Observer-based Predictive Repetitive Control with Experimental Validations

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Abstract: This paper develops a new design method for predictive repetitive control systems to track periodic reference signals or reject disturbances with bandlimited frequency content. In the presence of input disturbances acting on the system, the new design estimates these disturbances using an observer. This approach is complementary to the predictive repetitive control system designs previously reported where the periodic disturbance model was embedded in the controller. Supporting experimental results from application of the new design to a two joint robotic arm are given.

Keywords: repetitive predictive control, disturbance estimation, observer, robot arm

1. INTRODUCTION

The internal model principle states that to follow a periodic reference signal or to reject a periodic disturbance signal with zero steady-state error, the generator for the reference must be included in the stable closed-loop control system (Francis and Wonham, 1976). Repetitive control (RC) has been developed as a control system design approach which harnesses the internal model principle by embedding a suitable signal generator, and hence provides tracking of periodic signals or rejection of periodic disturbances. See, e.g., (Hara et al., 1988) and, for related areas such as iterative learning control, (Rogers et al., 2007; Longman, 2000)).

In the design of repetitive systems, the control signal is often generated by a controller described by a transfer-function with the required coefficients, see, e.g., (Hara et al., 1988). If there are a number of frequencies contained in the periodic reference signal, the repetitive controller will contain all periodic modes, and their number is proportional to the period and inversely proportional to the sampling interval. The result can be a very high order control system, especially under fast sampling, which could then lead to numerical sensitivity, noise amplification, sensitivity to modeling errors and other undesirable problems commonly encountered in practical applications.

Recent advances in repetitive control system have used frequency analysis techniques to determine the dominant frequency components in the reference signal or the disturbance signal and include only these in the design (Wang et al., 2010, 2012, 2013, 2016). The key idea in this recent work is modelling the reference signals using frequency sampling filter models (Wang and Cluett, 2000) and identifying the dominant frequencies required for the design of a repetitive control system. In a similar manner to model predictive control, multi-input and multi-output repetitive control systems have been designed and experimentally verified with operational

constraints in place. Such designs have been successfully implemented and experimentally tested on a 2-joint robot (Wang et al., 2013, 2016), industrial electrical drives and a power converter (Wang et al., 2015).

An alternative approach to the design of predictive control systems is via the estimation of a disturbance model, see (Goodwin et al., 2000) as a starting point for the literature. In the current paper, this alternative approach to embed the reference generator will be developed for repetitive predictive control. In the presence of a periodic input disturbance, the basis of the new design is to estimate the periodic disturbance signal using a suitably structured observer, followed by subtracting it from the optimized control signal.

2. OBSERVER BASED PREDICTIVE-REPETITIVE CONTROL SYSTEM DESIGN

In the design of a predictive repetitive control system, it is assumed that the disturbance model $D(q^{-1})$ is available, where q^{-1} denotes the backward shift operator. Moreover, $D(q^{-1})$ is selected to reflect the frequency characteristics of the reference or disturbance signals. For example, if the reference signal is piece-wise constant, $D(q^{-1}) = 1 - q^{-1}$. If $D(q^{-1})$ is not available, methods have been developed and experimentally tested to construct it from frequency response analysis (see, e.g. (Wang et al., 2010, 2012, 2013, 2016)).

Suppose that the plant to be controlled has m inputs and m outputs and is represented by the state-space model

$$x_m(k+1) = A_m x_m(k) + B_m u(k) + B_m \mu(k),$$
 (1)

$$y(k) = C_m x_m(k), (2)$$

where $x_m(k)$ is the $n_1 \times 1$ state vector, u(k) is the $m \times 1$ input vector, y(k) is $m \times 1$ output vector, and $\mu(k)$ is a vector representing the input disturbance signals. In the design of

a repetitive control system, the system matrices (A_m, B_m, C_m) together with $D(q^{-1})$ are assumed to be available.

Since $\mu(k)$ is a vector that has the same dimension as the control signal, it is assumed that the *i*th element has the form

$$\mu_i(k) = \frac{\varepsilon_i(k)}{D(q^{-1})} \tag{3}$$

where $\varepsilon_i(k)$ is a white noise sequence with zero mean and variance σ_i $(1 \le i \le m)$.

The polynomial $D(q^{-1})$ is central in the design of predictive repetitive control systems. This polynomial can be found through frequency response analysis of the reference signals or disturbance signals, depending on actual application considered (see (Wang et al., 2010, 2012, 2013, 2016)). In general, this polynomial must contain the dominant periodic modes found from either the reference or disturbance signals. In this paper it is assumed that $D(q^{-1})$ can be expressed as

$$D(q^{-1}) = 1 + d_1 q^{-1} + d_2 q^{-2} + d_3 q^{-3} + \dots + d_\gamma q^{-\gamma}$$
 (4) where γ is the order of the polynomial.

In observer based predictive repetitive control, the input disturbance $\mu(k)$ is estimated together with the state vector $x_m(k)$. Given (3) and the $D(q^{-1})$ defined by (4), the estimation procedure starts from the input disturbance $\mu(k)$ written as

$$\mu(k+1) = -d_1\mu(k) - d_2\mu(k-1) - \dots - d_{\gamma-1}\mu(k-\gamma+2) - d_{\gamma}\mu(k-\gamma+1) + \varepsilon(k)$$

where $\varepsilon(k)$ is a vector that has the same dimension as the control signal u(k) and entries that are zero mean white noise processes.

Introducing the state vector

$$z(k) = \left[x_m^T(k) \ \mu^T(k) \ \dots \ \mu^T(k - \gamma + 1) \right]^T,$$

gives the following augmented model for observer design

$$z(k+1) = A_o z(k) + B_o u(k) + \bar{B_o} \varepsilon(k)$$

$$y(k) = C_o z(k)$$
(5)

where

$$A_{o} = \begin{bmatrix} A_{m} & \bar{B}_{m} \\ O_{1} & A_{d} \end{bmatrix},$$

$$A_{d} = \begin{bmatrix} -d_{1}I & -d_{2}I & \dots & -d_{\gamma-1}I & -d_{\gamma}I \\ I & O & O & \dots & O \\ O & I & O & \dots & O \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ O & O & \dots & I & O \end{bmatrix}.$$

$$\bar{B}_{m} = \begin{bmatrix} B_{m} & O_{2} \end{bmatrix}$$

where O_1 , O_2 , O and I denote the zero and identity matrices, respectively, of compatible dimensions. In addition

$$B_o = \begin{bmatrix} B_m \\ O \end{bmatrix}, \ ar{B}_o = \begin{bmatrix} O \\ I \\ O \end{bmatrix}$$
 $C_o = \begin{bmatrix} C_m \ O \end{bmatrix}$

The pair of matrices (A_o, C_o) is observable if the original system (A_m, C_m) is observable and an observer is designed to estimate the augmented state variable z(k). Assume that an

observer gain K_{ob} is chosen such that the closed-loop observer error system $(A_o - K_{ob}C_o)$ is stable with a desired response speed. Then the augmented state variable z(k) is estimated as:

$$\hat{z}(k+1) = A_o \hat{z}(k) + B_o u(k) + K_{ob}(y(k) - C_o \hat{z}(k)) \tag{6}$$

Given the estimated state vector $\hat{z}(k)$, the estimated disturbance vector $\hat{\mu}(k)$ is obtained and used in the model predictive controller as detailed next.

In many applications, the state variables are measured. For instance, in the applications of electrical drives and power converters, the current and voltage variables are measured by the respective sensors, see, e.g., (Wang et al., 2015). For these applications, it is only required to estimate the disturbance vector $\mu(k)$. This will reduce the complexity in both observer design and implementation. The observer in such cases is developed next.

Using (1) and (2) gives:

$$y(k+1) = C_m x_m(k+1)$$

= $C_m A_m x_m(k) + C_m B_m u(k) + C_m B_m \mu(k)$ (7)

and it follows that

$$C_m B_m \mu(k) = y(k+1) - C_m A_m x_m(k) - C_m B_m \mu(k) = \zeta(k)$$
 (8)

The left-hand side of (8) is constructed under the assumption that all states, outputs and control vectors are measured and denoted by $\zeta(k)$ and the following equations will be used to estimate $\mu(k)$ without estimating the state variable $x_m(k)$

$$\mu(k+1) = A_d \mu(k) + \varepsilon(k) \tag{9}$$

$$\zeta(k) = C_m B_m \mu(k) \tag{10}$$

Assume that (9) is observable and an observer gain K_{ob}^r is designed using the pair of matrices $(A_d, C_m B_m)$ such that the observer error system is stable with a desired closed-loop response. Then, the disturbance vector $\mu(k)$ is estimated using the following observer:

$$\hat{\mu}(k+1) = A_d \hat{\mu}(k) + K_{ob}^r(\zeta(k) - C_m B_m \hat{\mu}(k))$$

$$= A_d \hat{\mu}(k) + K_{ob}^r(y(k+1) - C_m B_m u(k) - C_m B_m \hat{\mu}(k))$$
(11)

Since(11) uses y(k + 1), which is one step ahead of the measurement at the sampling instant k, it is not convenient for the observer implementation. Instead, we introduce

$$\hat{\beta}(k) = \hat{\mu}(k) - K_{ob}^r y(k) \tag{12}$$

and by adding and subtracting the term $(A_d - K_{ob}^r C_m B_m) K_{ob}^r y(k)$ to the left-hand side of (11), it is be easily verified that (11) becomes

$$\hat{\beta}(k+1) = (A_d - K_{ob}^r C_m B_m) \hat{\beta}(k) + (A_d - K_{ob}^r C_m B_m) K_{ob}^r y(k) - K_{ob}^r C_m A_m x_m(k) - K_{ob}^r C_m B_m u(k)$$
(13)

This last observer equation is ready for implementation. At sampling time k=0, an initial condition for $\hat{\beta}(0)$ is selected and the disturbance $\hat{\mu}(0) = \hat{\beta}(0) + K^r_{ob}y(0)$. Together with the measurements of states, outputs, and control signals at sampling instant k, the estimation of $\hat{\beta}(k+1)$ is performed.

Additionally, $\hat{\mu}(k) = \hat{\beta}(k) + K_{ob}^r y(k)$ is calculated for repetitive predictive control system.

3. PREDICTIVE REPETITIVE CONTROL SYSTEM DESIGN

To design the predictive repetitive controller, introduce

$$\tilde{u}(k) = u(k) + \mu(k)$$

and (1) then becomes

$$x_m(k+1) = A_m x_m(k) + B_m \tilde{u}(k) \tag{14}$$

If the pair (A_m, B_m) is controllable, there exists a state feedback controller K such that the closed-loop control system

$$x_m(k+1) = (A_m - B_m K) x_m(k)$$
 (15)

is stable, where $\tilde{u}(k) = -Kx_m(k)$. This means that the future trajectory of the control vector $\tilde{u}(k)$ can be modelled using a sequence of the pulse functions or a set of Laguerre functions (Wang, 2009) since, for a stable closed-loop system described by (15) with bounded initial conditions, $\tilde{u}(k) \to 0$ as $k \to \infty$. This feature is essential if a limited number of parameters is to be used to parameterize the future control trajectories in model predictive control design.

Assume that at sampling instant k, k > 0, the state vector $x_m(k)$ is available either through measurement or estimation as detailed in the previous section, the state vector $x_m(k)$ provides the current plant information. The future optimal control vector \tilde{U} is defined as

$$\tilde{U} = \left[\tilde{u}(k)^T \ \tilde{u}(k+1)^T \ \dots \ \tilde{u}(k+N_c-1)^T \right]^T$$

where N_c is the control horizon dictating the number of parameters used to capture the future control trajectory. With this given information, the future state vectors are predicted for N_p samples, where N_p is termed the prediction horizon $(N_c \leq N_p)$. It is assumed that after N_c samples, the control vector $\tilde{u}(k)$ is zero for all future samples $(k \geq N_c)$. The state vectors so obtained are written as

$$X = \left[x_m(k+1 \mid k)^T \dots x_m(k+N_p \mid k)^T \right]^T.$$

Using the state-space model (14), the future state variables are calculated sequentially using U_s as

$$X = F_x x_m(k) + \Phi_s \tilde{U}, \tag{16}$$

where

$$F_{x} = \begin{bmatrix} A_{m} \\ A_{m}^{2} \\ \vdots \\ A_{m}^{N_{p}} \end{bmatrix} \Phi_{s} = \begin{bmatrix} B_{m} & 0 & \dots & 0 \\ A_{m}B_{m} & B_{m} & \dots & 0 \\ A_{m}^{2}B_{m} & A_{m}B_{m} & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ A_{m}^{N_{p}-1}B_{m} & A_{m}^{N_{p}-2}B_{m} & \dots & A_{m}^{N_{p}-N_{c}}B_{m} \end{bmatrix}$$

The design criterion for the predictive repetitive controller is to find the control parameter vector \tilde{U} such that the following cost function is minimized

$$J = X^T \bar{Q} X + \tilde{U}^T \bar{R} \tilde{U},$$

where \bar{Q} is a block diagonal matrix with identical block diagonal matrix entries Q, where Q is a symmetric positive semi-definite matrix, likewise \bar{R} is block diagonal matrix with identical block diagonal matrix entries R, where R is a positive definite matrix. Substituting (16) into the cost function gives

$$J = \tilde{U}^T (\Phi_s^T \bar{Q} \Phi_s + \bar{R}) \tilde{U} + 2 \tilde{U}^T \Phi_s^T \bar{Q} F_x x_m(k_i)$$

+ $x_m(k)^T F_x^T Q F_x x_m(k).$ (17)

The solution of this optimal control problem is

$$\tilde{U} = -(\Phi_s^T \bar{Q} \Phi_s + \bar{R})^{-1} \Phi_s^T \bar{Q} F_x x_m(k). \tag{18}$$

If the state variable $x_m(k)$ is not measurable, then the observer given in (6) is used to estimate the augmented state vector $\hat{z}(k)$, from which $\hat{x}_m(k)$ is obtained and used instead of $x_m(k)$. The actual control vector at k is given by

$$u(k) = \tilde{u}(k) - \hat{\mu}(k)$$

where $\hat{\mu}(k)$ is calculated using the observer given by (6) or (13).

In this design the disturbance model is embedded through the observer and therefore the reference vector should enter the system as an output disturbance with a negative sign. Otherwise, the predictive repetitive control system will have steady-state errors when it is used for reference tracking. Assuming that the reference vector r(k) is specified, the observer (6) is rewritten as

$$\hat{z}(k+1) = A_o \hat{z}(k) + B_o u(k) + K_{ob}(y(k) - r(k) - C_o \hat{z}(k))$$
 (19)

When the state vector $x_m(k)$ is measured, then the reference vector enters the system through the reduced order observer

$$\hat{\mu}(k) = \hat{\beta}(k) + K_{ob}^{r}(y(k) - r(k))$$

$$\hat{\beta}(k+1) = (A_{d} - K_{ob}^{r}C_{m}B_{m})\hat{\beta}(k)$$

$$+ (A_{d} - K_{ob}^{r}C_{m}B_{m})K_{ob}^{r}(y(k) - r(k))$$

$$- K_{ob}^{r}C_{m}A_{m}x_{m}(k) - K_{ob}^{r}C_{m}B_{m}u(k)$$
(21)

3.1 Predictive Repetitive Control with Constraints

A critical feature of predictive repetitive control is the ability to include input and output operational constraints in the design. In the current design, the constraints are incorporated via the solution of real-time optimization problem, which effectively handles constrained control of multi-input and multi-output systems. The central idea is to minimize the objective function J as in (17) subject to linear inequality constraints.

There are two types of constraints commonly encountered in control applications. The first type is input constraints and the second is the output constraints. Input constraints have been successfully implemented in applications. If output and state constraints are also required then problems can arise, particularly when they are implemented together with input constraints.

When embedding the integrator into the observer, the input constraints are somewhat more complicated because the constraints will involve the estimated disturbance $\hat{\mu}(k)$. For example, a control amplitude constraint can be expressed as

$$u^{min} \le u(k) \le u^{max}$$

or, in terms of $\tilde{u}(k)$,

$$u^{min} + \hat{\mu}(k) < \tilde{u}(k) < u^{max} + \hat{\mu}(k)$$

and the rate of change on the input is expressed as

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} G_{11}G_{12} \\ G_{21}G_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \tag{22}$$

$$G_{11}(s) = \frac{0.16s^9 + 14.51s^8 + 578.2s^7 + 1.392e4s^6 + 2.26e5s^5 + 2.58e6s^4 + 2.09e7s^3 + 1.17e8s^2 + 4.21e8s + 7.6e8}{5.25e - 5s^{12} + 0.01463s^{11} + 0.91s^{10} + 31.2s^9 + 714.1s^8 + 1.19e4s^7 + 1.45e5s^6 + 1.4e6s^5 + 1.01e7s^4 + 5.7e7s^3 + 2.3e8s^2 + 5.9e8s + 7.6e8}, \\ G_{12}(s) = \frac{-0.022s^7 - 3.24s^6 - 88.3s^5 - 1347s^4 - 1.06e4s^3 - 4.52e4s^2}{5.25e - 5s^{10} + 0.014s^9 + 0.72s^8 + 20s^7 + 363s^6 + 4645s^5 + 4.3e4s^4 + 2.9e05s^3 + 1.4e6s^2 + 4.18e6s + 6.323e6}, \\ G_{21}(s) = \frac{-0.16s^7 - 8.7s^6 - 194s^5 - 2498s^4 - 1.78e4s^3 - 6.64e4s^2}{5.25e - 5s^{10} + 0.014s^9 + 0.67s^8 + 17.9s^7 + 316s^6 + 3963s^5 + 3.6e4s^4 + 2.42e5s^3 + 1.1e6s^2 + 3.5e6s + 5.3e6}, \\ G_{22}(s) = \frac{0.027s^9 + 4.95s^8 + 264s^7 + 7394s^6 + 1.3e5s^5 + 1.69e6s^4 + 1.5e7s^3 + 9.4e7s^2 + 3.8e8s + 7.6e8}{5.25e - 5s^{12} + 0.014s^{11} + 0.9s^{10} + 31s^9 + 714.1s^8 + 1.19e4s^7 + 1.48e5s^6 + 1.4e6s^5 + 1.04e7s^4 + 5.7e7s^3 + 2.3e8s^2 + 5.9e8s + 7.6e8}.$$

$$\Delta u^{min} + u(k-1) + \hat{\mu}(k) \le \tilde{u}(k) \le \Delta u^{max} + u(k-1) + \hat{\mu}(k)$$
(23)

Since $\tilde{u}(k)$ is the first component in \tilde{U} , these input constraints are converted into inequality constraints in terms of \tilde{U} .

The constraints for the output or states are formulated using the prediction equation (16). Once the constraints are formulated, a quadratic programming algorithm can be used to solve the constrained predictive control problem.

4. EXPERIMENTAL EVALUATION

This case study is for a two-input and two-output model obtained from frequency domain tests on an an anthropomorphic robot arm undertaking a 'pick and place' task in a horizontal plane using two joints, as shown in Fig. 1. Its end-effector travels between the pick and place locations in a straight line using joint reference trajectories which minimize the end-effector acceleration. Having reached the place location, the robot then returns back to the starting location. The overall system model has been identified by combining experimentally-derived models of its constituent components as described by the transfer-function matrix (22).

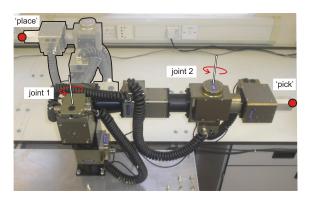


Fig. 1. Anthropomorphic robot arm showing pick and place locations.

The continuous time system is sampled using the sampling interval $\Delta t = 0.05$ (sec). The reference trajectories for output y_1 and y_2 appear in the upper plots of Figures 2a) and b), and the control objective is for each output to follow the corresponding reference signal as closely as possible in presence of measurement noise and model uncertainty. The reference signals for the outputs y_1 and y_2 each have a period of 20 seconds. With the sampling interval chosen to be 0.05 second, the number of samples for each period is $\frac{20}{0.05} = 400$. From our previous analysis (see (Wang et al., 2013)), the polynomial $D(z^{-1})$ is selected as

N_p	20
N_c	6
r_c	0.1
r_{ob}	10000

Table 1. Choice of performance parameters

$$D(z) = (1 - z^{-1})(1 - 2\cos(\frac{2\pi}{400})z^{-1} + z^{-2}) \times (1 - 2\cos(\frac{4\pi}{400})z^{-1} + z^{-2})$$
 (24)

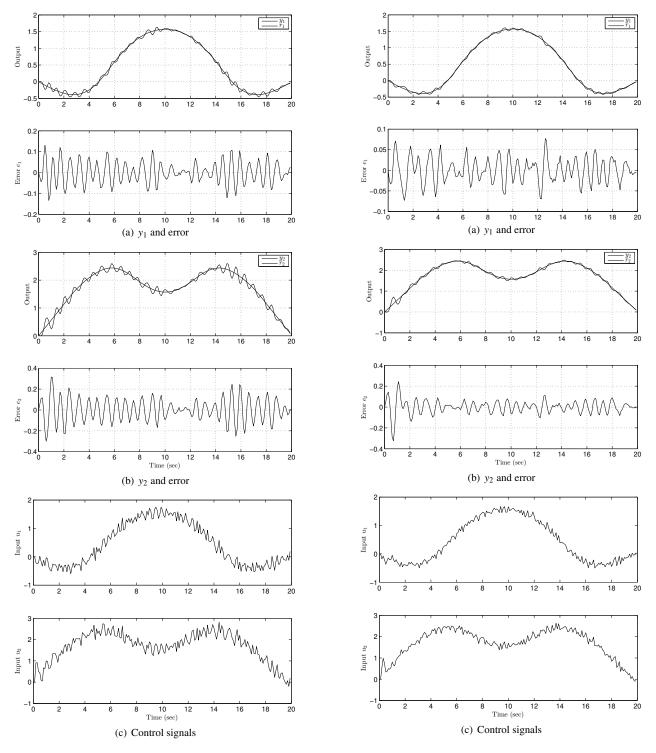
The mathematical model is a transfer function and when it is converted into a state space model, the state variables have no physical meaning. Thus, a full order observer is required to estimate both the state variable vector $x_m(k)$ and the disturbance signal $\mu(k)$. In the design of the observer, we select $Q_{ob} = I$ and $R_{ob} = r_{ob}I$ with r_{ob} being adjustable. For the predictive repetitive controller design, we select $Q = C_m^T C_m$ and $R = r_c I$ with r_c being adjustable. Table 1 shows the performance parameters used in the predictive repetitive control system design. Figure 2 shows the closed-loop output signals, error signals and control signals. The performance parameters for the observer must be chosen carefully to provide satisfactory performance. For instance, increasing the parameter r_{ob} produces improved tracking accuracy, as confirmed by results shown in Figure 3. However, when the weighting coefficient r_{ob} is reduced to 1000, the closed-loop predictive repetitive control system becomes unstable as shown in Figure 4. When the parameter r_{ob} is reduced, the dynamic response speed of the observer error system is increased, which consequently reduces the robustness of the observer error system because there are inevitable modelling errors in the robotic system.

5. CONCLUSIONS

This paper presented a novel observer based predictive repetitive control system design. This design is based on the assumption of the existence of a input periodic disturbance and an observer is designed to estimate such a periodic disturbance. Together with the model predictive controller, the resultant control system is shown experimentally to have the capability to track complex reference signals.

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in Table 1.

Fig. 2. Experimental results using the performance parameters

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Fig. 3. Experimental results using the performance parameters in Table 1 except $r_{ob} = 100000$.

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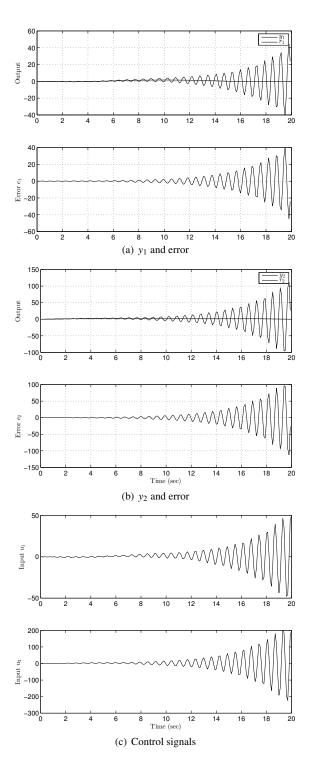


Fig. 4. Experimental results using the performance parameters in Table 1 except $r_{ob} = 1000$.

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