Checking that the sound speed from Eq. (28) approaches the low-frequency limit of the EDFM.

In order to provide a partial validation test for the calculations in the manuscript, Figure E1 plots the results of calculations using the effective fluid density model (EDFM) (as described in article [34] in the reference list of the paper to which this report is an Electronic Supplement).

Figure E1a is a reproduction of Figure 2 in Ref. [34] and the data plotted are identical to those in the original paper. The plots in Figure E1b are also based on the EDFM, but use for input the values cited in the Appendix of the article for which this is the Electronic Supplement [H. Dogan, P. R. White and T. G. Leighton, 'Acoustic wave propagation in gassy porous marine sediments: the rheological and the elastic effects', *Journal of the Acoustical Society of America* (2017)]. The parameter values used in Figure E1b are only slightly different from those used in Ref. [34]: Figure E1b uses K_r = 5 x 10^{10} Pa instead of the value 3.6 x 10^{10} Pa in Ref. [34], and K_f = 2.3 x 10^{10} Pa instead of the value 2.25 x 10^{10} Pa in Ref. [34]. The porosity values in this manuscript are 0.75 (for mud) and 0.68 (for silt), instead of the value 0.4 in Ref. [34]. Consequently, slightly lower sound speed values are calculated in this supplement, and the manuscript for which it is a supplement, when considering the water-saturated sediment. This is shown in Fig E1b.

Note that although the models are capable of plotting up to 1 MHz, any observations made at such high frequencies might be affected by features not included in the model, such as sediment inhomogeneity, refraction, and departure from the assumptions inherent in the long-wavelength limit. The choice is made to plot up to 1 MHz (note that the original graph in Ref. [24] on which figure E1a is based only plots up to 100 kHz) only because to stop at a lower frequency would fail to reveal the high-frequency asymptotic behaviour of Fig. E1b.

Having established this, we can proceed to compare the low and high frequency sound speed limits in water-saturated sediment (calculated with EDFM), and in gassy sediment, i.e. the results presented in the manuscript for which this is an Electronic Supplement. Figure E2a shows the results when using the gassy mud modelled in Figure 3a of the paper which this is an Electronic Supplement. Figure E2b shows the results when using the gassy silt modelled in Figure 3b of the paper which this is an Electronic Supplement.

The high frequency limits are identical, as shown in Figure E2. The low frequency limits obtained with the present formulation are lower than the EDFM results because of the relatively large void fraction values chosen in the examples. Refs. [66] and [68] of the paper for which this forms an Electronic Supplement, derive and use a preliminary formulation for estimating the gas void fraction at low frequencies. The derivation in Ref. [68] follows the same path as the Mallock-Wood equation for gas bubbles in water, and then continues with a binomial expansion for the sound speed at low frequencies. Using Eqs. 5, 8-10 of Ref. [68], together with resonance frequency expression (Eq. 27f) derived in the manuscript for which this is an Electronic Supplement, we have calculated the low frequency limit of the sound speed. For instance, the sound speed value at 100 Hz for muddy sediment is calculated as 597 m/s compared to the value of 635 m/s in Figure E2a, and the sound speed value at 100 Hz for silt example is calculated as 1325 m/s compared to the value 1344 m/s in Figure E2b.

When there are large discrepancies between the sound speed in water-saturated and gassy sediment, as in Fig. E2a, the binomial expansion should be avoided. Therefore, as noted in the manuscript for which this is an Electronic Supplement, the Mallock-Wood equation serves as a good approximation for the low frequency sound speed value, if the bulk modulus of the gas bubble is calculated through the linearized equations in Sec. II.C of the present manuscript.

References

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[68] T. G. Leighton, "A method for estimating sound speed and the void fractions of bubbles from sub-bottom sonar images of gassy seabeds," *ISVR Technical Report No. 320,* pp.30, Southampton UK, 2007.

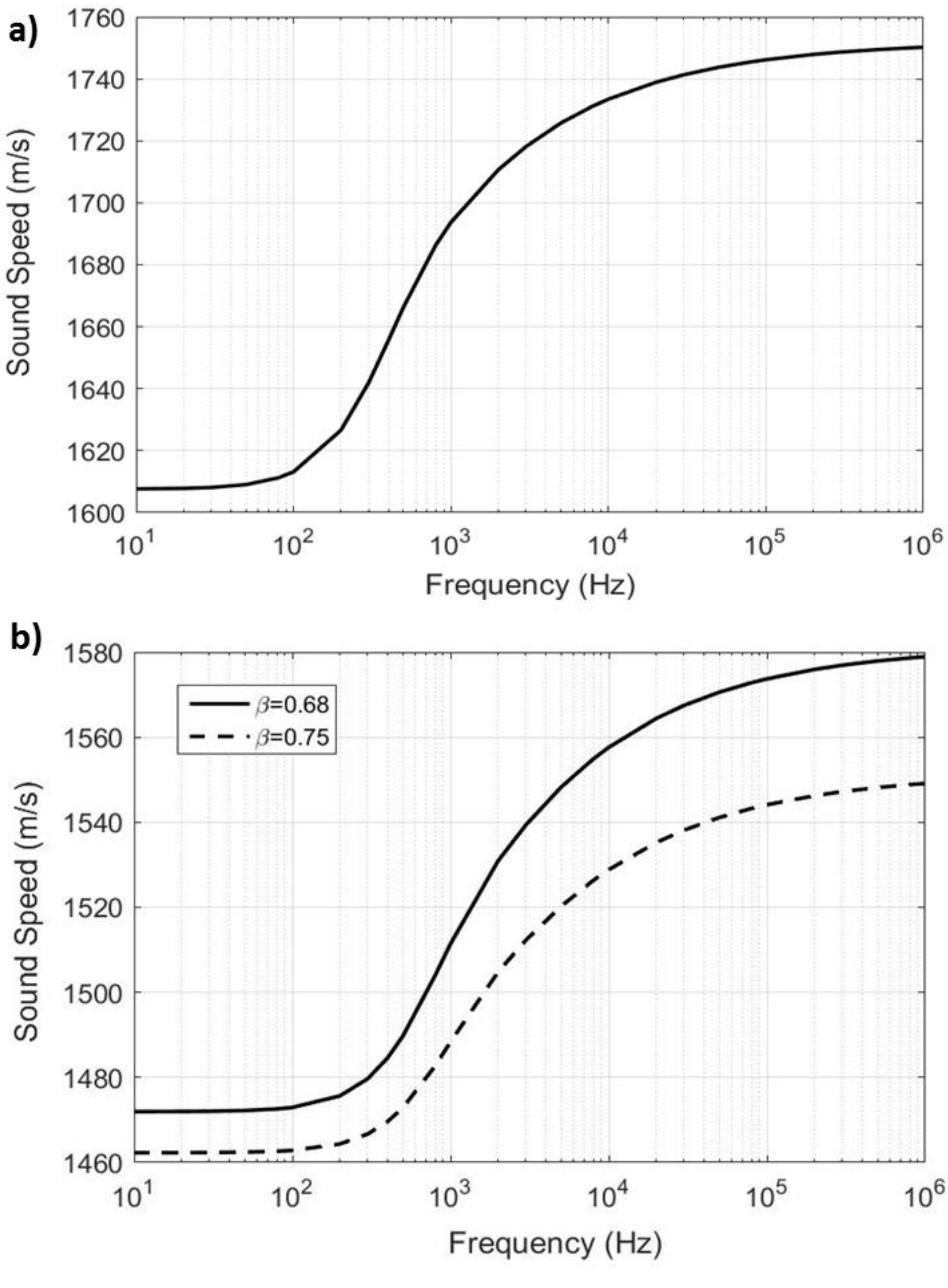

Figure E1: Sound speed curves calculated with the EDFM theory [34]. In (a), the parameter values are identical to the ones given in Ref. [34]. In (b), the parameter values are taken from the Appendix of the present paper, and porosity values are taken from Table 1 of the present paper (0.75 is the porosity of the mud and 0.68 is the porosity of silt).

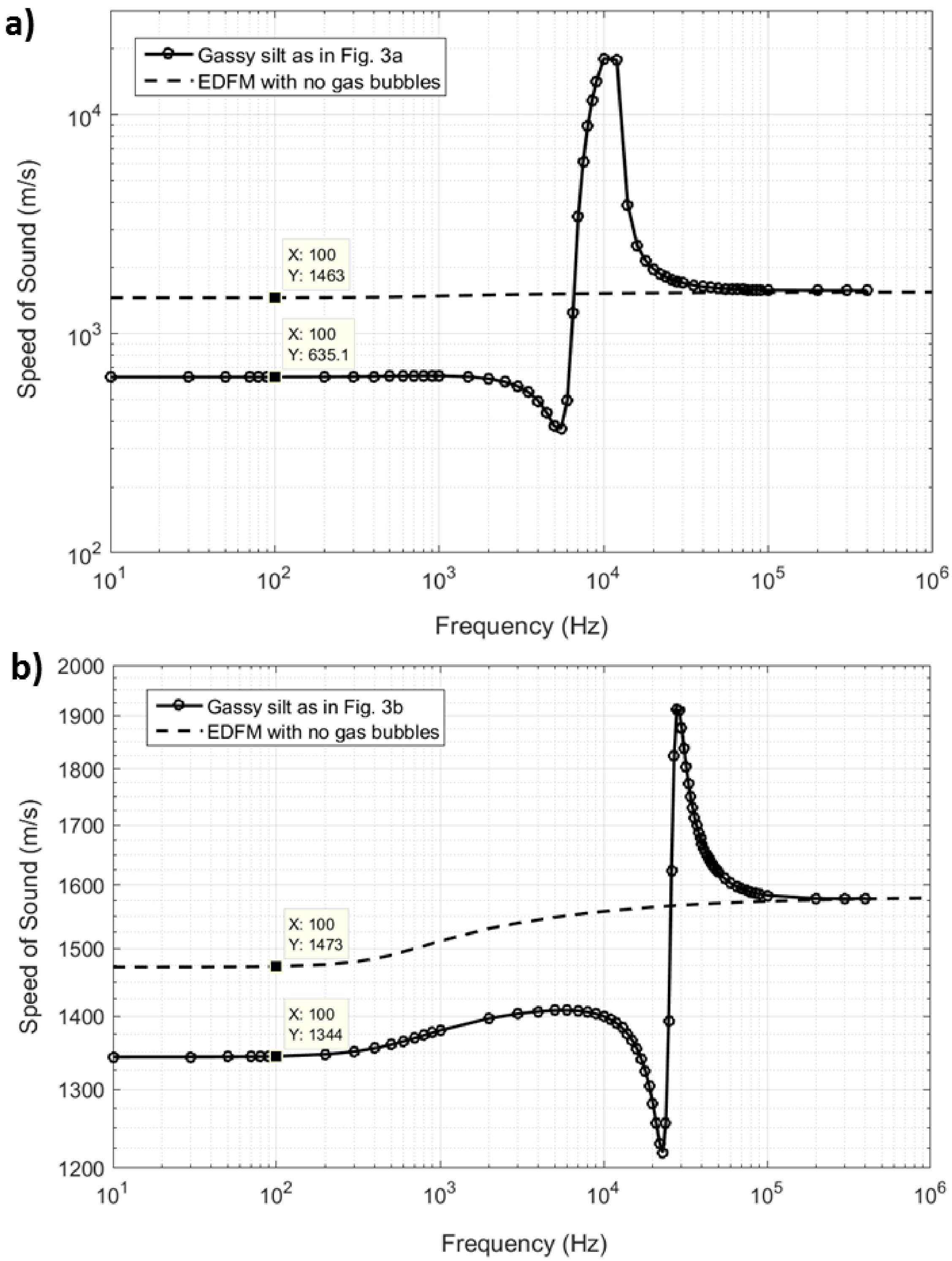

Figure E2: Comparison of the low and high frequency sound speed limits of the EDFM theory [34] and the formulation in the present manuscript. The parameter values are identical to the ones given in the Appendix and Table 1 of the manuscript for mud (a), and for silt (b).