

# Delay and Throughput Analysis of Cognitive Go-Back-N HARQ in the Face of Imperfect Sensing

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**Abstract**—In order to mitigate spectrum scarcity, the cognitive radio (CR) paradigm has been invoked for improving the overall exploitation of the licensed spectrum by identifying and filling the free spectrum holes without degrading the transmission of primary users (PUs). Hence, we conceive a CR communication scheme, which enables a cognitive user (CU) to sense the activity of the PUs over a primary radio (PR) channel, which is exploited to transmit data using the modified Go-Back-N hybrid automatic repeat request (GBN-HARQ) protocol, when PR channel is free from the PUs. This arrangement is termed as the cognitive GBN-HARQ (CGBN-HARQ), where the activity of PUs on the PR channel is modelled as a two-state Markov chain having ‘ON’ and ‘OFF’ states. However, the CU may wrongly detect the ‘ON’/‘OFF’ activity of the PUs in the channel, hence resulting in false-alarm or mis-detection. Therefore, the two-state Markov chain is extended to four states by explicitly considering all the wrong sensing decisions. In this paper, we analytically modelled the CGBN-HARQ scheme with the aid of a Discrete Time Markov Chain (DTMC). Explicitly, an algorithm is developed for deriving all the legitimate states and for eliminating the illegitimate states, which assists us in reducing both the dimensionality of the state transition matrix and the associated computational complexity. Furthermore, based on DTMC modelling, we derive closed-form expressions for evaluating the throughput, the average packet delay and the end-to-end packet delay of CGBN-HARQ in realistic imperfect sensing environment. The results are also validated by our simulations. Our performance results demonstrate that both the achievable throughput and

the delay are significantly affected by the activity of the PUs, as well as by the reliability of the PR channel and by the number of packets transmitted per TS. To attain the maximum throughput and/or the minimum transmission delay, the number of packets transmitted within a TS should be carefully adapted based on the activity level of the PUs and on the quality of the PR channel.

**Index Terms**—ARQ, GBN-ARQ, primary radio, PR channel, cognitive radio, spectrum sensing, number of packets, cognitive GBN-HARQ, Discrete time Markov chain, throughput, delay.

## LIST OF ACRONYMS

ACK	Positive Acknowledgement
ARQ	Automatic Repeat ReQuest
CGBN	Cognitive Go-Back-N
CR	Cognitive Radio
CSW	Cognitive Stop and Wait
CU	Cognitive User
DTMC	Discrete Time Markov Chain
DSA	Dynamic Spectrum Access
FEC	Forward Error Correction
GF	Galois Field
HARQ	Hybrid Automatic Repeat ReQuest
NACK	Negative Acknowledgement
OFF	Channel free from PU
ON	Channel occupied by PU
$P_e$	Packet Error Probability
PMF	Probability Mass Function
PR	Primary Radio
PU	Primary User
RS	Reed-Solomon
RTT	Round-Trip Time
SW	Stop and Wait
TS	Time-slot

## LIST OF SYMBOLS

$\alpha$	Probability of traversing from ‘ON’ to ‘OFF’ state
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$\beta$	Probability of traversing from ‘OFF’ to ‘ON’ state
$I$	Identity matrix
$k$	time duration used for sensing a TS
$K_d$	Information Bits of the codeword
$L_i$	Total number of new packets transmitted in state $S_i$
$M_c$	Total number of packets
$M_T$	Maximum delay
$N$	Number of packets in a TS
$N_d$	Coded symbols
$n_p(S_i)$	Function for storing number of new packets associated with state $S_i$
$P$	Transition matrix
$p$	Stores probabilities of all state in a TS
$P_d$	Distribution of end-to-end delay packet delay by simulation
$P_{i,j}$	$\{i, j\}$ th element of the transition matrix
$P_{MF}$	Distribution of end-to-end packet delay by analysis
$P_{off}$	Channel free probability
$P_{on}$	Channel busy probability
$\Phi$	Steady-state vector
$\Phi_i$	$i$ th element of the Steady-state vector
$\pi$	steady-state vector
$\pi_i$	$i$ th element of steady-state vector
$R_T$	Throughput by analysis
$R'_T$	Normalized throughput by analysis
$R_S$	Throughput by simulation
$\mathbb{S}$	Sample set
$S_i$	Represents state $i$
$\mathbb{S}_N$	a set of states containing new packets
$\mathbb{S}_i$	Subset of $\mathbb{S}_N$ , which contains states associated with new packet
$[ ]^T$	Represents matrix transpose
$T$	Total duration of the time-slot
$T_d$	duration for data transmission in a TS
$T_D$	Theoretically derived Total average packet delay
$T'_{DS}$	Total average packet delay obtained by simulation
$T_p$	Time duration for packet delivery
$T_s$	during for sensing a TS
$\tau$	Theoretically derived Average end-to-end packet delay
$\tau_s$	Average end-to-end packet delay obtained by simulation

## I. INTRODUCTION

Since the germination of the cognitive radio concept [1], it has been considered to be one of

the favourable regimes for mitigating the spectrum shortage that resulted from the inefficient spectrum utilization. The studies in [2]–[5] show that the inefficient utilization stems from the classic static spectrum allocation policy. Based on this policy, a certain bandwidth is exclusively allocated to a network, which is only available for the primary users (PUs). By contrast, CR relies on dynamic spectrum access (DSA), which allows both the PUs and CUs to share the licensed spectrum. For example, the CUs can sense and exploit the earmarked, but momentarily unoccupied licensed spectrum, which is referred to as a spectrum hole, and utilize it for their own transmissions [6]–[10]. However, the CUs have to evacuate the spectrum as soon as the PUs are reclaiming it. These capabilities of CR have encouraged the regulatory bodies to officially allow the deployment of CR systems in order to maximize the overall spectrum exploitation without having to make any changes to the existing PR systems operated under the classic fixed spectrum allocation approach [2], [10], [11]. As a result, various IEEE standards have incorporated the CR concept, including IEEE 1900, 802.11y, 802.16h, 802.22, etc., as shown in [12] and the references in it.

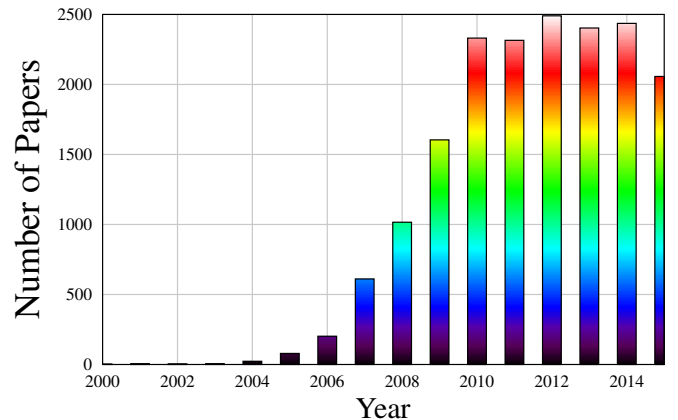


Figure 1. The number of contributions performed in the context of CR per year as per IEEE Xplore database.

In recent years, CR has attracted substantial research attention and has been systematically studied from diverse perspectives, as evidenced by the number of contributions dedicated to it, as shown in Fig. 1 [6]–[9], [13], [14]. Specifically, Haykin [8] has outlined the fundamental operating principles of CR systems in the context of spectrum sensing, channel estimation, cooperative communications, power control and dynamic spectrum management. In [6], [7], [9], the authors have addressed the spectrum sensing and resource allocation problems of CR. The architecture of the CR network and its impact on

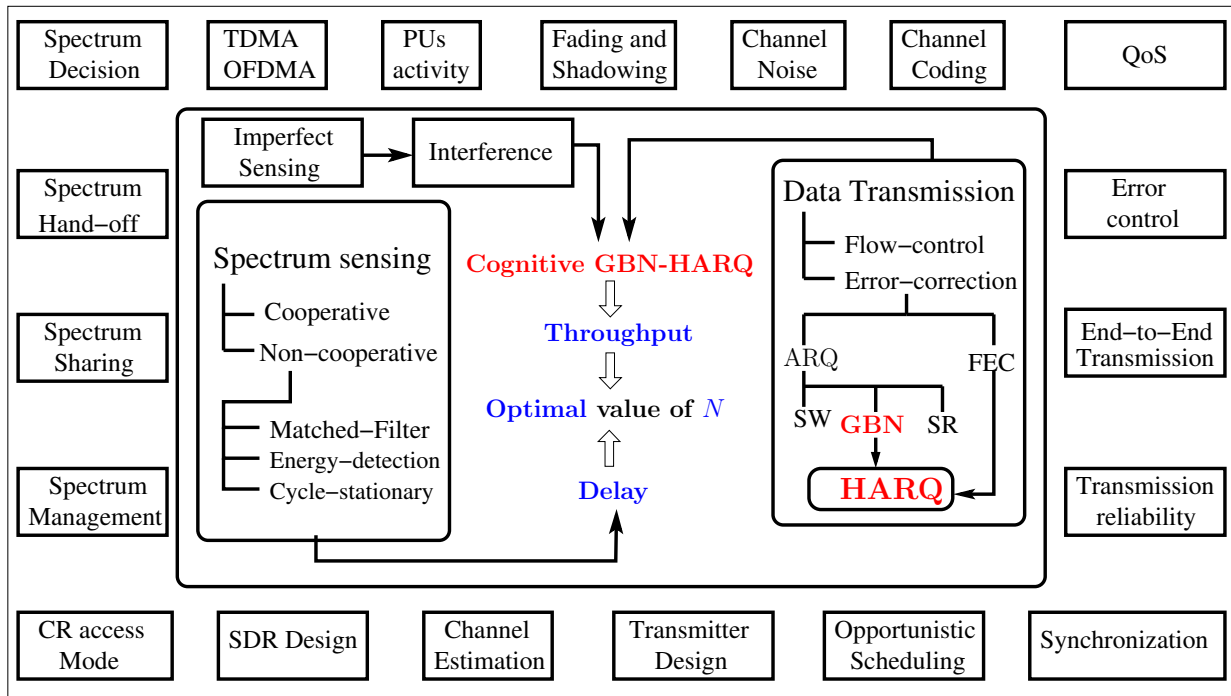


Figure 2. An overview of factors affecting the data transmission in CR systems.

dynamic resource allocation have been studied in [13]. By contrast, Tragos *et al.* [14] have investigated the resource allocation techniques of both centralized and distributed CR networks, while considering the factors of interference, power, delay, etc. In CR, in addition to the challenges imposed by both spectrum sensing and DSA, CR systems also face the extra challenges, when aiming for a low BER. Therefore, powerful error-control approaches are required for achieving reliable communications, similarly to the traditional wireless communication systems [15], [16]. Some of the challenges associated with achieving reliable communication in CR systems are presented in the Fig. 2.

Similar to previous studied [17]–[21] and [22]–[26], the activity of the PUs is modelled using a Markov chain having the ‘ON’ and ‘OFF’ states. Specifically, in the ‘ON’ state, the PR channel is considered to be occupied by the PUs, whereas in the ‘OFF’ state, it is free from PUs. In order to determine the status of the PR channel, the CR system invokes spectrum sensing and then accesses the channel, but only when it is deemed to be in the ‘OFF’ state. In practice, the sensing operation can never be perfect, therefore, the CR system may erroneously identify the PR channel’s state, hence generating either a false-alarm or a mis-detection event. In this case, the channel perceived by the CR system may be described by a four state Markov chain by considering both the actual states of the

PR channel and the states obtained after the sensing operation. Moreover, similar to [27]–[29], the PR’s channel is organized in time-slots (TSs), where each TS has a duration of  $T$  seconds, which is divided into two sections of  $T_s$  and  $T_d$  seconds [27], [29]. The first  $T_s$  seconds of a TS are used for sensing the channel, while the remaining  $T_d$  seconds are utilized for data transmission, which follows the Go-Back-N hybrid automatic repeat request (GBN-HARQ) principles [15], [19], [20], [30], provided that the PR channel is deemed to be in the ‘OFF’ state.

In this paper, we specifically extend the classic GBN-HARQ protocol to the CR system, when imperfect sensing is considered. Here the GBN protocol has been favoured over other ARQ schemes, because its performance is better than that of the classic stop-and-wait HARQ (SW-HARQ), while its complexity is lower than that of the classic selective-repeat HARQ (SR-HARQ). Hence, we are interested in the characteristics and performance of the GBN-HARQ operated in CR communication environments, which is hence referred to as CGBN-HARQ. As mentioned above, in our CGBN-HARQ scheme, first the CR transmitter senses the status of the PR channel. If it is sensed to be free from the PUs, it transmits a number of packets based on the GBN-HARQ protocol. The CR receiver detects the packets one after another. Then the corresponding feedback is generated for each packet and fed back to the trans-

mitter. When the transmitter receives a feedback, it (re)transmit the packets based on the feedback signal. Particularly, after each ACK flag, the transmitter sends a new packet, whereas for each NACK flag, the erroneous packet and all the subsequent packets are retransmitted in the next free TS.

This paper further develops our research on cognitive HARQ [17]–[21]. In a nutshell, in [17], [20], [21] we have studied both the throughput and delay of cognitive SW-HARQ (CSW-HARQ), cognitive Go-Back-N HARQ (CGBN-HARQ) and that of cognitive SR-HARQ (CSR-HARQ), all under the assumption of perfect spectrum sensing. By contrast, in [18], [19], we have characterized the throughput and delay of the CSW-HARQ and CGBN-HARQ schemes both for perfect and imperfect sensing. In more detail, in [18], we have conceived the throughput and delay of the CSW-HARQ scheme. On the other hand, in [19], we have investigated the performance of a CGBN-HARQ scheme in terms of its achievable throughput and delay. We assumed that the feedback flags are always received after the sensing duration of a TS, hence resulting in a simplified operational model for analysing the CR transmitter, but at the cost of a longer round-trip delay and increased number of packet retransmissions. By contrast, in this paper, we relinquished the above constraint by enabling the transmitter to receive feedback flags any time during a TS (namely both during sensing as well as in the transmission epoch). Since the feedback information is well-protected and typically relies on a single-bit information [31], [32], the feedback overhead does not affect the sensing process and the PU's communication. By eliminating the above simplifying assumptions, the analytical modelling of the CGBN-HARQ scheme associated with imperfect sensing became challenging, due to: 1) the reception of the feedback flags both with in the sensing duration and data transmission duration; 2) the imperfect sensing decisions seen in our four-state Markov chain. By doing so, the transmitter circumvents the retransmission of packets for which ACK flags are received in the sensing period. As a result, both the overall throughput and the delay of the system are enhanced. Moreover, in contrast to [19], [20], in this treatise, the CGBN-HARQ is modelled by a DTMC, based on which closed-form expressions are derived both for the throughput as well as for the average packet delay and for the end-to-end packet delay. Our performance studies show that for achieving an increased throughput and a reduced delay, the number of packet transmission per TS requires a

careful consideration for the sake of maximizing both the utility of PR channel and of the CR aided data transmission.

#### A. Related Work

The performance of the classical ARQ protocols has been extensively characterized [15], [30], [58]. Specifically, the conventional GBN-ARQ has been modelled with the aid of DTMC and its throughput and delay was investigated in [59]–[66]. In more detail, Turin as well as Ausavapattanakun and Nosratinia [61], [64] invoked hidden Markov modelling and studied the throughput of GBN-ARQ, when considering both perfectly reliable and practically unreliable feedback in fading environments. In [62], Chakraborty and Liinaharja introduced an adaptive GBN-ARQ scheme operating in a time-varying fading channel and characterized its throughput. In this adaptive GBN-ARQ, the transmitter switches its transmission mode based on the reception of contiguous ACK's and NACK's. Moreover, Hayashida and Komatsu [59] as well as Zorzi and Rao [60] have studied the delay of the classic GBN-ARQ regime. As further advances, Zorzi [63] has designed block-coded error-control scheme operating under delay constraints and studied its performance. Qin and Yang [67]–[70] invoked ARQ techniques for a butterfly network topology and analyzed both its throughput as well as the probability mass function of its delay and its average packet delay. Similarly, Chen *et al.* proposed an algorithm based on Markov modelling for characterizing both the probability distribution and the average packet delay [65]. Li *et al.* proposed a cooperative GBN-HARQ scheme relying on a limited number of retransmissions [71]. The HARQ technique constitutes a promising design for achieving reliable communication, therefore it has been widely employed in wireless systems, as seen in the Timeline 1.

In the context of ARQ aided CR systems, there are only a limited number of references [79], [80], [93]–[103], which are illustrated in Timeline 2. However, none these references have provided the detailed theoretical throughput and delay analysis of ARQ-assisted CR systems. Specifically, in [93]–[95], it is assumed that the CUs infer the activity of the PUs via observing their feedbacks flags. Touati *et al.* [96] have performed the seminal performance analysis of ARQ-aided and relay-assisted CR systems. Yue *et al.* [97], [98] have proposed an improved HARQ scheme for the family of CR systems by

TIMELINE 1: *Milestones of the conventional HARQ-aided wireless systems.*

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- 1975 – The family of ARQ techniques was studied in satellite communications [33].
- 1984 – ARQ and HARQ techniques were studied by Lin *et al.* [34].
- 1988 – De *et al.* studied the performance of Type-II HARQ techniques [35].
- 1990 – Kallel introduced the code combining technique [36].
- 1994 – Decker investigated the family of HARQ techniques in GSM [37].
- 1997 – Zorzi *et al.* characterized the ARQ regimes in fading mobile radio channels [38].
- 1999 – Hamorsky and Hanzo studied the performance of turbo coded Type-II HARQ [39].
- 2000 – Sozer *et al.* characterized various ARQ paradigms in underwater acoustic networks [40].
- 2001 – Caire and Tuninetti investigated the performance HARQ in Gaussian fading channels [41].
- 2003 – Kim *et al.* quantified the performance of HARQ protocols in mobile communication [42].
- 2005 – Zhao and Valenti studied *ad hoc* networks, when HARQ was employed [43].
- 2007 – Beh *et. al* investigated the HARQ techniques of LTE OFDMA systems [44].
- 2009 – Vuran and Akyildiz provided the cross-layer analysis of error control techniques [45].
- 2010 – Zhang and Hanzo proposed HARQ aided superposition coding [46], [47].
- 2011 – Chen *et al.* introduced multi-component turbo coded HARQ techniques [48]–[50].
- 2013 – Chen *et al.* discussed the challenges of concatenating HARQ and turbo codes [51], [52].
- 2014 – Ngo and Hanzo presented the state-of-the-art of HARQ techniques in cooperative communications [53].
- 2015 – Chen *et al.* introduced an adaptive HARQ scheme by amalgamating Raptor codes and HARQ [54].
- 2015 – Xu *et al.* proposed Type-III HARQ based on network coding and distributed turbo coding [55].
- 2016 – Zhu *et al* introduced truncated HARQ technique for reducing the video distortion [56].
- 2016 – Makki *et al.* characterized the performance of HARQ-aided and relay assisted networks [57].

invoking anti-jamming coding for achieving reliable communication. Furthermore, Liang *et al.* and Hu *et al.* [99], [100] studied the combination of spectrum interweave and underlay sharing access modes for the sake of maximizing the performance of the CUs. Specifically, Hu *et al.* [100] proposed ARQ techniques for maximizing the transmission reliability. On the other hand, in [101]–[103] comprehensive surveys have been presented for various communication techniques employed in CR systems.

However, to the best of our knowledge, no published treaties has theoretically analyzed the delay and throughput of ARQ protocols in context of CR systems. The dynamic activities of PUs and the unreliable nature of spectrum sensing make the modelling and analysis of the ARQ protocols in CR systems challenging. Therefore, our objective is to model the family of HARQ-aided CR systems by

deriving closed-form expressions for the throughput and delay of CGBN-HARQ schemes when imperfect spectrum sensing is assumed.

### B. Contributions and Paper Structure

Against the above background, the contributions of this paper are summarized as follows,

- (a) *The cognitive protocol proposed in [20] is extended by enabling the transmitter to simultaneously sense and receive the feedback flag of the packet transmitted in the previous free TS in a realistic imperfect sensing environment. To do so, the CR transmitter requires significant improvements for reformulating the transmission principles of the classic GBN-HARQ, when incorporating it in CR systems.*
- (b) *In contrast to [19], in this paper, the CGBN-HARQ has been theoretically modelled and anal-*



## TIMELINE 2: Milestones in CR-aided HARQ systems.

- 2001 – Ramos and Madani studied software defined radio systems, especially their protocol layer [72].
- 2005 – Cabric and Brodersen introduced a transmission scheme based on wide-band power control and spectrum shaping [73].
- 2006 – Fujii *et al.* focused on space-time-block-coding assisted distributed ARQ in *ad hoc* CR networks [74].
- 2006 – Devroye *et al.* investigated the communication limits of CR systems [75].
- 2007 – Weingart *et al.* investigated the parameters of the physical-network-and-application-layer for the sake of improving both the spectrum efficiency and reliability [76].
- 2008 – Cheng and Zhen combined power controlled modulation with truncated ARQ for improving the overall throughput of CR systems [77].
- 2009 – Yue and Wang introduced an anti-jamming coding technique in the context of CR systems for improving the reliability [78].
- 2010 – Ao and Chen introduced HARQ assisted amplify-and-forward cooperative relaying into CR systems [79].
- 2010 – Yang *et al.* invoked network coding for CR systems for improving the efficiency of classic ARQ techniques [80].
- 2011 – Cheng *et al.* focussed their attention on improving the data rate of CR system by observing the ARQ feedback flags of PUs [81].
- 2012 – Makki *et al.* characterized the throughput and outage probability of CR aided HARQ systems [82].
- 2013 – Andreotti *et al.* introduced a link adaptation paradigm for increasing the goodput, whilst reducing the complexity [83].
- 2014 – Ozcan *et al.* characterized the error rate performance of CR systems [84].
- 2014 – Chen *et al.* focused their efforts on the end-to-end transmission improvements of CR-aided *ad hoc* networks [85].
- 2015 – Zou *et al.* proposed an opportunistic relay assisted scheme for improving the reliability of CR systems [86].
- 2015 – Makki *et al.* invoked finite-length codewords in spectrum sharing CR networks and studied their throughput [87], [88].
- 2016 – Khan *et al.* reviewed the networking techniques, applications and CR communication protocols employed in smart grids [89].
- 2016 – Zhang *et al.* introduced energy-efficient DSA protocols for improving the overall channel utilization without degrading the PUs performance [90].
- 2016 – Patel *et al.* focused on maximizing the achievable rates of CR systems [91], [92].

*used by a DTMC-based approach in imperfect sensing environments. Based on this model, closed form expressions are derived for the throughput, average packet delay and end-to-end packet delay. For the case of end-to-end packet delay, both its probability distribution and average end-to-end packet delay are analysed*

(c) *Finally, the theoretical analysis has been verified by the simulations.*

The rest of this paper is organized as follows. In Section II, we model the primary radio systems

as well as the cognitive systems. In Section III, we outlined the transmission principles of CGBN-HARQ, whereas the operation of the CR transmitter and receiver is presented in Subsections III-A and III-B, respectively. Then, we detailed our DTMC-based analytical model in Section IV. Then, closed-form expressions are derived for the throughput and delay in the Subsections IV-D and IV-E, which are then verified by our simulations presented in Section V. Finally, the conclusions of this contribution are provided in Section VI.

## II. SYSTEM MODEL

In this section, we will model both the PR and CR systems. Furthermore, the main assumptions used are described.

### A. Modelling of Primary Radio Systems

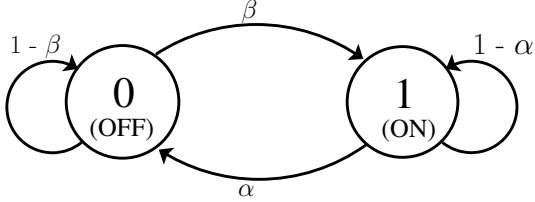


Figure 3. Two-state discrete-time Markov chain model for PR systems [23]–[25].

Similar to our previous studies [19], [20], a two-state Markov chain having ‘ON’ and ‘OFF’ states is invoked for modelling the activities of PUs in the PR channel, as shown in Fig. 3. In the ‘ON’ state, a PU is assumed to be utilizing the PR channel, while in the ‘OFF’ state, the PR channel is free from the PUs. The transition probability from the ‘OFF’ to the ‘ON’ and that from ‘ON’ to ‘OFF’ state are represented by  $\beta$  and  $\alpha$ , respectively. Let  $P_{on}$  and  $P_{off} = 1 - P_{on}$  represent the probabilities of the PR channel being in the ‘ON’ and ‘OFF’ states. Then, when the Markov chain reaches its steady state, we have [30]

$$P_{on}\alpha = P_{off}\beta, \quad (1)$$

which results in

$$P_{on} = \frac{\beta}{\alpha + \beta}, \quad P_{off} = \frac{\alpha}{\alpha + \beta}. \quad (2)$$

Similar to our previous contributions [18]–[20] and as shown in Fig. 4(a), the PUs utilize the channel on the basis of TSs, whereas the activation of each TS is independent and has the same probability. Similarly, we also assume that when a TS is found in the ‘OFF’ or ‘ON’ state, the PUs activity remains the same until the end of that TS. For instance, when a TS is deemed to be in the ‘OFF’ state, the CU is allowed to access the TS and send its own data. Otherwise, the CU waits until the next TS [22].

### B. Modelling of Cognitive Systems

In CR systems, each TS is further partitioned into the sensing time duration of  $T_s$  seconds and data transmission in the remaining of  $T_d = T - T_s$  seconds, as presented in Fig. 4(b). As mentioned above, in the sensing duration, the CUs perform

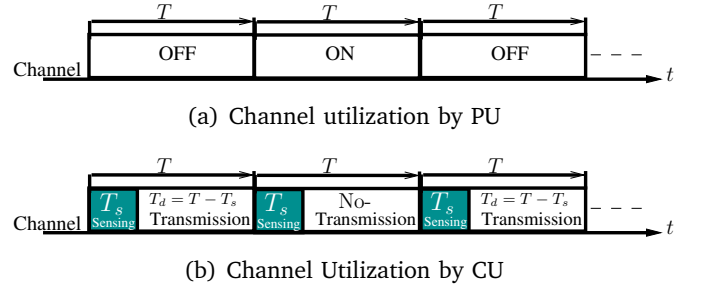


Figure 4. Time-slot structure of PR and CR systems, where a CR TS consists of a sensing duration of  $T_s$  and a transmission duration of  $T_d = T - T_s$ , when given the total duration  $T$  of a time-slot.

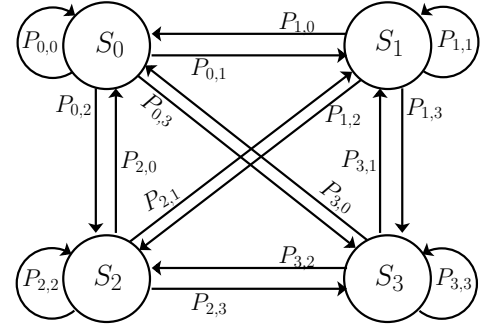


Figure 5. Modelling the CR system in the face of realistic imperfect sensing with the help of a four-state Markov chain, where  $S_0 = 00$  represents the case in which the TS is free in reality and the CU correctly sensed it as being free.  $S_1 = 01$  illustrates that the channel is free in reality, but the CU detected it to be busy. Likewise  $S_2 = 10$  and  $S_3 = 11$ .

sensing for the sake of determining the ‘ON’ and ‘OFF’ activity of the PUs in the channel considered. If the PR channel is deemed to be free, then in the remaining  $T_d$  seconds the CU embarks on data transmission using the classic GBN-HARQ protocol, which will be detailed below. Otherwise, it waits until the next TS, during which the above procedure is repeated.

It is widely recognized that the sensing operation cannot be perfectly reliable in practice, hence there is always a chance of incorrect sensing, due to numerous problems, such as for example the channel-induced shadowing and fading, aggravated by the background noise. Similarly, in the context of CR systems, it may also wrongly detect the presence/absence of PUs in the PR channel, hence resulting either in false-alarm ( $P_{fa}$ ) or in mis-detection ( $P_{md}$ ). To elaborate a little further, false-alarm represents the scenario, in which the PU is inactive but the CR system deems it to be active. On the other hand, mis-detection represents the case, in which the PU is active, yet the CR system deemed the channel to be free. In summary, the states of this realistic CR

system are shown in Fig. 5, where the index of each state is the combination of a two-digit sequence, where the first digit illustrates the real status of the PR channel, whereas the second digit represents the sensed status of the channel. Additionally, the transition probability between states can be readily arranged in a matrix form as

$$\mathbf{P} = \begin{bmatrix} P_{0,0} & P_{0,1} & P_{0,2} & P_{0,3} \\ P_{1,0} & P_{1,1} & P_{1,2} & P_{1,3} \\ P_{2,0} & P_{2,1} & P_{2,2} & P_{2,3} \\ P_{3,0} & P_{3,1} & P_{3,2} & P_{3,3} \end{bmatrix}.$$

Then, with the aid of (2) and Fig. 5, we obtain

$$\begin{aligned} P_{0,0} &= P_{1,0} = (1 - \beta)(1 - P_{fa}) \\ P_{0,1} &= P_{1,1} = (1 - \beta)P_{fa} \\ P_{0,2} &= P_{1,2} = \beta P_{md} \\ P_{0,3} &= P_{1,3} = \beta(1 - P_{md}) \\ P_{2,0} &= P_{3,0} = \alpha(1 - P_{fa}) \\ P_{2,1} &= P_{3,1} = \alpha(1 - P_{fa}) \\ P_{2,2} &= P_{3,2} = (1 - \alpha)(P_{md}) \\ P_{2,3} &= P_{3,3} = (1 - \alpha)(1 - P_{md}). \end{aligned} \quad (3)$$

Let us express the steady-state probabilities of the Markov chain as  $\Phi = [\Phi_0, \Phi_1, \Phi_2, \Phi_3]^T$ . Then, we have

$$\Phi = \mathbf{P}^T \Phi, \quad (4)$$

which demonstrates that  $\Phi$  is the right eigenvector of  $\mathbf{P}^T$  having an eigenvalue of one. Therefore, based on (4), we have

$$\begin{aligned} \Phi &= [\Phi_0 \quad \Phi_1 \quad \Phi_2 \quad \Phi_3]^T \\ &= \lambda \times \begin{bmatrix} \alpha(1 - P_{fa}) & \alpha(P_{fa}) & (P_{md}) \\ \beta(1 - P_{md}) & \beta(1 - P_{md}) & (1 - P_{md}) \end{bmatrix}^T, \end{aligned} \quad (5)$$

where  $\lambda \in \mathbb{R}$  is applied to satisfy

$$\sum_{i=0}^3 \Phi_i = 1, \quad (6)$$

giving

$$\lambda = \frac{\beta(1 - P_{md})}{\alpha + \beta}. \quad (7)$$

Upon substituting (7) into (5), the steady-state probabilities of the CR system at states  $S_0, S_1, S_2$  and  $S_3$  are given by

$$\Phi_0 = \frac{\alpha(1 - P_{fa})}{\alpha + \beta}, \quad \Phi_1 = \frac{\alpha(P_{fa})}{\alpha + \beta} \quad (8)$$

$$\Phi_2 = \frac{\beta(P_{md})}{\alpha + \beta}, \quad \Phi_3 = \frac{\beta(1 - P_{md})}{\alpha + \beta}. \quad (9)$$

### III. PRINCIPLES OF COGNITIVE GO-BACK-N HYBRID AUTOMATIC REPEAT REQUEST

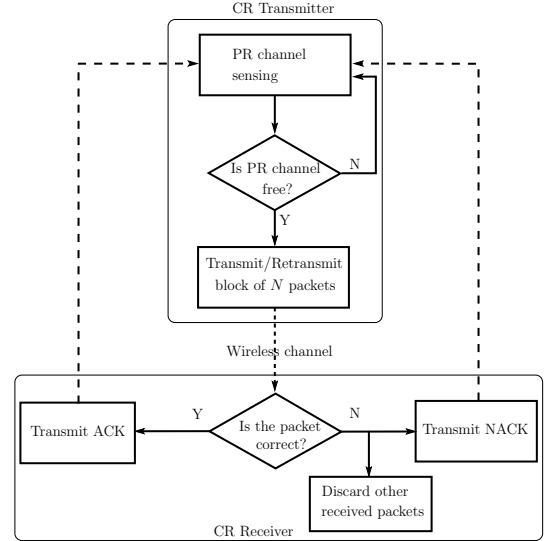


Figure 6. Flow chart showing the operations of the proposed CGBN-HARQ scheme.

Similar to our previous studies [19], [20], in this treatise, the CR system also transmits its data in the form of packets, where each packet is assumed to be encoded by Reed-Solomon (RS) codes defined over the Galois Field of  $\text{GF}(q) = \text{GF}(2^m)$ . Each RS coded packet is comprised of  $N_d$  and  $K_d$  symbols. Specifically,  $N_d$  represents the information symbols, whereas  $K_d$  denotes the coded symbols. Moreover, we assumed that each RS coded packet requires a duration of  $T_p$  seconds for its transmission. As a result, the transmitter is capable of sending  $N = T_d/T_p$  packets in each free TS. Furthermore, we also assumed that the RS code is capable of perfectly detecting and rectifying  $t$  errors. However, when more than  $t$  symbol errors are received in a packet, it discards the packet.

Following the above assumptions and as illustrated in Fig. 6, the data packets are transmitted between a pair of CR users, namely the CR transmitter and CR receiver, while relying on the principles of the CGBN-HARQ scheme. Similar to the classic GBN-HARQ [15], [30], [58], the CR transmitter seamlessly transmits  $N$  packets one after another in a free TS, without waiting for their feedback flags, where the feedback for each packet is programmed to be received after the round-trip-time (RTT) of  $T_d$  seconds. The RTT can be defined as the time duration spanning from the commencement of packet transmission until the reception of its feedback flag [15], [30], [58]. Note that during the RTT of a packet, the subsequent  $(N - 1)$  packets are also



transmitted.

Below we provide more details concerning the operational principles of the CGBN-HARQ based CR transmitter and CR receiver, which was described by Algorithm 1 of [20] in a perfect sensing scenario. By contrast, the additional consequences imposed by imperfect sensing are discussed as follows.

- When a TS is mis-classified, the transmitter sends  $N$  packets to the receiver following the normal procedure described in Algorithm 1 of [20].
- In the case of false-alarm, even though the channel is free, the CR transmitter does not transmit packets.
- At the receiver side, in the mis-detection scenario, the receiver discards the packet with a probability of one, since the probability of the packet being in error is too high both due to its collision with the PUs transmission and the channel noise.

#### A. Operation of the CR transmitter

In the conventional GBN-HARQ protocol, the packets are continuously transmitted by the transmitter until it receives a negative feedback flag (NACK). In this case, the retransmission of the corrupted and of all subsequent packets has to take place. In the CGBN-HARQ scheme of this contribution, however, the PR channel is first sensed by the CR transmitter for  $T_s$  seconds. If the TS is detected to be free from PUs, the corresponding packets are transmitted, otherwise the transmitter waits for the next TS. Additionally, in realistic imperfect sensing environments the transmission of packets takes place, when the PR channel is correctly detected to be in the ‘OFF’ state, or when it is deemed to be in the ‘OFF’ state but it is actually ‘ON’. On the other hand, if the PR channel is in the ‘ON’ state or falsely deemed to be in the ‘ON’ state, the transmitter waits for the next TS and repeats the process of sensing. In summary, in each free TS, the transmitter sends  $N$  packets. These packets may be either new or those that require retransmission.

Moreover, as in the classical GBN-ARQ, it is assumed that the CR transmitter has a buffer size of  $N$  packets and it follows the first-in-first-out principle [15], [30], [58]. In the transmitter buffer the copies of the transmitted packets are stored until it receives a positive feedback (ACK) flag. For example, if an ACK is received for a packet, the CR transmitter removes its copy from the buffer and replaces it with a new

packet. On the other hand, if a NACK flag is received, the transmitter buffer remains unchanged. Furthermore, it is assumed that the transmission of feedback flags is always error-free. This may be justified, since a single bit of feedback information is sufficient for the CGBN-HARQ, which can be strongly protected from errors [31], [32]. This justifies our assumption of allowing the CR transmitter to receive feedback flags at any time in a busy or free TS, without hampering the sensing process or the PU transmission, as shown in Fig. 7 and 8.

It is worth noting that when feedback flags are received during sensing time, the transmitter only updates its buffer and waits until the decision about the TS takes place. For instance, as shown in Fig. 7, the ACK flag for packet 1 is received during sensing time, hence the transmitter replaces the copy of packet 1 by packet 5 and waits for the sensing result. Soon after the sensing decision took place and ACK flags were received in the transmission period, the transmitter sends the new packets 5, 6, 7, 8 in the free TS, as illustrated in TS  $T_2$  of Fig. 7. The process continues until a NACK flag is received, as shown in Fig. 8. The transmitter immediately stops sending new packets and prepares for retransmitting both the corrupted and the subsequent packets. Moreover, in the case of a busy TS, the transmitter prepares the sequence of packets in the buffer for transmitting them in the next free TS, as shown in TS  $T_3$  and  $T_4$  of Figs. 7 and 8.

It is worth noting that in Fig. 7, TS  $T_2$  is mis-detected, which implies that the TS is actually occupied by the PU, but due to the wrong sensing decision, it is declared to be free. As a result, new packets, i.e., packets 5, 6, 7 and 8, of Fig. 7 are transmitted. However, due to their collisions with the PU transmissions, these packets are corrupted and hence all  $N$  packets are received in error. Moreover, in the case of false-alarm, the free TS is sensed to be busy, hence no transmission takes place.

#### B. Operation of the CR receiver

In this contribution, the CR receiver works in a similar way as that proposed in our previous studies [19], [20]. This is because, the receiver has no information about the sensing or the PU’s activity in the channel. However, it only performs RS decoding of the received packets and generates the respective feedback flags, which are then sent to the transmitter. For example, as shown in Fig. 7 and 8, if a packet is declared to be error-free after error-correction, the

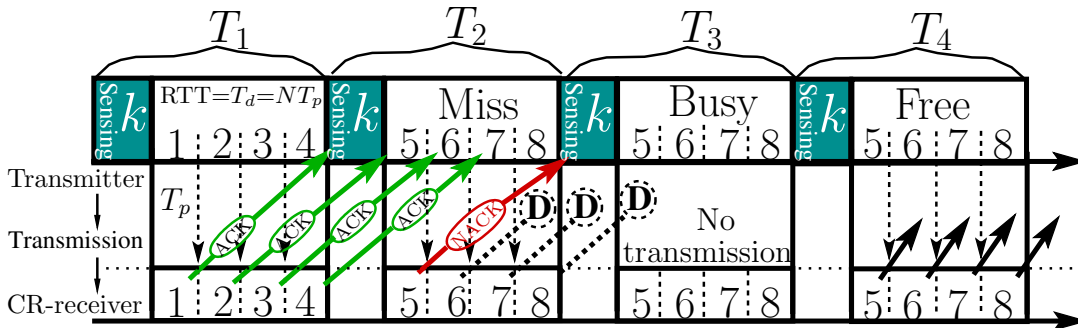


Figure 7. Packet transmission using the CGBN-HARQ scheme, when  $k = 1$  and  $N = 4$  in the case of imperfect sensing.

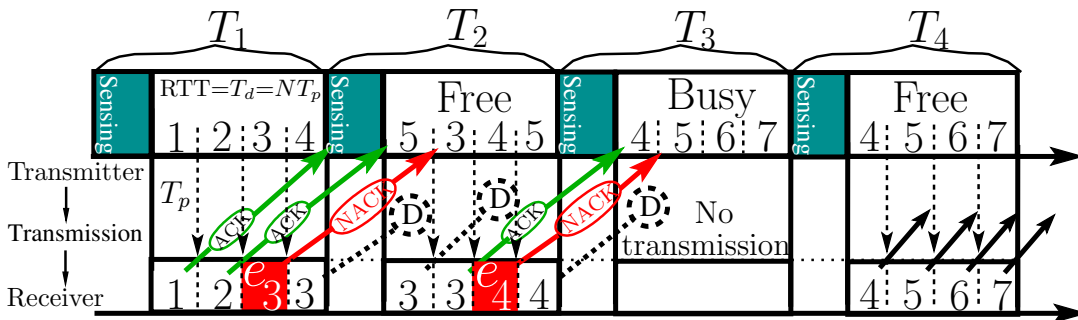


Figure 8. Packet transmission using the CGBN-HARQ scheme, when  $k = 1$  and  $N = 4$ , illustrating the effect of the NACK flag in both free and busy TSs.

receiver transmits an ACK flag to inform the transmitter that the packet is correctly received, hence a new packet is required. On the other hand, if the NACK flag is transmitted due to erroneous reception of a packet, the transmitter sends both the erroneous and the subsequent packets again. Moreover, in the case of mis-detection, the CR receiver considers the packet to be erroneous and feeds back a NACK flag to the transmitter's control channel receiver for the respective packet.

#### IV. DISCRETE TIME MARKOV CHAIN BASED ANALYSIS

In contrast to the classic GBN-HARQ protocol, the modelling of the CGBN-HARQ transmission scheme in imperfect sensing environments hinges on the dynamic activity of the PU and on the potentially incorrect sensing of the PR channel. In this paper, the CGBN-HARQ transmission scheme has been modelled by DTMC in a realistic imperfect sensing environment. Based on the DTMC modelling, we then evaluate the steady-state throughput, average packet delay and the end-to-end delay considering both its probability distribution and the average end-to-end packet delay.

To commence the analysis, the states of the CGBN-HARQ system can be jointly determined by the following factors:

- The actual status of the PR channel.
- The sensed status of the PR channel.
- The status of the  $N$  packets stored in the transmitter buffer, (which may be 'new', 'old' and 'packets transmitted twice in a TS').

Let us formulate the list of states as

$$\mathbb{S} = \{S_0, S_1, \dots, S_i, \dots, S_T\}, \quad (10)$$

where  $S_T$  represents the total number of states, i.e we have  $S_T = |\mathbb{S}|$ , and  $S_i$  is the  $i$ th state in  $\mathbb{S}$  and has an  $(N + 2)$ -length base-3 digit formulated as

$$S_i = \{S_{i0}S_{i1}S_{i2} \dots S_{i(N+1)}\}, \quad i = 0, 1, \dots \quad (11)$$

For convenience,  $S_i$  represents the  $i$ th state sequence. Specifically, in the state sequence of Eq (11), the first digit  $S_{i0}$  illustrates the real status of the PR channel,

$$S_{i0} = \begin{cases} 0, & \text{if the PR channel is free in real,} \\ 1, & \text{if the PR channel is busy in real.} \end{cases} \quad (12)$$

The second digit  $S_{i1}$  represents the sensed status of

the PR channel, which is given by

$$S_{i1} = \begin{cases} 0, & \text{if the PR channel is sensed free;} \\ 1, & \text{if the PR channel is sensed busy.} \end{cases} \quad (13)$$

Finally, the digit  $S_{ij}$  for  $j = 2, \dots, N + 1$ , is determined as

$$S_{ij} = \begin{cases} 0, & \text{if the } j\text{th packet is a new one;} \\ 1, & \text{if the } j\text{th packet is a retransmitted one,} \\ & \text{when the TS is free; or the one to be} \\ & \text{retransmitted, when the TS is busy;} \\ 2, & \text{if the } j\text{th packet is a repeated replica} \\ & \text{of a previous packet in the same TS.} \end{cases} \quad (14)$$

Having determined  $S_{ij}$  according to (12), (13) and (14), we can find a one-to-one mapping between  $i$  and  $S_i$  as

$$i = \sum_{j=0}^{N+1} S_{ij} 3^{N-j+1}, \quad (15)$$

which gives the subscript  $i$  of  $S_i$ , representing the  $i$ th state of the DTMC.

For the sake of deep understanding, a pair of examples are provided below, which explains the DTMC-based modelling in the face of imperfect sensing in terms of the associated state transitions.

#### A. Example 1

Let us examine the CGBN-HARQ scheme using a single  $T_p$  for the sensing process, which is only capable of transmitting a single packet (i.e.  $k = 1$  and  $N = 1$ ). In other words, the transmitter buffer stores the copy of a single packet. In this scenario, a packet will never be repeated in the same TS and therefore digit 2 of Eq. (14) will never appear in the state sequence. According to the above definitions, we can show that the DTMC of this system has  $S_T = 8$  states, which are explicitly shown in Table I. Corre-

State	$(S_{i0} \ S_{i1} \ S_{i2})$	State	$(S_{i0} \ S_{i1} \ S_{i2})$
$S_0$	(0 0 0)	$S_9$	(1 0 0)
$S_1$	(0 0 1)	$S_{10}$	(1 0 1)
$S_3$	(0 1 0)	$S_{12}$	(1 1 0)
$S_4$	(0 1 1)	$S_{13}$	(1 1 1)

Table I

POSSIBLE STATES OF THE CGBN-HARQ, WITH  $N = 1$  AND  $k = 1$ .

spondingly, the state transitions are shown in Fig. 10, where the transitions and their transition probabilities are detailed below. Let us assume that the

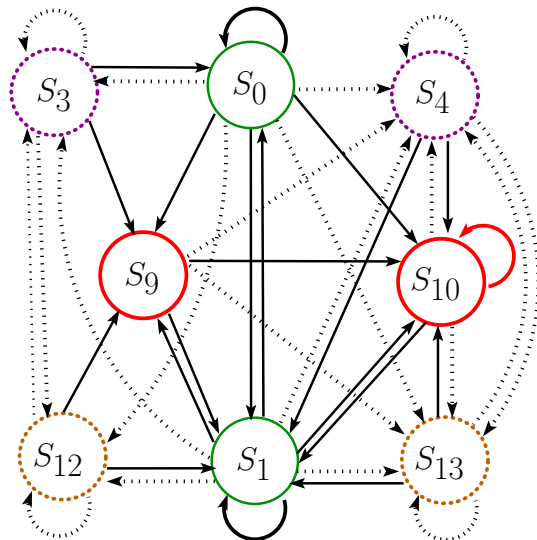


Figure 10. State diagram illustrating the modelling of the CGBN-HARQ scheme for  $N = 1$  and  $k = 1$ . The *dashed* lines denote the transitions to the busy states only, due to the detection of a busy TS. On the other hand, *solid* lines represent the transitions to the free states during the detection of free TSs. The *solid* circles illustrate the free states due correct detection and mis-detection, whereas *dashed* circles denote the busy states due to correct detection and false-alarm.

transmitter is initially in state  $S_0 = 000$ . This state is defined by the events that the TS is actually free (i.e.  $S_{i0} = 0$ ) and it is also sensed to be free by the CR system (i.e.  $S_{i1} = 0$ ). Therefore, the transmission of a new packet takes place ( $S_{i2} = 0$ ). Moreover, when the transmitter receives an ACK flag related to the transmitted packet and the next TS is again correctly detected to be free, the transmitter remains in the same state  $S_0$ . This gives the transition probability of  $P_{0,0} = P_{off}(1 - P_{fa}) \times (1 - P_e)$ , where  $P_e$  is the packet's error probability. By contrast, if the transmitter receives a NACK flag and the TS is found to be free from PUs, the system traverses to state  $S_1 = 001$ . Correspondingly, the transition probability is  $P_{0,1} = P_{off}(1 - P_{fa}) \times P_e$ . In a similar manner, if the next TS is correctly detected to be busy and an ACK (or a NACK) flag is received, then the transmitter changes its state from  $S_0$  to  $S_{12}$  (or  $S_{13}$ ), with a transition probability of  $P_{0,12} = P_{on}(1 - P_{md}) \times (1 - P_e)$  (or  $P_{0,13} = P_{on}(1 - P_{md}) \times P_e$ ). Likewise, if the next busy TS is mis-detected and an ACK (or NACK) flag is received, then the transmitter traverses from  $S_0$  to  $S_9$  (or  $S_{10}$ ), with a transition probability of  $P_{0,9} = P_{on}P_{md} \times (1 - P_e)$  (or  $P_{0,10} = P_{on}P_{md} \times P_e$ ).

When the transmitter is in state  $S_9$  or  $S_{10}$ , in the next TS the transmitter can only make a transition to one of the following states  $S_1$ ,  $S_4$ ,  $S_{10}$  and  $S_{13}$ . This is because the transmissions associated  $S_9$  and  $S_{10}$  are always assumed to be incorrect due to mis-detection.

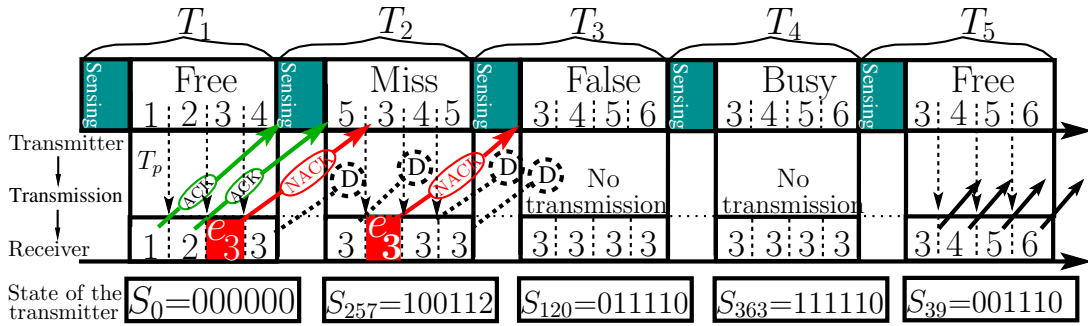


Figure 9. Packet transmission using CGBN-HARQ, where, Miss, and, False, represents mis-detection and false-alarm, respectively. At the end of the TS, the transmitter state is observed.

If the next TS yields false-alarm or it is correctly detected to be busy, then the transmitter moves to state  $S_4$  or  $S_{13}$ , associated with the transition probability of  $P_{9,4} = P_{off}P_{fa}$  or  $P_{9,13} = P_{on}(1 - P_{md})$ . Similarly, based on Fig. 10, the remaining state transitions can be readily analyzed, where in Table II, we presented the corresponding state to state transition probabilities.

### B. Example 2

In the second example, we considered the CGBN-HARQ scheme having the parameters of  $k = 1$  and  $N = 4$ . For clarity, we consider the scenario shown in Fig. 9, which illustrates that the transitions take place across five TSs, with the following justification.

- Let us consider in TS  $T_1$  that the PR channel is correctly detected to be free. Hence, we have  $S_{i0} = 0$  and  $S_{i1} = 0$ . In this case, we assume that  $N = 4$  new packets are transmitted, which give  $S_{ij} = 0$  for  $j = 2, 3, 4, 5$ . Therefore, at the end of TS  $T_1$ , the state observed by the transmitter is  $S_0 = \{0, 0, 0, 0, 0, 0\}$ . As shown in the figure, the first two packets are received without errors, therefore ACK flags are transmitted. By contrast, packet 3 is erroneously received, hence the receiver feeds back a NACK flag to the transmitter.
- Due to mis-detection, the TS  $T_2$  is deemed to be free, although it is actually busy. Therefore, the transmitter receives an ACK after the sensing duration and it transmits the new packet 5. However, after the reception of a NACK flag at the end of the first  $T_p$  of the transmission duration of  $T_2$ , both the corrupted as well as the subsequent packets are retransmitted, where the indices of the 4 packets transmitted during  $T_2$  are 5, 3, 4, 5. It is worth noting that packet 5 is transmitted twice in the same TS, while packets 3 and 4 are old packets

transmitted in the previous TS, as depicted in Fig. 9. All the above-mentioned events result in the state associated with the index of  $S_{257} = 100112$  in TS  $T_2$ . Correspondingly, the transition probability from state  $S_0$  to  $S_{257}$  is  $P_{0,257} = P_{on}P_{md} \times (1 - P_e)^2 P_e$ . Moreover, since the TS  $T_2$  is mis-detected, the packets transmitted in TS  $T_2$  are all received in error. Hence, the receiver feeds back a NACK flag, while packets 4 and 5 are discarded, regardless whether they are received correctly or incorrectly.

- During TS  $T_3$ , the channel is falsely detected to be busy. Since all packets transmitted during the TS  $T_2$  are erroneous, they require retransmission. Hence, the transmitter changes its state from  $S_{257}$  to  $S_{120} = 011110$  with a probability of  $P_{257,120} = P_{on} \times P_{md}$ .
- Following the same pattern, the state associated with TS  $T_4$  is  $S_{363}$  and the corresponding transition probability from state  $S_{120}$  to  $S_{363}$  is  $P_{120,363} = P_{on} \times (1 - P_{md})$ .
- Finally, the TS  $T_5$  is correctly detected to be free. Therefore, all the  $N$  packets waiting in the buffer are transmitted, which results in a transition from state  $S_{363}$  to the state  $S_{39}$  with a transition probability of  $P_{363,39} = P_{off} \times (1 - P_{fa})$ .

According to the principles of our CGBN-HARQ, the characteristics of the DMTC derived for the CGBN-HARQ relying on realistic imperfect sensing are given below.

- In a busy TS, there will no transmission or retransmission of a packet, therefore the respective state sequence does not contain digit 2. This busy state can be encountered either due to correct sensing of an occupied PR channel, or owing to the false-alarm encountered in a free PR channel.



- When a transition from a busy state to a free state occurs, only the first two digits of the current state sequence  $S_{i0}$  and  $S_{i1}$  may change in the new state sequence. This is explicitly shown in TS  $T_5$  of Fig. 9. Hence, digit 2 does not appear in the new state sequence.
- The first two digits of a state sequence, i.e.  $S_{i0}$  and  $S_{i1}$ , never assume a value of 2.

With the aid of these properties, large fractions of the states can be eliminated from consideration. However, there is still a large number of states. In this context, the mathematical formulation required for deriving total number of states and for generating state sequences with the aid of general expression remains an open challenge. However, the transition probability for any pair of state can be readily derived. Hence, we propose the algorithm shown in Fig. 11 for finding the number of states and the state sequences. The algorithm is conceived as follows.

Fundamentally, this algorithm uses a search technique for finding the legitimate states. Initially, the transmitter is set to the state  $S_0 = 000000$ . Let the legitimate states form the set  $\mathbb{S}$ , which initially contains only  $S_0$ . In the process for generating new states, we arrange for an erroneous transmission to occur at each possible position of the packets sent in the TS for the current state  $S_0$ . Let us simultaneously assume that the next TS is either free or busy, which is correctly sensed, mis-detected or sensed subject to a false-alarm.

For example, during the TS  $T_1$  of Fig. 7, only new packets are transmitted, which are correctly received. Hence ACK flags are fed back. In the algorithm, this case is represented by  $e = 0$ . Continuing this example, when the next TS is busy, but it is sensed to be free, 4 new packets are transmitted in TS  $T_2$ . Consequently, the state is changed to  $S_{243} = 100000$ . Moreover, in the same case of  $e = 0$ , it is possible that TS  $T_2$  may be sensed to be busy due to false detection of a free PR channel or owing the correct detection of a busy PR channel. If this is the case, the current state changes to  $S_{81} = 010000$  for the false-alarm or to  $S_{324} = 110000$  for the correct detection, respectively. When a new state is generated, for instance  $S_{324}$ , it is then checked against the state-set  $\mathbb{S}$ . If state  $S_{324}$  is already in  $\mathbb{S}$ , it is ignored. Otherwise,  $S_{324}$  is added to the set  $\mathbb{S}$ . Similarly, by considering the cases of  $e = 1$  to  $e = N$ , the algorithm will generate all legitimate states that the transmitter may encounter upon emerging from state  $S_0$ . Once all the error-free transmission to

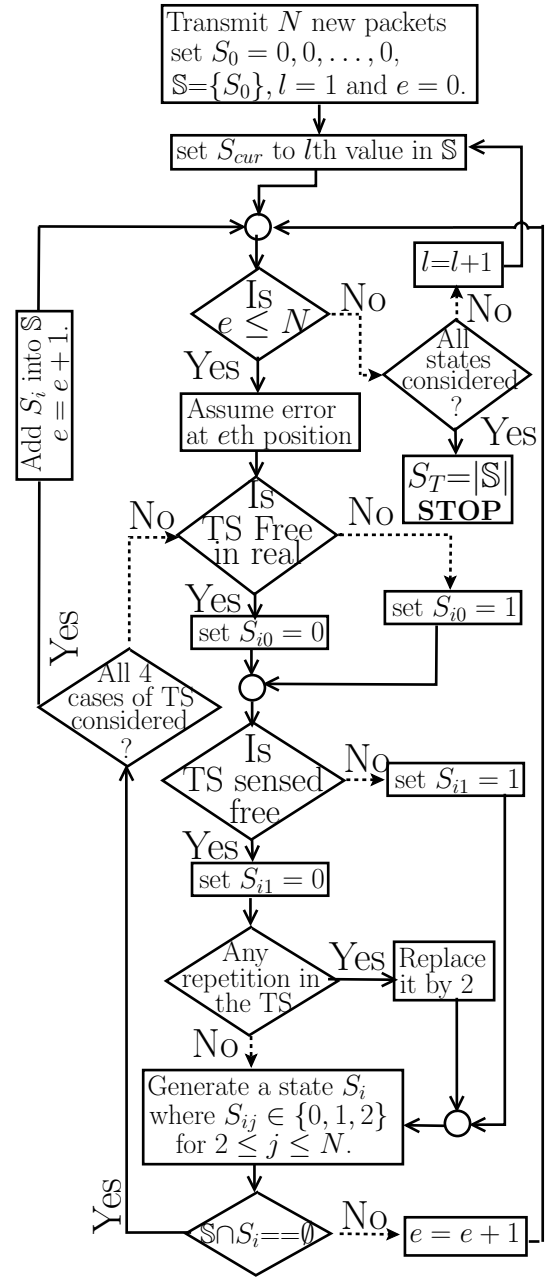


Figure 11. Illustration of the process of deriving states relying on  $(N+2)$ -length base-3 digits. For each value of  $e$ , the following four cases are considered: 1) correct detection of a free PR channel 2) correct detection of a busy PR channel 3) mis-detection; and 4) false-alarm. A special case of a completely error-free packet is represented by  $e = 0$ , while in a TS,  $e$  denotes the position of the corrupted packet.

erroneous transmission scenarios of each packet in a TS have been considered, the algorithm is then activated for the next state in  $\mathbb{S}$ , which is  $S_{243}$  in the example considered in Fig. 7. The algorithm repeats all the above steps for the sake of deriving new legitimate states. Moreover, due to the properties of irreducibility and the aperiodic nature of the DTMC, the state generating algorithm stops, once each state



in  $\mathbb{S}$  has been examined as current state and no new state is generated. Finally, at the end of the algorithm, the cardinality of the set  $\mathbb{S}$  gives the total number of states  $S_T$ , i.e. we have  $S_T = |\mathbb{S}|$ .

### C. State Transition Probability Matrix

Let the state transition probability matrix be expressed as  $\mathbf{P}$  and the state at the  $i$ th TS be represented by  $S(i)$ . Then, for the example of Fig. 9, we have  $S(1) = S_0$ ,  $S(2) = S_{257}$ ,  $S(3) = S_{120}$ ,  $S(4) = S_{363}$  and  $S(5) = S_{39}$ . Given the state  $S_i$  in TS  $m$ , we have  $S(m) = S_i$ . Then the probability  $P_{i,j}$  of transition to the next state is  $S_j$ , i.e.  $S(m+1) = S_j$  can be expressed [30], [104] as:

$$\begin{aligned} P_{i,j} &= P\{S(m+1) = S_j \mid S(m) = S_i, \dots, S(1) = S_0\} \\ &= P\{S(m+1) = S_j \mid S(m) = S_i\}, \end{aligned} \quad (16)$$

which is the  $(j, i)$ th element of  $\mathbf{P}$ . According to the properties of the DTMC, we have

$$\begin{aligned} 0 &\leq P_{i,j} \leq 1 \\ \sum_{S_j \in \mathbb{S}} P_{i,j} &= 1, \quad \forall S_i \in \mathbb{S}. \end{aligned} \quad (17)$$

It is worth noting that the transition probability  $P_{i,j}$  is set to zero, if the transmitter cannot make a transition from  $S_i$  to  $S_j$  in a single step (i.e. in the next TS). To continue our modelling, let the probability  $P_i(m)$  represent the transmitter state in TS  $m$ . Then we have  $\sum_{i \in \mathbb{S}} P_i(m) = 1$ . Again, let  $\mathbf{p}(m) = [P_0(m), P_1(m), \dots, P_{S_T}(m)]^T$  store the  $S_T$  probabilities in TS  $m$ . Furthermore, we assume that the transmission commences from state  $S(1) = S_0$  associated with the probabilities of

$$\mathbf{p}(1) = [1, 0, \dots, 0]^T. \quad (18)$$

Then, with the aid of the law of total probability [30], [105], the specific probability that the Markov chain transitions into state  $j$  during the TS  $(m+1)$  may readily be expressed as

$$P_j(m+1) = \sum_{\forall S_i \in \mathbb{S}} P_i(m) P_{i,j}, \quad \forall S_j \in \mathbb{S} \text{ and } m > 1. \quad (19)$$

By considering all the states in  $\mathbb{S}$ , we readily arrive at the recursive equation of

$$\mathbf{p}(m+1) = \mathbf{P}^T \mathbf{p}(m), \quad m = 1, 2, 3, 4, \dots \quad (20)$$

given the above equation, we know that

$$\mathbf{p}(m+1) = (\mathbf{P}^T)^m \mathbf{p}(1). \quad (21)$$

Eq (17) shows that the state transition probability matrix  $\mathbf{P}^T$  is a left stochastic matrix [105], because the sum of each column is equal to one. Consequently, based on the *Perron-Frobenius theorem*, the limit of  $\lim_{m \rightarrow \infty} (\mathbf{P}^T)^m$  exists [105], and when we have  $m \rightarrow \infty$ , the DTMC reaches its steady state [30], implying that

$$\mathbf{p}(m+1) = \mathbf{p}(m). \quad (22)$$

Let us represent the steady-state probabilities by  $\boldsymbol{\pi}$ , i.e.  $\boldsymbol{\pi} = \lim_{m \rightarrow \infty} \mathbf{p}(m)$ . Then, based on (20) and (22), we have [30], [104],

$$\boldsymbol{\pi} = \mathbf{P}^T \boldsymbol{\pi}, \quad (23)$$

where  $\boldsymbol{\pi}$  is the right eigenvector of  $\mathbf{P}^T$ , having an eigenvalue of one. As a result, the steady-state vector  $\boldsymbol{\pi}$  can be found by determining the eigenvector of  $\mathbf{P}^T$  [30], [104], [105].

### D. Throughput of CGBN-HARQ

The throughput of the CGBN-HARQ scheme is quantified in terms of the average number of packets successfully transmitted per TS. The successful transmission of a packet in a TS depends on two factors: 1) a free PR TS is correctly sensed by the CR system, and 2) the packet is delivered error-freely. As shown in (23), when the DTMC reaches its steady state, the throughput is determined by those states, which have one or more new packets successfully transmitted. Let us analyze the throughput in detail as follows.

As mentioned above, when the DTMC reaches its steady state, the additional throughput generated during the TS considered depends on two factors: 1) the PR channel is sensed free, corresponding to the scenario, when the first and second digits in the state sequence are either 00 or 10; and 2) the transmission of new packets in a respective state, provided that the channel is sensed to be free. Let us assume that  $n_p(S_i)$  is the number of new packets transmitted in association with state  $S_i \in \mathbb{S}_N$ . Explicitly,  $n_p(S_i)$  equals the number of zeros in the state sequence of  $S_i$  minus two, when the first two digits are 00 or minus one, if the first two digits are 10. Considering the first event, let us collect the states with the first two digits being 00 or 10 into a set denoted as  $\mathbb{S}_N$ . Then, when we obtained the steady state probabilities  $\boldsymbol{\pi}$ , the throughput of the CGBN-HARQ scheme can be evaluated as

$$R_T = \sum_{S_i \in \mathbb{S}_N} \pi_i \times n_p(S_i), \quad (\text{packets / TS}). \quad (24)$$

Current State $S_i$	State $S_j$ probability	State $S_j$ probability	State $S_j$ probability	State $S_j$ probability
$P_{i,j}=i=0,1$ & $j=0,1,3,\dots,13$	$P_{i,0}=P_{off}(1-P_{fa})(1-P_e)$ $P_{i,9}=P_{on}P_{md}(1-P_e)$	$P_{i,1}=P_{off}(1-P_{fa})P_e$ $P_{i,10}=P_{on}P_{md}P_e$	$P_{i,3}=P_{off}P_{fa}(1-P_e)$ $P_{i,12}=P_{on}(1-P_{md})(1-P_e)$	$P_{i,4}=P_{off}P_{fa}P_e$ $P_{i,13}=P_{on}(1-P_{md})P_e$
$P_{i,j}=i=3,12$ & $j=0,1,3,\dots,13$	$P_{i,0}=P_{off}(1-P_{fa})$ $P_{i,9}=P_{on}P_{md}$	$P_{i,1}=0$ $P_{i,10}=0$	$P_{i,3}=P_{off}P_{fa}$ $P_{i,12}=P_{on}(1-P_{md})$	$P_{i,4}=0$ $P_{i,13}=0$
$P_{i,j}=i=4,8,9,13$ & & $j=0,1,\dots,7$	$P_{i,0}=0$ $P_{i,9}=0$	$P_{i,1}=P_{off}(1-P_{fa})$ $P_{i,10}=P_{on}P_{md}$	$P_{i,3}=0$ $P_{i,12}=0$	$P_{i,4}=P_{off}P_{fa}$ $P_{i,13}=P_{on}(1-P_{md})$

Table II

STATE TO STATE TRANSITION PROBABILITIES WITH RESPECT TO TS FOR  $N = 1$  AND  $k = 1$  CASE.

Furthermore, if we express the throughput in terms of the number of packets per  $T_p$  (packet duration), we have

$$R'_T = \frac{T_p}{T} \times R_T = \frac{1}{k+N} \times R_T \quad (\text{packets} / T_p). \quad (25)$$

Let us now analyze the delay imposed by our CGBN-HARQ scheme.

### E. Delay of CGBN-HARQ

In the traditional GBN-HARQ, the transmission delay of a packet is contributed by the time required for its first transmission as well as the time associated with its retransmission. By contrast, in the proposed CGBN-HARQ, in addition to the above delay, extra delay may be imposed by the unavailability of the PR channel, which may either be due to the correct sensing of the busy PR channel or owing to the false sensing of the free PR channel. Therefore, in our CGBN-HARQ scheme [20], it is desirable to consider two types of delays, namely, the average packet delay and end-to-end packet delay. Specifically, the average packet delay may be defined as the total time required from the instant of sensing a TS until the error-free transmission of all packets divided by the total number of packets that had to be transmitted during this time. On the other hand, the end-to-end delay is the delay of a packet from its first transmission attempt until the instant that it is finally correctly received. Below we consider both types of delays.

1) *Average Packet Delay*: In the previous section, we have obtained the throughput quantified in terms of the average number of the packets per TS (or  $T_p$ ). Hence, given the throughput as expressed in (24) or (25), we can readily infer that the average packet delay can be evaluated as

$$T_D = \frac{1}{R_s} \quad (\text{TS per packet}) \quad (26)$$

$$= \frac{k+N}{R_T} \quad (T_p \text{ per packet}). \quad (27)$$

2) *End-To-End Packet Delay*: In this subsection, we will first commence by studying the probability mass function (PMF) of the end-to-end packet delay and then investigate the average end-to-end packet delay.

Let us define  $\mathbb{S}_N$  for storing all the states in which one or more new packets are transmitted. In other words,  $\mathbb{S}_N \subset \mathbb{S}$ . Given a state  $S_i \in \mathbb{S}_N$ , we define another set  $\mathbb{S}_i^{(m)}$  for accumulating the specific states  $\{S_j\}$  in which  $l_{i,j} \geq 1$  new packets transmitted from state  $S_i$  are correctly received with a delay of  $mT_p$ , i.e. we have:

$$\mathbb{S}_i^{(m)} = \{S_j \mid l_{i,j} \geq 1 \text{ new packets transmitted at state } S_i \text{ are correctly received at state } S_j \text{ with exactly the delay of } mT_p\}. \quad (28)$$

Let  $L_i$  represents the number of new packets transmitted from  $S_i$ . Then, the PMF of the end-to-end packet delay can be evaluated as:

$$P(m) = \frac{1}{c} \sum_{S_i \in \mathbb{S}_N} \sum_{S_j \in \mathbb{S}_i^{(m)}} \frac{\pi_i \times l_{i,j} \times P_{i,j}^{(m)}}{L_i}, \quad m = 1, 2, \dots \quad (29)$$

where  $c = \sum_{S_i \in \mathbb{S}_N} \pi_i$  and  $P_{i,j}^{(m)}$  denotes the probability of transition from state  $S_i$  to state  $S_j$  with the delay of  $mT_p$ .

Given  $\pi, \mathbf{P}$  and  $\mathbb{S}$ , the PMF of end-to-end packet delay can be obtained as follows. Let

$$\mathbf{P}_{MF} = [P(1), P(2), \dots, P(M_T)]^T, \quad (30)$$

where  $M_T$  represents the maximum delay having a very high value such as,  $10^{-8}$ . Using this approximation, the probability of events having a delay higher than  $M_T$  can be ignored. Let us first initialize  $\mathbf{p}^{(0)} = \mathbf{I}_i$ , where  $\mathbf{I}_i$  is the  $i$ th column of the identity matrix  $\mathbf{I}_{S_T}$ . Then, based on the properties of the DTMC, we have

$$\mathbf{p}^{(j)} = \mathbf{P}^T \mathbf{p}^{(j-1)} = (\mathbf{P}^T)^j \mathbf{I}_i, \quad j = 1, 2, \dots \quad (31)$$

From (31) we can obtain the following insights:

- If a packet originally transmitted in state  $S_i$  is successfully received in state  $S_j$ , the effective end-to-end delay is  $jT_p s$ ;
- The probabilities of transition from state  $S_i$  to any other states in  $\mathbb{S}$  are given by  $\mathbf{p}^{(j)}$  in (31);

Therefore, with the aid of the above information, we can update  $\mathbf{P}_{MF}$  using the following formula:

$$P(m) \leftarrow P(m-1) + \frac{\pi_i \times l_{i,j} \times P_{i,j}^{(m)}}{L_i}, \quad (32)$$

for  $m = 1, 2, \dots, M_T, S_j \in \mathbb{S}_i^{(m)}, S_i \in \mathbb{S}_N$ . Having formulated the PMF of the end-to-end packet delay, we now evaluate the average end-to-end packet delay as follows

$$\tau = \sum_{i=1}^{M_T} iT_p \times P(i). \quad (33)$$

Below we proceed by providing our performance results and validate our theoretical analysis by comparing the analytical results to those obtained from our simulations.

## V. PERFORMANCE RESULTS

In this section, the performance of the CGBN-HARQ scheme is characterized, when both perfect and imperfect sensing are assumed. We quantify the throughput, the average packet delay and the average end-to-end packet delay. More specifically, we quantify the impact of false-alarm probability ( $P_{fa}$ ), mis-detection probability ( $P_{md}$ ), channel busy probability ( $P_{on}$ ), packet error probability ( $P_e$ ), as well as the number of packets  $N$  transmitted per TS. Note that in the perfect sensing environment of [20], both  $P_{fa}$  and  $P_{md}$  are zero.

For our simulations, we configure the CGBN-HARQ transmission scheme in Matlab in a realistic imperfect sensing environment, where the transmitter seamlessly transmits  $N$  packets in each free TS and the receiver generates feedback flags for the received packet. The PU's activity is based on a uniformly distributed random process. To analyse the performance of the CGBN-HARQ transmission scheme, we performed 50,000 Monte Carlo simulations for all the parameters used in this study. Specifically, the parameters used in our simulations and in our theoretical modelling as well as their impact on the performance are summarized in Table III. Furthermore, the observation period commences from the first TS and continues until the CR receiver

successfully receives all packets. In our simulations, the period of observation commences from sensing the first TS and continues until the successful reception of all the packets considered. Correspondingly, the throughput is evaluated as

$$R'_S = \frac{N_s}{N_t \times (k + N)} \quad (\text{packets per } T_p), \quad (34)$$

where  $N_t$  denotes the overall number of TSs required for the error-free transmission of  $N_s$  packets.

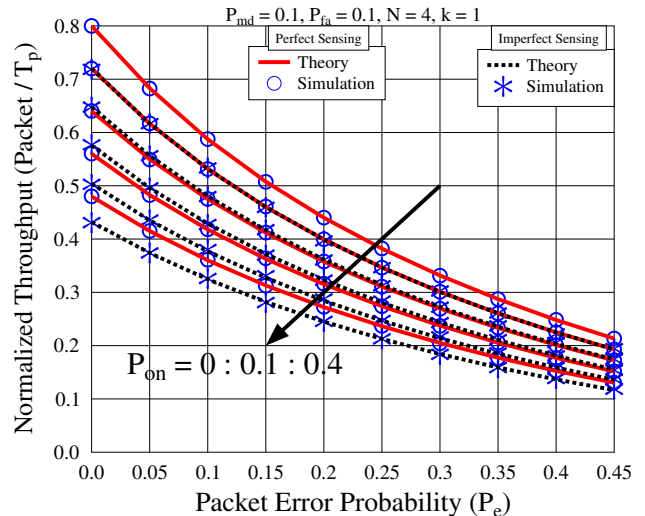


Figure 12. Throughput of the CGBN-HARQ versus  $P_e$  for various values of  $P_{on}$ , when  $k = 1$  and  $N = 4$ . The theoretical results were calculated from (25) and the simulation results were obtained from (34).

Fig 12 shows that for a given probability of  $P_{on}$ , the achievable throughput of CGBN-HARQ is at its maximum in both idealistic perfect and realistic imperfect sensing scenarios, when the channel used by the CU is perfectly reliable, yielding  $P_e = 0$ . However, when the channel becomes less reliable, i.e. when  $P_e$  increases, the throughput reduces with  $P_e$  due to the increase in the number of packet retransmissions. As shown in Fig 12, at a given  $P_e$ , the highest throughput is attained, when the PR channel is always free to use, corresponding to  $P_{on} = 0$ . By contrast, when  $P_{on}$  increases, the throughput of the CGBN-HARQ system decreases, since less time is available for the CU to transmit its data. Moreover, Fig 12 demonstrates the effect of unreliable sensing on the throughput, exhibiting a substantial drop due to the inaccurate sensing at a given  $P_e$  and/or  $P_{on}$ . Finally, as demonstrated in Fig 12, the results obtained from our analysis are validated by the simulation results, which confirms the accuracy of our analytical modelling.

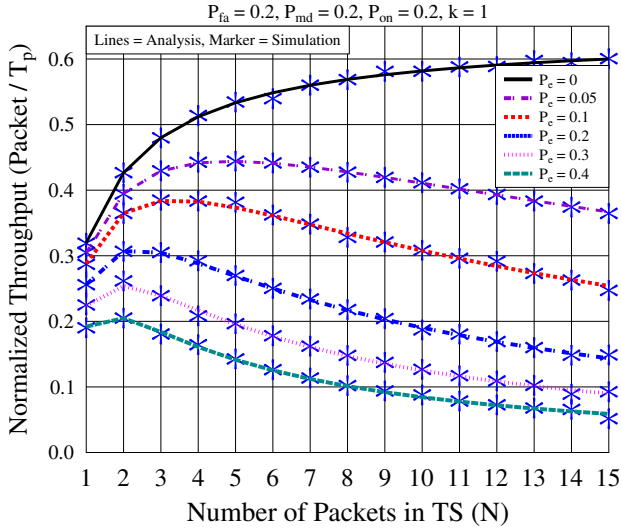


Figure 13. Investigating the optimal throughput of the CGBN-HARQ scheme based on the number of packet transmission ( $N$ ) per TS, for various values of  $P_e$  in a perfect sensing environment.

Continuing with the throughput analysis now the optimal value of  $N$  is investigated in Fig. 13 versus  $P_e, P_{fa}, P_{md}$  and  $P_{on}$ . It can be seen that the throughput always increases upon increasing the value of  $N$ , provided that the channel is perfectly reliable, i.e. we have  $P_e = 0$ . When  $P_e$  increases, an increased value of  $N$  may result in a reduced throughput. This is because for a large value of  $N$  the number of retransmissions increases, once there is even a single packet in error. Furthermore, as seen in Fig. 13, for a given packet error probability  $P_e \geq 0$ , there exists an optimum value for  $N$ , which results in the highest throughput. The optimum value of  $N$  reduces, as the packet error probability  $P_e$  increases.

Having characterized the attainable throughput, we now continue by quantifying the delay imposed. In our simulations, the average packet delay is computed as the total time duration required for the error-free transmission of  $N_s$  packets, divided by  $N_s$ , which is formulated as

$$T'_{DS} = \frac{N_t(k + N)}{N_s} \times T_p \quad (\text{seconds}). \quad (35)$$

The results presented in Figs. 14 and 15 are normalized by  $T_p$ .

Specifically, Fig. 14 shows the average packet delay versus the packet error probability  $P_e$ , parametrized by the channel busy probability  $P_{on}$ . As shown in Fig. 14, the minimum delay is observed, when the channel is ideal, i.e. when  $P_e = 0$  and/or when the channel is always free for the CU to use, corresponding to  $P_{on} = 0$ . The average packet delay increases upon increasing  $P_e$  and/or of  $P_{on}$ , as a

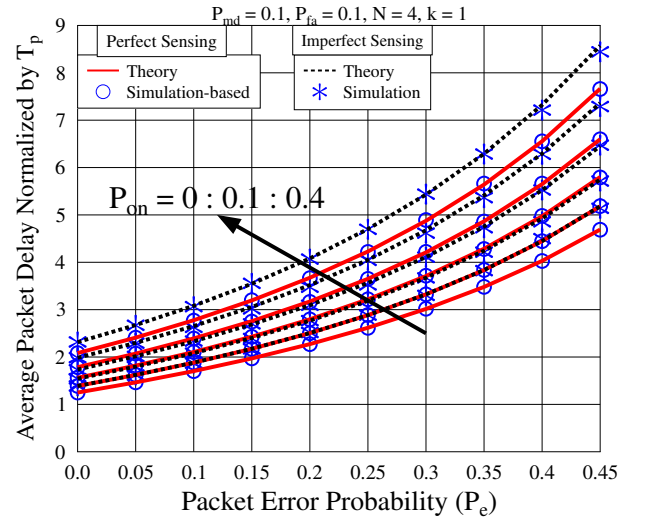


Figure 14. Average packet delay of the CGBN-HARQ system versus packet error probability for various channel busy probabilities, when both perfect and imperfect sensing are considered. The theoretical results were calculated from (27) and the simulation results were obtained by taking into account the total number of TS used for transmission (35).

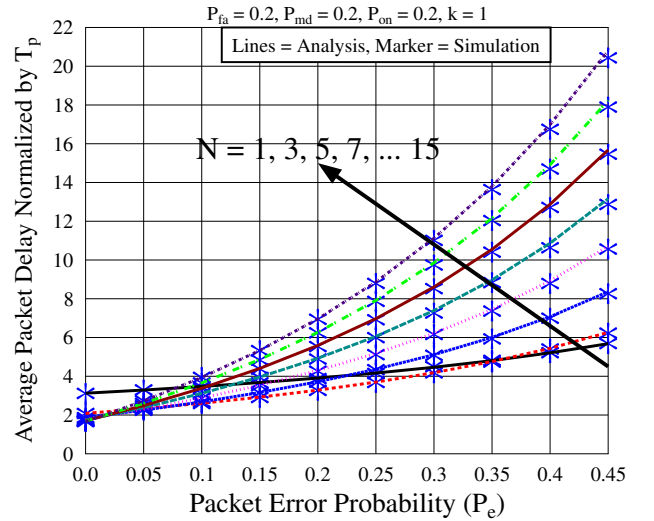


Figure 15. Investigating the optimal average packet delay of the CGBN-HARQ system based on the number of packets ( $N$ ) per TS versus  $P_e$  in a realistic imperfect sensing environment, when  $P_{on} = 0.2$  and  $T_s = T_p$ .

result of the increased number of retransmissions and/or the reduced transmission opportunities for the CU. Furthermore, similar to the above discussions, the unreliable sensing also increases the average packet delay by deeming a free TS to be busy. By contrast, activating packet transmissions in a busy TS also increases the average packet delay due to the increase in the number of erroneous receptions and retransmissions. Additionally, as shown in Fig. 14 and also in Fig. 15, the results of our analytical



approach are validated by the simulation results.

In Fig. 15, the average packet delay versus  $P_e$  relationship is characterized in terms of the number of packets transmitted in a single TS, given  $P_e, P_{fa}, P_{md}$  and  $P_{on}$ . As shown in the figure, at  $P_e = 0$ , a higher value of  $N$  gives a lower delay. However, when  $P_e$  is sufficiently large, a higher value of  $N$  results in a higher delay. For a given value of  $N$ , the average packet delay increases, as the channel becomes less reliable, i.e. when  $P_e$  increases. Again, as shown in Fig. 15, the simulation results validate our analytical results.

In Fig. 16, the PMF of the *end-to-end packet delay* is quantified for  $N = 4, 7$  and  $9$ , when realistic unreliable sensing is considered. In our simulation-based study the end-to-end delay of a packet is quantified in terms of the time duration spanning from its first transmission to the instant, when it is correctly received. Specifically, let the vector  $\mathbf{d}$  of length  $N_s$  store the end-to-end delay in terms of  $T_p$  of the  $N_s$  packets, where  $N_s$  is a large number and  $d(j)$  denotes the end-to-end delay of the  $j$ th packet. Hence, the PMF of the end-to-end packet delay presented in Fig. 16 is evaluated as

$$P_d(i) = \frac{\sum_{j=1}^{N_s} \delta[d(j) - i]}{N_s}, \quad 1 \leq i \leq \max(\mathbf{d}), \quad (36)$$

where  $\max(\mathbf{d})$  denotes the maximum delay of the  $N_s$  packets. Note that in Fig. 16 the PMF associated with a delay beyond  $120T_p$ 's is not shown, since the values are all close to zero.

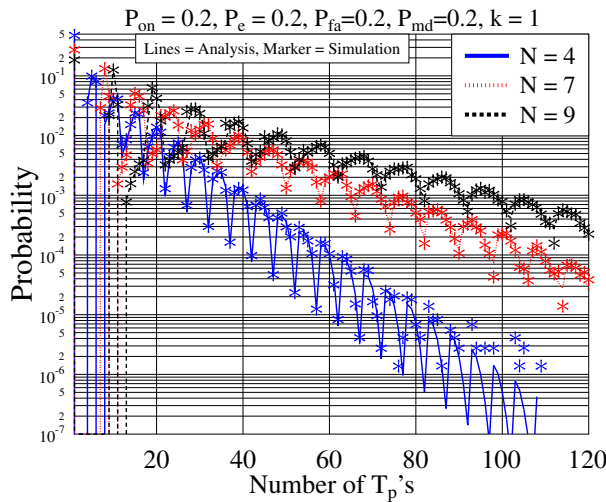


Figure 16. Probability distribution of end-to-end packet delay of the CGBN-HARQ systems in the case of imperfect sensing, when  $P_{on} = 0.2$ ,  $P_e = 0.2$ , and  $N = 4, 7$  and  $9$ . The theoretical results were calculated from (32) and the simulation results were obtained by taking into account (36).

We observe from Fig. 16 that for the given parameter values and  $N = 4$ , 47.7% of the packets are successfully received after their first transmission. For the packets that are successfully received after their second transmissions, there are two cases. In the first case, if the TS following the first transmission is free, on average 3.5%, 10% and 8.3% of the packets are successfully received with the delays of  $4T_p$ ,  $5T_p$  and  $6T_p$ , respectively. In the second case, if the TS during the first transmission is busy, then approximately 1.5%, 2.3%, 4.3% and 4.2% of the packets are successfully received with the delay of  $8T_p$ ,  $9T_p$ ,  $10T_p$  and  $11T_p$ , respectively. Similarly, we can find the probabilities for the cases, when  $N = 7$  and  $9$  are considered.

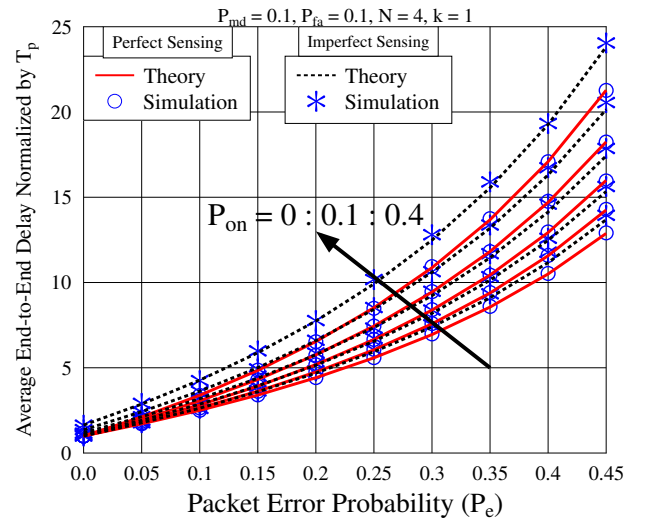


Figure 17. Average end-to-end packet delay versus  $P_e$  for various channel busy probabilities  $P_{on}$ , when both perfect and imperfect sensing are considered. The theoretical results were calculated from (33) and the simulation results were obtained by taking into account (37)

Having characterized the PMF of the end-to-end packet delay, we will now characterize the average end-to-end packet delay of the CGBN-HARQ scheme. Here, the average end-to-end packet delay is formulated as:

$$\tau = \sum_{i=1}^{\max(\mathbf{d})} P_d(i) \times i(T_p). \quad (37)$$

Fig. 17 depicts the average end-to-end packet delay for  $N = 4$  versus  $P_e$  and  $P_{on}$ , when both perfect and imperfect sensing are considered. Similar to the trends observed in the analysis of the average packet delay, the average end-to-end packet delay increases with  $P_e$  and/or  $P_{on}$ . However, when comparing Fig.



14 and 17, there are two main noticeable characteristics associated with the minimum delays and with the delay-gradient. Firstly, in terms of the minimum delays, observe in Fig. 17 at  $P_e = 0$  that the minimum end-to-end packet delay is one  $T_p$  in the case of perfect sensing, regardless of the channel's busy probability of  $P_{on}$ . In the case of imperfect sensing, the minimum delay increases to an average of  $1.6 T_p$ , when  $P_{on} = 0.4$ . By contrast, we observed from Fig. 14 that the average packet delay has a higher than a one  $T_p$  interval minimum delay, which further increases upon increasing in  $P_{on}$ , regardless of using perfect or imperfect sensing. The reason for this observation is that the average packet delay includes the busy time before a packet's transmission. By contrast, the end-to-end packet delay is calculated for each of packet individually, spanning from the instant of its first transmission attempt until its successful reception. Secondly, upon comparing Fig. 17 to Fig. 14, we observe that the average end-to-end packet delay gradient is higher than that of the average packet delay as a function of  $P_e$  and/or  $P_{on}$ . This is because in the CGBN-HARQ scheme the delay of every transmitted packet is dependent on the success of  $(N - 1)$  packets transmitted a head of it. Therefore, if any of the  $(N - 1)$  previous packet is corrupted, then all the respective packets have to be retransmitted, regardless whether they are error-free. This phenomenon results in longer end-to-end packet delays.

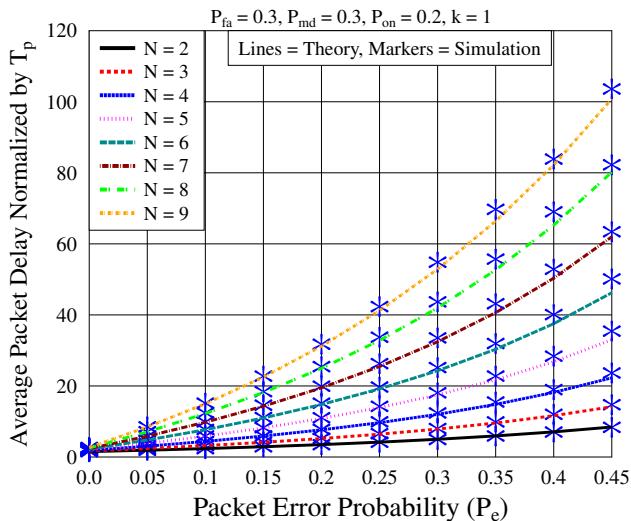


Figure 18. The optimal average end-to-end packet delay of the CGBN-HARQ system based on the number of packets ( $N$ ) versus  $P_e$  in the scenario of imperfect sensing.

Moreover, in Fig. 18 we depict the effect of increasing the number packets per TS on the average

end-to-end packet delay. Explicitly, for a given value of  $P_e$ , the average end-to-end packet delay increases, as the value of  $N$  increases. Furthermore, at a given  $P_e$ , the gradient of the average end-to-end delay curve increases, as the value of  $N$  becomes higher.

## VI. CONCLUSIONS

In this paper, we investigated a CGBN-HARQ transmission scheme conceived for CR systems, when both reliable and unreliable sensing are considered. Both the throughput and delay of CGBN-HARQ has been investigated both by analysis and simulations. The CGBN-HARQ system was modelled using DTMC-based approach, based on which closed-form expressions have been derived both for the throughput and for the delay. Both the average packet delay and the average end-to-end packet delay have been studied. Finally, our analytical results have been validated by our simulation results. The performance results of the CGBN-HARQ recorded for idealistic and realistic sensing environments were compared. The results presented in the above figures are summarized in Table III, which reveals that the reliability of spectrum sensing, the activity of the PUs and the reliability of the CR communication have a substantial impact on the throughput and delay of the CGBN-HARQ transmission scheme. When the CR communication become less reliable or when the PU has a high probability of exploiting the channel, the throughput reduces significantly, which ultimately results in an increased transmission delay. Additionally, the optimal value of the throughput and delay of CGBN-HARQ have been quantified in terms of the number of packets transmitted per TS. This quantitative analysis reveals that when the propagation environment is time-variant, the number of packets transmitted within a TS should be appropriately adapted for achieving the highest possible throughput and the shortest average transmission delay.

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(a) For $P_{on} = 0$					(b) For $P_{on} = 0.2$					(c) For $P_{on} = 0.4$				
$P_e$	$N$	$R_T$	$T_D$	$\tau$	$P_e$	$N$	$R_T$	$T_D$	$\tau$	$P_{on}$	$N$	$R_T$	$T_D$	$\tau$
0	1	0.4	2.5	1	0	1	0.32	3.13	1.18	0	1	0.24	4.14	1.5
	2	0.53	1.87	1		2	0.43	2.34	1.3		2	0.26	3.12	2
	3	0.6	1.67	1		3	0.48	2.1	1.4		3	0.36	2.8	2.2
	4	0.64	1.56	1		4	0.51	1.95	1.5		4	0.38	2.6	2.5
	5	0.67	1.5	1		5	0.53	1.87	1.6		5	0.4	2.5	2.8
	6	0.69	1.45	1		6	0.55	1.82	1.64		6	0.41	2.4	3.1
	7	0.7	1.42	1		7	0.56	1.78	1.7		7	0.42	2.3	3.4
0.2	1	0.32	3.1	1.6	0.2	1	0.26	3.9	1.9	0.2	1	0.17	5.23	2.6
	2	0.38	2.6	2.35		2	0.31	3.25	2.3		2	0.18	4.34	4.2
	3	0.37	2.63	3.4		3	0.30	3.28	4.4		3	0.23	4.36	6.3
	4	0.35	2.7	5.1		4	0.28	3.46	6.5		4	0.22	4.6	9.3
	5	0.33	3	7.3		5	0.27	3.7	9.3		5	0.2	4.9	13.12
	6	0.31	3.2	10.13		6	0.25	4	12.8		6	0.19	5.25	17.8
	7	0.28	3.5	13.5		7	0.23	4.3	17		7	0.18	5.6	23.3
0.4	1	0.24	4.15	2.66	0.4	1	0.19	5.2	3.3	0.4	1	0.14	7	4.4
	2	0.25	3.91	4.8		2	0.2	4.9	6.1		2	0.13	6.5	8.4
	3	0.23	4.35	7.8		3	0.18	5.44	10.1		3	0.14	7.25	14.1
	4	0.2	5	12.2		4	0.16	6.2	15.8		4	0.12	8.3	22.13
	5	0.17	5.6	18.1		5	0.14	7.1	23.4		5	0.1	9.4	32.6
	6	0.15	6.4	25.5		6	0.125	8	33		6	0.09	10.6	45.7
	7	0.14	7.12	34.6		7	0.11	9	44.4		7	0.08	12	61.4

Table III

THE PERFORMANCE PARAMETERS INVOKED IN BOTH THE ANALYTICAL AND SIMULATION-BASED MODELLING FOR THE SAKE OF QUANTIFYING THE THROUGHPUT ( $R_T$ ), AVERAGE PACKET DELAY ( $T_D$ ) AND AVERAGE END-TO-END PACKET DELAY ( $\tau$ ) OF THE CGBN-HARQ SCHEME IN TERMS OF PACKETS TRANSMISSION ( $N$ ) PER TS. THE RESULTS SHOWN ARE NORMALIZED TO THE UNIT OF  $T_p$  AND TO THE SENSING DURATION OF  $T_p$  WHILE  $P_{md} = 0.2$  AND  $P_{fa} = 0.2$ .

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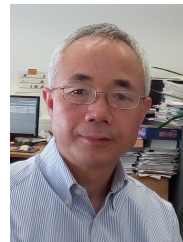
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