hat{P} + \hat{P}\hat{X} \rangle \\ \langle \hat{X}\hat{P} + \hat{P}\hat{X} \rangle & 2\langle \hat{P}^2 \rangle \end{pmatrix}, \quad (7)$$

where $\langle \hat{A} \rangle \equiv \text{Tr}[\rho \hat{A}]$ for any operator \hat{A} . Note that in our convention the covariance matrix of the vacuum state has unit determinant. We assume an initial thermal state of mode a , characterized by the covariance matrix

$$\sigma_a(0) = \begin{pmatrix} 2N_1 + 1 & 0 \\ 0 & 2N_1 + 1 \end{pmatrix}, \quad (8)$$

$$N_1 \equiv \frac{1}{e^{\frac{\hbar\omega_1}{k_B T}} - 1}, \quad (9)$$

where T is the absolute temperature and k_B Boltzmann's constant. The subsequent dynamics of the covariance matrix, as induced by Eq. (6), is easily determined by noticing that $\langle \hat{b}^\dagger \hat{b} \rangle$ does not depend on time, while $\langle \hat{b}(\tau)^2 \rangle = \langle \hat{b}(0)^2 \rangle e^{-i\omega_2 \tau - 2\gamma \tau}$. Exploiting the known relationship between \hat{a} and \hat{b} , we thus obtain the following matrix elements for $\sigma_a(\tau)$

$$[\sigma_a(\tau)]_{11} = (2N_1 + 1) \left[\frac{1 + c(\tau)}{2} + \frac{\omega_1^2}{\omega_2^2} \frac{1 - c(\tau)}{2} \right], \quad (10)$$

$$[\sigma_a(\tau)]_{22} = (2N_1 + 1) \left[\frac{1 + c(\tau)}{2} + \frac{\omega_2^2}{\omega_1^2} \frac{1 - c(\tau)}{2} \right], \quad (11)$$

$$[\sigma_a(\tau)]_{12} = (2N_1 + 1) s(\tau) \frac{\omega_1^2 - \omega_2^2}{2\omega_1 \omega_2}, \quad (12)$$

$$[\sigma_a(\tau)]_{21} = [\sigma_a(\tau)]_{12}, \quad (13)$$

where $s(\tau) \equiv e^{-2\gamma \tau} \sin(2\omega_2 \tau)$, $c(\tau) \equiv e^{-2\gamma \tau} \cos(2\omega_2 \tau)$. To assess the amount of squeezing that has been applied to our mechanical oscillator, we define the quantity

$$\mu_{\min}(\tau) \equiv \text{smallest eigenvalue of } \sigma_a(\tau), \quad (14)$$

which quantifies the variance in the squeezed quadrature. This has to be compared with $\mu_{\min}(0) = 2N_1 + 1$, the initial variance in the oscillator quadratures at thermal equilibrium. We can thus define a squeezing parameter (in dB units):

$$\lambda = -\frac{1}{2} 10 \log_{10} \left(\frac{\mu_{\min}(t)}{2N_1 + 1} \right), \quad (15)$$

where the factor of 1/2 is due to the fact that squeezing is defined in terms of standard deviations, rather than variances.