Performance of Cognitive Stop-and-Wait Hybrid Automatic Repeat Request in the Face of Imperfect Sensing

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ABSTRACT The cognitive radio (CR) paradigm has the potential of improving the exploitation of the electromagnetic spectrum by detecting instantaneously unoccupied spectrum slots allocated to primary users (PUs). In order to support the process of spectrum reuse, we consider a CR scheme, which senses and opportunistically accesses a PU’s spectrum for communication between a pair of nodes relying on the stop-and-wait hybrid automatic repeat request (SW-HARQ) protocol. This arrangement is represented by the cognitive SW-HARQ (CSW-HARQ), where the availability/unavailability of the PU’s channel is modeled as a two-state Markov chain having OFF and ON states, respectively. Once the cognitive user (CU) finds that the PU’s channel is available (i.e., in the OFF state), the CU transmits data over the PU channel’s spectrum, while relying on the principles of SW-HARQ. We investigate both the throughput and the delay of CSW-HARQ, with a special emphasis on the impact of the various system parameters involved in the scenarios of both perfect and imperfect spectrum sensing. Furthermore, we analyze both the throughput as well as the average packet delay and end-to-end packet delay of the CSW-HARQ system. We propose a pair of analytical approaches: 1) the probability-based and 2) the discrete time Markov chain-based. Closed-form expressions are derived for both the throughput and the delay under the perfect and imperfect sensing environments that are validated by simulation. We demonstrate that the activity of PUs, the transmission reliability of the CU, and the sensing environment have a significant impact on both the throughput and the delay of the CR system.

INDEX TERMS Cognitive radio, primary users, stop-and-wait, HARQ, DTMC, spectrum sensing, imperfect sensing, transmission reliability, throughput, delay, PMF.

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<td>MD</td>
<td>Mis-detection</td>
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<td>NACK</td>
<td>Negative Acknowledgement</td>
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<td>OFF</td>
<td>Markov chain in OFF state</td>
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<td>PEP</td>
<td>Packet Error Probability</td>
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<td>Probability Mass Function</td>
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<td>PR</td>
<td>Primary Radio</td>
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<td>RS</td>
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LIST OF SYMBOLS

- $\alpha$: Transition probability from ‘ON’ to ‘OFF’ state
- $\beta$: Transition probability from ‘OFF’ to ‘ON’ state
- $E$: Average
- $k$: Duration for sensing the TS
- $K_d$: Information Bits
- $\lambda$: Rational Constant
- $M$: Total number of packets to be transmitted
- $M_T$: Maximum delay
- $\mu$: Real status of the PU’s channel
- $N$: Number of packets in a TS
- $v$: Status of the PU’s channel sensed by the CU
- $N_d$: Coded symbols
- $N_{DP}$: Average delay due to busy channel occurring before transmission
- $P$: Transition matrix
- $P_A$: Probability that TS is actually free
- $P_B$: Probability that TS is found free due to mis-detection
- $P_{bus}$: Probability of finding a busy TS
- $P_d$: PMF of end-to-end delay obtained through simulation
- $P_e$: Packet error probability
- $P_f$: Probability of false-alarm
- $P_f$: Probability of finding a free TS
- $P_{i,j}$: $(i,j)$th element of the transition matrix
- $P_{md}$: Probability of channel being mis-detected
- $P_{MF}$: Probability distribution of end-to-end packet delay
- $P_{off}$: Probability of the PU’s channel being free from the PUs
- $P_{on}$: Probability of the PU’s channel being occupied by the PUs
- $\Phi$: Steady-state vector
- $\Phi_i$: $i$th element of a steady-state vector
- $\pi$: Express steady-state probabilities
- $\pi_i$: $i$th element of steady-state vector
- $S$: Sample set
- $S_i$: Represents state $i$
- $S_N$: Subset of $S$
- $S_i$: Subset of $S_N$, which contains states associated with new packets
- $R_s$: Throughput
- $R_s'$: Normalized throughput
- $R_{SP}$: Simulation throughput
- $[ ]^T$: Transpose of matrix
- $T$: Duration of the TS
- $T_d$: Data transmission epoch
- $T_D$: Total average packet delay obtained from theory
- $T_{DP}$: Delay due to busy channels
- $T_{DP}$: Total average packet delay obtained by simulation
- $T_{DS}$: Normalized average packet delay obtained by simulation
- $T_P$: Duration of packet transmission
- $T_s$: Sensing epoch
- $T_w$: Waiting epoch
- $\tau$: Average end-to-end packet delay
- $\tau_s$: Average end-to-end packet delay obtained by simulation
- $\xi$: Status of a specific packet
- $1$: Column vector containing 1

I. INTRODUCTION

The 21st century has witnessed an exponential growth in wireless applications, which has significantly increased the electromagnetic spectrum demand and led to spectrum scarcity in the most desirable low-attention frequency bands [1]. In order to identify the cause of the spectrum shortage and to conceive corresponding solutions, the Federal Communication Commission (FCC) in the USA and the European Telecommunications Standards Institute (ETSI) conducted surveys in different parts of the world and at different time instances [2]–[4]. The results of these studies reveal that under the conventional static spectrum allocation policy, substantial segments of the earmarked electromagnetic spectrum are actually under-utilized. For example, the spectrum measurements of [4] demonstrated that the typical spectrum occupancy in the United States varies between 15% to 85%, whereas, the measurements taken in downtown Berkley illustrated in Fig. 1 suggest that the spectrum utilization below 3 GHz is approximately 30%, while in the range of 3–6 GHz is only 0.5% [5], [6]. As a result, the available bandwidth cannot be efficiently exploited, whilst the readily available spectrum remains insufficient for innovative bandwidth-thirsty wireless applications [3], [7]. This inefficient spectrum exploitation motivates the concept of dynamic spectrum access (DSA), which allows the cognitive users (CUs) to access and utilize the unoccupied spectrum holes that have traditionally been exclusively assigned to the primary users (PUs) [7]–[9].


The concept of CR system introduced by Mitola and Maguire [10] emerged as a promising paradigm for maximizing the under-utilized spectrum for efficient spectrum utilization, the CUs continuously sense the licensed spectrum with the aim of detecting the unoccupied portions (holes) in the spectrum and then use them for their own...
data transmission, as shown in Fig. 2 [7], [9], [11]. Moreover, the CU has to relinquish the licensed spectrum as soon as the PRUs wish to access them. In order to improve the exploitation of the licensed spectrum, the regulatory bodies officially allow CUs to access and opportunistically exploit the licensed spectrum. Hence, this concept has been incorporated by various wireless standards, such as IEEE 1900, 802.11y, 802.16h and 802.22, which have been critically appraised in [12].

Automatic Repeat reQuest (ARQ) constitutes an efficient technique of reliable data transmission over noisy channels. The concept of ARQ was originally introduced by Chang [13], which was then classified into three popular ARQ protocols: Stop-and-Wait ARQ (SW-ARQ), Go-Back-N ARQ (GBN-ARQ) and Selective-repeat ARQ (SR-ARQ) [14]–[16]. The principle of ARQ is appealingly simple. After transmitting a packet, if the original transmitter node fails to receive a positive acknowledgement within the defined time duration or if it receives a negative acknowledgement, the packet is retransmitted. The ARQ protocols are capable of achieving reliable data transmission, provided that the channel-induced error rate remains moderate. However, beyond a certain error rate both the throughput and the delay may become inadequate. Hence, for the sake of enhancing the performance, hybrid Forward Error Correction (FEC) and ARQ (HARQ) schemes [17], [18] may be employed. In addition to detecting errors, in HARQ, the FEC scheme also has the capability of correcting a number of errors and the ARQ mechanism is activated for the retransmission of a packet, when residual errors are detected after FEC decoding. As a benefit, HARQ schemes are typically capable of providing a better throughput/delay performance than the corresponding ARQ schemes. Hence, they have been widely used in wireless communication systems [19], [20], which motivated us to study and incorporate the HARQ schemes in CR systems.

Similar to our previous studies [21], [22], we focus our attention on the opportunistic spectrum access in CR systems [7], [9], where a CU senses and occupies a PU’s channel for its own transmission, provided that the PU’s channel is not occupied at the instant of the demand [23]–[25]. In our studies, the activity of the PU is modelled using a two-state Markov chain, having the ‘ON’ and ‘OFF’ states [24], [26], [27]. The channel is considered to be occupied by the PUs in the ‘ON’ state and to be free in the ‘OFF’ state. However, when the sensing is unreliable, the Markov chain has four states [22], including the false-alarm and mis-detection events. In this scenario, a time-slot (TS) of duration $T$ is divided into two portions: a sensing epoch ($T_s$) and a transmission epoch ($T_d$) [23], [28]. The sensing epoch $T_s$ is used for detecting spectrum holes, while the transmission epoch $T_d = T - T_s$ is used for data transmission. In our CR system, the data transmission relies on the principles of classic stop-and-wait hybrid automatic repeat request (SW-HARQ) [14]–[16].

Our proposed cognitive stop-and-wait (CSW-HARQ) scheme intrinsically amalgamates CR system with the classic SW-HARQ regime in both perfect and imperfect sensing environments. We opted for the SW-HARQ protocol as a benefit of its low complexity at both the transmitter and receiver. However, it wastes time between the end of transmitting a packet and reception of its feedback acknowledgement [14], [15]. Again, in the CSW-HARQ, the CU transmitter first senses a PU’s channel and only transmits data if the channel is deemed to be free. Otherwise, it waits until the next TS. After the transmission of a data packet, the transmitter waits for the feedback. At the receiver side, when the CU’s receiver receives the packet, it starts its decoding and generates a feedback flag [29], [30]. A positive feedback (ACK) is generated and sent to the transmitter, if an error-free packet is received. Otherwise, a negative feedback (NACK) is generated and sent to the transmitter. Then, the CU transmitter sends a new packet after the reception of an ACK. Otherwise, the previous packet is retransmitted in the next free TS, if it receives a NACK. Based on the above arrangements, we investigate the throughput and delay of the CSW-HARQ scheme both by analysis and by simulation, when assuming either perfect or imperfect sensing.

A. RELATED WORK

Since, the germination of the CR concept, a substantial amount of research has been dedicated to the CR architecture, to its operating principle, spectrum sensing, reconfiguration, spectrum sharing, mobility etc [47], [48]. For instance, the throughput of CR networks has been widely studied in the context of perfectly detecting the activity of the PUs [49], relying on the assumption of the optimal sensing time [23], [26], on cooperative sensing [50] and using the optimal frame length [51]. More particularly, in [23] and [51], the authors have proposed various approaches of maximizing the throughput of the CUs by finding both the optimal sensing duration and the TS duration both in perfect and realistic imperfect sensing scenarios. By contrast, in [28], the trade-off between the sensing duration and throughput has
been optimized by proposing a hybrid spectrum sensing and data transmission technique.

Following the spectrum sensing operation, the CUs access the channel for data transmission. The data transmission in CR systems is faced with the usual hostile wireless communication channels, hence powerful error-correction and detection techniques are techniques have been proposed [15], [52]. For example, hybrid automatic repeat request HARQ protocols have been conceived for achieving reliable communication in underwater acoustic networks [37], in satellite communication [53], in audio video transmission over the Internet [54] and in multi-relay environments for transmission over orthogonal TSs [55]. HARQ-aided superposition coding has been proposed for improving the cell-edge coverage, while improving the energy efficiency [56]. The authors of [41], [57], and [58] proposed sophisticated strategies for a multi-component turbo coded HARQ environment where the complexity imposed was reduced by postponing the activation of the turbo iterations until sufficient redundancy was received at the destination. This was also combined with curtailing any further turbo-iterations, when a reliable decision was deemed to be likely. Following that, the authors of [59] proposed channel-adaptive stop-and-await retransmission schemes in the environment of short-range wireless links for reducing the energy consumption as compared to the classic SW retransmission schemes. Moreover, HARQ has also been involved in various IEEE standards [60]–[62]. Some of the related studies are presented at a glance in Fig. 3.

There is a paucity of studies on the analysis of ARQ protocols in the context of CR. Based on our observations, the dynamic nature of PUs makes the analytical modelling of the ARQ protocols in the context of CR more challenging. The studies performed in this direction include [29], [66], [67], and [69]–[81]. However, these studies do not provide the complete theoretical throughput and delay analysis of the CR system employing ARQ protocols. Specifically, in [69]–[71], the CUs sense the presence of the PUs by listening to their ARQ feedback. However, the authors did not investigate the performance of CUs using ARQ techniques. The throughput of the selective-repeat ARQ protocol evaluated in the context of CR systems has been studied in [66], where an efficient resource allocation scheme has been proposed for multi-hop relaying systems. Then, the authors of [29] and [72] have performed the seminal analysis of ARQ-aided relay-assisted CR systems. Moreover, the authors of [76] analyzed the performance of spectrum sharing networks using HARQ feedback in terms of the attainable throughput and outage probability. Following that, in [77] the efficiency of data transmission was studied in the context of finite-length codewords in spectrum sharing networks.

The authors of [73] and [74] have proposed an improved HARQ scheme employing an anti-jamming coding approach for achieving reliable communication in CR systems. As a further development, a network coding assisted ARQ scheme has been proposed in [75], which aims for improving the efficiency of the conventional ARQ schemes. A hybrid of the spectrum interweave and overlay sharing paradigms has been proposed in [78] and [79] for improving the performance of the CU. The authors of [79] have also involved ARQ for improving the attainable reliability. Moreover, in [82] energy-efficient dynamic spectrum access protocols were proposed for CR nodes in order to enhance the overall channel utilization without affecting the performance of the PU. On the other hand, the authors of [67], [80], and [81] comprehensively surveyed the various techniques employed in the context of CR systems.

**B. CONTRIBUTION AND PAPER STRUCTURE**

This paper constitutes an evaluation of our prior contributions [21], [22], in which we studied both the throughput and delay of CSW-HARQ, and of cognitive Go-Back-N (CGBN) HARQ in the context of a CR system. Particularly, in [22] a simulation based study was provided for characterizing the performance of CGBN-HARQ in both perfect and in imperfect sensing environments. However, in this contribution, we extend the work presented in [21] to the imperfect sensing environment and derived closed-form analytical expressions. It is observed that the respective parameters have to be carefully adapted according to the specific communication environment considered for maximizing the throughput and for minimizing the delay of the system. Against the above background, the contributions of this paper can be summarized as follows:

(a) The proposed CSW-HARQ scheme intrinsically amalgamates the CR capability with the conventional SW-HARQ protocol in order to achieve reliable data transmission in realistic imperfect sensing. Our protocol enables the CR transmitter to sense the channel before using it and to receive feedback at all times, regardless of the PUs activity.

(b) Firstly, the CSW-HARQ scheme is modelled and theoretically analysed using a probability-based approach considering both perfect and imperfect reusing. Using this approach, closed-form expressions are derived for the a) average packet delay, b) for the throughput of the CU system and c) for the end-to-end packet delay. Both the probability distribution and the average of the end-to-end packet delay are formulated.

(c) Secondly, based on Discrete Time Markov Chain (DTMC) approach, closed-form expressions are derived for the a) throughput of the CU system b) for the average packet delay and c) for the end-to-end packet delay, including its probability distribution and average end-to-end packet delay.

(d) Finally, we validate theoretical results by simulations.

The rest of this paper is organized as follows. We model CR system considered in Section II. The basic principles of the proposed CSW-HARQ transmission scheme are discussed in Section III, followed by the operation of the CR transmitter and receiver in Sections III-A and III-B, respectively. In Section IV and V, a probability-based approach as well as
II. SYSTEM MODEL

In this section, we describe both the primary radio (PR) and the CR systems, as well as the assumptions invoked in our analysis and for obtaining the results of Section VI.

A. MODELING THE PRIMARY USER

As in [22], we assume that there is a PU’s channel, which is sensed and used by the CR system considered. We assume that the PUs become active during each TS of duration $T$ independently with the same probability. Specifically, the activation of the PU’s channel by the PUs is modeled as a two-state Markov chain having the state transitions shown in Fig. 6. The state ‘OFF’ represents that the channel is free for the CU to use, whereas the state ‘ON’ indicates that the channel is...
occupied by the PUs; $\alpha$ and $\beta$ represent the transition probabilities from the ‘OFF’ and ‘ON’ to the ‘ON’ and ‘OFF’ state, respectively. Let the probabilities of the PUs channel being in the ‘ON’ and ‘OFF’ states be $P_{on}$ and $P_{off}$, respectively. Then, it can be readily shown that if the Markov chain is in its steady state, we have [14]

$$P_{on}\alpha = P_{off}\beta,$$

(1)

and from $P_{off} = 1 - P_{on}$ we have:

$$P_{on} = \frac{\beta}{\alpha + \beta}, \quad P_{off} = \frac{\alpha}{\alpha + \beta}.$$

(2)

Furthermore, as shown in Fig. 7(a), we assume that if the PU’s channel is found in the ‘ON’ state at the start of a TS, it remains in the ‘ON’ state until the end of that TS, and using this TS should be avoided by the CU. On the other hand, if a TS is deemed to be free from the PUs, then the CU may use it [26].

**B. MODELING THE COGNITIVE USER**

In the CR system, each TS of duration $T$ is divided into two phases: the sensing duration of $T_s$ seconds and the data transmission duration of $T_d = T - T_s$ seconds, as shown.
The structure of this paper is outlined in Fig. 5. The system model is discussed in Section II, where two main sections are covered - II-A Modeling the Primary user and II-B Modeling the Cognitive user. Section III focuses on the Cognitive Stop-and-Wait Hybrid Automatic Repeat Request (SW-HARQ) operation, divided into III-A Operation of the CU Transmitter and III-B Operation of the CU Receiver. The performance analysis of the SW-HARQ scheme is presented in Section IV, which is further divided into IV-A Average Packet Delay, IV-B End-to-End Packet Delay, IV-B1 Probability Mass Function, and IV-B2 Average End-to-End Packet Delay. The Markov chain-based analysis of the SW-HARQ scheme is described in Section V, with V-A Throughput and V-B Delay Analysis of SW-HARQ. Section VI presents the performance results, and Section VII concludes the paper.

The two-state discrete-time Markov chain model of the PU system is shown in Fig. 6. The states represent the channel usage as ‘OFF’ (0) and ‘ON’ (1), with transition probabilities α and β, respectively. The time-slot structure of PU and CU is depicted in Fig. 7. A CU TS consists of a sensing duration of $T_s$ and a transmission duration of $T_d = T - T_s$, where $T$ is the total duration of a time-slot. The pattern of channel usage by PU is shown in (a), and by CU in (b).

The discrete-time Markov chain model of the CR system is illustrated in Fig. 8. The states $S_0$, $S_1$, $S_2$, and $S_3$ represent the channel states, with the transitions shown in the diagram. The state $S_0$ corresponds to the channel being sensed free, $S_1$ to the channel being sensed busy, $S_2$ to the channel being sensed ‘ON’ but detected ‘OFF’, and $S_3$ to the channel being sensed ‘OFF’ but detected ‘ON’.

When the channel is ‘OFF’, but detected to be ‘ON’, and vice versa, mis-detection and false-alarm probabilities can be expressed. The transition probabilities are given by Eq. (3):

$$ P = \begin{bmatrix} P_{0,0} & P_{0,1} & P_{0,2} & P_{0,3} \\ P_{1,0} & P_{1,1} & P_{1,2} & P_{1,3} \\ P_{2,0} & P_{2,1} & P_{2,2} & P_{2,3} \\ P_{3,0} & P_{3,1} & P_{3,2} & P_{3,3} \end{bmatrix} \quad (3) $$

Let the false-alarm and mis-detection probabilities be expressed as $P_{fa}$ and $P_{md}$, respectively. Then from Fig. 8,
we obtain

\[ P_{0,0} = P_{1,0} = (1 - \beta)(1 - P_{fa}) \]
\[ P_{0,1} = P_{1,1} = (1 - \beta)P_{fa} \]
\[ P_{0,2} = P_{1,2} = \beta P_{md} \]
\[ P_{0,3} = P_{1,3} = \beta(1 - P_{md}) \]
\[ P_{2,0} = P_{3,0} = \alpha(1 - P_{fa}) \]
\[ P_{2,1} = P_{3,1} = \alpha(1 - P_{fa}) \]
\[ P_{2,2} = P_{3,2} = (1 - \alpha)P_{md} \]
\[ P_{2,3} = P_{3,3} = (1 - \alpha)(1 - P_{md}). \] (4)

Furthermore, let the steady state probabilities of the Markov chain be expressed as \( \Phi = [\Phi_0, \Phi_1, \Phi_2, \Phi_3]^T \). Then, we have [14], [83]

\[ \Phi = \mathbf{P}^T \Phi. \] (5)

Explicitly, \( \Phi \) is the right eigenvector of \( \mathbf{P}^T \) associated with an eigenvalue of 1. Therefore, when substituting the items in (4) into (3) and solving Equation (5), we obtain

\[ \Phi = [\Phi_0 \Phi_1 \Phi_2 \Phi_3]^T = \lambda \begin{bmatrix} \alpha(1 - P_{fa}) & \alpha P_{fa} & (P_{md}) & (1 - P_{md}) \beta(1 - P_{md}) & \beta(1 - P_{md}) & 1 \end{bmatrix}^T, \] (6)

where \( \lambda \in \mathbb{R} \). Upon exploiting the property of

\[ \sum_{i=0}^{3} \Phi_i = 1, \] (7)

gives

\[ \lambda = \frac{\beta(1 - P_{md})}{\alpha + \beta}. \] (8)

Consequently, the steady state probabilities of the CR system in the state \( S_0, S_1, S_2 \) and \( S_3 \) are

\[ \Phi_0 = \frac{\alpha(1 - P_{fa})}{\alpha + \beta}, \quad \Phi_1 = \frac{\alpha P_{fa}}{\alpha + \beta}, \]
\[ \Phi_2 = \frac{\beta P_{md}}{\alpha + \beta}, \quad \Phi_3 = \frac{\beta(1 - P_{md})}{\alpha + \beta}. \] (9)

III. COGNITIVE STOP-AND-WAIT HYBRID AUTOMATIC REPEAT REQUEST

In our CSW-HARQ system, data are encoded using a Reed-Solomon (RS) code \( RS(N_d, K_d) \) [15], defined over the Galois Field of \( GF(q) = GF(2^m) \), where \( K_d \) and \( N_d \) represent the number of information and coded symbols, respectively, and \( m \) is the number of bits per symbol. We assume that every packet consists of a RS codeword, which is transmitted within \( T_p \) seconds. Let \( N = T_d/T_p \). Then, within a free TS, the CU transmitter can transmit a packet using the \( T_p \) seconds and then waits for the feedback. We assume that the RS code is capable of correcting up to \( t \) random symbol errors and can ideally detect the uncorrectable errors, which is time for sufficiently long codes.

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Given the above assumptions, data are transmitted between a pair of CUs over the PU channel based on the principles of our CSW-HARQ scheme, which is characterized in Fig. 9 and formally stated in Algorithm 1, as detailed below. For clarity, Algorithm 1 is divided into two parts, namely the transmitter part and the receiver part. The transmitter always senses the PR channel and (re)transmits a packet, when the channel is free from PRUs. On the other hand, at the receiver side, RS decoding and error correction is performed, based on which a feedback signal is generated for the received packet. We assume that the receiver has perfect knowledge concerning the success or failure of decoding and updates its buffer accordingly.

A. OPERATION OF THE CU TRANSMITTER

In the classic SW-HARQ, the transmitter sends a single packet in a TS and then waits for its feedback, which is expected to be received after a specified round-trip time (RTT). By contrast, in the CSW-HARQ, the CU transmitter first has to sense the PU’s channel before the transmission or retransmission of a packet. If the PU channel is correctly detected to be ‘OFF’, or when the PU channel is mis-detected as being ‘OFF’ state when it is actually ‘ON’, the CU transmitter transmits a packet. Otherwise, if the PU channel is correctly sensed to be ‘ON’ or when it is falsely
Algorithm 1 CSW-HARQ Algorithm

1: **Initialization**: \( M_c = \) number of packets, \( T_d = N \), \( T_s = k, i = 1, \) TS = 1.
2: **Input**: \( T_d, T_s \) packets.
3: while \( i \leq M_c \) do
4:   CU transmitter senses a time-slot (TS).
5:   if TS is free and no false-alarm OR TS is busy but mis-detected then
6:     transmits the \( i \)th packet and, then,
7:     waits for \( T_w \) duration to receive feedback.
8:     if the \( i \)th packet is received error-free then
9:       receiver sends ACK signal.
10:       \( i = i + 1 \).
11:     else
12:       receiver sends NACK. It could be because of mis-detection or channel noise.
13:       end if
14:   else
15:     The TS may be found busy due to correct detection or false-alarm,
16:     hence, the transmitter waits until the next TS.
17:   end if
18: end while

FIGURE 10. The transmission flow of the proposed CSW-HARQ scheme. The total duration of each time-slot is \( T = T_s + T_d \), where \( T_d \) consists of a packet’s transmission duration and its waiting epoch \( T_w \).

detected to be ‘ON’, while it is actually ‘OFF’, then the transmitter has to wait until the next TS and has to sense the channel again. The CSW-HARQ operations are summarized both in the Algorithm 1 and in Fig. 9. Similar to the classic SW-HARQ scheme, the CU transmitter in CSW-HARQ has a buffer of size one, which is updated based on the feedback flag of each transmitted packet.

We assume that all the packets are of the same length and that the CU transmitter is always ready to transmit these packets, provided that there are free TSs. As shown in Fig. 10, each CU packet consists of a RS coded codeword, which is transmitted within the duration of \( T_p \) seconds. After transmitting a packet, the CU transmitter waits for a duration of \( T_w \) seconds in order to receive its feedback. We assume that the RTT is \( T_d \), which is the time interval between the transmission of a packet, and the instant when its feedback acknowledgement is received. Therefore, in our CSW-HARQ scheme, the feedback flag of each packet should be received within the RTT duration of \( T_d \), as shown in Fig. 10. Specifically, if a positive feedback (i.e., ACK) of a packet is received by the CU transmitter within the RTT, this packet is then deleted from the transmitter’s buffer and a new packet is transmitted in the next free TS which is also stored in the transmitter buffer. However, if a NACK is received, then the transmitter retransmits the erroneous packet in the next free TS. It is worth mentioning that, when there is a mis-detection of the PU channel, the packet transmitted will become erroneous with a high probability, due to collision with the PU’s transmitted signals. On the other hand, when there is a false-alarm, the CU will not transmit, even though the PU channel is free, which results in a reduced throughput.

B. OPERATION OF THE CU RECEIVER

When the CU receiver receives a packet from the CU transmitter, it invokes RS error-correction/detection and then generates a feedback flag accordingly. Specifically, if a packet is correctly recovered by the RS decoder, an ACK signal is fed back to the CU transmitter. Otherwise, a NACK signal is sent to the CU transmitter in order to request a retransmission. In this paper, we assume that the feedback channel is perfect and that the receiver has a buffer size of one packet, which is updated only when a packet is correctly received [14]–[16].

IV. PERFORMANCE ANALYSIS OF THE CSW-HARQ SCHEME: PROBABILITY-BASED APPROACH

In this and the next sections, we analyze the performance of the proposed CSW-HARQ scheme. We consider three performance metrics, namely, 1) the Average packet delay; 2) the End-to-end packet delay; and 3) the Throughput. They are defined as follows. **Average packet delay**: the average number of TSs (or \( T_p \)'s) required for the successful transmission of a packet. **End-to-end packet delay**: the average time duration from the first transmission of a packet to the instant when it is finally successfully received.
Throughput: the error-free transmission rate of the CR system [15], [42].

In this paper, we introduce two approaches in our analysis, which are the probability-based approach employed in this section, and the discrete time Markov chain (DTMC)-based approach employed in Section V. Our analytical approaches will be validated in Section VI by comparing the results obtained from the numerical evaluation of the derived formulas with those obtained from simulations. Let us first use the probability-based analysis to analyze the average packet delay.

### A. AVERAGE PACKET DELAY

In the traditional SW-HARQ scheme, the delay is generated by the data transmitted over unreliable channels, which results in erroneous transmission and thereby requires retransmission, in addition to the basic transmission delay.

In our proposed CSW-HARQ scheme, the unreliable channels also introduce delay similarly to the traditional SW-HARQ. Furthermore, in the CSW-HARQ, there is an extra delay both owing to the unavailability and due to the false detection of the PU’s channel. In order to analyze the packet delay of the CSW-HARQ scheme, let us denote the average delay due to the busy PU’s channels as $T_{DP}$. It can be shown that the PU’s channel is found busy in the following two scenarios:

(a) The PU channel is ‘ON’ state and the state is correctly detected.

(b) The PU channel is ‘OFF’, but false-alarm occurs, resulting in the CU not using the channel.

Therefore, according to the definitions in Section II-B, the probability that the CU finds that a TS is busy can be expressed as

$$P_{bs} = P_{on} \cdot (1 - P_{md}) + P_{off} \cdot P_{fa},$$

$$= \frac{\beta(1 - P_{md})}{\alpha + \beta} + \frac{\alpha P_{fa}}{\alpha + \beta},$$

$$= \frac{1}{\alpha + \beta} \left[ \beta(1 - P_{md}) + \alpha P_{fa} \right].$$  \text{(10)}

On the other hand, there are only two scenarios for the CU to access the PU’s channel for its own transmission:

(a) The PU’s channel is in the ‘OFF’ state, which is correctly detected by the CU.

(b) The PU’s channel is in the ‘ON’ state, but it is mis-detected by the CU.

Correspondingly, the probability of the above events can be expressed as

$$P_{fr} = P_{off} \cdot (1 - P_{fa}) + P_{on} \cdot P_{md},$$

$$= \frac{1}{\alpha + \beta} \left[ \alpha(1 - P_{fa}) + \beta P_{md} \right].$$  \text{(11)}

Explicitly, we have $P_{fr} = 1 - P_{bs}$. Moreover, let $T_{DP}(i)$ be defined as the delay imposed by $(i - 1)$ busy TSs prior to a free TS, yielding

$$T_{DP}(i) = (i - 1)T.$$  \text{(12)}

Then, the average delay $T_{DP}$ required by the CU to find a free TS may be formulated as

$$T_{DP} = E[T_{DP}(i)] = E[(i - 1)T].$$

$$= \sum_{i=1}^{\infty} (i - 1)TP_{bs}^{i-1}P_{fr},$$

$$= \frac{(P_{bs} \cdot P_{fr})T}{(1 - P_{bs})^2} = \frac{P_{bs}T}{(1 - P_{bs})},$$

$$= \frac{\alpha P_{fa} + \beta(1 - P_{md}) + T \cdot \alpha P_{fa} + \beta P_{md}}{\alpha(1 - P_{fa}) + \beta P_{md}}.$$  \text{(15)}

Let us assume that a packet is transmitted over a TS that is supposed to be free, but a delay is introduced by transmission over an unreliable channel. If a packet is successfully delivered in the first attempt, it induces a delay of a $T/T_p$ seconds. By contrast, each retransmission will impose a delay of $T$ seconds. As the analysis of Section IV-A shows, erroneous transmissions may take place in the actually free TSs, or in busy TSs misclassified by the CU’s transmitter.

Let $T_D(i)$ denote the delay given by the event that the CU transmitter uses a total of $i$ transmissions for successfully delivering a packet to the CU receiver. Here, $T_D(i)$ includes both the delay imposed by finding free TSs and the delay due to the packet’s transmission. Furthermore, some of the TSs are free and correctly sensed by the CU system, while others are actually busy TSs but they may be mis-detected by the CU. Therefore, given that a free TS is identified by the CU, the probability that the TS is actually free can be expressed as

$$P_A = \frac{P_{off}(1 - P_{fa})}{P_{fr}}.$$  \text{(16)}

By contrast, given that a free TS is discovered by the CU, the probability that it is resulted from a mis-detection is

$$P_B = \frac{P_{on}P_{md}}{P_{fr}}.$$  \text{(17)}

Remembering that each transmission attempt requires an average time of $T_{DP}$ seconds for finding a free TS, plus a subsequent delay of $T$ seconds for the actual round trip transmission, we hence have

$$T_D(i) = i(T + T_{DP}),$$  \text{(18)}

when $i$ transmissions are used for the successfully delivery of a single packet. According to the principles of the CSW-HARQ, the transmitter sends a packet in each free TS. Hence, the average packet delay $T_D$ can be evaluated by

$$T_D = \frac{1}{N} \sum T_D(i) = \frac{1}{N}E[i(T + T_{DP})],$$  \text{(19)}

where the multiplier $1/N$ is because each packet is transmitted within $N$ packet durations. Let us denote the packet error probability (PEP) for the packets sent in the free TSs by $P_e$, after RS decoding. We assume that the PEP of those packets that were transmitted because of mis-detection in the busy
TSs is as high as one. Then, it can be shown that we have

\[ T_D = \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} T_i (T_{DP}^j \cdot (P_B)^j \cdot (P_A)^j) - (1 - P_e)^j \cdot (P_e)^j - (1 - P_e) \]

where \( T_{DP} = T_{DP}/T \). Moreover, after normalizing the average packet delay \( T_D \) using the packet duration \( T_p \), \( T_D \) can be expressed as

\[ T_D = \left( \frac{k + N}{N} \right) \left( P_A(1 - P_e)(1 + N_{DP}) \right) T_p^s, \tag{21} \]

where \( T = (k + N)T_p \) is applied.

### B. END-TO-END PACKET DELAY

In this section, we derive the probability mass function (PMF) of the end-to-end packet delay, as well as the average end-to-end packet delay. Here the end-to-end delay of a packet represents the time duration spanning from the first transmission of the packet to the time that the packet is successfully received.

1) PROBABILITY MASS FUNCTION

First, it is worth restating that the delay between the first transmission and the final error-free reception of a packet depends on two factors: 1) the delay for retransmissions, and 2) the delay imposed by the occurrence of busy TSs.

Below we first derive the PMF of the end-to-end packet delay incurred purely due to retransmissions, when there are no sensed busy TSs. Then, the delay of the general cases including both sensed busy and free TSs between the first transmission and the final successful reception of a packet is considered.

Firstly, when the sensing results at the CU indicate that there is no busy TS between the first transmission and the final reception of a packet, the probability that \( n \) TSs are used for successfully delivering a packet can be expressed as

\[ P_{MF}(n) = \sum_{i=0}^{n-1} \left( \frac{n-i}{2} \right) (P_{onP_{md}}^j) \cdot (P_{off}(1 - P_{fa})) \cdot (P_e)^{n-i-1} - (1 - P_e) A(1 - P_e)(1 + N_{DP}) P_{fa} = (P_{bfa})^i \cdot (P_{onP_{md}}^j) \cdot (P_{off}(1 - P_{fa}))^j \cdot (P_e)^{n-j-1} - (1 - P_e) A(1 - P_e)(1 + N_{DP}) P_{fa} = (P_{bfa})^j \]

where, in addition to the terms that are similar to those in (22), \( (n-2)^{12} \) represents the probability that there are \( i \) sensed busy TSs between the first transmission and the final reception of a packet. Note that the largest value for \( i \) is \( n - 2 \), as the first and the last TSs are sensed to be free.

2) AVERAGE END-TO-END PACKET DELAY

Having obtained the PMF of the end-to-end packet delay, which is given by Eq. (23), the average end-to-end packet delay quantified in terms of TSs can be expressed as

\[ \tau = \sum_{n=1}^{\infty} n \cdot P_{MF}(n) \approx \sum_{n=1}^{M_T} n \cdot P_{MF}(n), \text{ (TSs)} \tag{24} \]

where \( M_T \) is set as the maximum delay to be considered. Note that \( M_T \) can be rendered of ensuring that the unconsidered components becomes negligible. For example, we may choose \( M_T \) to satisfy \( \sum_{n=1}^{M_T} P_{MF}(n) = 1 - 10^{-8} \).

### C. THROUGHPUT

Given the average packet delay expression of (20), we can readily obtain the throughput of the CU system operated under the CSW-HARQ, which can be expressed as

\[ R_s = \frac{1}{T_D} = \frac{N}{T} \left( \frac{(P_B + P_A A^2)}{P_A(1 - P_e)(1 + N_{DP})} \right) \text{ (PPS)} \tag{25} \]

\[ = N \left( \frac{(P_B + P_A A^2)}{P_A(1 - P_e)(1 + N_{DP})} \right) \text{ (PPTS)} \tag{26} \]

\[ = N \left( \frac{(P_B + P_A A^2)}{P_A(1 - P_e)(1 + N_{DP})} \right) \text{ (PPTs)} \tag{27} \]

where PPS, PPTS and PPTs denote packet per second, packet per TS and packet per \( T_p \), respectively. Furthermore, let us assume that a \( (N_D, K_d) \) RS code is employed denote by \( B \) the
The throughput can also be expressed in terms of bits per second (bps) as

\[ R_s = \frac{1}{T_d} \cdot K_d \cdot B \]  

(bps). (28)

Above we have analyzed the performance of the proposed CSW-HARQ scheme using the probability-based approach. In the following section, we also provide a Markov-chain based approach for analysing the CSW-HARQ scheme.

V. MARKOV CHAIN-BASED ANALYSIS OF THE CSW-HARQ SCHEME

In this section, we first model the CSW-HARQ scheme relying on realistic imperfect sensing using the DTMC. Then, we analyze the stationary throughput of the CSW-HARQ.

Let us first define the states of the CSW-HARQ, where a state can be jointly defined by 1) the real status of the PU’s channel \((\mu)\), 2) the status of the PU’s channel sensed by the CU \((v)\), and 3) the status of a specific packet \((\xi)\), which is either a new or an old packet stored in the transmitter’s buffer.

The state is observed and updated at the end of each TS. Let us express the eight legitimate states as

\[ S = \{S_0, S_1, S_2, \ldots, S_7\} \]  

(29)

where each state is a binary number of length 3, expressed as

\[ S_i = \{\mu, v, \xi\} \]  

(30)

where the index \(i\) represents the decimal value given by \(\mu v \xi\). In (30), \(\mu\), \(v\) and \(\xi\) are defined as

\[ \mu = \begin{cases} 0, & \text{the PU’s channel is free,} \\ 1, & \text{the PU’s channel is busy} \end{cases} \]  

(31)

\[ v = \begin{cases} 0, & \text{the PU’s channel is sensed to be free,} \\ 1, & \text{the PU’s channel is sensed to be busy} \end{cases} \]  

(32)

\[ \xi = \begin{cases} 0, & \text{if the packet stored in the transmitter buffer is a new packet,} \\ 1, & \text{if the packet stored in the transmitter buffer is an old packet.} \end{cases} \]  

(33)

As an example, Fig. 11 illustrates the transitions between states in the context of five TSs. The number in the boxes of the CU transmitter and the receiver represents the packet that is transmitted and received, respectively. In more detail, Fig. 11 is interpreted as follows.

(a) Assume that during TS \(T_1\), the PU’s channel is free and it is correctly sensed to be free corresponding to \(\mu = 0, v = 0\). Assume furthermore that a new packet which gives \(\xi = 0\) is transmitted. Hence, the state observed by transmitter in this TS is \(S_0 = \{0, 0, 0\}\). As shown in the figure, we assume that the packet is correctly received and hence an ACK flag is sent by the receiver to the transmitter as seen in Fig. 11.

(b) As a benefit of the error-free transmission in the TS \(T_1\), the transmitter buffer is updated to a new packet. In TS \(T_2\), the PU’s channel is busy, but it is mis-detected by the CU transmitter. Therefore, a new packet waiting in the buffer is transmitted in TS \(T_2\). Hence, the state during TS \(T_2\) is \(S_4 = \{1, 0, 0\}\). Furthermore, it can be shown that the transition from \(S_0\) to \(S_4\) takes place with a probability of \(P_{0,4} = (1 - P_e)P_{on}P_{md}\). Note that, due to the mis-detection of the busy PU’s channel, the packet sent in TS \(T_2\) is received in error and hence, a NACK flag is fed back.

(c) Still referencing to Fig. 11 in TS \(T_3\), the PU channel is free but it is found busy due to false-alarm. Since the transmission in TS \(T_2\) is erroneous and the packet requires retransmission, hence the state of the transmitter in TS \(T_3\) is \(S_3 = \{0, 1, 1\}\) with the transition probability \(P_{4,3} = P_{off}P_{fa}\).

(d) Similarly, we can find that the state during TS \(T_4\) is \(S_7 = \{1, 1, 1\}\). The transition from \(S_3\) to \(S_7\) takes place with a probability of \(P_{3,7} = P_{on}(1 - P_{md})\).
(e) Finally, in TS $T_5$, the PU’s channel is correctly detected to be free and therefore the old packet stored in the transmitter buffer is transmitted, which results in the transition from state $S_7$ to $S_1$ in Fig. 11, with the transition probability of $P_{7,1} = P_{\text{eff}}(1 - P_{fa})$.

**FIGURE 12.** The state transition diagram for the DTMC modelling the proposed CSW-HARQ scheme, where dashed lines correspond to the transitions towards busy states, while solid lines illustrate the transitions towards free states. The solid green and red circles represent the states in which the PU’s channel is free due to correct detection and mis-detection, respectively, while dashed brown and magenta color circles represent the state in which the PU’s channel is found busy due to correct detection and false-alarm, respectively.

Following the above analysis, we can see that the legitimate state to state transitions can be summarized, as shown in Fig. 12. Let the state-transition matrix be expressed as $P$, where the $(i,j)$th element of has the transition probability $P_{i,j}$, which can be found as shown in the above example. Furthermore, according to the properties of the DTMC [14], [83], we have

$$0 \leq P_{i,j} \leq 1,$$

$$\sum_{j=0}^{7} P_{i,j} = 1, \quad \forall S_i \in \mathcal{S},$$

In TS $(n)$, the transmitter is in one of the eight legitimate states with the probability of $p(n) = [P_0(n), P_1(n), \ldots, P_7(n)]^T$. Then, in TS $(n+1)$, the probabilities of the eight possible states can be found from [14], [84]

$$p(n+1) = P^T p(n).$$

According to [84], when $n \to \infty$, the Markov chain reaches its steady-state condition [14], and in this case we have

$$p(n+1) = p(n).$$

Let the steady-state probabilities be expressed as $\pi = \lim_{n \to \infty} p(n)$, where we have $\pi = [\pi_0, \pi_2, \ldots, \pi_7]^T$, and $\pi_1$ represents the steady state probability that the transmitter is in state $S_1$. Then, from (35) and (36) we have

$$\pi = P^T \pi,$$

which shows that the steady state probabilities of the different states which can be obtained by computing the right eigenvector of $P^T$ corresponding to the eigenvalue of 1 [14], [83], [84]. Note that the steady state probabilities satisfy the relationship

$$\sum_{j \in \mathcal{S}} \pi_j = 1 \quad \text{or} \quad \pi^T \times 1 = 1,$$

where $1$ represents a unit of column vector.

**A. THROUGHPUT OF CSW-HARQ**

As defined in Section IV, the throughput of the CSW-HARQ scheme is the rate of successful transmission per TS. Carrying out a successful transmission depends on two factors: 1) a free TS is successfully sensed, and 2) error-free transmission is a achieved. Hence, as shown in (37), when the DTMC reaches its steady state, the throughput of the CSW-HARQ scheme can be obtained from specific state transitions yielding a successful transmission. According to the definition of states in (30) and Fig. 12, the transmission of new packet is only possible when the CU transmitter is in state $S_0$ or $S_4$. Therefore, the achievable throughput of the CSW-HARQ is given by

$$R_s = \pi_0 + \pi_4 \quad \text{(packets / TS)}.$$  

Additionally, using $T = (T_d + T_s)T_p$, the throughput quantified in terms of the number of packets per $T_p$ can be expressed as

$$R'_s = \frac{1}{T} \cdot R_s \quad \text{(packets / $T_p$)}.$$  

Let us now analyze the delay of the CSW-HARQ scheme under the DTMC framework.

**B. DELAY ANALYSIS OF CSW-HARQ**

In this subsection, we analyse both the average packet delay and the end-to-end packet delay with the aid of the DTMC model of the CSW-HARQ. For the end-to-end packet delay, we derive both its probability distribution and the average end-to-end packet delay.

1) AVERAGE PACKET DELAY ($T_D$)

The average packet delay is defined as the average number of TSs or $T_p$’s needed for the successful transmission of a packet. After obtaining the achievable throughput of Eqs. (39) or (40), the average packet delay can be readily obtained, which can be expressed as

$$T_D = \frac{1}{R_s} \quad \text{(TS per packet)}$$

$$= \frac{k + N}{R_s} \quad \text{($T_p$’s per packet)}.$$  

2) END-TO-END PACKET DELAY

We first analyze the probability distribution of the end-to-end packet delay. Let $S_N$ be the specific subset of $\mathcal{S}$,
where the probability mass function of the end-to-end packet delay can be formulated as

\[ P(m) = \frac{1}{(\pi_0 + \pi_4)} \sum_{S_j \in S_N} \pi_i \cdot P_{i,j}^{(m)}, \quad m = 1, 2 \ldots \] (44)

\[ = \frac{\pi_0}{\pi_0 + \pi_4} \sum_{S_j \in S_0} p_{0,j}^{(m)} + \frac{\pi_4}{\pi_0 + \pi_4} \sum_{S_j \in S_4} p_{4,j}^{(m)}, \] (45)

where \( p_{i,j}^{(m)} \) denotes the transition probability from state \( S_i \) to state \( S_j \) after a delay of \( m \) TSs.

Let us express the PMF of the end-to-end packet delay as

\[ P_{MF} = [P(1), P(2), \ldots, P(M_F)]^T, \] (46)

where \( M_F \) is the largest delay considered, beyond which the probability of occurrence becomes negligible. From the properties of the DTMC, we know that given a state \( S_i \), after \( q \) transitions, we have

\[ p^{(q)} = (P^T)^q e_i, \quad q = 1, 2, \ldots \] (47)

where \( e_i \) is the \( i \)th column of the identity matrix. From (47) we can see that whenever we multiply \( P^T \) on a current \( p^q \), we can obtain the following information:

a) The end-to-end packet delay is \( q \) TSs, if the packet is firstly transmitted in state \( S_i \) is correctly received in some states after \( q \) TSs.

b) The transition probabilities from state \( S_i \) to any of the states in \( S \), which contains the states generating the end-to-end delay of \( q \) TSs.

c) The transition probabilities from state \( S_i \) to any other states without resulting in correct reception of the packet firstly sent in state \( S_j \).

Using the above information, we can update \( P_{MF} \) by the following formula.

\[ P(m) \leftarrow P(m) + \pi_i \cdot P_{i,j}^{(m)}, \quad m = 1, 2 \ldots, M_F, \quad S_j \in S, \ S_i \in S_N. \] (48)

Finally, when \( P_{MF} \) does not change, we can compute the average end-to-end packet delay, which can be formulated as

\[ \tau = \sum_{i=1}^{M_F} i \cdot P(i) \text{ (TSs)} \] (49)

\[ = \sum_{i=1}^{M} i(k + N)P(i)T_p^s. \] (50)

After completion of our theoretical analysis of the proposed CSW-HARQ scheme, we proceed to validate the accuracy of both the approaches discussed in Sections IV and V by comparing the analytical results to those obtained through simulations.

VI. PERFORMANCE RESULTS

In this section, we demonstrate the performance of the CSW-HARQ system in terms of three performance metrics, namely, 1) throughput 2) average packet delay and 3) end-to-end packet delay, when both idealized perfect sensing and practical imperfect sensing environments are considered. We will characterize the impact of the false-alarm probability \( (P_{fa}) \), of the mis-detection probability \( (P_{md}) \), the channel’s busy probability \( (P_{on}) \) and of the packet error probability \( (P_e) \) on the performance. Note that, in the case of perfect sensing, \( P_{fa} \) and \( P_{md} \) are always zero. In our studies, Matlab-based simulations are used, and fifty thousand packets are transmitted for every specific condition. The observation period commences from the first TS, which continues until all packets are successfully received by the CU receiver.

Fig. 13 shows the correct detection probability \( \Phi_0 \) of a free channel, the false-alarm probability \( \Phi_1 \), the mis-detection probability \( \Phi_2 \) and the correct detection probability \( \Phi_3 \) of a busy channel, as seen in (9) of Section II-B, with respect to the parameter \( \alpha \). It can be seen from Fig. 13 that when the transmission chances for a CU over those of a PU channel increase, i.e., when \( \alpha \) increases, \( P_{off} = \Phi_0 \) also significantly increases, while \( P_{fa} = \Phi_1 \) increases only slightly. The slight increase in \( P_{fa} \) is the consequence of imperfect sensing. On the other hand, when \( \alpha \) increases, \( P_{on} \) is dramatically reduced, while \( P_{md} \) exhibits only a moderate reduction. As shown in Fig. 13, the analytical results evaluated from Eq. (9) closely agree with the simulation results.

Fig. 14 depicts the throughput achieved by the CSW-HARQ scheme both for perfect and imperfect sensing versus the packet error probability \( P_e \) and with respect to the
probability $P_{on}$ of the channel being busy. In our simulations, the throughput is calculated as

$$R' = \frac{N_s}{N_t} \cdot \frac{T_p}{T_s + T_d} \quad \text{(packets per $T_p$)}, \quad (51)$$

where $N_t$ represents the total number of TSs used for the successful transmission of $N_s$ packets by the CU.

As shown in Fig. 14, the throughput is at its maximum, when the channel is perfectly reliable, i.e. when $P_e = 0$ for both the perfect and imperfect sensing scenarios. As $P_e$ increases, implying that the channel becomes less reliable, the achievable throughput reduces nearly linearly with $P_e$, which is a direct consequence of retransmissions. At a given $P_e$, the throughput attains its maximum, when the channel is always free to use, i.e. when $P_{on} = 0$. However, as $P_{on}$ increases, the attainable throughput significantly reduces, since the CU has to wait longer for finding free channels for its data transmission. Furthermore, Fig. 14 shows that realistic imperfect sensing may result in a significant throughout drop, implying that indeed it is important to achieve reliable sensing in cognitive radios.

Additionally, Fig. 14(a) shows that the analytical results evaluated from Eqs. (27) and (40) agree well with the simulation results.

Having characterized the throughput, let us now elaborate on the delay of the CSW-HARQ scheme. Firstly, we consider the average packet delay, as shown in Fig. 15. Note that for the results obtained by simulations, the average packet delay is given by the total number of TSs $N_T$ used for the successful transmission of $N_s$ packets, divided by $N_s$, which is expressed as

$$T_{DS} = \frac{N_s \cdot (T_s + T_d)}{N_s} \quad \text{(seconds)}. \quad (52)$$

The results depicted in Fig. 15 are normalized by $T_p$, yielding:

$$T'_{DS} = \frac{T_{DS}}{T_p} \quad (T_ps). \quad (53)$$

In Fig. 15, the average packet delay of the CSW-HARQ system is studied, both for perfect and imperfect sensing. For a given $P_{on}$, the average packet delay is at its minimum, when $P_e$ is zero and it increases, when $P_e$ or/and $P_{on}$ increases. As discussed above, the increase of $P_e$ triggers more retransmissions, while the increase of $P_{on}$ reduces the transmission opportunities for the CU’s, which hence result in the increase of the average packet delay.
Additionally, imperfect sensing results in the mis-use of the channel, which increases the average delay. Finally, as shown in Fig. 15, the results obtained both from our probability-based and from the DTMC-based approaches are accurate, closely agreeing with our simulation results.

In Fig. 16, we depict the end-to-end packet delay of the CSW-HARQ scheme in terms of both perfect and imperfect sensing. In the simulations, the end-to-end delay is evaluated as the total time duration from each individual packet’s first transmission until its successful reception, divided by the total number of packets, \( N_s \). In detail, let a vector \( d \) of length \( N_s \) be used to store the end-to-end delay experienced by each of the \( N_s \) transmitted packets. Specifically, \( d(j) \) represents the end-to-end delay of the \( j \)th packet. Then, the PMF of the end-to-end packet delay illustrated in Fig. 16 is given by

\[
P_d(i) = \frac{\sum_{j=1}^{N_s} \delta(d(j) - i)}{N_s}, \quad 1 \leq i \leq \max(d).
\]  

Observe from Fig. 16 that in the case of perfect sensing, 90% of the packets are successfully received with an end-to-end delay of one TS, when \( P_e = 0.1 \), as accurately shown in the zoomed-in portion of the figure. To elaborate further, Following is 7.2% of packets are correctly received with an end-to-end delay of 2 TSs, and 2% of the packets are correctly received with an end-to-end delay of 3 TSs, as shown in Fig. 16. On the other hand, in the case of imperfect sensing, at \( P_e = 0.1 \), the number of packets having an end-to-end delay of one TS is reduced to 84%, and the number packets having an end-to-end delay of 2 and 3 TSs is increased to 8.8% and 3.7%, respectively. By contrast, when \( P_e \) is increased to 0.4, the ratio of the packets having an end-to-end delay of one TS is reduced to 60% and 54% in the perfect and imperfect sensing scenarios, respectively. In a similar manner, the ratio of the packets having an end-to-end delay of two or more TSs also reduces correspondingly. Hence, the tail of the PMF curves increases as \( P_e \) increases, implying an increase of the end-to-end packet delay.

The average end-to-end packet delay is shown in Fig. 17, where the results gleaned from our simulations are evaluated from the formula

\[
\tau_s = \sum_{i=1}^{\max(d)} P_d(i) \times i(T) \text{ (seconds)}.
\]  

It can be observed from Fig. 17 that the average end-to-end packet delay increases, as \( P_e \) and/or \( P_{on} \) increases. More importantly, it can be seen that in the case of perfect sensing, the average end-to-end packet delay has a minimum of one TS when \( P_e = 0 \), regardless of the value of \( P_{on} \). This is because, when the channel is highly reliable and when the probability of mis-detection is zero, correct delivery of a transmitted packet is ensured in the first attempt. By contrast, when \( P_e \) and/or \( P_{on} \) increases, the delay increases significantly, although it remains still lower than the corresponding delay in the case of imperfect sensing. In the case of imperfect sensing, there is a delay caused by the false-alarm and mis-detection of the PU’s channel.

Finally, we compare the average packet delay to the average end-to-end packet delay in Fig. 18. Explicitly, for a given scenario, the average packet is always higher than the average end-to-end delay. Furthermore, the average packet delay increases faster than the end-to-end packet delay, when \( P_e \) or/and \( P_{on} \) increases. This is because, the average packet delay considers all the time spanning from the start of transmitting the first packet until the successful reception of the last packet. By contrast, the end-to-end delay only considers

\[
P_{md} = 0.45, P_{fa} = 0.45. \text{ Markers = Simulation, Lines = Analysis}
\]
the delay of each individual packet from the instant of its transmission to its correct reception, but it does not consider the time between two successful packets. For more clear insight, we present the performance results in Table 1.

### VII. CONCLUSIONS

In this paper, we have investigated the performance of the CSW-HARQ transmission scheme both in the perfect and imperfect sensing scenarios. Both the throughput and delay of the CSW-HARQ scheme have been investigated both by analytical and simulation techniques. Furthermore, two analytical approaches, namely the probability-based approach and the DTMC-based approach have been involved for deriving the closed-form formulas of the throughput and delay. Both analytical approaches have been validated by our simulation results. Based on our studies and performance results, we can conclude that the achievable throughput and delay performance of the CSW-HARQ are substantially affected by the activity of the PUs, by the reliability of the CU channels and by the reliability of sensing. When the PU channel becomes busier, the CSW-HARQ’s throughput becomes lower and the average packet delay increases, even when the CU sensing and transmission are reliable. When the sensing is reliable, the CSW-HARQ system achieves a higher throughput and a lower delay compared to the cases of imperfect sensing. Given that in CR the opportunity for data transmission is limited, it is vitally important to employ high-reliability sensing approaches for improving the CU’s throughput and delay, in addition to minimizing the interference imposed on the PUs.

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