Is “Morphodynamic Equilibrium” an oxymoron?

Zeng Zhou\textsuperscript{a,b}, Giovanni Coco\textsuperscript{b}, Ian Townend\textsuperscript{c}, Maitane Olabarrieta\textsuperscript{d}, Mick van der Wegen\textsuperscript{e,f}, Zheng Gong\textsuperscript{g}, Andrea D’Alpaos\textsuperscript{h}, Shu Gao\textsuperscript{i}, Bruce E. Jaffe\textsuperscript{j}, Guy Gelfenbaum\textsuperscript{k}, Qing He\textsuperscript{l}, Yaping Wang\textsuperscript{k}, Stefano Lanzoni\textsuperscript{l}, Zhengbing Wang\textsuperscript{f,m}, Han Winterwerp\textsuperscript{f,m}, Changkuan Zhang\textsuperscript{g}

\textsuperscript{a}Jiangsu Key Laboratory of Coast Ocean Resources Development and Environment Security, Hohai University, Xikang Road 1, Nanjing, 210098, China.
\textsuperscript{b}School of Environment, University of Auckland, New Zealand.
\textsuperscript{c}Ocean and Earth Sciences, University of Southampton, UK.
\textsuperscript{d}Department of Civil and Coastal Engineering, University of Florida, USA.
\textsuperscript{e}UNESCO-IHE, Delft, Netherlands.
\textsuperscript{f}Deltares, Delft, Netherlands.
\textsuperscript{g}College of Harbour, Coastal and Offshore Engineering, Hohai University, Nanjing, China.
\textsuperscript{h}Department of Geosciences, University of Padova, Padova, Italy.
\textsuperscript{i}State Key Laboratory of Estuarine and Coastal Research, East China Normal University, Shanghai, China.
\textsuperscript{j}Pacific Coastal and Marine Science Center, United States Geological Survey, USA.
\textsuperscript{k}School of Geography and Oceanography, Nanjing University, Nanjing, China.
\textsuperscript{l}Department of Civil, Architectural and Environmental Engineering, University of Padova, Padova, Italy.
\textsuperscript{m}Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, Netherlands.

Abstract

Morphodynamic equilibrium is a widely adopted yet elusive concept in the field of geomorphology of coasts, rivers and estuaries. Based on the Exner equation, an expression of mass conservation of sediment, we distinguish three types of equilibrium defined as static and dynamic, of which two different types exist. Other expressions such as statistical and quasi-equilibrium which do not strictly satisfy the Exner conditions are also acknowledged for their practical use. The choice of a temporal scale is imperative to analyse the type of equilibrium. We discuss the difference between morphodynamic equilibrium in the “real world” (nature) and the “virtual world” (model). Modelling studies rely on simplifications of the real world and lead to understanding of process interactions. A variety of factors affect the use of virtual-world predictions in the real world (e.g., variability in environmental drivers and variability in the setting) so that the concept of morphodynamic equilibrium should be

\*Corresponding to: zhouzeng@hhu.edu.cn

Manuscript submitted to Earth-Science Reviews (26/05/2016)
mathematically unequivocal in the virtual world and interpreted over the appropriate spatial and temporal scale in the real world. We draw examples from estuarine settings which are subject to various governing factors which broadly include hydrodynamics, sedimentology and landscape setting. Following the traditional “tide-wave-river” ternary diagram, we summarize studies to date that explore the “virtual world”, discuss the type of equilibrium reached and how it relates to the real world.

*Keywords:* morphodynamic equilibrium, estuaries and coasts, sediment transport, static equilibrium, dynamic equilibrium, numerical modelling
1. What is morphodynamic equilibrium?

Morphodynamic equilibrium is a common concept used in the field of morphodynamics which, in the coastal realm, can be defined as “the mutual adjustment of topography and fluid dynamics involving sediment transport” according to Wright and Thom (1977). Equilibrium refers to the condition where forces exerted over a system cancel each other out. In the case of morphodynamics, the balance of forces constitutes only one aspect of the problem since the term “morphodynamic equilibrium” is invoked to describe specific conditions of the system mass balance that lead, in its most intuitive and simple manner, to no net sediment accumulation or erosion. The concept of morphodynamic equilibrium on coasts, rivers and estuaries is pertinent but somehow elusive. It is pertinent because natural systems are shaped as a result of the balance between the internal processes (physical, chemical, biological, etc.) and the external drivers (primarily climatic and anthropogenic). The external drivers are changing as a result of climatic variations and technological advances, so that addressing the above balance has critical implications for a variety of fields, ranging from ecological to economic and even social. For example, large-scale anthropogenic activities displace large amounts of sediment directly (through engineering works) and indirectly (as a result of the modified balance between depositional and erosive processes), so that prediction of new equilibrium morphological configurations is vital to management and sustainability. Similarly, projected changes in the relative mean sea level could alter the morphology and profoundly affect the fragile balance that sustains many ecosystems (Kirwan and Megonigal, 2013; Lovelock et al., 2015). Within this context it is perhaps understandable that the interest in understanding and predicting “morphodynamic equilibrium” has so rapidly increased over the past decade (Figure 1).
Figure 1: Number of citations in Scopus searching for “morphodynamic equilibrium”. Results of the search have been verified for unrelated occurrences of the term. Although the number of publications using only the term “morphodynamics” has been growing faster over the same years, the figure shows how the term “morphodynamic equilibrium” has become popular and indicative of a specific line of research.

Understanding whether equilibrium exists implies quantitatively assessing what forces operate and if/when/how they balance. For some problems, for which equilibrium is well defined, the range of possible solutions and their behaviour can be readily explored. However, the concept of equilibrium is also elusive, primarily because of the separation between the “real” and the “virtual” world (Figure 2). In our endeavours to understand the real world, the ability to predict future conditions is constrained by incomplete knowledge of the structure of the systems we seek to represent and the dynamics if their behavioural response to external forcing conditions; all of which operate over a broad range of temporal and spatial scales. This is further complicated because processes simultaneously operating are difficult to disentangle. These shortcomings can be overcome by creating a “virtual” world where only selected processes operate under controlled conditions. We link these two worlds through the process of abstraction, usually through some combination of inductive and deductive approaches (abstraction here refers to a conceptual idealisation of the real world). In the “virtual” world numerical, analytical and physical studies developed through the processes of abstraction and implementation can evaluate the equilibrium of, for example, a tidal
network (Figure 2). Whereas in the “real” world, the design of engineering
and management actions are now informed by modelling studies, which lead
to changes in the “real” world. This sequence feeds back onto both worlds.
In the case of the “virtual” world, implementation involves simulating process
interactions and hopefully showing results that are robust, reliable and real-
istic (if not, new abstractions are required). In the case of the “real” world,
the implementation stage can give rise to new observations on the effect of
engineering/management actions which can highlight differences between ex-
pected (as simulated in the “virtual” world) and observed (in the “real” world)
behaviour, leading to new abstractions and possibly even new implementation
stages. In general, the link between implementation and the “real” world
represents our increasing knowledge, which always seems to generate further
questions to better understand how the “real” world works and to predict its
evolution. Testing and observations remain critical to assess the validity of the
predictions and the distance between the “real” and the “virtual” world, and
improve process description. But no matter the level of detail in the descrip-
tion of physical processes, these studies will always refer to the “virtual” world
where the underlying structure is known, the types of environmental drivers
and the modes of interaction are pre-defined and the presence (or lack) of an
“equilibrium” is almost a direct consequence of the system of equations used
to describe the “world”.
Therefore, it becomes immediately necessary to distinguish between the real and the virtual world, and how the concept of equilibrium differs for the two cases. As an example, we will consider the topic of estuarine morphodynamics but we notice that similar examples can be made for rivers, or open coasts. In a real estuary, external forcing (e.g., tides, river flows, wind and wave climate, and extreme events) operates over a variety of scales. If we only considered tidal forcing, whose characteristics mainly depend on planetary motions and so can be easily predicted (especially if compared to waves and river flows, which depend on atmospheric and climatic processes), even small tidal range variations with decadal cycles can still affect the morphodynamic equilibrium (Wang and Townend, 2012). Examples of other sources of variability that control the morphodynamic evolution and equilibrium include sediment sources (Jaffe et al., 2007; Gelfenbaum et al., 2015) and characteristics (Orton and Reading, 1993), geological controls, human-driven perturbations such as restoration activities, storms, presence of vegetation and biological activity. Attempting to account for all these sources of variability becomes an impossible task ultimately limiting the efficacy of the learning feedback (Figure 2). In contrast, a learning feedback based on the “virtual world” facilitates insight into the role of individual processes and interactions under a variety of conditions.
ditions. The immediate implication is that in the real world morphodynamic equilibrium might never exist while in the virtual world we more readily control the external forcing and the processes of abstraction and implementation (Figure 2). As a result, in the virtual world, we expect to be able to understand when morphodynamic equilibrium is possible, when it is not, and how it develops.

This contribution primarily deals with the notion of equilibrium in the virtual world which is a fundamental step towards the understanding of how real systems evolve under natural conditions. For example, recent advances on shoreline evolution along beaches show that improved predictability of shoreline position can be achieved using phenomenological models based on a concept of equilibrium (e.g., Yates et al., 2009) that can only exist in the virtual world. However, before proceeding any further, it is helpful to define the different types of equilibrium. The concept has been widely debated in the field of geomorphology (see the insightful review by Thorn and Welford, 1994) and for our purposes we will focus on the search for “stable equilibrium” configurations leaving aside, for now, a discussion on neutral, unstable and metastable configurations. The presence of a plethora of equilibrium definitions leads to much confusion and there have been many attempts to establish some definitions of direct relevance to geomorphology from a process or landform perspective (Gilbert, 1876; Chorley and Kennedy, 1971; Howard, 1988; Renwick, 1992; Ahnert, 1994; Thorn and Welford, 1994) and from an energetic or thermodynamic perspective (Leopold and Langbein, 1962; Zdenkovic and Scheidegger, 1989; Rodríguez-Iturbe and Rinaldo, 1997; Whitfield, 2005; Savenije, 2012; Kleidon et al., 2013).

Here, we restrict our consideration to mass flux equilibrium, as originally proposed by Gilbert (1876) and elaborated by Thorn and Welford (1994), which may depend on the geomorphological form. However, we acknowledge that there is a parallel discussion to be had from a thermodynamic perspective. In this context, thermodynamic equilibrium is of little interest and the focus is on steady states in non-equilibrium systems. This has been tenta-
tively explored but is comparatively much less well developed as an area of study (Leopold and Langbein, 1962; Scheidegger and Langbein, 1966; Townend and Dun, 2000; Nield et al., 2005). As yet, the equivalence with mass flux equilibrium has not been well defined (Thorn and Welford, 1994).

In essence we focus on equilibrium configurations where negative feedbacks dominate and lead to stable equilibrium. Given that our focus is morphodynamics, we approach equilibrium using the widely adopted Exner equation for conservation of mass of sediment. A comprehensive derivation of the Exner equation (Leliavsky et al., 1955) has been proposed by Paola and Voller (2005). This derives a form of the equation applicable to a basement (rock), sediment layer and fluid layer which has a total of ten terms, namely (i) rock basement subsidence and uplift; (ii) changes to the basement-sediment interface; (iii) compaction or dilation of the sediment column; (iv) divergence of any particle flux within the sediment layer (e.g., soil creep); (v) creation or destruction of particulate mass within the sediment column; (vi) changes to the sediment-water interface; (vii) loss or gain of particulate mass in the water column; (viii) horizontal divergence of particle flux within the flow; (ix) gain or loss through the water surface; and (x) creation or destruction of particulate mass within water column.

For most uses, a number of these terms play only a small part and can be ignored, although this can vary with the timescale of interest. For fluvial and marine applications, the Exner equation is commonly written as a volumetric balance:

\[(1 - p) \frac{\partial \eta}{\partial t} + \frac{\partial (CD)}{\partial t} + \nabla \cdot q_s = \sigma, \text{ with } C = \frac{1}{D} \int_{\eta}^{\eta+D} c \, dz, \]  

where \( p \) is the porosity of the bed, \( \eta \) is the bed level, \( D \) is the water depth, \( C \) and \( c \) are respectively the depth-averaged and local volumetric sediment concentration in the water column, \( q_s \) is the total volumetric sediment flux, and \( \sigma \) is any other relevant source/sink term, such as compaction, tectonic subsidence or uplift (note that the source/sink term here is used in a general sense and is not restricted to sediments). The second term in equation (1)
may be of interest for problems that involve short-term changes, but can be neglected when longer timescales are considered because of the limited capacity of the water column to act as a source/sink. The most common form for geomorphological studies is therefore:

\[(1 - p) \frac{\partial \eta}{\partial t} + \nabla \cdot q_s = \sigma \]  \hspace{1cm} (2)

On the basis of this equation, three conditions for equilibrium can be considered, whereby \(\partial \eta/\partial t\) equals zero:

(i) \(q_s = 0\) and \(\sigma = 0\). No sediment is transported, injected/extracted, such that a \textit{static equilibrium} is locally attained.

(ii) \(q_s \neq 0\), \(\nabla \cdot q_s = \sigma\), and \(\sigma = \text{constant}\). The sediment flux divergence is balanced by some constant source/sink term (e.g., a uniform rate of consolidation or tectonic uplift), such that the bed level locally does not change. This condition is referred to as \textit{dynamic equilibrium of type I}. A special case of this type of equilibrium occurs when there is no sediment flux divergence and no sources or sinks. Consequently, the bed level locally does not change even in the presence of a non-vanishing sediment transport (e.g., a flux through the system), or a net flux over the time period used to evaluate the sediment fluxes (e.g., a tidal cycle).

For the dynamic equilibrium case, we can replace the fixed reference frame with a moving reference system, where the origin moves vertically at a rate of \(-\sigma\). In equations (1) and (2), \(\eta\) is replaced by \(\zeta = \eta + \int \sigma dt\) and \(\sigma\) on the right hand side becomes zero. We discuss below the consequences for equilibrium of translation at the scale of the wider landscape setting (see also Kleidon et al., 2013).

These definitions of static equilibrium and dynamic equilibrium (type I) can be combined with the conventional hydrodynamic and sediment transport equations to derive solutions at some arbitrary time, \(t\). However, for morphology, we are often interested in historical and geological timescales (decades to millennia). Changes on the short timescale of a tide or storm event become subsumed in the longer-term patterns of change. For this case, there are two
important considerations: the frame of reference and the timescale of interest. The frame of reference relates to the geomorphological feature of interest. Whereas, in the first two definitions, this is simply the vertical elevation of the sea bed, over longer timescales we may be interested in changes to the “system”, such as the estuary or inlet. In such cases, the aspect of interest becomes the bed changes relative to the sea surface. For an inlet subject to the settlement, this will be a fixed frame of reference, whereas when subject to sea level rise, this will need to be considered in a moving frame of reference.

The second consideration is the timescale of interest. This has been a key consideration in research focused on aggregated-scale changes, aimed at understanding the longer-term response (de Vriend, 2003; Nicholls et al., 2016). The focus shifts from the instantaneous timescale used in process-based models, to the characteristic or geomorphological timescale, to consider the net bed level changes over the period of interest (e.g. a spring-neap cycle or the lunar tidal cycle). We therefore group these time dependent responses together, as an alternative view of dynamic equilibrium:

\[(iii) \ n_s \neq 0, \nabla \cdot q_s = \sigma(t), \text{and} \ \sigma(t) \text{ is a function of time. The flux divergence is balanced by some source/sink term that depends on time. Whilst the bed may adjust locally to accommodate the changing conditions, there is no net change when considered in the relevant frame of reference and integrated over a suitable timescale. We define this condition as } \text{dynamic equilibrium of type II.} \]

If the rate of change defined by the source term is sufficiently slow relative to the characteristic rate of morphological adjustment, any lag in the response will be small. As the rate of change of the source term increases, so the lag becomes more and more evident. In real systems with a constant rate of change, detecting such a lag can be extremely difficult unless there is a way of determining the morphological response time of the system a priori. In contrast, the more rapid change often exhibited by non-linear forcing conditions, such as the lunar nodal tide, can result in morphological response with a lag that is clearly identifiable in real systems. These two types of response are illustrated
schematically as II(a) and II(b) in Figure 3. Some examples of these types of response to (a) linear sea level rise and (b) the lunar nodal cycle in both the virtual and real world can be seen in Figures 4 and 5 of Townend et al. (2016).

In this morphological context, it is worth mentioning other types of equilibrium conditions that do not strictly refer to a solution of the Exner equation but that are of practical interest. For example, if over long timescales bed level changes exhibit small variations around a mean value that remains constant, the expression statistical equilibrium is invoked. This is illustrated schematically in Figure 3 for the different types of equilibrium.

![Figure 3: Equilibrium conditions defined in the text, where the solid blue line is sediment flux or sediment flux divergence, the dashed red line is the variation in the source/sink term and the thin black line illustrates stochastic variations as are more likely to be observed in the real world.](image-url)

The same expression has been used when looking at macro-characteristics of a system, for example when individual channels split or merge in a channel network but the overall statistical characteristics of the system, drainage density and size distribution of tidal channels, remain unchanged (D’Alpaos et al., 2005), or when in meandering rivers repeated cutoffs remove older, well developed meanders, limiting the planform complexity of the channel and, consequently, ensuring the establishment of statistically steady planform
configurations with a constant mean sinuosity (Frascati and Lanzoni, 2009).

Also, when bed level changes approach zero but remain non-zero (this is often the case in numerical simulations), the expression quasi-equilibrium is used (changes should be infinitesimally small in comparison to real world measurements of bed level changes).

Finally, there is a case where the whole system is moving within a landscape and we need to distinguish between equilibrium within the system and translation at the large scale. An example is the so-called “Bruun rule” where an equilibrium beach profile subject to sea level rise undergoes landward transgression (Bruun, 1962). The equilibrium form is maintained relative to the free surface and a cross-shore mass balance is achieved by erosion landwards and accretion in the lower portion of the profile. Consequently, there is a possible equilibrium of a geomorphic system as a whole, in a moving reference frame relative to the surrounding landscape, although the bed levels may be changing in a fixed frame of reference. Under conditions of marine transgression and sea level rise, Allen (1990) has suggested that an estuary could simply move upwards and landwards to maintain its form relative to the tidal frame. In this conceptual model, the equilibrium form is maintained relative to the free surface and a cross-shore mass balance is achieved by erosion landwards at the head and along the sides, and accretion on the bed of the estuary, as shown using a simple 3D form model by Townend and Pethick (2002). Importantly, there exists a rate of upwards and landwards migration within a coastal plain that does not require any import or export of sediment. The rate of change in elevation for the estuary as a whole is distinct from internal changes in the shape of the estuary that may occur, as described by the partial derivative. However, if the argument is that, in any such marine transgression, the estuary maintains its form (subject, as ever, to any imposed constraints, such as the underlying geology) then in the moving reference frame we require \( \partial \eta / \partial t = 0 \), so that any one of equilibrium conditions (i)-(iii) must be met for this to be the case. Consequently, the basis of exploring morphodynamic equilibrium outlined above is valid in a real world context provided that one accounts for
the potential movement of the system as a whole.

Geomorphologists are very conversant with the idea that different systems can have different spatial and temporal scales and that these scales are interrelated (Schumm and Lichty, 1965; Cowell and Thom, 1994; Coco et al., 2013). In the context of morphological equilibrium, we are generally interested in what might be regarded as relatively ‘long’ timescales for any given spatial scale. This means that the timescale will typically be long in relation to the morphological response time. Just what this is will depend on the system of interest. For example, the equilibrium profile of a beach might be defined in relation to the timescale of storm events, of the order of months to years, whereas the response timescale of a whole estuary might range from tens to hundreds of years, depending on the size of the system. We are therefore seeking to define equilibrium on the basis of equations (1) and (2) over a timescale that is consistent with the primary space and timescales of the system being considered.

For the case of estuarine settings, the conditions for equilibrium have been carefully explored by Seminara and co-workers (e.g., Seminara et al., 2010) for the case when the transport rate goes to zero at any instant of the tidal cycle, and the slight relaxation of this condition when there is a constant flux through the system (Toffolon and Lanzoni, 2010). They refer to the latter as dynamic equilibrium, which is consistent with our use of dynamic equilibrium of type I. Many researchers have considered the case where there is a zero or constant sediment flux gradient - dynamic equilibrium of type II(a) - in particular to consider the forms of equilibrium established under monotonically increasing sea levels or, more or less, uniform rates of subsidence (e.g., van der Wegen and Roelvink, 2008; D’Alpaos et al., 2011; Zhou et al., 2015). The variability of both forcing conditions (winds, waves, tides and river flows) and sediment supply make exploration of the equilibrium under non-stationary boundary conditions particularly difficult to address. One form of predictable variation, operating on a time-scale comparable to morphological response timescales, is the lunar nodal variation in tidal range. This provides an observable response
with a well-defined phase lag to the forcing condition, suggesting a dynamic equilibrium of type II(b) (Wang and Townend, 2012). Finally, the broader view, that considers dynamic equilibrium in a landscape context, has also been explored by Allen and others, as already noted, where the estuary system is seen as Lagrangian moving within a landscape reference frame. A detailed review of equilibrium studies in estuarine settings will be presented in the discussion (i.e., Section 2.3).

2. From the virtual to the real world

2.1. The role of variability and scale

The most immediate and striking difference between the virtual world (founded on analytical solutions and numerical models) and the real world is probably the simplified characterization of environmental forcing and characteristics. The observed variability in environmental forcing of morphodynamics (for the case of an estuary, limiting the example to the case of hydrodynamics, forcing could include river flow, tides and waves) and environmental characteristics (distribution of sediment types and vegetation) are reduced into one or two parameters (e.g., mean river flow, uniform vegetation cover, one sediment size for the bed material). This approach is almost a necessary condition to obtain equilibrium in the virtual world (and allows for easier interpretation of mechanisms and feedbacks) and should not necessarily be seen as a shortcoming, or as an argument to question the relevance and applicability of studies in the virtual world (Murray, 2007). In studies dealing with morphological equilibrium, the two worlds tend to reconcile over the long timescales, while at shorter timescales the real world will experience very different short-term variability. This variability results in transient configurations that are out of equilibrium with respect to the average value of the drivers, the ones that tend to be used in the virtual world. It is worth reiterating that in this contribution we focus on stable equilibrium conditions and the underlying assumption is that short-term variability in the forcing, or differences in the
sequences of forcing events, cannot result in alternative equilibrium configura-
tions. We obviously recognize that the possibility of alternative stable states
exists and it is certainly an area of research that deserves more attention. In
the case of estuarine settings, the problem has, so far, been approached with
several modelling studies (e.g., Schuttelaars and Swart, 2000; Marani et al.,

The use of sediment flux divergence as an indicator of morphodynamic
equilibrium implies the use of an interval in time over which the divergences
are evaluated. For tidally-forced systems, the choice of a tidal cycle might seem
a logical condition but it would neglect spring-neap variations or even longer
oscillations (Wang and Townend, 2012). Clearly, adding longer temporal scales
complicates the problem and poses practical limitations to numerical studies.
Overall, even the simplified virtual world requires attention in the choice of the
timescales analysed and the interpretation of the corresponding equilibrium
condition.

Finally, in the real world, the role of humans on long-term morphodynamic
evolution needs to be discussed. Nowadays, long-term configurations of natu-
ral systems are the result of intrinsically coupled natural, social and economic
feedbacks that have only begun to be explored. In this context, studies in the
“real world” that only focus on the equilibrium of natural systems, free of an-
thropogenic interventions, remain a useful tool particularly to understand the
effect of localized interventions that can cause a larger scale impact. For the
case of estuaries for example, the reclamation of a large area affects the tidal
prism and so the overall circulation and sediment transport patterns, which
will ultimately result in changes of the overall system geometry including, for
example, the cross-sectional area of the channels (e.g., D’Alpaos et al., 2010;
van der Wegen et al., 2010). This case and similar ones where the effect of
humans is simply limited to changes in boundary conditions can be certainly
studied by changing the same boundary conditions in the virtual world. On
the other hand, studies that fully couple anthropogenic drivers and natural
systems in the virtual world have only recently begun to be explored (e.g.,
2.2. Sedimentary and landscape setting

In the real world, equilibrium should prevail subject to the constraints imposed on the system. In real systems, such constraints might reflect large-scale geological, environmental or anthropogenic constraints that “fix” parts of the system. Whilst the intrinsic system dynamics (embodied by the feedback loop involving hydrodynamics, sediment dynamics and morphological change) may determine the morphology of a system, this is invariably conditioned by the overall landscape setting and sediment features (see, e.g., the estuarine system depicted in Figure 4). Despite wide variation in these three main factors (hydrodynamics, landscape setting and sedimentology), comparable equilibrium states can be identified. This implies that there is sufficient redundancy in possible morphological forms for equilibrium to be realised under a variety of constraints (e.g., the variation of width and depth in response to channel meanders).

Figure 4: Estuary classification that captures the three factors (hydrodynamics, landscape setting, sedimentology) that encapsulate the main characteristics of estuary systems (a). Each of these have their own subdivisions depending on the degree of detail required (b), where the red arrows indicate additional processes that mediate interactions within the system (e.g., vegetation). After Townend (2012).
The first component that determines the morphology is of course the prevailing hydrodynamics. This has been extensively explored in terms of the interaction and relative importance between waves, tides and river flows (e.g., Galloway, 1975; Geyer and MacCready, 2014) and how this is reflected in the emergent morphology. It is also the focus of much of the effort to explore the character of morphological equilibrium to date. However, we seek here to place this component in the broader context of landscape setting and sedimentology, as expressed in Figure 4, so that our “virtual world” constructs can be related to “real world” systems, taking proper account of the conditions and constraints that are present and help control the resultant morphology.

For the case of estuaries, many examinations of equilibrium are based on the prevailing hydrodynamics (and often just the tidal hydraulics). For example, Prandle (2004) and Seminara et al. (2010) provide estimates of estuary length assuming that this is a free parameter. In some real landscape settings this is undoubtedly the case but in others, such as those found in the UK and Taiwan, lengths are constrained by the relatively rapid rise of the land (Tow-nend, 2012). The role of landscape setting has also been clearly identified in the recent work by Dam et al. (2016). In this work, performance of the numerical model is shown to improve as a constrained planform increasingly limits the possible morphological changes. Consequently, the landscape setting can be an important constraint in determining which dimensions of the system can adjust. In addition, this can prevail as a system wide constraint, as already illustrated, or as a local constraint, such as changes in the underlying geology, e.g., the sill near Hull on the Humber, UK (Rees, 2006) and the numerous gorges on the Yangtze estuary, China (Yang et al., 2011).

Sediment availability and composition are another constraint on the formation and evolution of sedimentary systems that are the result of contemporary processes. At one extreme, there are systems with limited supply, such as Fjords and Fjards that are only able to adapt their morphology over geological time-scales. At the other extreme there are systems with very large (fluvial) supplies which control the channel network and delta formation, such
as the Lena River Delta, Russia. There are then a range of estuary types (e.g., Ria, Funnel shaped, Embayment and Tidal inlet) that occupy the state space between these extremes. Systems can also be altered as a result of changes in sediment availability, for example, hydraulic mining (Barnard et al., 2013) or dam construction (Yang et al., 2011). In addition to supply, the type of sediments available can also affect the morphology. For systems with both coarse and fine fractions available there can be a partitioning of the sediments (e.g., van Ledden et al., 2006; Zhou et al., 2015). It then follows that the various forms of equilibrium, outlined above, can be influenced by the nature of the sediments present on the bed (and in the subsoil, if erosion occurs) and in suspension.

In the next section, we introduce, in more detail, the variety of existing studies that explore morphodynamic equilibrium of different estuarine settings from the standpoint of the traditional “tide-wave-river” ternary diagram (Figure 4b).

2.3. Equilibrium in estuaries

The literature at the estuarine system scale is more limited than studies at the channel, creek or tidal flat scale. In this synthesis, we provide a brief summary of the former before examining a range of studies that focus on the equilibrium of estuaries from a predominantly hydrodynamic perspective. Hume and Herdendorf (1988) framed the landscape setting as a context for different types of estuarine systems. These settings can be reduced to three types, land surface deformations, river valleys, and marine embayments (Figure 4b). Of these, land surface deformation, with tectonic, volcanic and glacial origins, are typically associated with fjords and are of limited interest for the present discussion of equilibrium. River valleys are a common setting for estuaries that have evolved in response to contemporary processes over the Holocene. Marine embayments are widespread along the coastal regions of the world.

A particularly detailed consideration of rivers and associated catchment
basin morphology was compiled by Rodríguez-Iturbe and Rinaldo (1997) who examined how self-organisation, fractal structures and minimum energy concepts could be used to explain the dominant characteristics of river basins. More recently, Kleidon et al. (2013) considered even larger, continental scale, landscape development, founded on the principle of maximum power, to examine how the maximisation of sediment export leads to the depletion of topographic gradients back towards an equilibrium state. At the more local scale of the estuary itself, Dalrymple et al. (1992) considered the along-channel variations in energy due to waves, tide and river flow and how this was reflected in the sediment facies laid down and recorded in the geological record. Just how the setting and external forcing condition the resultant estuary was examined by Townend (2012), highlighting the relative influence of the three ternary diagrams reported in Figure 4b in defining the dominant characteristics of individual estuaries. In an interesting study of the transition from a marine basin to a range of enclosed and semi-enclosed systems, comprising lagoons, shoals, islands and estuary, Di Silvio et al. (2001) explored the extent to which these could be explained on the basis of sediment availability and primary forcing conditions, namely relative sea level and local wind conditions.

In most of the literature the morphodynamic equilibrium of estuarine systems has been investigated on the basis of hydrodynamic forcing (primarily tides, waves and rivers) that exert over and shape the landforms. Galloway (1975) proposed a hydrodynamics-based ternary diagram, demonstrating that coastal and estuarine morphologies have a strong link with their associated hydrodynamic forcing. Here, we summarize the typical studies that explore estuarine morphodynamic equilibrium following Galloways widely-adopted “tide-wave-river” ternary diagram. It is worth noting that these studies are mostly based on numerical modelling which makes it possible to cover the timescale from initial ontogeny to final equilibrium. Inevitably, a number of simplifications and abstractions have to be made and hence the morphodynamic equilibrium obtained in these modelling studies is a “virtual world” equilib-
rium strictly. There is also an open question as to whether model complexity influences the ability to identify equilibrium conditions. Some recent studies have suggested conflicting conclusions regarding equilibrium and this merits further research (Lanzoni and Seminara, 2002; Mariotti and Fagherazzi, 2010; Tambroni and Seminara, 2012).

In Figure 5 we list most of the references to date which either build models to predict morphodynamic equilibrium or use the equilibrium concept to build models. Commonly, these studies solve, either numerically or analytically, the governing equations describing several major components, such as hydrodynamics, sediment transport, biological processes and bed level change (Coco et al., 2013). For simplicity, below we just select some typical estuarine examples to further demonstrate the “virtual world” equilibrium concept as defined above.

Figure 5: Ternary diagram of studies that explore morphodynamic equilibrium. The colours of listed references correspond to the coloured dots representing the dominant forcing conditions (e.g., red colour indicates the studies deal with the joint influence of tidal and river forcing). Note that the literature listed in this figure aims to provide some typical examples and may not be inclusive.
A wide literature is found at the “tide” vertex of the ternary diagram, indicating the existence of morphodynamic equilibrium of “virtual” estuarine systems ranging from small-scale tidal flats and channels to large-scale estuaries and tidal inlets (Figure 5). Given the large number of existing studies, we will here only focus on this vertex. For example, Lanzoni and Seminara (2002) solved numerically the one-dimensional (1D) de Saint Venant and Exner equations for a friction-dominated tidal channel and found that the bottom profile evolved asymptotically toward a static equilibrium configuration which was characterised by a vanishing net sediment flux everywhere along the channel. A recent contribution by Lanzoni and D’Alpaos (2015) investigated the altimetric and planimetric evolution of a tidal channel flanked by intertidal flats, suggesting that, in the presence of a negligible external sediment supply, a static morphodynamic equilibrium was reached whereby the net sediment flux vanished everywhere. Roberts et al. (2000) developed a 1D morphodynamic model to investigate the equilibrium morphologies of tidal flats under different sediment supply conditions, indicating that the divergence of the residual sediment flux needs to balance the constant external sediment supply in order to reach equilibrium (i.e., a dynamic equilibrium of type I). This was also confirmed by Zhou et al. (2015) who used a different morphodynamic model to study sediment sorting dynamics on intertidal flats under both tides and waves. Their model results were consistent with the analytical solution by Friedrichs and Aubrey (1996) who found that tides and waves favour convex and concave equilibrium profiles, respectively. A good example of dynamic equilibrium of type II(a) is provided by the use of the ASMITA model (short for “Aggregated Scale Morphological Interaction between Tidal basin and Adjacent coast”) to explore the morphodynamic evolution of tidal inlet systems under a rising sea level varying linearly with time (van Goor et al., 2003). The response to a non-linear forcing (dynamic equilibrium of type II(b)) has been explored numerically and analytically (Wang and Townend, 2012) to examine the influence of the nodal tidal cycle, and to identify the main characteristics of the system scales and the along-estuary dynamic response. Another example of
this type of dynamic equilibrium is provided by some recent studies that have examined equilibrium conditions for combined river and tidal forcing with a fluvial supply of sediment (Guo et al., 2014; Bolla Pittaluga et al., 2015).

Many studies have also addressed the morphodynamic equilibrium of large-scale estuarine systems such as estuaries and tidal inlets. van der Wegen and Roelvink (2008) and van der Wegen et al. (2008) conducted both 1D and 2D numerical experiments to investigate the long-term evolution of a schematic estuary with a dimension similar to the Western Scheldt estuary. Without external sediment supply (either fluvial or marine), the estuary evolved over millennia asymptotically toward a state characterised by a vanishing residual sediment transport (dynamic equilibrium of type I). Using the same morphodynamic model (Delft3D), Guo et al. (2014) studied the role of a river (associated with sediment source) on the morphological development of a large-scale schematic estuary with a dimension comparable to the Yangtze estuary, suggesting that equilibrium could be approached over millennia given a constant river discharge (i.e., dynamic equilibrium of type II). The riverine influence was also investigated numerically by Zhou et al. (2014a) using a schematic tidal inlet setting and a similar equilibrium state was reached. George et al. (2012) modelled morphological change of a tide and river influenced estuary and found that the bed reached a dynamic equilibrium of type II within a few years. Modelling studies that included all the three components are rarely found to address the concept of equilibrium because of the complexity of the model, particularly in terms of the so-called “process-based” approach. However, using a different approach based on the existing equilibrium relationships, Townend (2010, 2012) developed a 3D form model which could implicitly include all the three components and model results agreed well with the field data, e.g., for a range of UK estuaries.

2.4. Alternatives to the Exner equation

In this review, we have focussed on mass flux balance as expressed by the Exner equation. However, there are a number of other approaches that have
been considered and it remains to be debated on the usefulness and applicability of different approaches (Griffiths, 1984; Seminara and Bolla Pittaluga, 2012). In a landscape and fluvial context the foremost among these are various considerations relating to energy and entropy within the system. In the 1960s-1980s numerous studies examined concepts such as minimum stream power, maximum flow efficiency and uniform energy dissipation. An extensive literature exists that discusses minimising or maximising various derivative properties, which is summarised in the context of hydraulic geometry by Singh et al. (2003) and synthesised for river basins by Rodríguez-Iturbe and Rinaldo (1997).

The application of these concepts to estuaries has not been as extensive. Langbein applied the concepts of uniform dissipation and minimum work for the system as a whole, to constrain the derivation of hydraulic geometry for an estuary (Langbein, 1963). Townend and Pethick (2002) used similar entropy based arguments to consider the most probable distribution of energy flux in an estuary. The possible existence of general geomorphic relations has also been explored in a “virtual world” either explicitly (Nield et al., 2005), or by examining the resultant properties from idealised long-term morphological simulations (van der Wegen and Roelvink, 2008), and in the “real world” based on measurements (Huang et al., 2014; Ensign et al., 2013), model analysis (Zhang et al., 2016) and combining both measurements and model results (D’Alpaos et al., 2010).

Even when using energy or entropy concepts, making the link to morphological form is not straightforward. It was for this reason that Thorn and Welford (1994) proposed adopting mass flux, whilst acknowledging that energy is an attractive alternative but needs a more formal basis for linking energy and form. When equilibrium is specified in energy terms, there is a loss of detail and a limited ability to make statements about subsystems. In an estuary context, this is alluded to by Savenije (2012) in his discussion of the 7th equation needed to derive a solution to the hydraulic equations. There are also numerous studies that assume an exponential plan form a priori, thereby
implicitly imposing one form of minimum work. Consequently, a more unified consideration of mass flux and energy would certainly merit further attention.

2.5. Linking the virtual and the real world

In the real world, measurements of sediment fluxes, or bed level changes, at the scale of an entire system are practically difficult to achieve (Jaffe et al., 2007; Gelfenbaum et al., 2015). At the same time, some large scale empirical relationships that relate geometrical aspects of the system have been identified for a wide variety of settings. For estuaries, relationships have been proposed between tidal prism and cross-sectional area (O’Brien, 1931), surface plan area and volume (Renger and Partenscky, 1974; Townend, 2005), and channel hypsommetry (Renger and Partenscky, 1974; Boon and Byrne, 1981; Wang et al., 2002; Townend, 2008). The most extensively explored is the empirical relationship between characteristic tidal prism, \( P \), and the cross-sectional area, \( \Omega \), in the form of \( \Omega = k P^n \), originally identified for tidal inlets in the USA (e.g., O’Brien, 1931; Jarrett, 1976). Subsequently, a number of researchers have endeavoured to derive an equation of a similar form using physical arguments (Marchi, 1990; Kraus, 1998; Hughes, 2002; Van De Kreeke, 2004), and argued that the prism-area relationship (indicated by PA relationship hereafter) is applicable along the length of the tidal channel (Friedrichs, 1995; van der Wegen et al., 2010; D’Alpaos et al., 2010; Guo et al., 2014, 2015).

The empirically-derived PA relationship is found to hold using field and laboratory observations (e.g., Stefanon et al., 2010; Zhou et al., 2014b), and has begun to be explored as a test for models. For example, Figure 6 shows how a morphodynamic model (Delft3D) evolves from its initial configuration towards the expected PA relationship (notice that Figure 6 uses the modified tidal prism, following Hughes, 2002). Other studies (van der Wegen et al., 2010; Tran et al., 2012; Lanzoni and D’Alpaos, 2015) show that numerical models can reproduce this type of relationship, providing some confidence in the use of numerical models to study real world morphodynamics. Experiments using numerical models have also been used to shed light on the
physical justification of the relationship (van der Wegen et al., 2010). However, there remains some debate about the extent to which such theoretical derivations match observations and numerical experiments (D’Alpaos et al., 2010; Stive et al., 2011). Furthermore, this relationship highlights the need to carefully define the limits of applicability of supposed equilibrium conditions in the landscape settings of the real world. Taking a broader view, Gao and Collins (1994) included estuaries from Japan and Hume and Herdendorf (1993) included the New Zealand estuaries, many of which were of tectonic, volcanic or glacial origin, rather than coastal plain and did not lie on the same line as the well documented US inlets. Estuaries from the UK are similarly diverse, and Townend (2005) argued that there was a progression from fjords to rias (partially infilled) to coastal plain systems that reflects the extent to which the system has responded to contemporary processes over the Holocene.

Figure 6: Modified tidal prism versus cross-sectional area. The linear regression line is obtained by fitting the data sets of both field observations and laboratory experiments. Evolution of the prism-area relationship from numerical simulations (red circles at the laboratory scale, blue circles at the natural system scale) is shown in detail in the insets. Physical model results (Stefanon et al., 2010) are similar but not shown here (Zhou et al., 2014b). Modified from Zhou et al. (2014b)
We have analysed the concept of morphodynamic equilibrium and its importance for the study and prediction of natural systems (e.g., coasts and estuaries). Although we have focused examples and interpretations on estuarine landforms, our discussion is equally applicable to open coast and river morphodynamics. The equilibrium conditions are based on the Exner equation, which is commonly used in morphodynamics studies of estuarine systems. We distinguish among four conditions of equilibrium that can be defined as static (one type) and dynamic (two types). We also acknowledge the use of other expressions like statistical and quasi-equilibrium which do not strictly satisfy the Exner equilibrium conditions but are a strong indication of the convergence of the system towards a specific configuration. The concept of equilibrium requires an a priori choice of the temporal and spatial scales over which equilibrium is analysed. It also requires a differentiation between the virtual world, where systems of equations are solved and the solution of the system is in fact the equilibrium configuration, and the real world, where variability in the environmental drivers and landscape settings often precludes the system from reaching an equilibrium condition. This leads to the title of this contribution “is morphodynamic equilibrium an oxymoron?” Certainly it appears so in the real world where, over short timescales, equilibrium is seldom observed. Paradoxically, it is also the basis of studies in the virtual world where processes are represented by a set of fundamental equations; unless a statistical or quasi-equilibrium approach is adopted. Overall, the study of equilibrium configurations remains a useful approach for discovering which processes, and usually which negative feedbacks, dominate the system. In this perspective, it is easy to predict that equilibrium will continue to remain a focus of morphodynamic studies. The challenge is bridging the gap between the virtual and the real world, and in doing so incorporating ecological, social and economic feedbacks into a geomorphic framework.
Acknowledgements

This work benefited from the open discussion during the 4th Estuary Day Workshop, with a theme of “Morphodynamic equilibrium in tidal environments”, held at Hohai University, Nanjing in October 2015. The authors are grateful to all the participants for the insightful and stimulating ideas. We wish to thank Brad Murray, the anonymous reviewer and Joan Florsheim (the Editor) for the insightful comments which helped to improve the original manuscript considerably. We also would like to thank Amy East (Coastal and Marine Geology group, USGS) for many useful comments and language edits. This research is supported by the National Natural Science Foundation of China (NSFC, Grant Nos. 41606104, 51620105005), the Jiangsu Provincial Natural Science Foundation (Grant No. BK20160862) and “the Fundamental Research Funds for the Central Universities” (Grant Nos. 2016B00714, 2015B24814), China.

References


Townend, I. and Dun, R. (2000). A diagnostic tool to study long-term changes


