

1                   Establishing the validity of cycle path  
2                   capacity assumptions in the Highway  
3                   Capacity Manual

4                   Chris Osowski and Ben Waterson

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6                   **Abstract**

7                   Cycle mode share increase is widely desired, but highway design practitioners lack the numerical tools to deliver infrastructure, instead relying on  
8                   design standards and intuition, with little literature basis. As a case in point,  
9                   the US Highway Capacity Manual (which is well-used internationally) has  
10                  developed levels of service for cycle infrastructure which are, at their core,  
11                  based on an assumption of non-interaction between multiple cyclists.

13                  This paper uses a modified implementation of the Social Force Model  
14                  to test the validity of this assumption. Necessary changes such as the  
15                  consideration of acceleration characteristics and minimum maintainable speed  
16                  are included. The resulting model produces valid outcomes in keeping with  
17                  established traffic flow properties reflecting three-phase traffic flow theory  
18                  and the ability for the stochastic elements in traffic flow to cause flow break-  
19                  down.

20                  The developed simulation indicates that there is a fundamental differ-  
21                  ence in outcome if cyclists are assumed to have a fixed-speed versus one  
22                  they can change given their surroundings. This difference in outcomes is  
23                  found to exist within the range of literature design flow capacities for bicycle  
24                  infrastructure and also yields emergent outcomes which align closely with  
25                  those known behaviours of highway vehicles which intuitively transfer to cy-  
26                  clists. These findings reinforce the standing need for large-scale empirical

27 studies to determine the basic numerical and behavioural parameters for  
28 cyclists, upon which all design ultimately rests.

29 **1 Introduction**

30 A desire to increase the mode share contribution of cyclists is widely-held  
31 (e.g. City of Copenhagen, 2011; Greater London Authority, 2013; New  
32 York City Department of Transport, 2008) both as a means to achieving  
33 CO<sub>2</sub> emissions reductions (e.g. Walsh, Jakeman, Moles, & O'Regan, 2008)  
34 and as a means to reduce the impact of congestion in urban areas (e.g.  
35 Buehler, 2012). If such objectives are to be realised then cycling, and sup-  
36 porting planning operational and economic evaluation tools, have an impor-  
37 tant underpinning role to play in the development of the built environment  
38 required to enable it (Willis, Manaugh, & El-Geneidy, 2014).

39 Modelling and simulation tools for highway motor traffic (e.g. SATURN,  
40 Atkins, 2013; Aimsun, Transport Simulation Systems, 2014) and pedes-  
41 trian spaces (e.g. Legion, Legion Ltd., 2013; Viswalk, PTV, 2013c) are  
42 well established in traffic engineering and are widely used. Consequently,  
43 when properly calibrated, they have become a standard expectation of the  
44 planning process (e.g. City of Portland Oregon, 2009; Transport for Lon-  
45 don, 2010; Transportation Research Board, 2010) and an important tool for  
46 the transport planner. Amongst other benefits, simulation tools allow the  
47 ex ante testing and quantification of scheme designs where implementing  
48 multiple scheme options is either infeasible or prohibitively expensive.

49 Yet by contrast, models for the analysis and development of cycle schemes  
50 are not similarly established. Empirically-based literature relating to the  
51 basic principles of cyclist–cyclist interaction is non-existent. Practitioners  
52 usually develop cycle schemes on an essentially qualitative basis (e.g.  
53 American Association of State Highway and Transportation Officials, 1999;  
54 CROW, 2007; Department for Transport, 2008) and often to arbitrary ge-  
55 ometric constraints, with operational value for money measures and the  
56 ability to rank comparable schemes by effectiveness, often absent. There  
57 is therefore, a standing need for the development of acceptable cycle nu-  
58 mercial modelling tools.

59 A wide review of the academic state-of-the-art of cycle modelling is pre-  
60 sented in Twaddle, Schendzielorz, and Fakler (2014). This will not be repli-  
61 cated here, suffice to note that the academic situation is similarly immature  
62 as the industrial. Examples referred therein, such as Schönauer, Stuben-  
63 schrott, Huang, Rudloff, and Fellendorf (2012), also serve to reinforce the  
64 view that the academic state of cycle modelling, whilst developing, is not  
65 yet ready for wide-scale practical use.

66 Despite this limited research grounding, the Highway Capacity Manual  
67 (Transportation Research Board, 2010) presents a level of service (LoS)  
68 measure for cyclists based, at its core, on Botma (1995). Whilst other (pri-  
69 marily subjective) data feed into the LoS measure (Barker, Biehler, Brown,  
70 Clark, & Ekern, 2008; Routhail, Hummer, Milazzo II, & Allen, 1998), the key  
71 quantitative measures remain those presented in Botma (1995). LoS are a  
72 concept based primarily on the emergent behaviour of traffic at increasing  
73 flow levels which results from the interaction of the vehicles with one an-  
74 other. However, as Botma (1995) is explicitly grounded in an assumption  
75 of non-interaction of cyclists with one another beyond a simple statistical  
76 quantification of meetings/passings (“events”), any qualitative adjustment  
77 is potentially going to be based on a core inaccuracy.

78 This paper takes the Social Force Model (SFM) of a form originally pro-  
79 posed for pedestrians by Helbing and Molnár (1995) and transfers the core  
80 principles to cyclists, developing them where necessary. The SFM is an es-  
81 tablished and validated model which underpins the operation of the major  
82 pedestrian microsimulation software packages on the market (i.e. VisWalk,  
83 and to a lesser extent Legion) and has been proven through validation and  
84 calibration (and subsequent use) to be an effective tool for the design and  
85 evaluation of pedestrian schemes. A valid model for cyclists developed in  
86 this manner could therefore be similarly calibrated against real cycle be-  
87 haviour and thus become an appropriate tool for the analysis and develop-  
88 ment of cycle schemes and interventions.

89 The application of the model presented here is to the situation consid-  
90 ered in Botma (1995). A key part of the (limited) body of quantitative practi-  
91 tioner literature indirectly depends upon this and whilst one could perhaps  
92 intuitively consider the issue at high flow levels (only so many bikes can

93 pass each other at once on a given width of roadway), this has not been  
94 shown in the literature. Indeed, we show here that even at trivial flow lev-  
95 els, there is a non-zero chance of being prevented from passing/overtaking.  
96 The inclusion of an ability of the rider to select speed (as in reality), shows a  
97 non-linear relationship with both hysteresis and a step-change toward con-  
98 gestion at high flow levels. This finding is in keeping with observed motor  
99 vehicle traffic and theory, and pedestrian observation and theory – both of  
100 which can be reasonably assumed to have some transferability to cycling.

101 Given the industrial reach of the Highway Capacity Manual and that it  
102 depends quantitatively on that non-interaction assumption and the criticality  
103 of a flow breakdown point in design terms, this paper provides a basis  
104 and direction for further research to establish (amongst other quantitative  
105 parameters) actual capacity bounds on cycle infrastructure.

## 106 **2 Background**

107 Simulation modelling in the field of transport is well-established and it is  
108 reasonable to assume that principles already validated for other modes are  
109 likely (to some degree) to be of use in informing the simulation of bicycle-  
110 only (and potentially mixed-modes) flows.

111 A number of well-studied highway traffic models involving vehicle speed,  
112 capacity and service quality are in existence (e.g. Atkins, 2013; PTV, 2013a,  
113 2013b; SIAS, 2000). However, vehicle traffic models generally operate by  
114 separating traffic into lanes and thus break the model down into a series  
115 of related but separate one-dimensional models. Other workstreams re-  
116 lating to heterogeneous traffic (e.g. Agarwal & Lämmel, 2016; Arasan &  
117 Koshy, 2005; Pandey, Ramachandra Rao, & Mohan, 2015), are in the early  
118 stages of refinement but are focused upon validation for mixed traffic, and  
119 so may have limited applicability to the operation of segregated homoge-  
120 neous cycle-only infrastructure. Such infrastructure is often considered  
121 more appropriate when motor volumes are high, and depends fundamen-  
122 tally upon the direct interactions of cyclists with one another.

123 Ideally, a robust literature background relating to cyclist behaviour/interaction  
124 would be established upon which a model could be constructed. However,

125 no such literature exists. Some literature exists relating to individual cyclists  
 126 (e.g. vehicle interaction, Parkin & Meyers, 2010; or speed profiles, Parkin  
 127 & Rotheram, 2010), however literature does not exist where the case of  
 128 the specific interaction of multiple cyclists with one another is concerned.  
 129 To take the simple example of bicycle infrastructure capacity (which we  
 130 will show in this paper depends fundamentally on the interaction of users),  
 131 compare the literature values for capacity presented in Table 1; note that  
 132 these vary by multiple orders of magnitude. The interaction of bicycle users  
 133 with one another is therefore an under-explored research area.

Table 1: Literature capacity measures of unidirectional cycle-only infrastructure

Source	Use	Type	Capacity (bicycles per metre per hour)
(CROW, 2007)	Design Std: Netherlands	Empirical	75-187.5 (width dependent; comfort); 2611–3300 (width dependent; capacity)
(Botma, 1995)	Design Std: USA	Theoretical	650
(Vejdirektoratet, 2012)	Design Std: Denmark	Theoretical	Path up to 2.0m: 1000 Path over 2.0m: 1500
(Navin, 1994)	None	Empirical (with theoretical extrapolation)	4000

134 In contrast to the highway traffic models noted above, bicycles share  
 135 some behavioural traits with pedestrians in that they essentially operate in  
 136 continuous two-dimensional space. In the area of pedestrian modelling,  
 137 and originally constructed by Helbing and Molnár (1995), the Social Force  
 138 Model has become an established tool for the modelling of pedestrian  
 139 movements. The two major commercial pedestrian microsimulation pack-  
 140 ages in wide use, PTV VisWalk (part of the Vissim package; PTV, 2013c)  
 141 and Legion (Legion Ltd., 2013) both depend on the principles established

142 by Helbing and Molnár (VisWalk directly, and Legion by way of distillation  
143 in Still, 2000). Their validity (when properly calibrated) has led to their wide  
144 use in industry and indeed often, an expectation of their use in informing  
145 scheme designs (e.g. Transport for London, 2010). Given this, a model  
146 based upon the SFM core principles is demonstrated in this paper.

147 Development of a simple model allows the testing of the basic assumption  
148 underpinning Botma (1995) which is that passing/overtaking cyclists do  
149 not impede/interact with one another, and that any such event is able to be  
150 ignored for all but the most extreme rates of flow. Furthermore, despite that  
151 paper presenting its capacity measures as a “proposal” with “preliminary  
152 character”, those measures have seen their adoption into the core literature  
153 used (internationally) by the practitioner – albeit with some caveats  
154 (Rouphail et al., 1998)<sup>1</sup>. Given that the only real quantitative capacity mea-  
155 sures in use by practitioners stem from that paper (the other main source  
156 being CROW, 2007, which focuses on user comfort not absolute capac-  
157 ity<sup>2</sup>), this is an important assumption to validate. Indeed, Botma makes  
158 the point that such testing is required, however no such consideration has  
159 occurred in the literature to date.

## 160 2.1 The Social Force Model

161 The model presented here adopts the core principles of the SFM (as pro-  
162 posed originally by Helbing & Molnár, 1995) whilst simultaneously making  
163 the necessary modifications to ensure applicability to bicycles. Given there  
164 are modifications being applied, a number of different models could have  
165 been chosen as a base. The SFM was chosen as it is widely-studied, un-  
166 derpins the VisWalk package of Vissim (PTV, 2013c) which is well-used for  
167 pedestrian modelling, and operates in continuous two-dimensional space.

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<sup>1</sup>Rouphail et al. (1998) note that the numbers considered are not absolute capacity measures but the interpretation is the equivalent of considering a highway maximum operational flow to not be the measure of moving vehicle capacity, in contrast to its capacity as a parking lot of the same dimension.

<sup>2</sup>Whilst a theoretical capacity of 3300 bicycles per hour for a cycle path of 1m width (and 4700 bicycles per hour for a 1.8m path) is noted at one point in CROW (2007), the parts of that document focus on the infrastructure, revolve around the comfort values.

168 The contribution of this paper is the principle of the systematic modelling of  
169 cycle flows at all, not simply our methodology.

170 The SFM operates based on agents navigating a force-field like quantity  
171 generated by the presence of the agents in that field (the ‘social field’,  
172 Lewin, 1951). The force represents a metaphorical “motivation to act”; for  
173 example, to avoid other agents, to stay with your group, or to avoid fixed  
174 objects in the environment. For a full discussion of the basic SFM, the  
175 reader is directed to Helbing, Farkas, and Vicsek (2000); Helbing and Johansson  
176 (2013); Helbing and Molnár (1995). As a summary, the details of  
177 the various aspects of the SFM model, the acceptance or rejection of their  
178 relevance, and signposting to the specific sections addressing each, are  
179 noted in Table 2.

### 180 **3 Model Theory**

181 As noted in Section 2, a literature backing for the fundamentals of cyclist in-  
182 teractions simply does not exist. The following Section therefore discusses  
183 and presents the theoretical basis upon which modifications to the core  
184 SFM have been made.

#### 185 **3.1 Bicycle Force Generation**

186 The spatial arrangement of the force generated by pedestrians is assumed  
187 in the SFM to take the form of an ellipse projected in the direction of travel  
188 with foci separation equivalent to one walking step length. Pedestrians  
189 themselves are considered point objects and make decisions based on an  
190 instantaneous perception of the current Social Force field which notionally  
191 reflects the probable future arrangement in the spatial distribution. How-  
192 ever, changing speed or direction is essentially trivial for the pedestrians;  
193 i.e. acceleration is unconstrained. For sufficiently long time steps (in the  
194 order of a second or more) this may be essentially true, but if time steps  
195 are small (especially if substantially <1s) then this starts to become less  
196 realistic.

197 For bicycles, the comparable definition of “sufficiently long” is likely to

Table 2: Comparison, acceptance and rejection of underlying properties of the pedestrian SFM

Principle	Acceptance	Rationale
Agents move through, and due to, a socially generated force field	Accept	Principle is transferable to cyclists if additional kinematic constraints are addressed.
Force field is vectoral	Reject	Vector forces result in potential ‘pushing’ force. Unlikely to be experienced by cyclists in usual operation. See Section 3.1.
Agents are attracted to a waypoint/destination	Partial accept	Agents in our model will continue in their direction of travel until perturbed otherwise. This does not necessitate a destination, though is not incompatible with one.
Agent speeds ‘relax’ to their desired velocity.	Reject	Relaxation implies unbounded acceleration/deceleration. The time frame for speed changes is non-trivial for cyclists. Acceleration is applied based on standard kinematics. See Section 3.4.
Social force spatial profiles are ‘near-future’ distributions of possible location.	Reject	Cyclist speeds and thus time considerations are longer. If accepted then spatial force distributions would be too diffuse to be useful. Future locations are therefore computed by the agents. See Sections 3.1 and 3.4.
Agents are compressible	Reject	Cyclists in contact would likely have crashed. Proximity of cyclists therefore exhibits a step-change. See Section 3.1.
Agents can be attracted to one another (e.g. family/friend groups)	Accept	Equally valid for cyclists as for pedestrians.
Agent perception is variable with angle of view	Partial accept	Concept in original SFM is variable with angle but always non-zero. Rearward visibility for cyclists is much more difficult. See Section 3.3.
Agent speeds vary continuously down to zero	Reject	Below a certain speed, maintaining balance on a bicycle is more difficult, imposing a de facto lower cap on speed with a step down to zero.
Agent speeds are bound at a maximum	Accept <sub>8</sub>	Given the lack of pushing forces noted above, this is accepted and is equal to the cyclist’s desired speed. See Section 3.4.

198 be longer than that for pedestrians (given higher speeds) and thus longer  
199 than is reasonable for the purposes of modelling the interactions. This time  
200 frame is limited (in the absolute) by the ability of the rider to deliver power  
201 but, given this time length is proportionally long compared to the time frame  
202 of interactions, the constraint of the rate of speed change (i.e. acceleration)  
203 must be considered for bicycles.

204 Furthermore, given the higher speeds of cyclists (relative to pedestri-  
205 ans), it is necessary to separate the future situation from instantaneous  
206 perception as cyclists need to realistically consider a greater time into the  
207 future than is the case for pedestrians. Consequently in this model, bi-  
208 cycles do not consider their exposure to forces at the point which they  
209 currently occupy, but consider the forces at a range of projected poten-  
210 tial future locations based on extrapolation of where other bicycles will be  
211 at the relevant time (given an assumed continuation of other bicycles' cur-  
212 rent behaviour until that time in the future) and the kinematic constraints  
213 of the considering agent's current speed and possible range of turn prior  
214 to that future point. Future force returns at those potential locations are  
215 discounted with increasing time to account for the uncertainty in future de-  
216 cisions of the other bicycles; essentially producing a net present value for  
217 the force on each considered path choice (in essence, a 'net present force';  
218 see Section 3.4). Force returns include whichever repulsive and attractive  
219 forces are present in the given simulation, although for simplicity all attrac-  
220 tive forces are set equal to zero (i.e. there is no grouping of cyclists or  
221 waypointing) in the demonstration that follows in Section 5 onwards. Cy-  
222 clists navigate the force field, seeking vectors that minimise their overall  
223 exposure to repulsive force as calculated in each time step.

224 Simply modifying the SFM's spatial force projections, which embody the  
225 uncertainty about the next few steps of the pedestrian, to fit the above pa-  
226 rameters (i.e. projecting through an extended time step) would not produce  
227 acceptable outcomes owing to the extensive and diffuse spatial distribu-  
228 tion of the projected force that would result given the speed, extended time  
229 consideration and potential range of motion of the cyclist.

230 Bicycle force generation is proposed to be generated with an exponen-  
231 tial distribution akin to the SFM but with an additional overlay reflecting the

232 cyclist envelope. Pedestrians in the SFM are treated as point objects (Hel-  
233 bing & Molnár, 1995); by contrast, bicycles cannot compress one another  
234 (Helbing, Buzna, Johansson, & Werner, 2005) nor rub against one another  
235 (Helbing et al., 2000) and simultaneously maintain the ability to cycle (com-  
236 pared to pedestrians which to some extent, are ‘compressible’). Physical  
237 contact between cycles is almost certainly, by definition, a crash and is not  
238 recoverable in trivial time. The equivalent in a pedestrian model would be  
239 pedestrians physically falling over one another. Clearly, such a mode of  
240 operation is not appropriate for ‘usual’ operation and thus is not proposed  
241 to be transferred to the bicycle model here. As an alternative, it is pro-  
242 posed to overlay a force equal to the bicycle maximum<sup>3</sup> over the spatial  
243 area of the bicycle (i.e. the rectangular area in Figure 1). For simplicity,  
244 this is approximated to a rectangle with dimensions equating to a bicycle.  
245 Such a dimension incorporates the rider, clearance for steering and po-  
246 tential panniers etc. For the remaining bicycle force, elliptical spatial force  
247 profiles are proposed centred on the cyclist (and orientated longitudinally;  
248 Figure 1b) reflecting the inappropriateness of a circular approximation for a  
249 bicycle/cyclist combination (see Section 4.4).

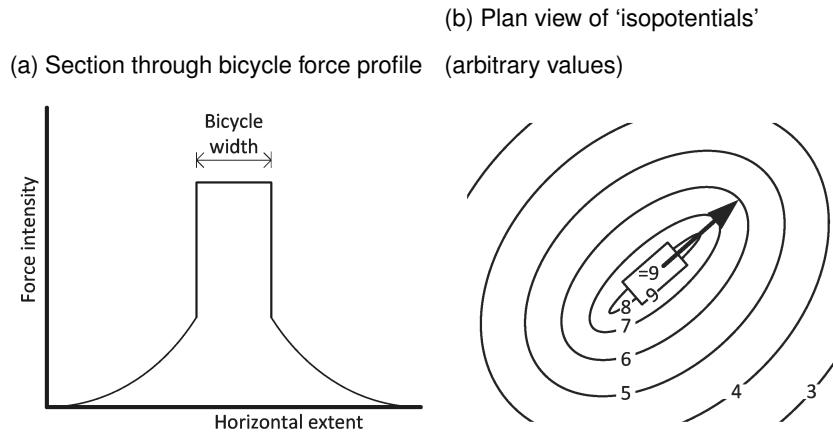
## 250 **3.2 Boundary Force Generation**

251 Boundary exponential force distributions (as per the original SFM) result in  
252 a model in which the ultimate calibrated balance of forces is sensitive to  
253 path width as an exponential distribution has infinite extent. To avoid this,  
254 and given that cyclist boundary effects are currently poorly understood, a  
255 simple linear function for force is proposed (Section 4.5) which decreases  
256 with distance from the boundary and reaches zero at a given distance (i.e.  
257 approximating edge-shyness; Department for Transport, 2008). Thus, the  
258 balance of the interactions with other bicycles and with the boundaries is  
259 broadly decoupled from the path width for the purpose of this model, with  
260 the exception of those in close proximity to the path edge where avoidance  
261 behaviour is as expected.

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<sup>3</sup>I.e. that which would be calculated at a 0.0m from the location of the bicycle. The exact number is a function of the parameters chosen.

Figure 1: Illustrative force profiles (arbitrary scales/values)



### 3.3 Force Perception

The SFM considers the potential of a view-cone and Helbing and Molnár (1995) proposed a reduction factor of 0.5 for pedestrians outside of it (i.e. to the rear). However, combined with the directionality of the force, the result is a ‘pushing’ force from a pedestrian approaching from behind. Helbing and Molnár were primarily concerned with crowd crushes wherein such pushing forces are explicitly non-trivial. Future development to the SFM (e.g. Helbing et al., 2005) revised this to a continuous anisotropic  $\lambda$  parameter, however, for the purposes used here, a simple augmented zonal view-cone is sufficient when considering the issue from first-principles.

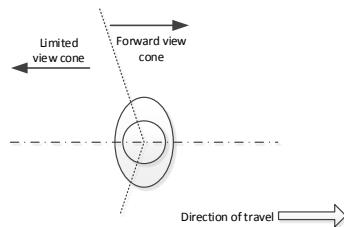
For bicycles in ‘usual operation’, a simple reduction by half of the perceived effect of other cyclists to the rear is not realistic for two reasons. Firstly, a reduction factor in the order of a half for anywhere outside the view-cone is suggested for pedestrians, but it requires substantially more effort to look directly behind safely whilst cycling than is the case for walking, thus reducing the ability to perceive such a force. Secondly, it is debatable whether a cyclist would feel any non-trivial requirement to cycle substantially faster due to a perceived social pressure of cyclist(s) behind them. The second issue is alleviated through the use of a scalar force field which, given the proposed directional choice algorithm, does not fundamentally change the operation of the model. With regard to the reduction

283 factor, three different perception profiles are proposed (see Figure 2):

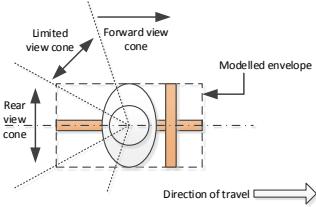
- 284 • If the perceived bicycle is within the forward view-cone, then the full  
285 force effect of that bicycle is perceived. This area requires no effort to  
286 view on behalf of the perceiving cyclist.
- 287 • If the perceived bicycle is within one of the side view-cones then no-  
288 tionally, this would require the cyclist to turn his or her head to observe  
289 them. This requires effort on behalf of the cyclist and may result in  
290 them not being observed if the cyclist is otherwise engaged. This  
291 equates with the side blind-spots for a motor vehicle driver. As a  
292 result, the effect of bicycles present within these view-sectors is re-  
293 duced.
- 294 • If the perceived bicycle is not within the side or front view-cones, then  
295 the cyclist would have to turn their head and torso simultaneously to  
296 look behind; a difficult task on a bicycle whilst maintaining full control.  
297 Consequently, the effect of these bicycles are perceived less than the  
298 side view-cones (if at all).

Figure 2: Illustrative view cones

(a) Original Social Force Model



(b) Proposed in this model



299 Perception of the force field ahead of the cyclist is condensed to a 'net  
300 present force' on each potential vector choice. The vector with the least  
301 repulsive (or most attractive) net present force is selected. This ensures  
302 that cyclists react most to obstacles close to them rather than those further  
303 away and that the greater certainty of the imminent situation is reflected  
304 in the force perception. As each cyclist makes this consideration once per

305 time step, the cyclist can react appropriately to the changing circumstances  
306 around them, as they unfold.

### 307 **3.4 Bicycle Speed Selection**

308 Pedestrian speed in the SFM is only bounded at the maximum; i.e. pedes-  
309 trians can move at very low speeds. Whilst this may be realistic for pedes-  
310 trians who can essentially reduce step length and walking cadence contin-  
311 uously to zero, cyclists are limited by a minimum speed below which they  
312 cannot maintain balance on the bicycle and (usually) react by lowering one  
313 or both feet to the ground.

314 Acceleration is proposed to be applied to the bicycle using the appro-  
315 priate kinematic equations of motion (assuming uniform linear acceleration  
316 during that time step) with resulting speeds capped at the desired maxi-  
317 mum and by the minimum sustainable (which if the speed would fall below,  
318 then a ‘foot down’ stopping state is activated). For this model, accelera-  
319 tion is simply assumed to be invariant with speed as there is a lack of any  
320 empirically-backed literature to the contrary.

321 Speed selection therefore takes place realistically rather than being ab-  
322 stracted to a “relaxation parameter” as in the original SFM. Acceleration in  
323 the model is modified by the cyclists’ perception of the force field ahead of  
324 them on their chosen path. This too replicates real behaviour where speed  
325 selection is achieved through the application of braking or by acceleration.

## 326 **4 Model Implementation**

### 327 **4.1 Model Structure**

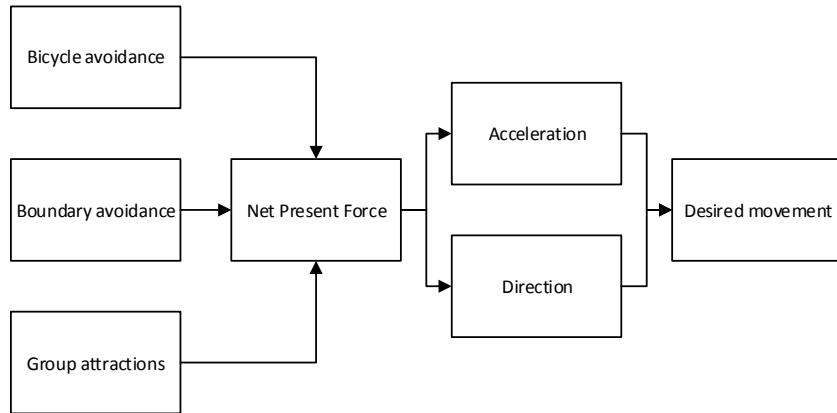
328 The model for each cyclist agent follows the structure detailed in Figure 3  
329 which parallels the SFM as formulated in Helbing and Molnár (1995). How-  
330 ever in structural terms, it should be noted that the complexities of cycle  
331 behaviour over those of pedestrians mean that the proposed model is not  
332 entirely built upon mathematical abstraction (as per the SFM); in common  
333 with many models, some aspects are algorithmic and/or iterative in nature<sup>4</sup>.

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<sup>4</sup>E.g. the rule-based lane selection algorithm in PTV (2009), detailed in Barcelo (2010).

334 Whilst this may impact upon the elegance, for an ultimate use case in sim-  
335 ulation, this does not impact the validity of the proposed model.

Figure 3: Structure of the proposed SFM



336 At the highest level, the model is designed to be run iteratively, where  
337 time is stepped by a fixed amount ( $t_{step}$ ) between iterations, every cyclist is  
338 iterated through once at each time step (according to the process defined  
339 in Figure 3), and the location/speed of each cyclist is subsequently updated  
340 once per time step (once all other calculations are complete).

## 341 4.2 Desired Movement

342 In the SFM, desired movement is defined numerically as a combined func-  
343 tion of the desired speed and direction of movement, and/or a relaxation to  
344 this. The resultant behaviour of the pedestrian then being affected by the  
345 social force. In this model, the concept of desired movement is emergent  
346 from the tendency, in the absence of any social force, for bicycles to travel  
347 in the desired direction and at the desired speed. Following perturbation,  
348 travel direction returns to the desired and speed returns to desired at a  
349 rate bounded by maximum acceleration. Speed and direction of travel are  
350 independently changed as follows:

351      **4.2.1 Speed**

352      As with the pedestrian SFM, this cyclist SFM gives the cyclist a desired  
 353      speed. For the purpose of modelling and in the absence of experimental  
 354      data, a normally-distributed speed distribution is assumed.

355      Resultant speed is defined as a function of acceleration and speed in  
 356      the previous iteration time step; itself calculated from the perception of so-  
 357      cial force on the chosen route. At each time step, acceleration is applied as  
 358      constant linear acceleration for the duration of the time step, and using the  
 359      relevant kinematic equation of linear motion (Equation 1). Consequently,  
 360      the need for a critical, yet practically abstract, 'relaxation' parameter is  
 361      avoided.

$$v(t) = \begin{cases} v(t - t_{step}) + (a(t) \times t_{step}) & \text{if } \leq v_{desired}; \text{ or} \\ v_{desired} & \text{if } > v_{desired}. \end{cases} \quad (1)$$

362      where  $v(t)$  is the new speed in the current time step,  $v(t - t_{step})$  the speed  
 363      in the previous time step,  $a(t)$  the current acceleration and  $t_{step}$  the length  
 364      of the simulation step.

365      Acceleration is equal to the desired acceleration ( $a_{max}$ ) which is reduced  
 366      based on the overall net present repulsive force perceived on the cho-  
 367      sen direction of travel ( $netPresentForce$ ) adjusted by calibration with a  
 368      *crowdingFactor*, which if sufficient, results in deceleration and thus slow-  
 369      ing (Equation 2). Such a deceleration is bounded by the maximum deceler-  
 370      ation parameter ( $a_{min}$ ). In the absence of any literature to the contrary, ac-  
 371      celeration is considered independently of the speed, though it is accepted  
 372      that this may not be the case in reality.

$$a(t) = \max \left( (a_{max} - (crowdingFactor \times netPresentForce)), a_{min} \right) \quad (2)$$

373      **4.2.2 Direction**

374      Desired direction of movement is defined in the SFM as the direction from  
 375      the current position to the next waypoint/destination. This model proposes

376 similar in the absence of any surrounding social force.

377 However, the pedestrian SFM modifies the direction of travel vector (i.e.  
378 the velocity) instantaneously according to the ambient social force. Here  
379 instead, speed and direction are decoupled and the social force is aggregated  
380 into a net present force, which is perceived on each available direction  
381 of travel. The direction of travel chosen at each time step is simply the  
382 available direction of travel with the most attractive (or least unattractive)  
383 social force exposure. In that sense, motion is an agent ‘choice’ based on  
384 planning, rather than an immediate reaction to the surroundings.

### 385 4.3 Net Present Social Force

386 The concept of a net present force is key to this implementation of the  
387 SFM. The discount to a net present value follows established economics  
388 principles and is defined by Equation 3:

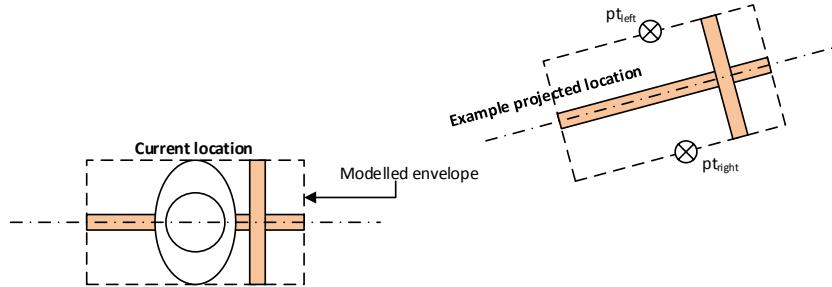
$$389 \text{netPresentForce} := \sum_{u=\text{plan}_{\text{step}}}^{u=\text{plan}_{\text{max}}} \exp \left( -\lambda \times \left( \frac{u}{\text{plan}_{\text{step}}} - 1 \right) \right) \times (\text{force}(\text{pt}_{\text{left}}) + \text{force}(\text{pt}_{\text{right}})) \\ (3)$$

390 where  $u$  is the current planning step ahead being considered,  $\text{plan}_{\text{step}}$  is  
391 the minimum planning step duration considered,  $\text{plan}_{\text{max}}$  is the maximum  
392 time ahead being considered and  $\lambda$  is the decay constant. Force at the  
393 current position is not considered.  $\text{force}(\text{pt}_{\text{left}})$  and  $\text{force}(\text{pt}_{\text{right}})$  are the  
394 returned forces at that planning time step for a point on the left and right  
395 extents of the bicycle envelope on the given vector (see Figure 4). This is  
396 included so the bicycles do not attempt to force their way through gaps that  
397 are too small to pass through, should a single point on that vector happen  
398 to fall between two close objects.

399 The force returned at a given point and time ( $\text{pt}$ ) is the sum of the re-  
400 pulsive force of all other bicycles ( $\text{force}_{\text{bicycle}}$ ), boundaries ( $\text{force}_{\text{boundary}}$ )  
401 and any attractive forces from other bicycles ( $\text{forceAttraction}_{\text{bicycle}}$ ; Equa-  
402 tion 4):

$$403 \text{force}(\text{pt}) := \sum_i^i \text{force}_{\text{bicycle}_i} + \sum_j^j \text{force}_{\text{boundary}_j} + \sum_k^k \text{forceAttraction}_{\text{bicycle}_k} \\ (4)$$

Figure 4: Indicative example of arrangement of  $pt_{left}$  and  $pt_{right}$  used in force computation



#### 4.4 Bicycle Repulsive Force Generation

403 The repulsive force of each bicycle is generated according to Equation 5:

$$404 \quad force_{bicycle}(pt) := \begin{cases} 405 \quad forceScale_{bicycle} \times \exp\left(\frac{bikeWidth - b}{forceSpread_{bicycles}}\right) \\ 406 \quad \text{if } pt \text{ is not within the bicycle spatial envelope; or} \\ 407 \quad forceScale_{bicycle} \times \exp\left(\frac{bikeWidth}{forceSpread_{bicycles}}\right) \\ 408 \quad \text{if } pt \text{ is within the bicycle spatial envelope.} \end{cases} \quad (5)$$

409 where  $pt$  is the point for which force is to be calculated and  $b$  is the semi-minor axis length of the elliptical ‘isopotential’ coincident with that point (Figure 1b).

$$410 \quad b := \frac{\sqrt{(df_1 + df_2)^2 - (df_{inter})^2}}{2} \quad (6)$$

411  $b$  is calculated as per Equation 6 where  $df_1$  is the distance from the point of interest to the rear focus,  $df_2$  is the distance from the same point to the front focus and  $df_{inter}$  is the inter-foci distance.

## 4.5 Boundary Force Generation

Boundary forces are generated according to a simple linear function (Equation 7):

$$force(pt) := \max \left( forceScale_{boundary} - (forceSpread_{boundary} \times d), 0 \right) \quad (7)$$

where  $d$  is the perpendicular distance from the point of interest to the boundary concerned. If repulsive forces are defined to increase in the positive (i.e. ‘more’ force is repulsive), then the maximum value is chosen as shown here. If repulsive forces are defined to increase negative (i.e. ‘more’ force is attractive), the reverse is true. Care must be taken to ensure consistency of signing (i.e. boundaries should always be repulsive), however the choice of signing for either repulsive or attractive force as positive, is the choice of the modeller.

## 4.6 Bicycle Attractive Force Generation

The attractive force of each bicycle is generated according to Equation 8:

$$forceAttraction_{bicycle}(pt) := \begin{cases} related_i \times forceAttractionScale_{bicycle} \times \exp \left( \frac{bikeWidth - b}{forceAttractionSpread_{bicycles}} \right) & \text{if } pt \text{ is not within the bicycle spatial envelope; or} \\ 0 & \text{if } pt \text{ is within the bicycle spatial envelope.} \end{cases} \quad (8)$$

Attractive forces are defined in a similar manner to the repulsive forces discussed above, however no attractive force is returned for the area within the spatial envelope of the cyclist. As noted above, this is not a desirable area for another cyclist to seek as it would result in a collision, thus no attractive force should be experienced at that point.

Parameters for the spatial distribution of the attractive force ( $forceAttractionScale_{bicycle}$  and  $forceAttractionSpread_{bicycle}$ ) are defined similarly to repulsive force, save that if repulsive force is defined positive, then attractive force must be negative, or vice versa.

Appropriate calibration ensures groups of ‘related’ cyclists remain apart yet still attempt travel in a proximate group. The existence of an attractive

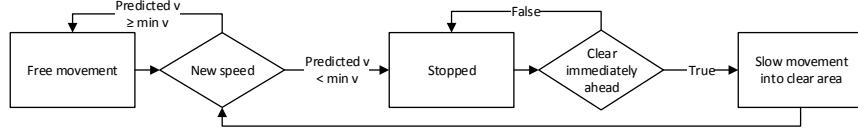
434 force between given bicycles, depends on them being defined in the same  
 435 group or not; *related*, being between 1.0 and 0.0 depending on if a rela-  
 436 tionship exists between the bicycle concerned and the other bicycle being  
 437 considered (i.e. no force is returned if the two cyclists do not have a group-  
 438 ing relationship) and allowing for a tunable strength to that relationship.

## 439 4.7 Stopping

440 The model presented thus far produces outputs (when appropriately cali-  
 441 brated) that appear valid at low densities of flow. However, there is an al-  
 442 ready identified issue (Section 3.4) with bicycles which is not yet included:  
 443 namely, that there is a minimum speed that cyclists can maintain before  
 444 they need to put one or both feet down, and stop.

445 A final consideration must therefore be overlaid upon the resulting new  
 446 speed calculated in each time step. In this regard, the cyclist can be con-  
 447 sidered as a ‘state machine’ illustrated in Figure 5.

Figure 5: Cycle state machine arrangement



448 A cyclist exhibits movement until such time as the speed of the cyclist  
 449 falls below the minimum speed threshold. At this time, the cyclist stops (by  
 450 lowering one or both feet), which given the low speed, can be assumed in-  
 451 stantaneous (i.e. stopping occurs within a single time step). Thus far (and  
 452 in reality), the distance considered ahead is (indirectly) a function of the  
 453 cyclist’s speed — the cyclist considers a given time ahead, a faster cyclist  
 454 therefore considers a greater distance. When the cyclist stops, such a con-  
 455 sideration is no longer appropriate. Instead, the cyclist considers, “Is the  
 456 area immediately ahead clear?” If this is the case then the cyclist moves off  
 457 at a minimum speed for as long as it takes to traverse that distance already  
 458 considered and known to be clear (in this time, the cyclist is regaining their

balance). Once that time has passed, consideration is as the standard behaviour where, if the space ahead is clear, the cyclist will accelerate away; if not, then the cyclist may again slow below the minimum threshold and stop again.

Algorithmically, this represents appropriate cycle behaviour relating to moving off or stop-go queuing and is included here in such a manner as this is an implementation aspect of the model, rather than mathematical. The model retains substantial validity without such a consideration, however as discussed (Section 3.4), the ability of a cyclist to travel at potentially infinitesimal speeds is unrealistic.

## 5 Simulation Design

### 5.1 Implementation and Parameters

An agent-based computer simulation was built which implemented the model described. For simplicity, no grouping of cyclists was assumed (i.e. only repulsive forces were present) and unidirectional flow in two-dimensional space was considered on paths of fixed widths (1.0m–10.0m; in steps 0.1m) and parallel sides and a length<sup>5</sup> of 60m. Bicycles arrive stochastically according to Poisson-distributed arrival intervals (based on a set overall rate for the run) and at random lateral positions in the pathway at rates of 100–5000 bicycles per hour (in steps of 100 bicycles per hour). To reduce the effect of stochastic noise, 25 runs per parameter combination were undertaken.

All bicycles are assumed equal in terms of parameters, the exception being their individually desired speed which is drawn from a Normal distribution with the specified parameters (Table 3). Those, and the other literature parameters used in this implementation, are taken from CROW (2007) which ensures that they have some element of empirical backing. Though perhaps intuitively low (e.g. a mean of  $4.02\text{ms}^{-1}$ ), this source also provide

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<sup>5</sup>Testing of the model indicated that beyond the length used in this paper for this particular scenario (parallel-sided path), output parameters were substantially unchanged, however the run-time of the simulation would rapidly become intractable.

values for standard deviation and acceleration/deceleration, so they can be considered to be at least internally consistent within the given (in this case Dutch) population.

Minimum speed is however taken from Navin (1994) as no such value is included in CROW (2007) and no other literature exists. Model parameters for the intensity of the social forces were chosen using visual observation of the simulation running, using informed approximations (such as the angles of sight utilised) in order to produce outputs commensurate with intuitive and empirical knowledge; e.g. forces were calibrated to allow virtual lanes of approximately 1.0m width to form, in keeping with the dynamic envelope of the cyclist specified in Department for Transport (2008) and elsewhere (though it should be noted the empirical basis for such a value is, in itself, unproven). All parameters used in the preparation of the data presented in this paper are detailed in Table 3.

## 5.2 Simulation Outcomes

The two parameter variables – path width and bicycle arrival rate – are essentially covariant if expressed in terms of flow density. For the purpose of the following, these are therefore combined into a bicycles per metre width per hour (bpmph) measure. Two sets of the parameter sweeps discussed above (Section 5) were completed: one with all agents set such that they could not vary their speed (i.e.  $v(t)$  was fixed to  $v_{desired}$  for each agent), and the other as described above. All other parameters remained fixed. The former scenario therefore uses the assumption from Botma (1995), whereas the latter does not. Should the assumption be valid, we would expect to observe a similar output from both sets across a range of input parameters. The result of this comparison is shown in Figure 6.

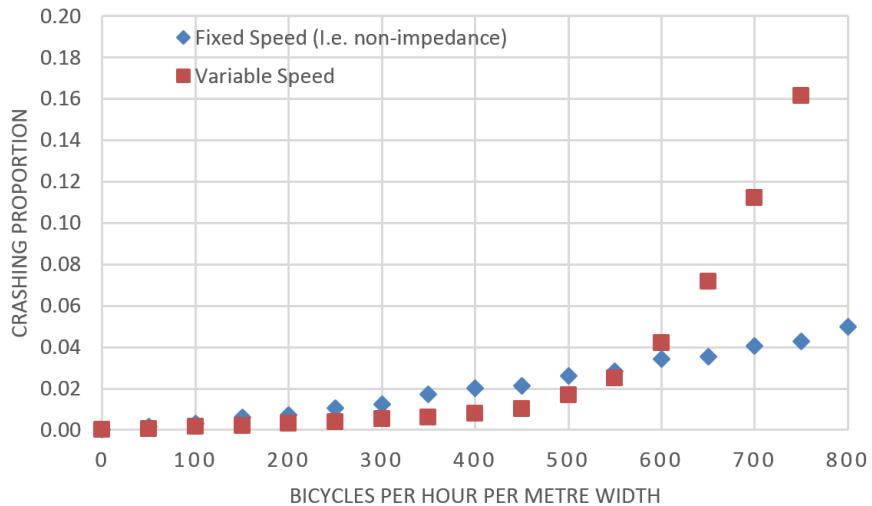
For fixed speed operation, the output indicates a proportion of agents are experiencing a ‘crashing’ state across essentially all arrival rates with proportions steadily increasing. The implication of this being that the interaction of bicycles is potentially non-negligible, even at the low end of flow rates.

For variable speed operation, a fundamentally different outcome is ob-

Table 3: Model Parameters

Parameter	Symbol	Value
Simulation time step	$t_{step}$	0.1s
Simulation length		300.0s
Bicycle traversal length		75.0m
Boundary force spread (Department for Transport, 2008)	$forceSpread_{boundary}$	500
Boundary force scaling	$forceScale_{boundary}$	10000
Bicycle force spread	$forceSpread_{bicycles}$	75
Bicycle force scaling	$forceScale_{bicycles}$	375
Bicycle attractive force spread	$forceAttractiveSpread_{bicycles}$	75
Bicycle attractive force scaling	$forceAttractiveScale_{bicycles}$	0
Bicycle force ellipse foci separation	$df_{inter}$	5000mm
Bicycle width		750mm
Bicycle length		1800mm
Bicycle fleet speed mean (CROW, 2007)		$4.02\text{ms}^{-1}$
Bicycle fleet speed standard deviation (CROW, 2007)		$0.21\text{ms}^{-1}$
Bicycle fleet maximum acceleration (CROW, 2007)	$a_{max}$	$+1.0\text{ms}^{-2}$
Bicycle fleet maximum deceleration (CROW, 2007)	$a_{min}$	$-1.5\text{ms}^{-2}$
Bicycle minimum speed (Navin, 1994)		$0.92\text{ms}^{-1}$
Bicycle maximum steering angle		$40.0^\circ$
Bicycle steering step		$4.0^\circ$
Bicycle forward planning step	$plan_{step}$	0.25s
Bicycle forward max planning time	$plan_{max}$	5.0s
Bicycle forward planning decay constant	$\lambda$	1.0
Bicycle angle of sight		$\pm 100.0^\circ$
Bicycle angle of reduced sight		$\pm 160.0^\circ$
Crowding reaction factor	$crowdingFactor$	0.4
Side reaction factor		0.1
Rear reaction factor		0.0

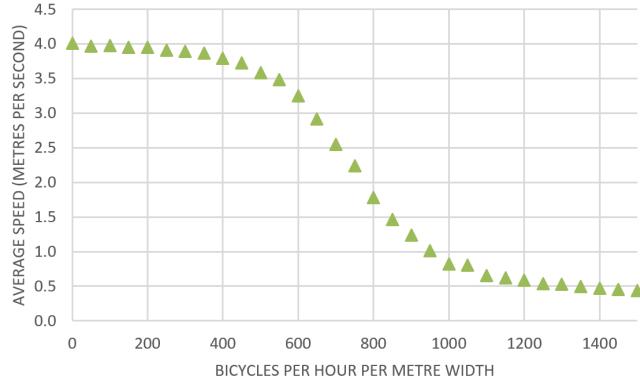
Figure 6: Crash proportions across a range of arrival rates for fixed and variable speed simulation runs



519 served. In this case, crash proportions are approximately zero until an  
 520 inflection point is reached at approximately 550bpmph. A state change  
 521 has occurred and the proportion of agents in a crashing state escalates  
 522 rapidly (much more so than for fixed-speed operation). This corresponds  
 523 with the visual observation of the simulation that congestion has occurred.  
 524 The inflection point therefore indicates a capacity limit to the arrangement.  
 525 Rather than the flow degrading slowly with increasing arrival, the flow de-  
 526 grades sharply. When this happens the shockwave propagates through the  
 527 flow with dissipation only occurring once inflow to the area of breakdown  
 528 has dropped sufficiently. This corresponds with equivalent observations of  
 529 highway traffic and pedestrian traffic.

530 Exploring this further and plotting the average speed against this mea-  
 531 sure, Figure 7 is produced. This clearly demonstrates outputs in keep-  
 532 ing with established three-phase traffic theory. Below inflow rates of ap-  
 533 proximately 500bpmph, speeds are high and close to the cyclists' desired  
 534 speed. Once densities increase, and up to an inflow of approximately  
 535 1000bpmph, flow breakdown is increasingly likely. Beyond 1000bpmph,  
 536 flow breakdown occurs consistently with average speeds being low. Be-

Figure 7: Average bicycle speeds across runs undertaken at given inflows



537      between these two values, some runs will complete without flow breakdown,  
 538      others will experience limited breakdown which resolves, and the remain-  
 539      der will experience flow breakdown which does not resolve. The balance  
 540      between these three states depends on the inflow rate which, as it is stochas-  
 541      tic in nature, means all three phases can potentially exist for a given param-  
 542      eter combination in different locations on the same path.

543      Breakdown tends to occur in the entry area, as inflow is not stopped  
 544      if the inflow area is not clear. In essence, the bicycles are arriving from a  
 545      notionally infinite width space into a fixed width one and consequently, a  
 546      de facto bottleneck. Breakdown at the entry in this manner is paralleled  
 547      in pedestrian behaviour, such as crowding around a door or at a corridor  
 548      entrance (Hoogendoorn & Daamen, 2005).

549      The results of each parameter combination's set of runs is consequently  
 550      bimodally distributed between those runs where speeds were high (i.e. no  
 551      breakdown occurred) and where speeds were very low (i.e. breakdown  
 552      was rapid and ongoing) as flow breakdown often takes substantial time  
 553      to resolve (as in reality). The resulting plot (Figure 7) averages out that  
 554      behavioural detail and thus it must be underlined that for a parameter  
 555      combination such as at 750bpmph, few bicycles will be found travelling  
 556      at  $2.0\text{ms}^{-1}$  at any given time step — instead, this is an average of runs  
 557      comprised mainly of slow/stopped bicycles and those travelling essentially  
 558      unimpeded. The only variable between those runs is the stochastic na-

ture of arrivals and therefore the occurrence of flow breakdown due to the 'noise' in the system, and in the absence of any exogenous factors, parallels real highway traffic behaviour. Similarly, no bicycle will be travelling at a speed less than their minimum but greater than zero (in a given time step), so indicated average speeds beyond 1000bpmph are results of the aggregation of stopped bicycles with those travelling at a speed greater than, in this case,  $0.92\text{ms}^{-1}$ .

Given the inputs, it is important to note that the exact numbers observed are unimportant. However, the inputs to the simulation are based on first-principles and the limited literature that exists, and yield simulation outcomes in the range of values which are expected from that literature (Table 1): at the low end CROW (2007) specifies a comfortable capacity in the range of 75–187.5bpmph; at the high end, Navin (1994) computes a maximum capacity of 4000bpmph.

Most importantly however is the fundamental difference that is observed by the inclusion of speed-changing behaviour. Speed-changing behaviour is explicitly ignored as an assumption in Botma (1995) but is shown here to result in a fundamental shift in outcomes within the range of values which a practitioner is likely to be concerned with.

## 6 Conclusions and Future Work

The lack of appropriate micro-scale bicycle modelling tools is a potential barrier to the assessment and quantification of schemes of various types. Such a barrier has been experienced by pedestrian scheme designers (and highway designers) in the past and the potential exists to utilise the basis of some successful pedestrian tools to produce models which are suitable for the modelling of bicycles. In addition, such bicycle models would have the obvious extension for interconnection with established pedestrian models and thus the modelling of more complex shared-space arrangements than is possible with current tools.

No literature exists establishing the behavioural traits of interacting cyclists and the limited literature that does exist is in disagreement by orders of magnitude over basic key values like infrastructure flow capacity. Simi-

591 larly, empirical data of the scale necessary to establish these parameters  
592 are not available and are expensive to obtain. The clear next step of work  
593 from this paper is for practitioners to validate the theoretically-justified pa-  
594 rameters and outcomes in the context of empirical data.

595 Work to deliver the Highway Capacity Manual took the work of Botma  
596 (1995) and folded those numbers into the fundamental level-of-service mea-  
597 sure often used by highway designers. In the process, the caveats attached  
598 to any assumption in their production have been lost. However, we have  
599 shown here though a modified implementation of the Social Force Model  
600 (as first proposed by Helbing & Molnár, 1995) that the core assumption  
601 that the Botma (1995) paper is founded upon, results in a fundamental  
602 step-change in outputs within the range of values that capacity is consid-  
603 ered.

604 There remains a wide range of research required in order to quantify cy-  
605 cle infrastructure. If there remains an aspiration to produce a robust mod-  
606 elling framework for cyclists (as, for example, the current wave of 'shared-  
607 space' research indicates is the case), then the underlying numerical basis  
608 must be similarly robust. Large-scale empirical data is required over a  
609 range of facilities, and similarly, large-scale studies of individual cycle be-  
610 haviour are required over a range of demographics and nationalities. These  
611 do not currently exist in any form in the literature as they are resource-  
612 intensive to collect, however, if cycle modelling is to become as robust as  
613 motor vehicle modelling, then they are a necessity.

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