An error indicator for finite difference methods using spectral techniques with application to aerofoil simulation

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Abstract

This work focusses on the development of an error indicator which can be used to determine areas of insufficient numerical resolution in unfiltered finite difference simulations. The approach adopts spectral techniques, applying small-domain Fourier transforms to a finite number of blocks which span the domain. The indicator is evaluated in the context of a NACA-0012 aerofoil simulation.

Keywords: Error indicators, Finite difference methods

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1. Introduction

Computational grids are at the core of many numerical models. One requirement of grid generation is that small-scale turbulent structures must be sufficiently well resolved, since any errors (introduced through numerical dispersion and dissipation, as well as nonlinear effects such as aliasing) can cause the simulation to become inaccurate and unstable. This work implements an error indicator for finite difference-based models, based on spectral techniques using small-domain Fourier transforms. It does not attempt to quantify the solution error, but instead estimates the severity of any under-resolution that occurs in the solution field. The indicator is implemented in the OpenSBLI modelling framework [1].

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2. Methodology

The domain is partitioned into blocks of N_e^3 points that span the whole domain. Fourier amplitudes for selected modes of a solution field are then computed for each block. With an increasing mode k, we desire the spectral energy $E(k)$ (and therefore the mode amplitude) to decrease at a certain rate, such that the smallest scales have the lowest energy content. An increase in $E(k)$ is likely to mean that the small scales are not well enough resolved, and instabilities are likely to occur. The indicator is based on detecting whether this spectrum decay is worse than some prescribed value.

3. Application: NACA-0012 Aerofoil

Error severity values, denoted I_i, from a NACA-0012 aerofoil simulation are shown in Figure 1. The uniform flow away from the aerofoil is well-resolved as suggested by I_i values of 0 or 1. However, a significant number of $I_i = 2$ values can be seen along the boundary layers either side of the aerofoil. Furthermore, a cluster of high values are present near the trailing edge where eddy shedding occurs. This suggests that more resolution would be required in these areas to adequately resolve the wake.

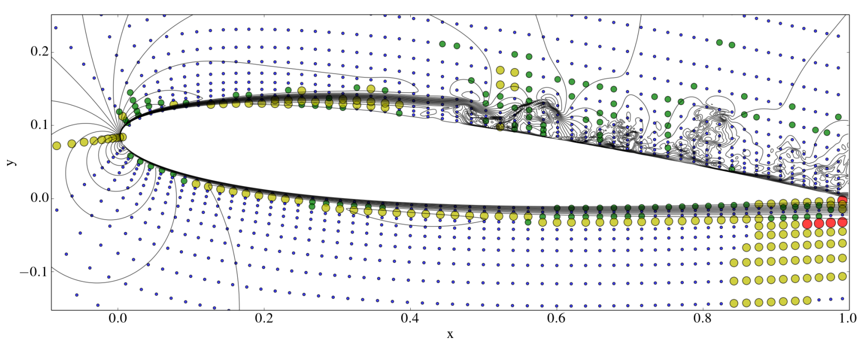

Figure 1: Contours of velocity magnitude in a two-dimensional slice (in the x–y plane at $z = 0$) from a NACA-0012 aerofoil simulation. Circles filled with blue, green, yellow and red indicate I_i error severity values of 0, 1, 2 and 3, respectively.

References

[1] C. T. Jacobs, S. P. Jammy, N. D. Sandham, OpenSBLI: A framework for the automated derivation and parallel execution of finite difference solvers on a range of computer architectures, Journal of Computational Science 18 (2017) 12–23. doi:10.1016/j.jocs.2016.11.001.