Rapid Electromagnetic Analysis and Design using Flux Tubes

A. Stuikys\textsuperscript{1}, M. H. Mohammadi\textsuperscript{2}, D. A. Lowther\textsuperscript{2} Fellow, IEEE, and J. K. Sykulski\textsuperscript{1} Fellow, IEEE

\textsuperscript{1}Electronics and Computer Science, University of Southampton, Southampton, SO16 1BJ, UK, a.stuikys@soton.ac.uk
\textsuperscript{2}Department of Electrical and Computer Engineering, McGill University, Montreal, Canada

A reduced order numerical computational method based on flux tube modelling is proposed for the rapid electromagnetic analysis and design of electromechanical energy transducers using an example of a synchronous reluctance machine. The flux tube method is applied to establish flux linkage functions facilitating fast and accurate inductance estimation. The practical advantage is that the approach does not require precalculation of the air gap flux functions using computationally expensive methods such as finite elements. Initial results indicate that the method can predict, reliably and accurately, the flux distribution in the magnetic circuit, thus ultimately enabling efficient estimation of the inductance, and is suitable for rotational and translational synchronous reluctance machines.

\textit{Index Terms}—Flux tube model, electric machine optimization, synchronous reluctance machine, nonlinear magnetic circuits.

I. INTRODUCTION

FLUX TUBES, a reduced order numerical method, are well suited to the magnetic field analysis of switched reluctance (SR) motors \cite{1}, \cite{2}. The AC counterparts of SR machines are referred to as synchronous reluctance machines (synchRM), while both operate on the reluctance torque principle and share many features. In particular, both require direct and quadrature axis inductances, \(L_{ds}\) and \(L_{qs}\), to quantify the electromechanical energy conversion capacity and power factor, as given by \cite{3}

\[
\text{torque index} = L_{ds} - L_{qs}, \quad \text{saliency ratio} = \frac{L_{ds}}{L_{qs}}. \tag{1}
\]

An analytical estimation of the inductances is difficult due to magnetic nonlinearity. However, once the quantities are found the torque of the synchRM machine can be expressed as \cite{3}

\[
T_e = \frac{3}{2} \cdot \frac{n_p}{2} (L_{ds} - L_{qs}) I_{ds} \cdot I_{qs} \tag{2}
\]

where \(n_p\) is the number of rotor poles and \(I_{ds}\) denotes direct and quadrature phase currents. It follows that the larger the torque index and/or the saliency ratio, the higher the torque \(T_e\).

In this paper, the flux tube method is applied to the magnetic analysis of a synchRM for rapid design purposes.

II. SYNCHRONOUS RELUCTANCE MACHINE DESIGN PROBLEM

Details of the machine under consideration will be given in the full version. For illustrative purposes, Fig. 1 shows the initial finite element method (FEM) results of the flux lines and magnetic field density at the direct axis rotor position.

![Fig. 1. FEM analysis contour plots for the fractional slot synchRM.](image)

For the purpose of this study, all design variables are fixed except for the rotor carrier and rotor barrier widths, \(W_c\) and \(W_b\) respectively. The machine can be analysed using commercial software; however, exploring the synchRM response surface of average torque using (2) in a two-parameter design space would require numerous FEM solutions. One way of reducing computational effort is to use the magnetic equivalent circuit (MEC) based method \cite{4}, although the reported approach was still reliant on FEM simulation of the air gap flux distribution. We propose an alternative approach, based on flux tubes, which requires no precalculated air gap flux functions.

In order to investigate the effects of varying the two design parameters on the saliency ratio (1) and torque (2), a full factorial sampling was performed using FEM to compute the average torque for each design with fixed current magnitude; the corresponding response surface is shown in Fig. 2.

![Fig. 2. Average torque response surface of the 33-slot 4-pole synchRM.](image)

The response surface of Fig. 2 is constrained so that the two design variables of Fig. 1 are positive and do not intersect with other rotor poles. Each of the 90 sample points was evaluated thoroughly using the Maximum-Torque-Per-Ampere control strategy to find the optimal advance angle. Therefore, the response surface is three-dimensional, bound by the convex feasibility triangle. It transpires that the maximum value of the average torque is skewed towards a higher \(W_b\). Unfortunately, the FEM based full-factorial exploration of the response surface requires substantial computational resources to achieve desired accuracy in a reasonably short time period.
III. FLUX TUBE MODELLING APPROACH

The general application of the flux tube method to the magnetic analysis of saturable SR machines was reported in [1], [2]. In this study, we extend the treatment to the synchRM machine, also analysed independently using FEM. The objective is to obtain a synchRM which would deliver similar performance to a PM synchronous machine. This machine is far more difficult to analyse using analytical or MEC methodology due to its lack of symmetry, as it is a fractional slot machine (33 slots, 4 poles). The symmetrical synchRM geometry was analysed previously using MEC techniques [4] and is not repeated here.

Proper application of the flux tube model to a particular geometry relies on the ability to recognize the existing flux function patterns; the FEM based analysis (see Fig. 1) may be helpful in this respect. Once the representative machine design FEM analysis is completed, the flux tube paths may be subdivided using equipotential slices – these will approximately coincide with the magnetic field density contours. The approximate slice positions are indicated as white dashed lines in Fig. 1. Subsequently, the slices are used as sets of coordinate points between which the smooth and continuous cubic splines are fitted.

The flux tube modelling starts with transforming the rotary to translating domain of the machine geometry as shown in Fig. 3, where the red line is the same symmetry line as in the chosen quadrant in Fig. 1.

Using the flux tube model, the machine can be analysed by varying the two rotor geometric parameters, using the same full factorial sampling plan as in Fig. 2, so as to establish the highest inductance region of the direct rotor position for all possible variations of the two design parameters.

IV. FLUX TUBE MODELLING RESULTS

The design space of possible proportion combinations of the flux carrier and flux barrier widths of the rotor were explored using the flux tube model. The results indicate that the model can consistently generate the flux distribution within the machine magnetic circuit. For example, the flux function distribution for an alternative design is shown in Fig. 5.

The above comparisons reveal that the flux tube method can roughly, but consistently, locate the highest direct inductance region with a very small computational effort (time needed to create all flux tube plots for full factorial sampling was 210s compared with 8.2 hours using 4 parallel processes of FEM simulations). Even extreme designs can be accurately mapped using the flux tube method as shown by Fig. 5. However, the reported analysis is not yet complete as the quadrature rotor axis flux tube results are also needed to compute the average torque using (2) – those will be reported in the full version together with the discussion of the saliency ratio.

V. CONCLUSIONS

The flux tube method has been applied to a fractional slot synchronous reluctance machine. Preliminary results suggest that the flux tube approach is a feasible and efficient alternative to FEM for modelling the complex flux functions.

REFERENCES