# Pinning Down the Superfluid and Nuclear Equation of State and Measuring Neutron Star Mass Using Pulsar Glitches

Wynn C. G. Ho<sup>1</sup>, Cristóbal M. Espinoza<sup>2</sup>, Danai Antonopoulou<sup>1</sup>, and Nils Andersson<sup>1</sup>

<sup>1</sup>Mathematical Sciences, Physics & Astronomy, and STAG Research Centre, University of Southampton, Southampton, SO17 1BJ, United Kingdom <sup>2</sup>Departamento de Física, Universidad de Santiago de Chile, Avenida Ecuador 3493, Estación Central, Santiago, Chile

E-mail: wynnho@slac.stanford.edu

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Pulsars are rotating neutron stars that are renowned for their timing precision, although glitches can interrupt the regular timing behavior when these stars are young. Glitches are thought to be caused by interactions between normal and superfluid matter in the star. We update our recent work on a new technique using pulsar glitch data to constrain superfluid and nuclear equation of state models, demonstrating how current and future astronomy telescopes can probe fundamental physics such as superfluidity near nuclear saturation and matter at supranuclear densities. Unlike traditional methods of measuring a star's mass by its gravitational effect on another object, our technique relies on nuclear physics knowledge and therefore allows measurement of the mass of pulsars which are in isolation.

KEYWORDS: equations of state of neutron-star matter, neutron stars, pulsars, quantum fluids

## 1. Pulsar Spin-down and Superfluid Model of Glitch Spin-up

Rotating neutron stars, or pulsars, are born in the collapse and supernova explosion at the end of a massive star's life. More massive than the Sun but only  $\sim 20$  km in diameter, pulsars are primarily composed of neutron-rich matter near and above nuclear densities. Pulsars emit beamed electromagnetic radiation, and this loss of energy comes at the expense of the pulsar's rotational energy, causing the star to spin more slowly over time. While most isolated pulsars are observed to rotate very stably with a spin period between  $\sim 1$  ms and 10 s, the timing behavior of many young pulsars is interrupted by sudden changes, so-called glitches. During glitches, the pulsar spin rate suddenly increases over a very short time and then usually relaxes to its pre-glitch rate over a longer time.

Glitches are thought to be the manifestation of a neutron superfluid in the pulsar's inner crust, which contains neutron-rich nuclei embedded in a sea of free neutrons. These free neutrons are expected to be in a superfluid state because the critical temperature  $T_c$  (below which neutrons become superfluid) is well above the typical crust temperature of pulsars. Unlike normal matter, superfluid matter in the inner crust rotates by forming vortices whose areal density determines the spin rate of the superfluid. To decrease its spin rate, superfluid vortices must move so that the areal density decreases. In the inner crust of a neutron star, these vortices are usually pinned to the nuclei of normal matter [1]. While the rest of the star slows down owing to electromagnetic energy loss, the neutron superfluid does not. As a result, this superfluid can act as a reservoir of angular momentum. Over many pulsar rotations, an increasing lag develops between the stellar spin rate and that of the neutron superfluid in the inner crust. When this lag exceeds a critical value, superfluid vortices unpin and transfer their angular momentum to the rest of the star, causing the stellar rotation rate to increase and producing what we observe as a glitch [1,2]. Thus glitches provide a valuable measure of the amount of superfluid angular momentum, or equivalently moment of inertia, in neutron stars. To probe superfluid and nuclear equation of state (EOS) properties with pulsar glitches, three questions need answering.

## 2. Observational and Theoretical Constraints on Pulsar Moment of Inertia

#### 2.1 Question 1: How much angular momentum or moment of inertia do glitches require?

For glitching pulsars,  $G \equiv 2\tau_c \langle A \rangle$  is the measured parameter that is to be compared to theoretical models. Here  $\tau_c = \Omega/2\dot{\Omega}$ ,  $\Omega$  and  $\dot{\Omega}$  are the pulsar spin rate and its time derivative, respectively, and  $\langle A \rangle = (1/t_{obs}) \sum \Delta \Omega/\Omega$  is the average activity parameter and the summation is over each glitch with spin rate change  $\Delta\Omega$  during an observation time span  $t_{obs}$ .  $G = 1.62 \pm 0.03\%$  for the Vela pulsar, and  $G = 0.875 \pm 0.005\%$  for PSR J0537–6910. The fractional moment of inertia of the superfluid must exceed G in order for there to be sufficient angular momentum to be transferred during glitches.

In our recent work [3], we examine a large new dataset of glitches [4, 5] and explore an idea proposed in [6] and illustrated in Fig. 1. The neutron star crust (core) is denoted by shaded (unshaded) regions. Vertical dotted lines indicate the density at which the moment of inertia exceeds G = 1.6%(for the Vela pulsar) for neutron star models of various mass. Low mass neutron stars have a thicker crust than high mass stars, and we see that a neutron superfluid confined to the crust can only provide a relative moment of inertia of 1.6% for neutron stars with a mass much less than 1  $M_{\text{Sun}}$ . Furthermore, all glitching pulsars with  $G \approx 1\%$  would have to be of low mass (< 1  $M_{\text{Sun}}$ ) as well. For typical neutron star masses of 1.2 to 2  $M_{\text{Sun}}$  [7], some small fraction of the core must contribute to the moment of inertia required by glitches seen in, for example, the Vela pulsar.



**Fig. 1.** Temperature as a function of baryon number density, using the BSk20 EOS. Curved lines are the superfluid critical temperature for nine models from [15] (left panel) and seven of eleven models discussed in Sec. 3 (right panel). The vertical solid line indicates the separation between the crust (shaded region) and the core. Vertical dotted lines denote the density at which the partial moment of inertia is Vela's G = 1.6% of the total stellar moment of inertia for neutron stars of different mass (labeled in units of solar mass). The (nearly horizontal) dashed line is the temperature of a  $1.4 M_{Sun}$  neutron star at the Vela pulsar's age of 11000 years.

#### 2.2 Question 2: How much superfluid moment of inertia do pulsars possess?

In the previous analyses of [6, 8–13], pulsar glitches are assumed to tap the angular momentum reservoir associated with superfluid neutrons in the inner crust of the star (and largely ignoring the temperature dependence of superfluidity). Therefore, by calculating the entire moment of inertia of the crust, it is possible to determine the maximum reservoir available for producing glitches, and this is found to be smaller than that needed to explain observed glitch activity [6, 9, 12, 13] due to the effect of entrainment [14], unless the crust is unusually thick [10, 11]. Here and in [3], we consider a superfluid reservoir that extends into the stellar core and account for the temperature dependence of superfluidity. Importantly, we use the observed temperature of pulsars to constrain the latter. Figure 1 shows models of critical temperature of neutron (singlet-state) superfluidity as a function of baryon number density  $n_b$ : models plotted in the left panel are from [15], while those in the right panel are

discussed in Sec. 3. For some superfluid models, the critical temperature, and hence allowed region for neutrons to become superfluid, is confined to the inner crust, i.e., in the shaded region to the left of the vertical solid line. Therefore, pulsar glitches can only involve the moment of inertia of the inner crust if one of these superfluid models is the correct one. However, there are superfluid models that extend into the core, e.g., solid curve labeled SFB or HFB-16nm. For superfluid models such as the SFB model, if the pulsar temperature is low enough so that neutrons in the inner crust *and* outer core are superfluid, then pulsar glitches could involve additional moment of inertia from the core.

2.3 Question 3: How much superfluid moment of inertia do observed pulsars have at present time? Finally, are enough neutrons in the crust and core actually superfluid, i.e., to the left of one of the vertical dotted lines and below the superfluid critical temperature  $T_c$  in Fig. 1? To answer this question, we need to determine the interior temperature  $T(n_{\rm b})$  of a neutron star and evaluate at what densities  $n_b$  the inequality  $T < T_c$  is satisfied. This will vary for each pulsar, depending on its age and/or measured temperature. Neutron stars are born in supernovae at very high temperatures but cool rapidly because of efficient neutrino emission. We perform neutron star cooling simulations using standard neutrino emission processes to find the interior temperature at various ages [16]. The dashed lines in Fig. 1 show the resulting temperature profile (at the age of the Vela pulsar) for a  $1.4 M_{\text{Sun}}$  neutron star. The temperature profile is different for different mass but not drastically so, unless neutrino emission by direct Urca processes occurs. We find that, among nine superfluid models which span a wide range in parameter space (see left panel of Fig. 1), only the SFB model provides a superfluid reservoir of the required level (see below for more recent results). For superfluid models that are confined to the crust, the reservoir is too small, whereas the reservoir is too large for models that extend much deeper into the core. The latter would be unable to explain the regularity of similarsized glitches, which requires the reservoir to be completely exhausted in each event.

#### 3. Measuring Pulsar Masses and New Results Since Ho et al. 2015 [3]

The intersection of the three lines in Fig. 1 [vertical dotted line at  $1.4 M_{Sun}$  for glitch requirement G = 1.6%, solid line for SFB model of superfluid critical temperature  $T_c$ , and horizontal dashed line for neutron star temperature  $T(n_b)$  at age = 11000 years] is one of our key findings: The Vela pulsar has a mass near the characteristic value of 1.4 M<sub>Sun</sub>, and the size and frequency of Vela's observed glitches are a natural consequence of the superfluid moment of inertia available to it at its current age. Our results using the BSK20 EOS and SFB superfluid models are summarized in the left panel of Fig. 2, which shows interior temperature T of a pulsar as a function of glitch parameter G (see [3] for results using APR and BSk21 EOS models). Note that G and T are directly determined from observational data. The former comes from radio or X-ray glitch measurements. The latter is obtained either from the age of the pulsar or by measuring the surface temperature of the pulsar through X-ray observations. The age gives the interior temperature via neutron star cooling simulations, whereas surface temperature is related to interior temperature via well-known relationships [16]. Thus for a given pulsar that has measured G and T, Fig. 2 allows one to determine the pulsar's mass. For example, using the BSk20 EOS and SFB superfluid models, we find that Vela is a  $1.51 \pm 0.04 M_{Sun}$ neutron star and PSR J0537-6910 is a  $1.83 \pm 0.04 M_{Sun}$  neutron star. Results for seven other glitching pulsars are also plotted in Figure 2.

Since [3], we tested an additional two EOS models and eleven superfluid models (see right panel of Fig. 1): two EOS models from [17], along with superfluid models calculated using the same interactions [18]; same HFB-16 symmetric and neutron matter superfluid models that are used to construct BSk EOS models [19]; and six superfluid models from [20]. In brief, the models of [17,18] imply low masses, while superfluid models HFB-16nm [19] and N3LOsrc+lrc [20] yield reasonable masses.

The ability to measure the mass of isolated pulsars has not been previously demonstrated. The



**Fig. 2.** Left panel: Neutron star mass from pulsar observables *G* and interior temperature *T*. Data points are for pulsars with measured *G* from glitches and *T* from an age or surface temperature observation [3]. Lines (labeled by neutron star mass, in units of solar mass) are the theoretical prediction for *G* and *T* using the BSk20 EOS and SFB superfluid models. Right panel: Neutron star mass and  $1\sigma$  uncertainty using the BSk20 EOS and SFB superfluid models; see Table 2 in [3] for masses derived using APR and BSk21 EOS models.

most precise neutron star mass measurements to date are by radio timing of pulsars that are in a binary star system [7]. Our method of using glitches to measure mass can greatly increase the number of known masses, thereby providing constraints on fundamental physics properties such as the nuclear EOS and superfluidity. Although there are currently relatively large systematic uncertainties, these will improve as the understanding of dense matter improves. The novelty of our approach is the combination of pulsar glitch data and the temperature dependence of superfluidity. The method is especially promising with upcoming astronomical observatories such as the Square Kilometer Array (SKA) in radio and Athena+ in X-rays. SKA could discover all observable pulsars in the Galaxy, and a program to monitor glitching pulsars could transform the fields of neutron star and nuclear physics.

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## References

- [1] P. W. Anderson, N. Itoh, Nature 256, 25 (1975).
- [2] G. Baym, C. Pethick, D. Pines, M. Ruderman, Nature 224, 872 (1969).
- [3] W. C. G. Ho, C. M. Espinoza, D. Antonopoulou, N. Andersson, Science Advances 1, e1500578 (2015).
- [4] C. M. Espinoza, A. G. Lyne, B. W. Stappers, M. Kramer, Mon. Not. R. Astron. Soc. 414, 1679 (2011).
- [5] M. Yu, et al., Mon. Not. R. Astron. Soc. 429, 688 (2013).
- [6] N. Andersson, K. Glampedakis, W. C. G. Ho, C. M. Espinoza, Phys. Rev. Lett. 109, 241103 (2012).
- [7] J. M. Lattimer, Jpn. Phys. Soc. Conf. Proc., this volume.
- [8] B. Link, R. I. Epstein, J. M. Lattimer, Phys. Rev. Lett. 83, 3362 (1999).
- [9] N. Chamel, Phys. Rev. Lett. 110, 011101 (2013).
- [10] J. Piekarewicz, F. J. Fattoyev, C. J. Horowitz, Phys. Rev. C 90, 015803 (2014).
- [11] A. W. Steiner, S. Gandolfi, F. J. Fattoyev, W. G. Newton, Phys. Rev. C 91, 015804 (2015).
- [12] T. Delsate et al., Phys. Rev. D 94, 023008 (2016).
- [13] A. Li, J. M. Dong, J. B. Wang, R. X. Xu, Astrophys. J. Suppl. 223, 16 (2016).
- [14] N. Chamel, Phys. Rev. C 85, 035801 (2012).
- [15] W. C. G. Ho, K. G. Elshamouty, C. O. Heinke, A. Y. Potekhin, Phys. Rev. C 91, 015806 (2015).
- [16] W. C. G. Ho, K. Glampedakis, N. Andersson, Mon. Not. R. Astron. Soc. 422, 2632 (2012).
- [17] M. Baldo, G. F. Burgio, H.-J. Schulze, G. Taranto, Phys. Rev. C 89, 048801 (2014).
- [18] X.-R. Zhou, H.-J. Schulze, E.-G. Zhao, F. Pan, J. P. Draayer, Phys. Rev. C 70, 048802 (2004).
- [19] S. Goriely, N. Chamel, and J. M. Pearson, Eur. Phys. J. A 42, 547 (2009).
- [20] D. Ding et al., Phys. Rev. C 94, 025802 (2016).