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TEACHING GEOMETRY IN LOWER SECONDARY SCHOOL IN SHANGHAI, CHINA

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This paper reports on a study of geometry teaching at the lower secondary school level in Shanghai, China. Through an analysis of data from observing a variety of Year 9 (Grade 8) lessons, and utilising data from the students' performance in school examinations, the study suggests that teachers in this region of China use classroom strategies that attempt to reinforce visual and deductive approaches in order to develop students' thinking in the transition to deductive geometry education.

INTRODUCTION

The teaching and learning of geometry has been the subject of considerable international interest, with many questions remaining about appropriate teaching methods and curriculum design (Mammana& Villani, 1998; Royal Society, 2001). With a view to informing the development of better pedagogical models and instructional strategies in geometry, this paper reports some findings from a study of geometry teaching at the lower secondary school level in Shanghai, China. The analysis is based on observations of geometry lessons, and data on students' performances in mid-term examinations, at Grade 8 (UK Year 9, students aged 13-14). The study suggests that an essential teaching strategy used by Chinese mathematics teachers at this Grade level is an approach that aims mutually to reinforce visual and deductive approaches in order to develop students' thinking, particularly in the transition to deductive geometry.

THEORETICAL FRAMEWORK

According to the van Hiele model, the development of students' thinking in geometry is not dependent upon age or biological maturation, but on the form of instruction received (for further detail, see Clements & Battista, 1992; Fuys, Geddes & Tischler, 1988; Hoffer, 1983; van Hiele, 1984). The model postulates a hierarchy of five levels of geometrical thinking, denoted as 1) visual; 2) descriptive/analytic; 3) abstract/relational; 4) formal deduction; and 5) rigor/meta-mathematical (Clements and Battista, 1992). To advance students' thinking to any subsequent level, the van Hiele model proscribes five sequential phases of instruction, summarised as: 1) Inquiry; 2) Directed orientation; 3) Expliciting; 4) Free orientation; 5) Integration. It is supposed that students would be able to think at a new level at the end of the fifth phase of instruction (for more details, see Hoffer, 1983).

While the van Hiele model suggests that the levels of students' geometric thinking are sequential and that progress through these levels can be enabled by the use of particular teaching techniques, there is little research on examining the relationship between instruction and the levels of thinking (Clements and Battista, 1992; Fuys *et al*., 1988). This is despite the fact that research continues to focus on the difficulties that pupils have in developing an understanding of geometrical theory and making the transition to formal proofs in geometry (Herbst, 2002; Jones, 2000). In this context, Fujita and Jones (2002) suggest that a bridge between practical and deductive geometry could be built up by developing students' 'geometrical eye', defined as "the power of seeing geometrical properties detach themselves from a figure" (p385). Their analysis indicates that intuitive and visual approaches might play an essential role in developing students' geometrical thinking, particularly in the transition from practical geometry to theoretical geometry.

CONTEXT AND METHOD

In Shanghai, there are four grades at the lower secondary school level, from Grade 6 (students' age, 11-12 years old) to Grade 9 (students' age, 14-15 years old). In this study, two school districts, Xuhui and Yangpu, were sampled. Two types of lesson were observed during the project, "open" lessons and "regular" lessons (an "open" lesson is a lesson open to other teachers in the city for improving teaching or for teaching competitions; a "regular" lesson is an ordinary lesson at a school). In this paper, the data is from a sample of three "open" lessons given by three teachers from three schools in Yangpu district, and 14 "regular" lessons given by 9 teachers from three schools in both Yangpu and Xuhui districts. As teachers in Shanghai share the same school curriculum, it was possible to observe the same topic taught by different teachers in different schools during the period of the research project.

For students at lower secondary school in Shanghai, the geometry curriculum is currently divided into three parts: intuitive geometry (first term, Grade 6); experimental geometry (second term Grades 6 and Grade 7); deductive geometry (at Grades 8 and 9), see Mathematics Textbook, 1996.

Thus Grade 8 (UK Year 9) is a particularly critical year for students to begin making the transition from practical geometry to theoretical geometry. The analysis that follows focuses on the relationship between the lesson structure and the development of students' geometrical thinking at this grade level.

RESULTS

In each of the schools there were five mathematics lessons per week, each between 40 and 45 minutes long. One of the lessons is usually devoted to doing exercises. A common lesson structure was observed across teachers and schools:

Introduction/ Review➔New content➔Exercises➔Summary➔Homework.

For lessons introducing new theorems, reviewing previous knowledge and introducing new content and exercises generally took the major part of the lesson. Observation data, for example, shows that the teachers devoted an average of 17 minutes to new content and 21 minutes to exercises.

The instructional model the teachers used for teaching new geometrical theorems is summarised as follows: 1) do experiment to observe, guess or verify a possible fact of a geometric figure; or pose a question or problem; 2) draw the figure to illustrate the known and the possible fact and write down the problem in mathematical language on the blackboard; 3) prove the fact; 4) use words to present the theorem; 5) use accurate mathematics language to present the theorem; and show the basic figure to represent the theorem; 6) read and highlight the key words in the theorem; recite the theorem for writing proof.

This instructional model, used strictly by the teachers observed, may be due to the same structure found in the textbook. For instance, the model used by three different teachers to teach a theorem of a right triangle was observed, as follows:

1. Students are asked to draw a right triangle. Next, they measure the length of the three sides of the right triangle and the length of the bisector of its hypotenuse, and then guess the possible fact from the various results they obtain.

2. Teacher states what is known and what needs to be proved: in Rt triangle ABC, angle ACB=90º, CD is a bisect line on AB. Prove: CD=AB/2 (see figure 1).

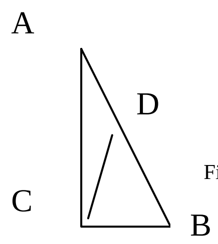


Figure 1

3. Teacher reviews the properties of a bisect line in a triangle, and the way to add an auxiliary line (see figure 2).

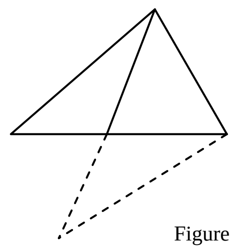


Figure 2

4. Next, students are guided to do proof by analysing Figure 3.
5. The teacher writes on the blackboard:‘In a right triangle, the bisect line on the hypotenuse is equal to half of the hypotenuse.”
6. The teacher highlights Figure 1 again as the basic figure of the theorem. Precise mathematical language is used to present the theorem, and this is written on the blackboard.

   In Rt triangle ABC, angle ACB=90º, CD is a bisect line on AB (known)

   CD=AB/2 (in a right triangle, the bisect line on the hypotenuse is equal to half of the hypotenuse.)

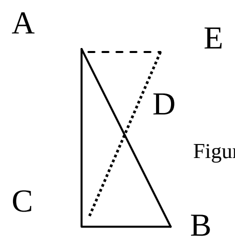


Figure 3

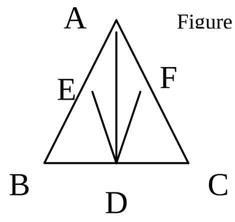
7. Students read the theorem. In the later part of the lesson, this theorem was used in writing a proof (for example, used in Figure 4, below).

In the observed lessons, based on judgments of students’ responses to, and explanations of, questions set by the teacher, students’ thinking levels mostly appeared to be between van Hiele levels 2 and 3. For instance, students could:

1. Formally tell the differences between definition, theorems and properties learned of the right triangle; transform a proposition into formal mathematical language and draw correctly its basic figure.

2. Use theorems to prove and compare different proofs of theorems. For example (see figure 4): Known: In triangle ABC, angle B= angle C, AD is a bisect line of angle BAC, E and F are respectively the bisectors of AB and AC. Prove: DE=DF.

Figure 4



Students considered two ways to prove:

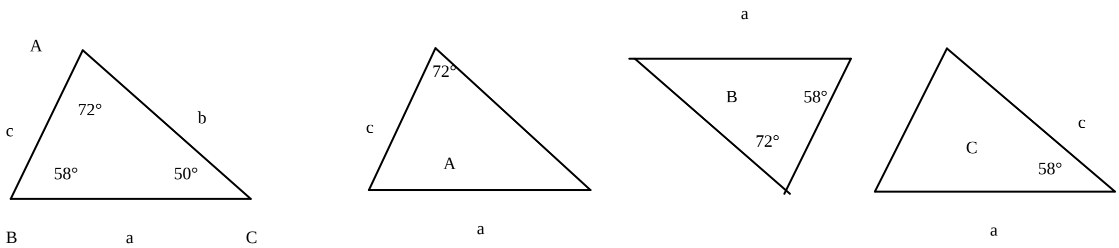
(a) Most students used the property of congruent triangles

(b) A few students used the property of a right triangle just learned (ED=AB/2, FD=AC/2) to prove DE=DF.

Nevertheless, it is worth noting that, in tackling this problem (Figure 4), most students had great difficulty in adding an auxiliary line for use in the proof (as in Figure 2). Even though the new theorem of a right triangle (proved in Figure 3) had just been introduced to students, most of them still preferred to use the property of congruent triangles to prove the example of Figure 4. In addition, it was difficult for most students to write a full proof, particularly the multiple steps of a proof. They often ignored certain necessary conditions of a theorem in writing their proof.

In addition, while during the lessons the responses of students appeared to suggest that the thinking levels of many students appeared to be between van Hiele levels 2 and 3, other evidence indicates that some students' thinking levels could be in the transition between van Hiele level 1 and 2. As an example, in a mid-term examination paper at first term of Grade 8 of one of the schools observed, there was an item as follows: in the figures below, there are triangles A, B and C. Which one is congruent to triangle ABC?

a) A and B; b) A and C; c) B and C; d) only B.



While 78 Grade 8 students took this test paper, 23 out of the 78 students gave an incorrect response to this item. Out of these 23 students, 17 chose choice 'd'. According to the analysis of their teacher, most of these incorrect students appeared to conclude that figure C was not congruent and, as a result, they chose the option that only B is congruent to triangle ABC. This may be because, as Shaughnessy and Burger (1985, p.423) observed, 'if conflict occurred between the visual and the analytic levels of reasoning (levels 1 and 2), the visual usually won'. In some respects this is reminiscent of Schoenfeld's (1985, 1987, 1988) findings of students who appear capable of writing a formal proof and yet fail to make use of what they had just proved when trying to complete a construction problem. It could be that students' responses in class, when in interaction with the teacher, may be as much as one level above what they can do when unassisted in a problem-solving situation or when doing a test or unaided homework.

In terms of the teaching methods employed in the lessons observed, overall these could be classified as bearing the hallmarks of involving elements of the van Hiele learning phase between the 'Directed Orientation' stage and the 'Expliciting' stage. In all the observed lessons, the teacher played a central role in enhancing students' thinking in the transition from experimental geometry to deductive geometry (from van Hiele level 2 to level 3). On the one hand, the teachers usually offered students carefully sequenced learning materials, with direct teaching being the predominant method. On the other hand, new knowledge was usually learned by students through observation and investigation. Exercises regularly played a key part in the lessons; not a large quantity of one-step tasks to practice, but a few carefully sequenced multiple-step tasks. Open-ended problems were also used and students were encouraged to complete such tasks in various ways, with teacher encouragement.

CONCLUDING COMMENTS

At Grade 8 (UK Year 9), students in Shanghai start to learn deductive geometry in the formal sense. The teachers observed in this study used three main strategies. First, they emphasised the *process* of learning every theorem and frequently reviewed previous theorems and their basic figure in classes (for example, as shown in Figure 1 and its theorem). This is likely to train the students' 'geometrical eye' at van Hiele level 2. Next, two strategies are used to train the 'geometrical eye' at higher levels. One strategy observed was to draw a complicated figure gradually, while presenting a proof problem to students. The other strategy was that teachers often separated a complicated figure into several basic figures of theorems on the blackboard.

As Fuys *et al*. (1988) point out, a visual approach is likely not only to maintain students' interest, but also to assist students in creating definitions and conjectures and in gaining insight into new geometrical relationships and inter-relationships. The visual approach was largely used by Chinese teachers particularly in helping students to receive insights into interrelationships and observe the transformation of figures in deductive geometry. In this way, an essential teaching strategy used by the Chinese teachers was mutually reinforcing visual and deductive approaches in order to develop students' 'geometrical eye' in the learning of theoretical geometry. Further analysis of data from the project is continuing to focus on the relation between the lesson structures used by Chinese teachers and their students' geometrical thinking development.

The recent UK study of geometry teaching (Royal Society, 2001) concludes that "the most significant contribution to improvements in geometry teaching will be made by the development of good models of pedagogy, supported by carefully designed activities and resources" (p19). By studying lessons given by experienced mathematics teachers in China, this might inform the development of new pedagogical approaches to teaching geometry.

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