Variance-based Robust Optimization of Permanent Magnet Synchronous Machine

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This paper focuses on the application of the variance-based global sensitivity analysis for a topology derivative method in order to solve a stochastic nonlinear time-dependent magnetoquasistatic interface problem. To illustrate the approach a permanent magnet synchronous machine has been considered. Our key objective is to provide a robust design of rotor poles and of the tooth base in a stator for the reduction of the torque ripple, while taking material uncertainties into account. Input variations of material parameters are modeled using the polynomial chaos expansion technique, which is incorporated into the stochastic collocation method in order to provide a response surface model. Additionally, we can benefit from the variance-based sensitivity analysis. This allows us to reduce the dimensionality of the stochastic optimization problems, described by the random-dependent cost functional. Finally, to validate our approach, we provide the two-dimensional simulations and analysis, which confirm the usefulness of the proposed method and yield a novel topology of a permanent magnet synchronous machine.

***Index Terms*—Design optimization, Permanent magnet motors, Topology derivative, Robustness, Stochastic processes, Chaos Polynomials, Uncertainty quantification.**

I. Introduction

DUE to the several attractive features, such as high efficiency and power factor, high torque to weight ratio and brushless construction, PM synchronous machines have found recently a wide range of applications in the automotive industry, e.g., [1]. However, in spite of their unquestionable advantages, including also the field weakening capability of 1:5, the Electrically Controlled Permanent Magnet Excited Synchronous Machine (ECPSM) [4], served here as a case study, suffers inherently from the considerable level of the torque pulsation [4], [5]. From this perspective the mitigation of the torque fluctuations is a key issue for the design of a permanent magnet (PM) machine. In the literature, various methods for suppressing the ripple torque (RT) have been proposed. In particular, they are devoted to both the deterministic and stochastic optimization methods, e.g. [2] and [5]. Yet, in many engineering applications, physical models are very often affected by a relatively large amount of uncertainty [3]. Hence, there is a need to include uncertainty quantification (UQ) in order to provide a reliable numerical simulation. For this reason, in this paper we explore the stochastic collocation method (SCM) combined with the polynomial chaos expansion (PCE) [9]. A novel aspect of this work, in comparison with [6] and [7], is to attain the low RT design of the ECPSM, when taking the uncertainty of reluctivities into account. For this purpose, the robust variance-based gradient is used to improve the PM machine quality by minimizing variations of the output performance function.

II. Stochastic Forward Problem

The electromagnetic behavior of the ECPSM is analyzed here in a two-dimensional (2D) formulation, as in [5]. Thus, a 2D model can be described using the magnetic vector potential A for the stochastic quasi-linear system of PDEs, defined on $t \in (0,T]$ with $T>0$ and $\mathbf{x} = (x,y)^\top \in \mathrm{D} \subset \mathbb{R}^2$ as

$$\left\{\begin{array}{l} \nabla\cdot\left(\upsilon_{\text{Fe}}\left(\mathbf{x},|\nabla A(\theta)|^2,\xi_1\right)\nabla A(\theta)\right)+\sigma(\xi_4)\partial_t A(\theta)=J_i(\mathbf{x},t),\\ \nabla\cdot\left(\upsilon_{\text{air}}\left(\mathbf{x},\xi_2\right)\nabla A(\theta)\right)=0,\\ \nabla\cdot\left(\upsilon_{\text{PM}}(\mathbf{x},\xi_3)\nabla A(\theta)\right)=\nabla\cdot\upsilon_{\text{PM}}(\mathbf{x},\xi_3)\mathbf{M}(\mathbf{x}), \end{array}\right. \tag{1}$$

endowed with both boundary and initial conditions, where $\theta := (\mathbf{x},t;\boldsymbol{\xi}) \in D\times(0,T]\times\Omega$ with the domain D, which refers to the sextant region; $\sigma(\xi_4) = \sigma_{\text{Fe}}(1+\delta_4\xi_4)$ represents conductivity. $J_i(\mathbf{x},t)$, $i=1,2,3$ denotes an excitation density current and Ω is a sample space; $\mathbf{M}$ represents the remanent flux density of the PM, while $\boldsymbol{\upsilon}$ denotes the reluctivity. In particular, a stochastic model for $\upsilon(\cdot;\boldsymbol{\xi})$ is given by

$$\upsilon(\theta)=\left\{\begin{array}{ll} \upsilon_{\text{Fe}}(\mathbf{x},|\nabla A(\theta)|^2)(1+\delta_1\xi_1) & \text{for } \mathbf{x}\in \mathrm{D}_{\text{Fe}}\\ \upsilon_{\text{air}}(\mathbf{x})\,(1+\delta_2\xi_2) & \text{for } \mathbf{x}\in \mathrm{D}_{\text{air}}\\ \upsilon_{\text{PM}}(\mathbf{x})\,(1+\delta_3\xi_3) & \text{for } \mathbf{x}\in \mathrm{D}_{\text{PM}}, \end{array}\right. \tag{2}$$

where $\boldsymbol{\xi} = (\xi_1,\xi_2,\xi_3,\xi_4)$ is assumed to be random variables, defined on some probability space $(\Omega,\mathcal{F},\mathbb{P})$.

III. UQ Analysis & Polynomial Chaos Expansion

For the uncertainty quantification, we consider $\mathbf{p}(\boldsymbol{\xi}) = [\upsilon_{\text{Fe}}(\xi_1),\ \upsilon_{\text{air-gap}}(\xi_1),\ \upsilon_{\text{PM}}(\xi_1),\ \sigma(\xi_4)] \in \Pi$, where $\xi_j, j = 1,\ldots,4$ are independent and identically uniformly distributed in the interval $[-1,1]^4$ with the constant magnitude $\delta_j = 10\%$. Thus, we assume a joint probability density function $g:\Pi\to\mathbb{R}$, which is associated with $\mathbb{P}$, and that A is a square integrable function. Then, a response surface model of A is represented by a truncated series of the PCE [9] in the form

$$A(\mathbf{x},t;\mathbf{p}) \doteq \sum_{i=0}^{N}\alpha_i(\mathbf{x},t)\Psi_i(\mathbf{p}), \tag{3}$$

with a priori unknown coefficient functions α_i and predetermined basis polynomials Ψ_i with the orthogonality property

$\mathbb{E}[\Psi_i \Psi_j] = \delta_{ij}$. Here, $\mathbb{E}$ is the expected value, associated with $\mathbb{P}$. For the calculation of α_i, we applied the SCM with the Stroud-3 formula [7], which yields the solution at each quadrature node $\boldsymbol{\xi}^{(k)}$, $k = 1, \ldots, K$ of the problem (1). Next, the multi-dimensional quadrature rule with associated weights w_k is used for projecting function A_k into the basis Ψ_i as

$$\alpha_i(\mathbf{x}, t) \doteq \sum_{k=1}^{K} A\left(\mathbf{x}, t, \mathbf{p}^{(k)}\right) \Psi_i\left(\mathbf{p}^{(k)}\right) w_k, \tag{4}$$

Finally, the statistical moments are approximated by

$$\mathbb{E}\left[A\left(\mathbf{x}, t; \mathbf{p}\right)\right] \doteq \alpha_0(\mathbf{x}, t), \mathrm{Var}\left[A\left(\mathbf{x}, t; \mathbf{p}\right)\right] \doteq \sum_{i=1}^{N} |\alpha_i(t)|^2 \tag{5}$$

assuming $\Psi_0 = 1$ [9]. Based on (3), a variance-based sensitivity analysis can be performed.

IV. Stochastic Optimization Problem

Finally, we formulate the stochastic magnetoquastatic interface problem for the stochastic cost functional

$$F(t, \mathbf{p}(\boldsymbol{\xi})) = \int_0^T \left[w_1 W_{\mathrm{B}}(\theta) + w_2 P(\theta)\right] \mathrm{d}t, \tag{6}$$

where $W_{\mathrm{B}}(\theta)$ and $P(\theta)$ denote the stored magnetic energy and the electromagnetic losses, while w_i refer to the prescribed weights. Furthermore, in order to reduce the dimensionality of the optimization problem, defined by $\mathbb{E}[F(t, \mathbf{p}(\boldsymbol{\xi}))]$, we applied the Sobol decomposition [8]

$$S(\mathbf{x}, t)_j := \frac{\mathrm{V}_j^d}{\mathrm{Var}[A\left(\mathbf{x}, t; \mathbf{p}\right)]}, \quad \mathrm{V}_j^d := \sum_{i \in I_j^d} |\alpha_i|^2, \tag{7}$$

where $j = 1, \ldots, 4$, and sets $I_j^d := \{j \in \mathbb{N} : \Psi_j(\mathbf{p})$ is not constant in p_j and $\mathrm{degree}(\Psi_i) \leq d\}$ with d being the maximum degree of the Laguerre multivariate polynomials.

V. Numerical Results & Conclusions

We applied the proposed methodology to the optimization of the ECPSM structure under uncertainties. Both structures before and after the optimization are shown in Fig. 1.

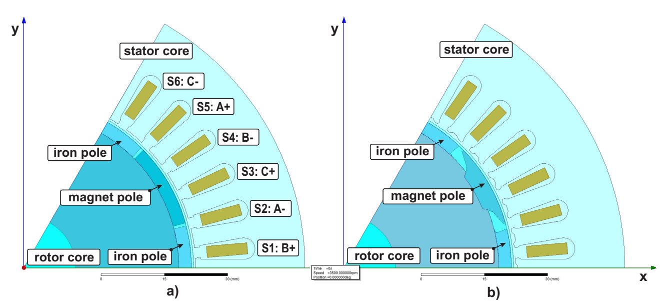

Fig. 1. ECPSM topology for (a) initial and (b) optimized configuration.

For the optimized configuration, the average of the RT mean value, shown in Fig. 2, has been reduced by 60%, while the variation has been minimized by 9%. Likewise, the mean value of total losses (and the standard deviation), depicted in Fig. 3, have been decreased by 28% in the average sense.

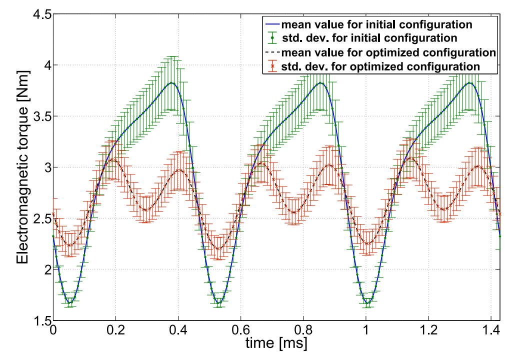

Fig. 2. Mean and standard deviation of electromagnetic torque for initial and optimized ECPSM topology.

However, the decrease of the mean value of the electromagnetic torque by 9% might be considered as a drawback of the proposed approach and needs to be further investigated.

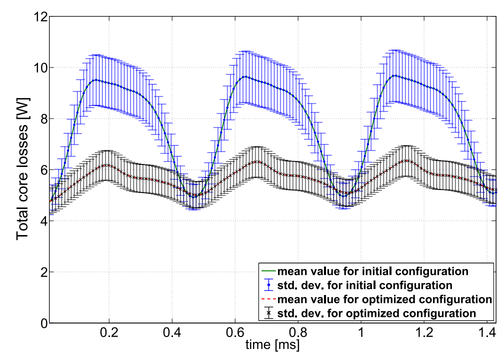

Fig. 3. Mean and standard deviation of total core losses for initial and optimized ECPSM topology.

References

[1] F. Gieras, and M. Wing, Permanent magnet motor technology, John Wiley & Sons Ltd., 2008.

[2] M.S. Islam, R. Islam, T. Sebastian, A. Chandy, S.A. Ozsoylu, "Cogging torque minimization in PM motors using robust design approach," *IEEE Trans. on Magn.* vol. 47, pp. 1661-1669, 2011.

[3] P. Sergeant, G. Crevecoeur, L. Dupre, and A. van den Bossche, "Characterization and optimization of a permanent magnet synchronous machine," *COMPEL*, vol. 28, pp. 272-284, 2008.

[4] H. May, R. Pałka, P. Paplicki, S. Szkolny, W.R. Canders, "New concept of permanent magnet excited synchronous machines with improved high-speed features," *Arch. of Electr. Engineer.*, vol. 60, pp. 531-540, 2011.

[5] P. Putek, P. Paplicki, and R. Pałka, "Low cogging torque design of permanent magnet machine using modified multi-level set method with total variation regularization," *IEEE Trans. on Magn.*, vol. 50, pp. 657-660, 2014.

[6] P. Putek, P. Meuris, R. Pulch, E. J. W. ter Maten, W. Schoenmaker, M. Günther, "Uncertainty quantification for a robust topology optimization of power transistor devices," *IEEE Trans. on Magn.*, vol. 52, 2016.

[7] P. Putek, R. Pulch, A. Bartel, E.J.W. ter Maten, M. Günther, K.M. Gawrylczyk, "Shape and topology optimization of a permanent-magnet machine under uncertainties," *J. Math. Industry*, vol. 6, pp.1-20, 2016.

[8] B. Sudret, "Global sensitivity analysis using polynomial chaos expansions," *Rel. Eng. Syst. Safety*, vol.93, pp. 964-979, 2008.

[9] D. Xiu, "Efficient Collocational Approach for Parametric Uncertainty Analysis," *Commun. in Comput. Phys.*, vol. 2, pp. 293-309, 2007.