Tournaments as a Response to Ambiguity Aversion in Incentive Contracts ☆

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Abstract

We study a principal-agent problem with multiple identical agents, where the action-dependent stochastic relationship between actions and output is perceived to be ambiguous, and agents are ambiguity averse. We argue that ambiguity, and particularly ambiguity aversion, make it more attractive for the principal to choose a tournament. If agents are risk neutral, but ambiguity averse, we show that the set of optimal incentive schemes contains a tournament. Moreover, if ambiguity is rich enough, all optimal incentive schemes must be such that realized output levels affect only the distribution of wages across agents and not the total wages paid out, as it is true for tournaments. When agents are both risk averse and ambiguity averse, tournaments need not be optimal, but ambiguity and ambiguity aversion still favor, in many cases, the use of tournaments or tournament-like schemes over e.g. incentive schemes that only depend on each agent's own output level.

Keywords: Ambiguity aversion, Incentive contracts, Tournaments

JEL: D81, D86, M52

1. Introduction

In principal-agent problems with multiple agents, a variety of incentive contracts can be observed. Sometimes the compensation of the agents is solely based on their own performance. Often, however, the relative performance of an agent in comparison to the other agents also determines compensation. One example of this are tournaments, in which the payment awarded to an agent depends only on the rank of her output contribution. In the context of wages in firms it

☆The author thanks Peter Klibanoff, Sandeep Baliga, Bard Harstad, Sujoy Mukerji, Matthias Lang, Sven Rady, Nicolas Roux, Marciano Siniscalchi and seminar participants at Northwestern University, the University of Bonn and the Institute for Advanced Studies in Vienna for valuable comments and discussions. This work extends a chapter of Kellner's Ph.D. dissertation submitted to Northwestern University in 2010.

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is often argued that wage determination can be understood as the result of a firm-internal labor-market tournament, where promotions (which are accompanied by a wage increase) constitute the prize for winning the tournament. Prendergast [1] even argues that at least for white-collar workers, tournaments seem to be the prevailing means of providing incentives in large firms.[1] Yet, as we will argue in more detail below, the reasons for the popularity of tournaments are not necessarily well-understood.

The question of finding the optimal incentive contract for multiple agents has so far been analyzed under the assumption that principal and agents maximize expected utility. In many situations, however, the agents may not have enough information about the stochastic process that links effort to performance to describe it with a single probability distribution. Hence, we depart from existing literature by treating the (effort-dependent) distribution over performance levels as subjectively uncertain. In line with recent literature (e.g. Ghirardato et al. [3], Klibanoff et al. [4]), we use the term ambiguity in reference to this uncertainty, and assume that agents are ambiguity averse.

Ambiguity and ambiguity aversion have important effects on the design of the optimal contract. We will show that tournaments do achieve first-best profits under ambiguity aversion and risk neutrality (while other types of incentive schemes usually fail to do so). Under risk aversion, we show that ambiguity aversion often ensures that the optimal contract is at least tournament-like, and makes it more likely that a tournament performs better than other commonly used incentive contracts.

More specifically, we analyze the problem of a principal who employs multiple agents. Each agent's performance is observable with noise, and contributes to the principal's payoff. Agents can improve their chance of performing well by exerting costly effort. The agent's choice of effort level cannot be observed by the principal. The principal needs to design a wage contract which conditions wages on the performance of the agents in the firm. Our baseline assumption is that there are identical agents who face the same tasks. We make this common assumption as it allows us to isolate most clearly the potential advantages of tournaments in the case of ambiguity aversion.

One motivation for taking ambiguity aversion into account is that in many contracting

[1] Note however, that the wage increase attributed to promotions could have other explanations besides being a reward for past performance to the winner of a tournament. For instance, Waldman [2] argues that promotions are accompanied by wage increases since they send positive signals about the worker's ability to other potential employers.

situations, it seems plausible to expect that the players perceive output distributions to be uncertain (so that ambiguity aversion could in fact make a difference). Certainly, a (noisy) performance measure representing the output of a worker cannot be considered to be objectively determined (like coin flips or lottery tickets), but is better understood as an example of subjective uncertainty. The subjective nature of distributions over performance levels may not matter equally in all situations. If the decision makers faces the same problem a large number of times in a stationary environment, the subjective performance measure will become free of uncertainty (about the distribution) over time (see Marinacci [5]). Thus, ambiguity aversion could explain why contracts differ between blue-collar workers (who typically work in a more stationary environment) and white-collar workers (who less often face routine tasks repeatedly). There is a lot of evidence, for instance obtained by experiments, that in fact many individuals violate the expected utility axioms in a way that is consistent with ambiguity aversion.[2]

We argue that if agents are risk neutral but ambiguity averse, tournaments can implement the first-best outcome, while other incentive contracts, including independent incentive schemes, typically do not. We establish that under a quite general assumption about the nature of ambiguity, all optimal contracts will have the property that sum of the two agents' utility does not change with the outcome levels, only the distribution of utility does.

The assumption that agents are risk neutral but ambiguity averse may appear very strong.[3] There are two directions in which our insights carry over to the case of risk aversion. First, observe that, as Magill & Quinzii [11] and Fleckinger [12] show, in many cases the optimal incentive contract is not tournament-like under ambiguity neutrality and risk aversion. We show that even if there is little ambiguity, but large ambiguity aversion, the optimal contract will remain tournament-like also in these situations, in the sense that an agent's wage will decrease in the performance of other agents. We also show that given infinite ambiguity aversion, irrespective of the degree of ambiguity, the optimal contract will still be a tournament for sufficiently small (but positive) degrees of risk aversion.

Second, we study and compare the profits resulting from tournaments with those from in-

[2]The classic thought experiment illustrating how ambiguity aversion is displayed has been described by Ellsberg [6]. Examples of actual implementations of this experiment are Fox & Tversky [7] and Halevy [8].

[3]However, the observation that risk aversion is decreasing may suggest that this assumption could be a good approximation for wealthy agents (if they remain ambiguity averse). Hence, our findings could apply to CEO compensation, for example. While we are not aware of any work that investigates how ambiguity aversion in fact changes with income, there is evidence that ambiguity aversion and risk aversion in general are not correlated significantly (Potamites & Zhang [9], Dimmok et al. [10]).

dependent wage schemes. As tournaments and independent wage schemes are the two most commonly considered wage schemes in a multiple-agent setting, it is interesting to compare their merits in the presence of ambiguity aversion even when neither of them is necessarily the optimal incentive scheme.[4] We show that in many cases, ambiguity and ambiguity aversion favor the use of tournaments. We compare the ambiguity neutral case to existing findings in the literature. We point out that the addition of an additive common shock to the output levels as in Green & Stokey [13] could be interpreted as increasing both ambiguity and riskiness at the same time. Moreover, we show that ambiguity aversion favors tournaments even in cases where ambiguity alone does not.

The paper is organized as follows. Section 2 discusses how we model ambiguity and ambiguity aversion. In Section 3 we analyze the case of binary outcomes. We describe first the optimal contract, dealing with the two cases of risk neutrality and risk aversion separately. Second, we compare tournaments with independent schemes. Section 4 discusses the case of multiple outcomes, focusing mostly on the optimal contract under risk neutrality. Section 5 investigates how robust our results are to the specifications of our model, such as our assumptions about the nature of ambiguity or the symmetry of the agents, and we compare our results with existing literature in Section 6. Section 7 draws conclusions.

2. The Model

We study the problem of providing incentives to multiple workers. The owner of a firm, the principal, employs more than one worker, the agents, to undertake a certain task. The effort of the agents determines, to a large extent, the payoff-relevant output of the firm, but it is also influenced by random events that are beyond the control of the workers. Inefficiencies may arise as the agents' effort cannot be observed (or otherwise inferred) by the principal. In the case of multiple agents the nature of the interactions between the agents deserves some attention. We focus on the case of two identical agents, who are working on exactly the same task, and any output that is produced can be clearly associated with the agent who produced it.[5] We rule out

[4] See Prendergast [1] for evidence on the use of these two kinds of contracts.

[5] Note that the assumption that agents are identical might apply particularly well in cases where choosing the high-cost action over the low-cost action stands for choosing a higher quality of inputs, which are equally available to both agents. This could be the case if the agent is in fact the head of an organizational unit. Tournaments between various kinds of organizational units can be observed widely. For instance university departments are often funded based on their ranking.

any considerations typically related to team work (synergies, free riding). When we introduce the model formally now, we seek to present it in a way that highlights the differences created by allowing for ambiguity and ambiguity aversion.

We will assume that there are only finitely many output levels that every worker might possibly achieve, which are elements of the finite set $Q = \{q_1, \ldots, q_I\}$.[6] As a convention we index the output levels so that $q_i < q_j$ if $i < j$. As the principal can reward each agent based on the output contribution achieved by the agent herself, as well as the output of the other worker, agent k's wage scheme can be represented by a function $w : Q_k \times Q_{-k} \to (\underline{w}, \infty)$. Here, $\underline{w}$ represents the minimum wage that the principal needs to pay to the workers (but we allow for the possibility that $\underline{w}$ is $-\infty$). As both workers are identical, we focus on the case of symmetric wage functions, where the labels of the two agents do not matter. Hence, if agent 1 produces output q_i and agent 2 produces q_j, the wage for agent 1 is given by $w(q_i, q_j)$ and agent 2's wage is given by $w(q_j, q_i)$. For simplicity, each of the two agents has only two actions available: The high action, H, represents high effort, and results in effort cost c^H to the agent. Choosing the low action, L, incurs lower effort costs, c^L, but will also, on average, result in a lower productivity level.

If we did not allow for ambiguity, we would conclude this discussion by stating that in addition to the effort cost, each action $a \in A = \{H, L\}$ is identified with a probability distribution $p^a \in \Delta(Q)$ over outcomes, which applies equally to both agents.

2.1. Preferences under ambiguity

To introduce ambiguity, we assume that both the principal and the agents have a common but imperfect understanding of the (stochastic) relationship between the actions and the outcomes. We represent this subjective uncertainty about the outcome distributions using the smooth ambiguity model [4]. Decision makers who satisfy the axioms imposed in this model appear to evaluate uncertain prospects in the following way: They are unable to assign a single probability distribution to describe the subjective likelihood of each outcome. Instead, a set of probabilities might better describe the implications of each action. However, they do not necessarily consider all members of that set equally likely, but assign different likelihoods to them, which are represented by a second-order belief. The utility function moreover reflects

[6] This is the same modeling approach as taken by Grossman & Hart [14] for the single-agent case and Mookherjee [15] for the case of multiple agents, while the seminal papers Holmstrom [16, 17] and Green & Stokey [13] allow for infinitely many outcomes.

their aversion against uncertainty about the probability distribution.

Formally, let f denote any act which specifies the decision maker's payoff for any possible contingency (i.e. f is a real valued function over a state space, S). In the smooth ambiguity model, preferences are represented by the function

$$U(f) = \int_{\Delta} \phi \left(\int_S u(f)dp \right) d\mu. \tag{1}$$

Here, p is a probability measure over S, Δ the set of all such probabilities. The second-order belief μ indicates the subjective likelihood that the agent attributes to each probability measure that she considers possible. The function u is a von-Neumann-Morgenstern utility function, representing risk attitude, and ϕ is a real-valued transformation which represents ambiguity attitude. In analogy to risk aversion, ambiguity aversion corresponds to a concave ϕ; if the function ϕ is linear the decision maker is ambiguity neutral, and thus an expected utility–maximizer.

To apply the smooth ambiguity model to the specific principal–agent problem we analyze, we assume that, in analogy to uncertainty about probabilities over an outcome space, agents perceive uncertainty about different (probabilistic) scenarios: A scenario is a pair of probability distributions over output levels, one for each action.

Let M denote the set of possible scenarios, so that if $(p^H, p^L) \in M \subseteq \Delta(Q) \times \Delta(Q)$, then p^H represents a possible output distribution for the high action, and p^L a possible output distribution for the low action. (We also denote by M^H and M^L the corresponding set of output distributions possible for H and, respectively, L.) The second-order belief $\mu \in \Delta(M)$ represents the likelihood over different scenarios.

Note that we assume that even if agents perceive the probabilistic scenario to be uncertain, they understand that the same scenario always applies for both agents. In doing so, we want to capture the idea that all ambiguity about the outcome distributions is solely about the likelihood of observing the output levels, and how effort changes them, but not about whether or not the two agents, and the problems they are facing, are in fact identical. This assumption might apply well for instance in a situation where workers or students are evaluated by a third party (a manager or a teacher), and there is uncertainty about how strict this person is, while there is little reason to believe that the evaluator is biased against certain individuals.

We assume that agent 1's preferences over wage schemes, dependent on both agents' action choices, can be represented by the utility function $U : A \times A \times (\underline{w}, \infty)^{I \times I} \to \mathbb{R}$ defined by

$$U(a, a', w) = \int_M \phi \left[\sum_{i,j=1}^{I} (p_i^a p_j^{a'})[u(w(q_i, q_j)) - c^a] \right] d\mu, \tag{2}$$

where $\phi(u) = -\exp[-\alpha u]$. (Agent 2's preferences are specified in a symmetric way.) Here $a \in A$ is the action chosen by agent 1, $a' \in A$ the action chosen by agent 2; $u : (\underline{w}, \infty) \to \mathbb{R}$ is an increasing real-valued and concave function, and $\alpha > 0$.[7] For a fixed pair of actions, these preferences fit into the smooth ambiguity model introduced in equation (1) as follows: The wage scheme w can be understood as an act over the possible set of states $Q \times Q$, which stands for all possible combinations of output levels.

Note that the term in the sum resembles the utility of an expected utility–maximizer who thinks that the *independent* probability distributions p^a and $p^{a'}$ (corresponding to one of the scenarios in M) determine the likelihood of any combination of output levels in $Q \times Q$. We make the usual assumption that effort costs contribute in an additive way to expected utility, and since u is weakly concave, agents are risk neutral or risk averse.

Ambiguity enters this utility function since the agent does not only evaluate the expected utility level that a wage scheme provides according to a single probability distribution (for each agent) over the output levels, but instead she computes various expected utility levels corresponding to a set of probability distributions.

When modeling ambiguity attitude, we chose the negative exponential form for ϕ for convenience.[8] We will replace it by a linear function to model the case of ambiguity neutrality, the limiting case as α approaches zero. Klibanoff et al. [4] classify such preferences as constant absolute ambiguity aversion (CAAA, in analogy to CARA risk aversion). Such preferences are also members of the class of variational preferences studied in Maccheroni et al. [19]. Increasing α corresponds to increasing the agent's degree of ambiguity aversion. We also allow for the case of infinite ambiguity aversion, which corresponds to the limit utility as α approaches infinity. Proposition 3 in Klibanoff et al. [4] shows that these preferences approach maxmin expected utility preferences (Gilboa & Schmeidler [20]) if α approaches infinity. In our context, decision

[7] As is standard, we assume that $\lim_{w \to \underline{w}} u(w) = -\infty$. This ensures the existence of an optimal contract, as shown in Grossman & Hart [14].

[8] As Kellner [18] argues, without this assumption it is possible that the individual rationality constraint is the binding constraint, as effort costs do not enter separably in U given equation (2) if ϕ is not of the CAAA form. Provided that the utility function U is specified in an alternative way such that effort costs enter separably, our results are robust to other forms of ϕ.

makers with infinite ambiguity aversion would evaluate every action according to the scenario in M that minimize the resulting expected utility.

Finally, we turn to the principal's payoff. Assuming the firm is both ambiguity neutral and risk neutral, the firm's payoff if actions a and a' are chosen under payment scheme w is given by the function $\Pi : A \times A \times (\underline{w}, \infty)^{I\times I} \to \mathbb{R}$ defined by

$$\Pi(a, a', w) = \int_M \left[\sum_{i,j=1}^{I} p_i^a p_j^{a'} (q_i + q_j - w(q_i, q_j) - w(q_j, q_i)) \right] d\mu$$

$$= \sum_{i,j=1}^{I} \bar{P}_{ij}(a, a')(q_i - w(q_i, q_j)) + \sum_{i,j=1}^{I} \bar{P}_{ij}(a, a')(q_j - w(q_j, q_i)). \tag{3}$$

Thus the principal maximizes expected profits, where the expectation is taken with respect to the distribution given by the action-dependent average distribution over outcome pairs, $\bar{P}_{ij}(a, a') \equiv \int_M p_i^a p_j^{a'} d\mu$. When looking for the optimal contract, we can restrict ourselves to deterministic wage schemes, since using a randomization device can never strictly improve the principal's profits due to the agents' risk aversion.[9]

2.2. The principal's problem

As the formulation of the principal's problem differs from, e.g., Mookherjee [15] only in the way the uncertainty is modeled, we will be brief. It is generally useful to think about the principal's problem in two stages: First, the principal tries to find the cheapest way to implement each possible combination of effort levels. Second, she compares the implementation costs to the resulting expected output, and decides to implement the effort level that offers the largest spread between costs and benefits.

To solve the first part of the problem, we look for a Nash equilibrium solution. An agent accepts a wage contract and carries out the effort level the principal intends him to do, if it meets two constraints: The individual rationality constraint states that the agent must be better off when she accepts the contract offered by the principal than with her outside option (which results in the utility level u_0), assuming that the other agent accepts the contract and chooses the intended effort level. The incentive constraint (IC) requires that the agent prefers choosing the action the principal intends to the other action, again assuming that the other agent does not deviate.[10]

[9] Replacing any random wage component by its certainty equivalent yields weakly lower implementation costs.
[10] Note that there may be multiple equilibria. We focus on the equilibrium that is best for the principal.

As we consider only incentive schemes that treat the two agents in a symmetric way, it suffices to consider costs and incentives for agent 1: they are the same for agent 2. The solution to the first stage (compare equation (3)) becomes:

$$C(a) \equiv \min_{w} \sum_{i,j=1}^{I} \bar{P}_{ij}(a,a) w(q_i, q_j), \tag{4}$$

subject to the two constraints

$$U(a,a,w) \geq u_0 \tag{IR}$$

$$U(a,a,w) \geq U(a',a,w) \text{ for } a' \neq a. \tag{IC}$$

The second stage is to choose $a \in \{H, L\}$ to maximize the principal's total profits

$$\Pi(a) = 2\left(\sum_{i=1}^{I} \bar{p}_i^a q_i - C(a)\right),$$

where $\bar{p}_i^a = \int p_i^a d\mu$, the $\mu-$average probability of observing output level i under action a.

To make the problem interesting, we assume that at least in a first best world it would be optimal to implement H for both agents. That is, $\mathrm{E}_{\bar{p}^H}[q] > \mathrm{E}_{\bar{p}^L}[q]$, and the difference in effort costs is sufficiently small. The problem of optimally implementing the high-cost action is the key part of our analysis, since the low-cost action is most cheaply implemented using a constant wage scheme.

It will turn out to be convenient to state the problem in terms of the utility levels attributed to each output combination. This is possible since for any wage level w there is a unique corresponding utility level $u(w)$. Thus we denote by $h(u)$ the inverse utility function that satisfies $h(u(w)) = w$. Consequently, h is increasing and convex. Thus, instead of choosing a wage scheme, the principal can equivalently choose a scheme of utility levels. This approach is common in the agency literature.

3. Main analysis: Two outcomes

To describe the set of optimal contracts, we introduce the following definition.

Definition 1. *A* tournament *is a wage scheme that increases iff the rank of the agent's output level increases, i.e.*

$$u_{ij} \equiv u(w(q_i, q_j)) = \begin{cases} u^W & \textit{if } i > j \\ u^T & \textit{if } i = j \\ u^L & \textit{if } i < j \end{cases}$$

and $u^W > u^T > u^L$.

Thus, for any tournament, the agent's compensation depends only on the rank of their output. Under risk neutrality these compensation levels can be directly interpreted as wage levels (since u can be assumed to equal the identity function, scaling costs accordingly). In this section, we consider the case of two output levels, success (S) and failure (F). This fits our framework if we set $S = 2$ and $F = 1$. Section 4 considers the case of I outcomes.

Throughout this section, to simplify the exposition, we assume that M is finite, there is $\epsilon > 0$ such that $\max_{p^H \in M^H} p_S^H = \bar{p}_S^H + \epsilon$, $\min_{p^H \in M^H} p_S^H = \bar{p}_S^H - \epsilon$ and $(\bar{p}_S^H, 1 - \bar{p}_S^H) \in M^H$, reflecting a minimal degree of symmetry about the ambiguity of the high action. Moreover, we assume $\bar{p}_S^H - \epsilon > \max_{p^L \in M^L} p_S^L$, which ensures that it is unambiguous that the high action achieves a higher success rate than the low action.[11]

3.1. Optimal contract under ambiguity aversion and risk neutrality

We can now describe the optimal contract for the case of risk neutrality:

Proposition 1. *Suppose agents are risk neutral but ambiguity averse, and $I = 2$. The set of optimal contracts includes a tournament, and implementation costs are at the first-best level. Moreover, if M^H contains at least three priors, all optimal schemes are tournaments.*

Proof. See below for an outline, and the appendix for a full proof. □

Thus, ambiguity aversion provides an argument why the principal would prefer a tournament over most other incentive schemes, including independent wage contracts, or even more complex contracts that combine features of tournaments with features of independent contracts. Note that no comparable agency model can provide such a justification for the use of tournaments under risk neutrality. Without taking ambiguity aversion into account, many kinds of incentive scheme are optimal under risk neutrality.

We now give a brief outline of the proof, to explain the intuition behind the above result. First, any incentive scheme that implements the high action for both agents has to satisfy the IR constraint, which requires that

$$\sum_{i,j=1}^{2} \bar{P}_{ij}(H,H) w(q_i, q_j) \geq \mathcal{A}^H + u_0 + c^H,$$

[11]Without the latter assumption incentive contracts could be designed which use a negative bonus, i.e. reward lower output level instead of the higher (compare e.g. [18]). Also, a tournament could be designed where the prize is awarded based on achieving the lower output level.

where
$\mathcal{A}^H = \frac{1}{\alpha}\log\left(\int_M \exp\left[-\alpha\left(\sum_{i,j=1}^I \left[p_i^H p_j^H - \bar{P}_{ij}(H,H)\right] w(q_i,q_j)\right)\right] d\mu\right).$

The principal's implementation costs correspond to the left-hand side of this inequality. The (nonnegative) term $\mathcal{A}^H$ can be interpreted as an ambiguity premium, which the principal has to pay to compensate the agent for the ambiguity of the wage scheme. Thus implementation costs are at least $u_0 + c^H$. Tournaments with random tie-breaking are purely risky, so that $\mathcal{A}^H$ vanishes. These schemes result in the same expected utility irrespective of which possible output distribution (in M) is considered. This property can be understood in a very intuitive way: If ties are broken at random, there is one prize which is awarded to one of the two agents. (Hence, we call tournaments with random tie-breaking one-prize tournaments in what follows.) Since these agents are identical, and the tournament treats them symmetrically, both agents must have the same winning probability of one half. (Instead of breaking ties at random, the principal may also choose a deterministic wage scheme that pays the certainty equivalent of the tie-breaking lottery if a tie occurs. For risk neutral agents, this means the prize is split in half.) As we verify in the appendix, other incentive schemes will not eliminate all ambiguity from the wage scheme, unless the agents consider only two priors to be possible.

Given two outcomes, a tournament that uses a large enough prize $u^W - u^L$ will always meet the incentive constraint. Deviating to the low action would reduce the (average) chances to receive this prize. The deviation may even introduce ambiguity about the prospective wages, which makes it even more unattractive to deviate. Since for any purely risky wage scheme, $\mathcal{A}^H = 0$, ambiguity aversion does not change the IR constraint for one-prize tournaments, so that no ambiguity premium has to be paid. Thus, the first best implementation costs, $u_0 + c^H$, can be achieved.

3.2. Optimal contract under ambiguity aversion and risk aversion

We now want to look at the effect of ambiguity aversion in the presence of risk aversion. Uncertainty about the probability distributions may matter even under ambiguity neutrality if this uncertainty has a common component (as we have assumed). In this case, the optimal contract is complex, and combines features of independent schemes and relative performance evaluation (see Mookerjee [15] and Holmstrom [17]). This raises the question whether the optimal contract will be tournament-like, in the sense that an agent's wage decreases if the performance of an other agent increases. As Fleckinger [12] and Magill & Quinzii [11] have argued, under ambiguity neutrality this may not be the case even if uncertainty about the true probability distribution affects both agents in the same way. The way the probability distributions of the two actions are believed to vary together still matters. If the high action does well in situations where the low action does poorly, the optimal contract will not be tournament-like. We are going to show that if ambiguity aversion is high enough, already a small amount of ambiguity will suffice to establish the optimality of tournament-like schemes also in these situations.

Before we discuss the effects of ambiguity aversion, we review the case of uncertainty about success rates under ambiguity neutrality. The following definition introduces a more general class of incentive schemes, which includes the set of all tournaments.

Definition 2. *An incentive scheme is* tournament-like *if both $u_{SF} \geq u_{SS}$ and $u_{FF} \geq u_{FS}$.*

Recall that, e.g., $u_{SF} = u_{21} = u(w(q_2, q_1))$. The following proposition characterizes the set of principal-agent problems (within the class of problems discussed in this paper) in which the optimal incentive contract is tournament-like for ambiguity neutral agents.[12] To state the result, denote the variance of the success rate of the high action by $\sigma_H^2 \equiv \mathrm{E}_\mu[(p_S^H - \bar{p}_S^H)^2]$ and the covariance between the success rate of the high and the low action by $\rho_{HL} \equiv \mathrm{E}_\mu[(p_S^H - \bar{p}_S^H)(p_S^L - \bar{p}_S^L)]$.

Proposition 2. *(Fleckinger[12]) Assume strict risk aversion, ambiguity neutrality, and $I = 2$. In the optimal incentive scheme, $u_{SF} \geq u_{SS}$ iff $\frac{\rho_{HL}}{\bar{p}^L} \geq \frac{\sigma_H^2}{\bar{p}^H}$ and $u_{FF} \geq u_{FS}$ iff $\frac{\rho_{HL}}{1-\bar{p}^L} \leq \frac{\sigma_H^2}{1-\bar{p}^H}$. Thus, the optimal incentive scheme is tournament-like iff both conditions hold.*[13]

For the case of perfect negative correlation between the two actions (where for all $(p_S^H, p_S^L) \in M$, $p_S^H - \bar{p}_S^H = -(p_S^L - \bar{p}_S^L)$ such that $\rho_{HL} = -\sigma_H^2$) the result is particularly intuitive since the optimal contract can be interpreted as rewarding an outcome the more, the stronger it signals that the agent has in fact chosen the equilibrium action. Under perfect negative correlation success of the other agent occurs more often under circumstances where effort makes a difference. Hence, if the other agent succeeded as well, success is a stronger signal of the high action and

[12] Apart from our limitation to two outcomes, this result differs from Magill & Quinzii [11] in that they give a condition that guarantees an incentive scheme to be tournament-like no matter which probabilities are attributed to the realization of the common shock. Fleckinger [12] considers a setting similar to ours. His setting is slightly more general since outcome distributions may differ between the two agents even if they choose the same action. Neither paper considers ambiguity aversion.

[13] This proposition is a special case of Proposition 3 in Fleckinger [12]. Note that this author uses the term RPE schemes (relative performance evaluation schemes) instead of tournament-like contracts.

effort should be rewarded more strongly in these situations. (That is, $u_{SS} - u_{FS}$ should be larger than $u_{SF} - u_{FF}$ so that u_{SS} will exceed u_{SF}.)

An example of a situation with substantial negative correlation between the actions could be that the (sales) agents' task consists of advertising a new feature of a well-established product to potential costumers. In such a situation, it could be uncertain weather there is a relevant number of potential costumers who care about this feature at all. If there is little interest, effort to explain the implementation will make little difference. If there is interest in the feature, it will however be important to convince potential costumers that it is actually implemented in a better way than in the product of the main competitors. In such a situation, the fact that another agent makes a sale is an indication that costumers do care about the feature, and hence success should be rewarded more strongly.

We will argue now that these results are not robust to ambiguity aversion. We will show first that, if ambiguity aversion is infinite, for sufficiently small levels of risk aversion the optimal contract is a tournament. Second, we show that if risk aversion is more substantial, already for small levels of ambiguity, under sufficiently high ambiguity aversion the optimal contract will be at least tournament-like.

Proposition 3. *Suppose $I = 2$. If risk aversion is sufficiently small, and ambiguity aversion is infinite, the optimal contract is a tournament.*

See the appendix for a proof. To understand the intuition, recall that the costs of implementing the high action consist of the expected payments, a risk premium, and an ambiguity premium. Given risk aversion, it is clear that both constraints bind, so we only consider such contracts. Then there is a unique contract, a tournament, that eliminates all ambiguity aversion, and hence the need for an ambiguity premium. Implementing a contract other than a tournament increases the ambiguity premium, while it could reduce the risk premium. Given infinite ambiguity aversion, the ambiguity premium increases linearly in any direction in a neighborhood around the tournament. The effect of the decrease in the risk premium vanishes as risk aversion gets small, so that eventually there will be a global minimum in implementation costs at this tournament.

Under finite levels of ambiguity aversion, the effect of introducing a small amount of ambiguity to an unambiguous contract vanishes at a first-order level, while any effects related to reducing the degree of riskiness from an already positive level persist. Nevertheless one expects that, still, as risk aversion becomes sufficiently small, the optimal contract is arbitrarily close to a tournament, and hence tournament-like.

We now investigate the case where risk aversion is not necessarily small, so that one would not expect the optimal contract to be close to a tournament. We will show that also in such cases, it will often be tournament-like under high ambiguity aversion, irrespective if it is so under ambiguity neutrality or not.

Proposition 4. *Suppose $I = 2$. For small enough degrees of ambiguity the optimal incentive scheme will be tournament-like if ambiguity aversion is high enough.*

A proof can be found in the appendix. Here, we will provide some intuition for this result for the limiting case of infinite ambiguity aversion (where preferences can be represented by the maxmin expected utility model).

For maxmin preferences, agent 1 obtains the following utility if both agents choose the high action:

$$U(H,H) \equiv \min_{(p^H,p^L)\in M} p_S^H \left(p_S^H u_{SS} + (1-p_S^H)u_{SF}\right) + (1-p_S^H)\left(p_S^H u_{FS} + (1-p_S^H)u_{FF}\right) - c^H. \tag{5}$$

Denote by $\tilde{p}_S^H$ the value of p_S^H that solves this minimization problem. If agent 1 chooses the low action instead, her utility becomes

$$U(L,H) \equiv \min_{(p^H,p^L)\in M} p_S^L \left(p_S^H u_{SS} + (1-p_S^H)u_{SF}\right) + (1-p_S^L)\left(p_S^H u_{FS} + (1-p_S^H)u_{FF}\right) - c^L. \tag{6}$$

Denote by $(\check{p}_S^H, \check{p}_S^L)$ the solution to this minimization problem.

Assume ambiguity is small. Then the optimal contract will still be close to an independent contract, which is optimal in the absence of ambiguity. In this case $\tilde{p}_S^H$ will correspond to the lowest possible probability level. To understand generally why tournament-like schemes are now attractive, we will now demonstrate that the principal benefits from adding a small tournament-like relative performance component in two different ways. We focus on the case where the agent in question has succeeded herself (u_{SS} and u_{SF}). The highly ambiguity averse agent will attribute a probability of $\tilde{p}_S^H$ to the state u_{SS}, while the principal applies the conditional probability of $\bar{p}_S^H + \sigma_H^2/\bar{p}_S^H > \tilde{p}_S^H$. Hence, the agent views u_{SS} effectively to be less likely than the principal, the opposite holds for u_{SF}. With respect to fulfilling the IR constraint, u_{SF} becomes more attractive for the principal over u_{SS} (to an extent limited by the agent's risk aversion). Second, if for the high action an increase in u_{SF} is exactly balanced by a corresponding decrease in u_{SS}, this leaves the utility from the low action unchanged or makes it worse.[14] The case of perfect negative correlation between the actions illustrates most clearly the second possible advantage of tournament-like schemes. For a contract that is not very close from an independent contract, a very ambiguity averse agent will fear that the success rate of the high action is low (for both agents). In the case of perfectly negative correlation, a deviation to the low action would mean that this agent fears only his own success rate to be below average, while the success rate of the other agent is now believed to be above average. This has a positive effect on the incentive constraint only for tournament-like schemes, as then, under the low action, ambiguity averse agents consider it less likely to get the bonus for outperforming the other agent.

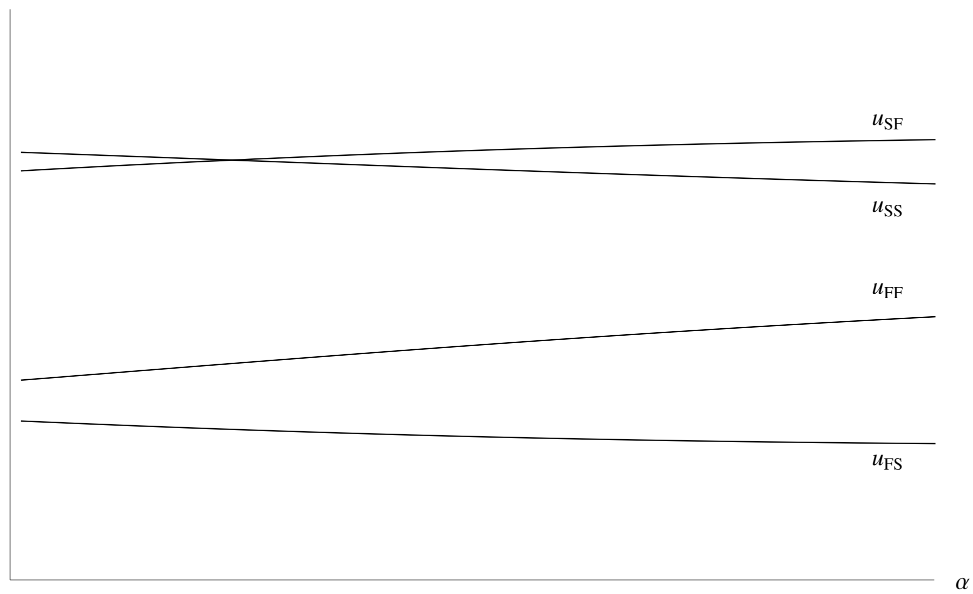
Note that one expects that, even if ambiguity is not small, and risk aversion is substantial, the worst-case success rate will typically be the lowest success rate, so that the optimal contract will still be tournament-like. Consider for reference the tournament where both constraints bind, where all success rates yield the same utility on equilibrium. While $u_{SF} - u_{SS} > 0$ and $u_{FF} - u_{FS} > 0$ (corresponding to relative performance incentives), $u_{FF} - u_{SS} = 0$ (which can be interpreted as independent incentives). Given the lowest success rate in M^H, the arguments above suggest that both kind of utility differences are suitable to provide incentives. In this case, it is well-known that tournaments perform poorly given strong risk aversion, as they neglect independent incentives entirely. However, introducing individual incentives in response to risk aversion makes one's own performance more relevant, and hence one expects the lowest prior to be the one which makes such a contract least favorable.

Figure 1 illustrates the proposition. It presents the optimal solution to a set of principal-agent problems which differ only by the agents' ambiguity aversion (obtained using numerical methods). In the figure, it is assumed that for all $(p^H, p^L) \in M$, $p^H - \bar{p}^H = -(p^L - \bar{p}^L)$. The optimal scheme is not tournament-like for ambiguity neutral agents: The payment of an agent who succeeds is higher if also the other agent succeeds ($u_{SS} > u_{SF}$). However, as ambiguity aversion increases, agent 1's compensation (measured in utility terms), given success, varies less whether agent 2 fails or not, up to some finite level of ambiguity aversion after which the optimal incentive scheme becomes tournament-like.

In conclusion, the results in [12] (summarized in proposition 2) establish circumstances that if ambiguity aversion is absent (or perhaps small, for fixed risk aversion), the optimal contract is not even tournament-like. If ambiguity aversion is low relative to risk aversion those results

[14] It becomes worse, if $\check{p}^H_S > \tilde{p}^H_S$ before the change, or if the change leads to a change in $\check{p}^H_S$.

Figure 1: Optimal incentive contract depending on ambiguity aversion



are robust to ambiguity aversion. If ambiguity aversion (but not necessarily ambiguity) is large relative to risk aversion, one expects however the optimal contract type to change (to a tournament-like contract) as ambiguity aversion is taken into account.

3.3. Effect of ambiguity and ambiguity aversion on popular contracts

As we argued in the introduction, it seems that in many applied contexts principals select either tournaments (implemented, for instance, via promotions) or independent contracts to provide incentives to the agents. As these schemes are not optimal in general, it might be the case that practical constraints prevent the principal from implementing the optimal contract, which may be quite complex. The principal might be forced to choose among sufficiently simple contracts, such as tournaments and independent schemes. Thus we want to investigate whether, keeping everything else equal, introducing ambiguity and ambiguity aversion makes it more likely that the principal prefers a tournament over independent schemes. To do so, we analyze how the profits of the principal change for tournaments and compare with the effects for independent incentive schemes.

We begin with the case of introducing ambiguity, assuming agents are ambiguity neutral. We will focus on one-prize tournaments (which have the constant-sum property). Such schemes arise naturally in a situation where the principal can only award a single indivisible prize (like a promotion).

Proposition 5. *Suppose $I = 2$. For ambiguity neutral agents, introducing mean-preserving ambiguity leaves profits that can be achieved using a one-prize tournament constant.*

Proof. It can be verified easily that a one-prize tournament results in an expected utility of $\frac{1}{2}u^W + \frac{1}{2}u^L$ on the equilibrium path. Given a deviation to L, for a scenario (p^H, p^L) the expected utility is $\frac{1}{2}u^W + \frac{1}{2}u^L - \frac{1}{2}(p_S^H - p_S^L)(u^W - u^L)$. Under expected utility, this becomes $\frac{1}{2}u^W + \frac{1}{2}u^L - \frac{1}{2}(\bar{p}_S^H - \bar{p}_S^L)(u^W - u^L)$, so that any (mean preserving) change in the degree of ambiguity is immaterial for both constraints. Also, expected wages $(h(u^W) + h(u^L))$ and output $(2\sum_i \bar{p}_i^H q_i)$ are unchanged for the principal. □

This result shows that for one-prize tournaments, ambiguity alone does not have any effect on implementation costs. The following proposition indicates that introducing ambiguity aversion will never decrease attainable profits when one-prize tournaments are used.

Proposition 6. *Suppose $I = 2$. Increasing ambiguity aversion can never decrease the profits that the principal can achieve if the best one-prize tournament is used.*

Proof. We will argue that the set of implementable one-prize tournaments cannot shrink, but may grow. (The result follows since ambiguity aversion changes nothing about implementation costs for a given tournament.) As we have argued in the context of Proposition 1, the agent's utility does not change with increases in ambiguity aversion when she chooses the high action, as one-prize tournaments are purely risky. Thus, the set of constant-sum tournaments which satisfy the IR constraint remains unchanged. If also the wages induced by the low action are unambiguous, the IC constraint remains unchanged. However, the wage scheme following a deviation may not be not purely risky. Then increases in ambiguity aversion make (only) the low action less attractive. In both cases, any one-prize tournament that satisfies both constraints under ambiguity neutrality still does so under ambiguity aversion, and hence the set of all such schemes which implement the high action pair cannot shrink. □

In short, one-prize tournaments eliminate all ambiguity about the agent's equilibrium wages. However, deviating to the low-cost action may still result in an ambiguous wage scheme. This might allow the principal to choose an incentive scheme that is less risky. Thus, for constant-sum tournaments, ambiguity aversion might increase the principal's profit, but it can never decrease it. This provides tournaments with an advantage over other schemes.

Note the combined effect of the last two propositions corresponds to the effect of introducing ambiguity for ambiguity averse agents. Hence, as far as one-prize tournaments are concerned, it is not in the interest of the principal to remove ambiguity even if this were possible in specific applications at a low cost (e.g. by providing evidence on the true success rates).

The effect of ambiguity and ambiguity aversion is quite different for independent schemes. For binary outcomes, independent schemes pay a base wage regardless of the performance and an additional bonus to the agent if she achieves the high outcome. Benefits from independent schemes will be the same as if there were only a single agent present. Mean-preserving ambiguity (without ambiguity aversion) can never have an effect on independent schemes. In the framework studied in Weinschenk [21], ambiguity aversion always makes the principal worse off. In general, it might be possible that ambiguity aversion makes the principal better off when she uses independent wage schemes. An intuitive example is the case where the high action is unambiguous, but the low action is not. Then, ambiguity aversion makes deviations less attractive, so a lower wage bonus will suffice to implement the high action.

To introduce a sufficient condition for profits to decrease under ambiguity aversion, we introduce a definition to compare the degree of ambiguity of two actions. To do so, denote by $\hat{\mu}^a$ the distribution of the difference between the success rate of action a and its average under μ, i.e. define $\forall \hat{p} \in [-\bar{p}^a_S, 1 - \bar{p}^a_S]$, $\hat{\mu}^a(\hat{p}) \equiv \mu(\bar{p}^a + (-\hat{p}, \hat{p}))$.

Definition 3. *Action a is at least as ambiguous as action a' if the distribution $\hat{\mu}^a$ is a mean preserving spread of $\hat{\mu}^{a'}$.*

Proposition 7. *Suppose $I = 2$. If the high action is at least as ambiguous as the low action, ambiguity aversion will reduce the profits that the principal can achieve using an independent scheme.*

See the appendix for a proof. To understand the result, consider the optimal contract in the absence of ambiguity aversion. First, for an ambiguity averse agent, it will fail to meet the individual rationality constraint, as an independent bonus contract exposes the agent to ambiguity on the equilibrium path. To correct this, the principal needs to pay an ambiguity premium, which reduces her profits. Second, if the high action is at least as ambiguous as the low action, ambiguity aversion (weakly) favors the low action relative to the high action, so that the principal needs to pay a (weakly) higher wage bonus in case of success to ensure the incentive constraint holds. In compensation for the resulting increased riskiness of the new contract, a higher risk premium must be paid, further lowering profits. If the low action however is more ambiguous, the second effect goes in the other direction. Provided that the high action is sufficiently close to being unambiguous, the first effect diminishes and the second effect will eventually dominate, so that profits may increase in this case.

In conclusion, ambiguity aversion increases profits for one-prize tournaments (at least weakly), while profits for independent schemes are strictly decreased if the high action is at least as ambiguous as the low action. In fact, if ambiguity aversion and ambiguity about the high action are sufficiently large, a tournament will always dominate all independent contracts.

Proposition 8. *Suppose $I = 2$. Fix the average outcome distributions $\bar{p}_S^H$ and $\bar{p}_S^L$. If ambiguity about the high action and ambiguity aversion both grow sufficiently large ($\min_{(1-p_S^H, p_S^H)\in M^H} p_S^H$ is small enough), the principal will strictly prefer some tournament over any independent bonus contract (or will choose a constant scheme to implement the low action).*

See the appendix for a formal proof. Note that the profits of a tournament are bounded below by the case where the low action is unambiguous. Increasing ambiguity or ambiguity aversion can never decrease the profits below this level. Regarding the bonus contract, if ambiguity aversion becomes large enough, the decision maker will essentially only be driven by the worst case scenario. As ambiguity increases, the agent will at one point attribute a success rate to the high action that is just marginally above (or even below) the one for the low action. This means that, if at all possible, to provide incentives the principal would need to pay an extremely high bonus. This would lead to extremely low profits if she insisted on implementing the high action with an independent bonus scheme.

In general, a tournament will eventually do strictly better than an independent contract in such situations. Under risk aversion, it might however be possible that the principal chooses to implement the low action using a constant contract, which is a degenerate version of both classes of incentive schemes. However, provided u is unbounded above, implementing H using a non-degenerate tournament will always be possible and better than implementing L if $q_S - q_F$ is sufficiently large. For the last argument, the assumption that u is unbounded is clearly stronger than necessary. It is only important that a possible bound on u is large enough so that a tournament can in fact provide incentives for H.

4. Extension: Multiple outcomes

We will now discuss how our results generalize to the case of more than two output levels. We begin with the optimal contract under risk neutrality and turn to the comparison of popular contracts in the next subsection.

4.1. Multiple output levels under risk neutrality

If agents are risk neutral, there are two main differences to the case of two outcomes. If there are more than two outcomes, it might sometimes be the case that no tournament in the narrow sense provides incentives for the high action. However, the principal can resort to a coarser type of tournament to provide incentives, which does not use all information about the agent's performance to rank them. Moreover, it will now be true that not only tournaments, but a larger class of incentive schemes can lead to first-best profits.

When there are more than two output levels, the principal might rank the agents using a coarser criterion than the set of all available output levels. For instance, if the revenue generated by each agent can be measured in dollars and cents, the principal could rank agents only based on the dollar-amount. We use the term coarse tournament to describe such tournaments.

Definition 4. *A* coarse tournament *is a tournament that ranks agents according to a connected partition of Q, i.e. there exists a weakly increasing function $f : \{1, \dots I\} \to \{1, \dots I\}$ such that*

$$u_{ij} \equiv u(w(q_i, q_j)) = \begin{cases} u^W & \text{if } f(i) > f(j) \\ u^T & \text{if } f(i) = f(j) \\ u^L & \text{if } f(i) < f(j) \end{cases}$$

and $u^W > u^T > u^L$.

Next, we define a property that the optimal incentive contract will satisfy under risk neutrality (except in a limited set of circumstances):

Definition 5. *An incentive scheme is* constant-sum *if total compensation does not vary with output ($u_{ij} + u_{ji} = u_{i'j'} + u_{j'i'}$ for all $i, i', j, j' \in \{1, \dots I\}$).*

Under risk neutrality, this implies that for constant-sum schemes total wages do not change with the realized combination of output levels, only the division of wages among the agents may change. For the natural class of one-prize tournaments, and their deterministic equivalents, the constant-sum property is satisfied.

However, there are also schemes other than tournaments who do so. One notable example are wage schemes that are linear in the performance difference: $w(q_i, q_j) = \alpha + \beta(q_i - q_j)$. As in a tournament, the agents are paid based on their relative performance only, absolute levels do not matter. Another example is the class of k-contracts introduced in Hvide [22]. While a tournament rewards agents based on there performance rank, such contracts rank the agents based on the difference of their performance to some benchmark level k. (Not all values of k would, however, provide incentives to exert high effort.)

Proposition 9. *Suppose agents are risk neutral but ambiguity averse.*

(a) The set of optimal contracts includes a coarse tournament, and profits are at the first-best level.

(b) Schemes with the constant-sum property eliminate all ambiguity from equilibrium wages. If M^H contains less than $I(I+1)/2$ priors, there are also wage contracts that eliminate all ambiguity from the equilibrium path which do not have the constant sum-property.

(c) Fix the average probability distribution $\bar{p}^H$ and $\bar{p}^L$. One can always find a corresponding set of $I(I+1)/2$ beliefs M^H such that only schemes with the constant-sum property achieve first-best profits.

See the appendix for a proof. We focus now on the differences to the case of two outcomes. Part a) of the proposition establishes that coarse tournaments can always implement the first-best. As is well-known, even in the absence of ambiguity, in some specific situations tournaments fail to provide incentives to choose the high action. The first best can always be achieved with a coarse tournament that partitions output levels in a "high" and a "low" group. The resulting coarse tournament parallels the case of $I = 2$.

Parts b) and c) discuss which type of schemes other than constant-sum schemes can implement the first best. More specifically, point b) states that if only few beliefs are considered possible, also other schemes are purely risky, and hence may result in first-best profits. Point c) identifies the minimum number of (suitable) beliefs sufficient to ensure that only constant schemes induce the first best.

To understand why also other schemes might now achieve the first best, observe that according to μ, it might be possible, for instance, that a (non-trivial) event is unambiguous. That is, all probability distributions agree on the likelihood to observe a particular set of outcomes. Then, even an independent wage scheme may achieve the first best. To provide incentives, the principal could pay a bonus whenever this event occurs, and a base wage otherwise. Such situations require that agents would consider either very few or a very specific set of outcome distributions. In the appendix we provide an example that illustrates that the number of $I(I+2)/2$ (suitable) priors is always enough to ensure that only constant-sum schemes can implement the first-best.

4.2. Multiple output levels under risk aversion

We now investigate whether, given risk aversion, tournaments become more attractive given ambiguity and ambiguity aversion in comparison to the best independent contract also if there are more than two outcomes. Given ambiguity, introducing ambiguity aversion continues to have weakly positive effects on profits given one-prize tournaments are used, since those continue to reduce all ambiguity on the equilibrium path. Hence, Proposition 6 generalizes to the case of multiple outcomes. However, it is easy to provide examples where, even for ambiguity neutral agents, introducing ambiguity alone now has an effect on tournaments, which could go in either direction. In many situations however, there is no such effect, so that the combined introduction of ambiguity and ambiguity aversion would not decrease profits if one-prize tournaments are used. This includes situations where uncertainty about the distribution of the high action is perfectly (positively or negatively) correlated with uncertainty about the distribution of the low action, or if there is no correlation between the two distributions.

In principle, Proposition 7 can also be generalized to show that also given many outcomes, ambiguity often has a negative effect profits under independent contracts. However, it is less obvious how to compare the degree of ambiguity attached to different actions when there are more than two outcomes. See Jewitt & Mukerji[23] and Kellner [18] for possible orders on the degree of ambiguity that would allow a generalization of Proposition 7.

5. Robustness

To show to what extent our results depend on our assumptions, we focus mostly on the question as to when the principal can still design a tournament that provides incentives and removes all ambiguity about wages.

As we have already discussed, we assumed that there are only two agents mainly for notational convenience. If there are many agents, it is still possible to design a purely risky wage scheme. If all agents exert the high effort, the principal could still break ties in a way that all agents have the same probability of winning the tournament, irrespective of the actual probability distribution. The same is true if there are more than two actions available to the agent. As long as all agents choose the same action, a one-prize tournament will be unambiguous on equilibrium.

In the remainder of this section we discuss to what extent our results remain valid in situations where there is a difference in abilities between the two agents. First, note that if we model changes in abilities as changes in the effort costs, our results will not be altered in important ways. It will still be possible to design an ambiguity-free tournament. Of course, a symmetric wage scheme will not be optimal in this case, as the agent with higher effort costs needs to be compensated more to choose the high action. The situation is different if the two agents differ in the actions that they have available, that is if the high action of agent 1 results in different output distributions than the high action of agent 2.

Even then, a tournament may result in an ambiguity-free wage scheme. We focus on the case where the skill difference is perceived not to be ambiguous. That is, the probability distribution over output levels for the high-ability agent differs from its average always to the same extent as the probability distribution of the low-ability worker (under the high action). Consider a one-prize tournament (where ties are broken at random). We begin with the case of two outcomes. If we reinterpret $\bar{p}^L$ as the success rate attributed to the agent with the *low* ability when she chooses the *high* action, the winning probability of the low skilled agent becomes $P(\text{prize}|\text{low ability}) = .5 - .5(\bar{p}_S^H - \bar{p}_S^L)$ for all possible success rates (while it is the complementary probability for the high-skilled agent). Hence, equilibrium wages are purely risky. Thus, in this case, tournaments remain a good response to ambiguity, at least if risk aversion is sufficiently small.

The situation becomes slightly more complicated if there are more than three outcomes. In this case, it can be easily verified that tournaments can result in ambiguous equilibrium wages. However, there is still a (binary) coarse tournament that induces unambiguous equilibrium wages. Thus, if ambiguity aversion is an important concern relative to risk aversion, a coarse tournament could be used to address differences in skills, leading the principal to ignore even more information she has about the performance of the agents. This increases even further the contrast to the Informativeness Principle, which guides contract design under risk aversion.

Our results however may change quite considerably, if differences in abilities are perceived to be ambiguous. Any incentive scheme that uses relative performance measures is likely to be vulnerable to such kinds of ambiguity. Similarly, if the agents do not face exactly the same task, or if they do not operate in the same environment, the agent may consider it to be ambiguous how their own outcome distribution compares to their competitors'. But even in these cases, tournaments can still do better than independent schemes, as independent schemes are exposed to ambiguity as well. Whether this will be the case in a given situation depends on whether ambiguity about aspects of the output distributions that apply to both agents is large in comparison to ambiguity about the difference between the two agents.

6. Related literature

The positive properties of tournaments have been documented in several papers, starting with the seminal work of Lazar & Rosen [24]. Not many papers contribute to answering the question: Why are tournaments, not other schemes, used to provide incentives? Lazar & Rosen [24] show that tournaments achieve the first best solution if risk-aversion is absent. But in this case, many other incentive schemes, for instance linear piece rates, achieve this goal as well. An advantage of tournaments could be that sometimes only the rank of the agents' contribution can be observed, but not the level. This argument is often considered unsatisfactory (e.g. Prendergast [1]). Tournaments, for instance implemented via promotions, also seem to prevail in situations where a measure of individual performance is available.

Mookherjee [15] and Holmstrom [17] consider more general incentive contracts based on relative performance. They show that the optimal incentive scheme in general is based on more information than just the performance-rank of the agents. Even if the optimal incentive scheme is more complex, it could still share one of the main characteristics of a tournament: Wages increase in the agent's own performance but decrease with the performance of other agents. Magill & Quinzii [11] and Fleckinger [12] show that in an expected utility context even this may not hold. One could indeed conclude that from an agency point of view, support for the use of tournaments or even tournament-like schemes is actually not very strong (see Prendergast [1]).

Our Section 3.1 suggests that in situations where risk aversion plays a limited role in comparison to ambiguity aversion, the use of tournaments can be justified also from the perspective of an agency model. Also, our Section 3.2 illustrates that even if the agents' risk aversion matters, the optimal contract under ambiguity aversion is more often at least tournament-like.

Green and Stokey [13] have argued that tournaments do at least better than independent contracts if uncertainty about the output levels contains a sufficiently important common shock (to output levels). They argue that tournaments are a way to filter common shocks, but this benefit comes at the cost of adding some of the other agents' idiosyncratic noise to the compensation scheme of any given individual.

Such a common shock could be interpreted as adding ambiguity about the mean of the outcome distribution, and in this sense, also Green & Stokey [13] suggest that ambiguity favors tournaments. However, in their model the common shock does not leave the average output distribution unchanged, but instead makes it more risky. In short, one can interpret [13] as showing that a common shock to the output *levels* makes tournaments relatively more attractive under ambiguity neutrality. We show that a common shock to output *distributions* (preserving the average distribution) often has no effect under ambiguity neutrality, while it favors tournaments if agents are ambiguity averse.

For the single-agent case, the effect of ambiguity aversion on incentive contracts has been discussed in the literature (Mukerji [25], Weinschenk [21], Lopomo [26]). It is often argued that ambiguity aversion can favor the use of simpler contracts in this context. An important difference is that in the single-agent case, even under risk-neutrality, the first best cannot be achieved under ambiguity aversion, while tournaments may allow this in a world with multiple agents.

Our results rely crucially on the idea that agents would indeed perceive tournaments as unambiguous. The results in Halevy [8], claiming that ambiguity averse agents also often do not reduce compound lotteries, cast some doubts on the empirical validity of this idea. In contrast, Kellner & Riener [27] provides preliminary experimental evidence that tournaments are indeed attractive under ambiguity.

7. Conclusions

We have argued that in many cases ambiguity aversion provides a strong justification why a principal might set up a tournament as a way to provide incentives. In the existing literature, which stays within the expected-utility framework, tournaments are either suboptimal (under risk aversion), or many possible incentive schemes are optimal (under risk neutrality). Under ambiguity aversion however, if agents are risk neutral, tournaments are always optimal, and many other schemes, like independent schemes, are not. Even if agents are risk averse, ambiguity aversion will often make it relatively more attractive for the principal to choose certain tournaments over independent contracts, or to choose incentive schemes that are at least tournament-like.

We conclude that, if agents are in fact ambiguity averse, one would expect that a principal chooses tournaments or schemes that are very similar to tournaments if the distribution of output levels is very ambiguous. In applications, this might more likely be true for white-collar workers, as opposed to blue-collar workers who routinely face the same task. Indeed, as we have demonstrated in the introduction, it has often been argued that tournament-like incentives (for instance via promotions) seem to play a more important role for white-collar workers.

8. Appendix

We begin with stating a short lemma.

Lemma 1. *The utility function $\tilde{U}(a, a', w)$, given below, represents the same preferences as the utility function $U(a, a', w)$.*

$$\begin{aligned} \tilde{U}(a, a', w) = \sum_{i,j=1}^{I} \bar{P}_{ij}(a, a') u(w(q_i, q_j)) - c^a - \\ \frac{1}{\alpha} \log \left[\int_M e^{\left[-\alpha \left(\sum_{i \in Q, j \in Q} [p_i^a p_j^{a'} - \bar{P}_{ij}(a,a')] u(w(q_i,q_j)) \right) \right]} d\mu \right]. \end{aligned} \tag{7}$$

Proof. Immediate since $\tilde{U}(a, a', w) = -\frac{1}{\alpha} \log(-U(a, a', w))$. □

Also, let μ^a denote the marginal distribution over possible probabilities for a given action $a \in \{H, L\}$ alone, so that M^a becomes the support of μ^a.

Proof of Proposition 1

Step 1 (Describing ambiguity-free wage schemes.). Using the utility function introduced in Lemma 1, the individual rationality constraint becomes

$$\sum_{i,j=1}^{I} \bar{P}_{ij}(H, H) u_{ij} \geq \mathcal{A}^H + u_0 + c^H,$$

where $\mathcal{A}^H = \frac{1}{\alpha} \log \left(\int \exp \left[-\alpha \left(\sum_{i,j=1}^{I} [p_i^H p_j^H - \bar{P}_{ij}(H, H')] u_{ij} \right) \right] d\mu \right)$.

The left hand side equals the principal's implementation costs (under risk neutrality, u can be assumed to be the identity function). Since the expectation (over μ) of $\sum_{i,j=1}^{I} [p_i^H p_j^H - \bar{P}_{ij}(H, H)] u_{ij} = 0$, and $\exp(0) = 1$, according to Jensen's inequality the integral evaluates to a number larger than 1, so that $\mathcal{A}^H \geq 0$.

Thus implementation costs are at least $u_0 + c^H$, and they reach this lower bound whenever $\mathcal{A}^H = 0$. We focus on the case where M is finite. Due to Jensen's inequality, $\mathcal{A}^H = 0$ iff $\sum_{i,j=1}^{I} [p_i^H p_j^H - \bar{P}_{ij}(H, H')] u_{ij} = 0$. As $\bar{P}_{ij}(H, H))$ does not vary between elements in M^H, the condition becomes that

$$\sum_{i,j \in Q} p_i^H p_j^H u_{ij} = \text{constant over } M^H. \tag{8}$$

Step 2 (Characterizing ambiguity-free wage schemes.). If there are only two outcomes, the previous requirement is equivalent to the condition that the expression

$$u_{FF} + p_S^H (u_{SF} - u_{FF}) - p_S^H (u_{FF} - u_{FS}) - (p_S^H)^2 [(u_{SF} - u_{SS}) - (u_{FF} - u_{FS})]$$

does not vary between members of M^H. If p^H and p'^H are two such members, it is required that

$$u_{SF} - u_{FF} + u_{FS} - u_{FF} = (p_S^H + p_S'^H)[(u_{SF} - u_{SS}) - (u_{FF} - u_{FS})].$$

If those are the only two members, this condition can be satisfied by multiple incentive contracts. When M^H has at least three elements, this can only be fulfilled if $u_{SF} - u_{FF} = u_{FF} - u_{FS} = u_{SF} - u_{SS}$, or equivalently $u_{SS} = u_{FF} = (u_{SF} + u_{FS})/2$. This condition describes one-prize tournaments (respectively their deterministic equivalents).

Step 3 (A certain one-prize tournament satisfies the IC). Now, we show that a certain one-prize tournament always provides the necessary incentives, and moreover results in first-best profits for the principal.

Now suppose w^W denotes the wages paid to the winner of this coarse tournament, w^L denotes the wages paid for losing under this scheme, and assume agent 2 chooses the high action. Then if agent 1 chooses the high action as well, her utility becomes $w^L + \frac{1}{2}(w^W - w^L) - c^H$. This follows from the fact that on the equilibrium path, the probability of winning is 1/2 and hence the wage scheme is purely risky. If she defects to the low action, wages may become ambiguous. In this case the agent's utility is equal to

$$\tilde{U}(L, H, w) = w^W + (1/2 - (\bar{p}_S^H - \bar{p}_S^L))(w^W - w^L) - \mathcal{A}^L - c^L$$

where $\mathcal{A}^L = \frac{1}{a} \log \left(\int_M e^{-\alpha((p_S^L - \bar{p}_S^L) - (p_S^H - \bar{p}_S^H))(w^W - w^L)} d\mu \right)$. This expression can be derived from simplifying equation (7) for the case of $I = 2$.

Comparing the utility under the two actions, the IC constraint becomes

$$\frac{1}{2}(\bar{p}_S^H - \bar{p}_S^L)(w^W - w^L) \geq c^H - c^L - \mathcal{A}^L.$$

Since A^L increases (weakly) in $(w^W - w^L)$, the right-hand side of the IC decreases in $(w^W - w^L)$. The left-hand side increases in a linear way with $(w^W - w^L)$. Therefore, for some large enough level of $(w^W - w^L)$ the IC constraint holds with equality.

The IR constraint becomes

$$w^L + \frac{1}{2}(w^W - w^L) \geq u^0 + c^H,$$

which can be easily satisfied by choosing w^L accordingly. Hence, a coarse tournament can always be designed to meet both constraints, and thus to implement the high action. Since under risk neutrality, our assumption that $\lim_{w \to \underline{w}} u(w) = -\infty$ corresponds to assuming $\underline{w} = -\infty$, it is clear that w^L can be chosen in a way where the IR constraint binds, which implies that the principal's expected profits are at the first-best level.

Proof of Proposition 3

Consider a family of utility functions u^r, where risk aversion is continuously parametrized by a parameter r, such that if $r' < r$, $u^{r'}$ is less risk averse than u^r, and if $r = 0$, $\forall w, u^{0''}(w) = 0$. As usual, let h_r be the inverse of u^r.

We now assume that agents display infinite ambiguity aversion, the highest level of aversion we allow. Proposition 3 in [4] establishes that such preferences can be represented by the maxmin model of ambiguity aversion, where the set of beliefs remain those implied by the set M. We will now argue that if r is sufficiently small, the optimal solution to the principal's problem is in fact a tournament.

Given risk aversion, it is clear from [18] that in an optimal contract both constraints must bind. Hence, it suffices to consider such incentive contracts. Let $\Delta u^S \equiv u_{SF} - u_{SS}$,$\Delta u^F \equiv u_{FS} - u_{FF}$ and

$$U(H, H, p) \equiv p(u_{SS} + (1-p)\Delta u^S) + (1-p)(u_{FF} - p\Delta u^F),$$

$$U(H, H, p^H, p^L) \equiv p^L(u_{SS} + (1-p^H)\Delta u^S) + (1-p^L)(u_{FF} - p^H\Delta u^F).$$

The principal's implementation costs for the high action under contracts with binding constrains correspond to the function $C_r(\Delta u^S, \Delta u^F)$ given by:

$$\begin{aligned} C_r(\Delta u^S, \Delta u^F) =& \bar{p}^H \left(h_r(u_{SS}) + (1 - \bar{p}^H - \frac{\sigma_H^2}{\bar{p}_S^H})(h_r(u_{SS} + \Delta u^S) - h_r(u_{SS}) \right) \\ &+ (1-\bar{p}^H) \left(h_r(u_{FF}) - (\bar{p}^H - \frac{\sigma_H^2}{1-\bar{p}_S^H})(h_r(u_{FF}) - h_r(u_{FF} - \Delta u^F)) \right) \\ &\text{s.t.} \min_{p^H \in M^H} U(H, H, p_S^H) = u_0 + c^H \\ &\text{and} \min_{(p^H, p^L) \in M} U(L, H, p_S^H, p_S^L) = u_0 + c^L. \end{aligned} \tag{9}$$

(A closed form solution is available but omitted.) Also, denote by $C_r(\Delta u^S, \Delta u^F, p_S^H, p_S^L)$ the case where the minimization over probabilities is removed from the *second* constraint. Observe that the constraints do not depend on r, only the objective function does. To determine the values of this function for various contracts, consider first C_0, the limit case of risk neutrality. Let

$$\begin{aligned} \tilde{C}(\Delta u^S, \Delta u^F, \tilde{p}_S^H, \check{p}_S^L, \check{p}_S^H) \equiv h_0'(0) \Bigg[& \bar{p}_S^H \left(u_{SS} + (1 - \bar{p}^H - \frac{\sigma_H^2}{\bar{p}_S^H})\Delta u^S \right) + \\ & (1-\bar{p}^H) \left(u_{FF} - (\bar{p}^H - \frac{\sigma_H^2}{1-\bar{p}_S^H})\Delta u^F \right) \Bigg] + h(0), \end{aligned}$$

where u_{FF} and u_{SS} are chosen to satisfy $U(H, H, \tilde{p}_S^H) = u_0 + c^H$ and $U(L, H, \check{p}_S^H, \check{p}_S^L) = u_0 + c^L$.

It is easy to see that $C_0(\Delta u^S, \Delta u^F)=\tilde{C}(\Delta u^S, \Delta u^F, \tilde{p}_S^H, \check{p}_S^L, \check{p}_S^H)$ iff $\tilde{p}_S^H$ is a maximizer of $C(\Delta u^S, \Delta u^F, \tilde{p}_S^H, \check{p}_S^L, \check{p}_S^H)$. Note that if $\overline{\Delta u^S}$ and $\overline{\Delta u^F}$ correspond to the tournament defined by $\check{p}_S^H$ and $\check{p}_S^L$, then, since $\tilde{C}$ is affine in its first two arguments,

$\tilde{C}(\overline{\Delta u^S}+ds, \overline{\Delta u^F}+df, \tilde{p}_S^H, \check{p}_S^L, \check{p}_S^H) = u_0 + c^H + ds\frac{\partial \tilde{C}(0,0,\tilde{p}_S^H,\check{p}_S^L,\check{p}_S^H)}{\partial \Delta u^S} + df\,\frac{\partial \tilde{C}(0,0,\tilde{p}_S^H,\check{p}_S^L,\check{p}_S^H)}{\partial \Delta u^F}$.

Omitted algebra reveals that

$$ds\frac{\partial \tilde{C}(0,0,\bar{p}_S^H,\check{p}_S^L,\check{p}_S^H)}{\partial \Delta u^S} + df\frac{\partial \tilde{C}(0,0,\bar{p}_S^H,\check{p}_S^L,\check{p}_S^H)}{\partial \Delta u^F} > 0 \text{ iff } ds < df.$$

Moreover,

$$ds\frac{\partial \tilde{C}(0,0,\bar{p}_S^H+\epsilon,\check{p}_S^L,\check{p}_S^H)}{\partial \Delta u^S} + df\frac{\partial \tilde{C}(0,0,\bar{p}_S^H+\epsilon,\check{p}_S^L,\check{p}_S^H)}{\partial \Delta u^F} > 0 \text{ iff}$$

$$ds > \left(1 - \frac{\epsilon(\check{p}_S^H - \check{p}_S^L)}{\bar{p}_S^H(\epsilon\bar{p}_S^H - 2\check{p}_S^L\epsilon + \epsilon^2 - \sigma_H^2) - \check{p}_S^L((\check{p}_S^H - \epsilon)\epsilon + \sigma_H^2) - \epsilon\sigma_H^2}\right) df,$$

and

$$ds\frac{\partial \tilde{C}(0,0,\bar{p}_S^H-\epsilon,\check{p}_S^L,\check{p}_S^H)}{\partial \Delta u^S} + df\frac{\partial \tilde{C}(0,0,\bar{p}_S^H-\epsilon,\check{p}_S^L,\check{p}_S^H)}{\partial \Delta u^F} > 0 \text{ iff}$$

$$ds < \left(1 - \frac{\epsilon(\check{p}_S^H - \check{p}_S^L)}{\bar{p}_S^H(\epsilon\bar{p}_S^H - 2\check{p}_S^L\epsilon - \epsilon^2 + \sigma_H^2) - \check{p}_S^L((\check{p}_S^H + \epsilon)\epsilon - \sigma_H^2) - \epsilon\sigma_H^2}\right) df.$$

Note that

$$1 - \frac{\epsilon(\check{p}_S^H - \check{p}_S^L)}{\bar{p}_S^H(\epsilon\bar{p}_S^H - 2\check{p}_S^L\epsilon - \epsilon^2 + \sigma_H^2) - \check{p}_S^L((\check{p}_S^H + \epsilon)\epsilon - \sigma_H^2) - \epsilon\sigma_H^2} < 0$$

and

$$1 - \frac{\epsilon(\check{p}_S^H - \check{p}_S^L)}{\bar{p}_S^H(\epsilon\bar{p}_S^H - 2\check{p}_S^L\epsilon - \epsilon^2 + \sigma_H^2) - \check{p}_S^L((\check{p}_S^H + \epsilon)\epsilon - \sigma_H^2) - \epsilon\sigma_H^2} < 0$$

and the former is larger than the latter if $2(\epsilon^2 - \sigma_H^2)(\bar{p}^H - \check{p}_S^L) > 0$, which is the case since $\epsilon^2 > \sigma_H^2$ given our assumptions.

This means that if the relevant combination of $(\check{p}_S^L, \check{p}_S^H)$ was known, one can find a minimal $\delta(\check{p}_S^L, \check{p}_S^H)$ such that for any (ds, df) of length k the implementation costs increase by at least $k\delta(\check{p}_S^L, \check{p}_S^H)$. Now consider the case where $r > 0$ but small. It is clear that we can restrict our search for optimal contracts to a compact set of candidate contracts: For $(\Delta u^S, \Delta u^F)$ to be optimal given off equilibrium probabilities $(\check{p}_S^L, \check{p}_S^H)$, if $(\overline{\Delta u^S}, \overline{\Delta u^F})$ represents the corresponding tournament contract, it is necessary that $C_r(\Delta u^S, \check{p}_S^L, \check{p}_S^H) \leq C_r(\overline{\Delta u^S}, \overline{\Delta u^F}, \check{p}_S^L, \check{p}_S^H)$, or equivalently

$$\begin{aligned}&C_0(\Delta u^S, \Delta u^F, \check{p}_S^L, \check{p}_S^H) - C_0(\overline{\Delta u^S}, \overline{\Delta u^F}, \check{p}_S^L, \check{p}_S^H) \leq \\ &C_r(\overline{\Delta u^S}, \overline{\Delta u^F}, \check{p}_S^L, \check{p}_S^H) - C_0(\overline{\Delta u^S}, \overline{\Delta u^F}, \check{p}_S^L, \check{p}_S^H) - \big(C_r(\Delta u^S, \Delta u^F, \check{p}_S^L, \check{p}_S^H) - C_0(\Delta u^S, \Delta u^F), \check{p}_S^L, \check{p}_S^H\big).\end{aligned} \tag{10}$$

The right hand side of the preceding inequality is bounded above by $C_r(\overline{\Delta u^S}, \overline{\Delta u^F}, \check{p}_S^L, \check{p}_S^H) - C_0(\overline{\Delta u^S}, \overline{\Delta u^F}, \check{p}_S^L, \check{p}_S^H)$, while the left hand side, as shown above, increases at least proportionally with the distance to the tournament. Taking the union over the respective off-equilibrium probabilities results the set of candidate contracts. Also, the set can be made arbitrarily small, as $C_r(\overline{\Delta u^S}, \overline{\Delta u^F}, \check{p}_S^L, \check{p}_S^H) - C_0(\overline{\Delta u^S}, \overline{\Delta u^F}, \check{p}_S^L, \check{p}_S^H)$ approaches 0 as r approaches 0. Provided the optimal contract is arbitrarily close to a tournament, it is now straightforward to determine which are the relevant off-equilibrium beliefs, and hence the relevant values of $\overline{\Delta u^S}, \overline{\Delta u^F}$. Taking the minimum of $\delta(\check{p}_S^L, \check{p}_S^H)$ over all remaining values of $(\check{p}_S^L, \check{p}_S^H)$ one can indeed conclude that there exists a $\delta > 0$ such that $C_0(\overline{\Delta u^S} + ds, \overline{\Delta u^F} + df) > u_0 + c^H + k\delta$.

Given this compact set of candidate contracts, and comparing the derivative of (9), with the corresponding derivative for $r = 0$, for r small enough, any directional derivative around $C_r(\overline{\Delta u^S}, \overline{\Delta u^F})$ of length k will be arbitrarily close to $k\delta > 0$, as the differences in h' for the for different reward levels will be small enough. Hence, the optimal contract will be a tournament.

Proof of Proposition 4

We will first specify how we parametrize the degree of ambiguity. Consider first an arbitrary set of action-dependent beliefs M, with a second-order belief μ. Define $M_\delta = \{(\bar{p}^H + \delta(p^H - \bar{p}^H), \bar{p}^L + \delta(p^L - \bar{p}^L)) : (p^H, p^L) \in M\}$ and μ_δ such that $\mu_\delta(\bar{p}^H + \delta(p^H - \bar{p}^H), \bar{p}^L + \delta(p^L - \bar{p}^L) = \mu(p^H, p^L)$. It is clear that reducing δ reduces the degree of ambiguity involved in the problem, but leaves the $\mu-$average success probabilities $\bar{p}^H$ and $\bar{p}^L$ constant. Moreover, since this change reduces both σ_H^2 and ρ_{HL} by to a factor of δ^2 of it original value, under ambiguity neutrality, a contract is tournament-like for $\delta = 1$ iff it is so for any other (positive) value of delta. As we study changes in uncertainty about the two success rates (corresponding to the high and the low action), this definition allows for both effects on ambiguity averse and ambiguity neutral decision makers (via the implied correlation). This is in contrast to the definition in Jewitt and Mukerji [23].

Again, we now assume that agents display infinite ambiguity aversion, where preferences can be represented by the maxmin model of ambiguity aversion. The set of beliefs corresponds to M_δ.

The principal's problem (given some M_δ and μ_δ) is then to minimize

$$\sum_{i,j\in\{S,F\}} \bar{P}_{ij}(H,H)h(u_{ij}) \text{ s.t. } \sum_{i,j\in\{S,F\}} \tilde{p}_i^H \tilde{p}_j^H u_{ij} \geq \bar{u} + c^H \quad \text{and} \quad \sum_{i,j\in\{S,F\}} \tilde{p}_i^H \tilde{p}_j^H u_{ij} - c^H \geq \sum_{i,j\in\{S,F\}} \check{p}_i^L \check{p}_j^H u_{ij} - c^L. \tag{11}$$

where

$$\tilde{p}^H = \underset{p^H \in M^H_\delta}{\arg\min} \sum_{i,j\in\{S,F\}} p_i^H p_j^H u_{ij}$$

and

$$(\check{p}^H, \check{p}^L) = \underset{(p^H,p^L)\in M_\delta}{\arg\min} \sum_{i,j\in\{S,F\}} p_i^L p_j^H u_{ij}.$$

Note that also $\bar{P}_{ij}(H,H)$ depends in a continuous way on δ. In analogy to the proof of proposition 1 in [14] one can artificially bound the constraint set due to the agent's risk aversion. Therefore, Berge's maximum theorem implies that the solution to this problem is upper hemi-continuous. Hence, we can find an $\delta > 0$ such that the solution to the principal's problem is arbitrarily close to the solution to the the principal's problem for the case where $\delta = 0$, which is an independent contract. In what follows, we will therefore omit the dependence on δ, but assume δ is indeed as small as needed.

The Lagrangian for the principal's minimization problem is given by:

$$\mathcal{L} = \sum_{i,j\in\{S,F\}} \bar{P}_{ij}(H,H)h(u_{ij}) - \lambda \sum_{i,j\in\{S,F\}} \tilde{p}_i^H \tilde{p}_j^H u_{ij} - \mu \sum_{i,j\in\{S,F\}} \left(\tilde{p}_i^H \tilde{p}_j^H u_{ij} - \check{p}_i^L \check{p}_j^H u_{ij}\right)$$

where

$$\tilde{p}_S^H = \underset{p^H\in M^H}{\arg\min}\, p_S^H(p_S^H u_{SS} + (1-p_S^H)u_{SF}) + (1-p_S^H)(p_S^H u_{FS} + (1-p_S^H)u_{FF}) \tag{12}$$

and

$$(\check{p}_S^H, \check{p}_S^L) = \underset{(p^H,p^L)\in M}{\arg\min}\; p_S^L(p_S^H u_{SS} + (1-p_S^H)u_{SF}) + (1-p_S^L)(p^H u_{FS} + (1-p_S^H)u_{FF}) \tag{13}$$

Since in the optimal independent scheme, $u_{SS} = u_{SF} > u_{FS} = u_{FF}$, in any contract that is sufficiently close to the optimal independent contract, it is true that $\tilde{p}_S^H = \min_{p^H\in M^H} p_S^H$. By similar arguments, $\check{p}_S^L = \min_{p^L\in M^L} p_S^L$ for all contracts that are sufficiently close to the optimal independent contract. The value of $\check{p}^H$ is harder to determine, but also of less importance.

At those points where the Lagrangian is differentiable, the first-order derivatives become:

$$\begin{aligned}
\partial\mathcal{L}(u_{SS},u_{SF},u_{FS},u_{FF},\lambda,\mu)/\partial u_{SS} &= ((\bar{p}_S^H)^2 + \sigma_H^2)h'(u_{SS}) - \left((\lambda+\mu)(\tilde{p}_S^H)^2 - \mu\check{p}_S^L\check{p}_S^H\right)\\
\partial\mathcal{L}/\partial u_{SF} &= ((1-\bar{p}_S^H)\bar{p}_S^H - \sigma_H^2)h'(u_{SF}) - \left((\lambda+\mu)\tilde{p}_S^H(1-\tilde{p}_S^H) - \mu\check{p}_S^L(1-\check{p}_S^H)\right)\\
\partial\mathcal{L}/\partial u_{FS} &= \left(\bar{p}_S^H(1-\bar{p}_S^H) - \sigma_H^2\right)h'(u_{FS}) - \left((\lambda+\mu)\tilde{p}_S^H(1-\tilde{p}_S^H) - \mu\check{p}_S^H(1-\check{p}_S^L)\right)\\
\partial\mathcal{L}/\partial u_{FF} &= ((1-\bar{p}_S^H)^2 + \sigma_H^2)h'(u_{FF}) - \left((\lambda+\mu)(1-\tilde{p}_S^H)^2 - \mu(1-\check{p}_S^H)(1-\check{p}_S^L)\right).
\end{aligned} \tag{14}$$

At points where the Lagrangian is not differentiable, the right hand sides of the above formulae still corresponds to directional derivatives, given the right choice of $\check{p}_S^H$. (Close to an independent contract, $\check{p}_S^H$ is the only potential source of non-differentiability. In many cases, also $\check{p}_S^H$ stays unchanged close to any independent contract, so that the Lagrangian is in fact differentiable. This is the case e.g. if p_S^H and p_S^L are perfectly positively or negatively correlated.)

Suppose first that $u_{SS} > u_{SF}$. Consider the vector $v_S \equiv (-\frac{1-\tilde{p}_S^H}{\tilde{p}_S^H}, 1, 0, 0, 0)$. The corresponding directional derivative can be computed to equal

$$\begin{aligned}\nabla_{v_S}\mathcal{L} = &-\frac{\bar{p}^H + \frac{\sigma_H^2}{\bar{p}_S^H}}{\tilde{p}_S^H}(1-\tilde{p}_S^H)\bar{p}_S^H h'(u_{SS}) - \mu\frac{\check{p}_S^H}{\tilde{p}_S^H}\check{p}_S^L(1-\tilde{p}_S^H) \\ &+ \left(1-\bar{p}_S^H - \frac{\sigma_H^2}{\bar{p}_S^H}\right)\bar{p}^H h'(u_{SF}) + \mu\check{p}_S^L(1-\check{p}_S^H).\end{aligned} \tag{15}$$

To see that this directional derivative is indeed negative for all $u_{SS} \geq u_{SF}$ consider first the terms multiplying μ. Since $\check{p}_S^H/\tilde{p}_S^H \geq 1$ and $1-\tilde{p}_S^H \geq 1-\check{p}_S^H$, these terms sum to at most zero. Since $\frac{\bar{p}_S^H+\sigma_H^2/\bar{p}_S^H}{\tilde{p}_S^H} > 1$, $(1-\tilde{p}_S^H) > (1-\bar{p}_S^H - \sigma_H^2/\bar{p}_S^H)$, and h' is increasing, for all $u_{SS} \geq u_{SF}$ the remaining terms sum to a strictly negative number. Hence, the principal's cost decreases in this direction, so that $u_{SS} \geq u_{SF}$ cannot be optimal. For $v_F = (0, 0, -\frac{1-\tilde{p}_S^H}{\tilde{p}_S^H}, 1, 0, 0)$, the directional derivative is given by

$$\begin{aligned}\nabla_{v_F}\mathcal{L} = &-\frac{\bar{p}_S^H - \frac{\sigma_H^2}{1-\bar{p}_S^H}}{\tilde{p}_S^H}(1-\tilde{p}_S^H)(1-\bar{p}_S^H)h'(u_{FS}) - \mu(1-\check{p}_S^L)\frac{\check{p}_S^H}{\tilde{p}_S^H}(1-\tilde{p}_S^H) \\ &+ \left(1-\bar{p}_S^H + \frac{\sigma_H^2}{1-\bar{p}_S^H}\right)(1-\bar{p}_S^H)h'(u_{FF}) + \mu(1-\check{p}_S^L)(1-\check{p}_S^H).\end{aligned} \tag{16}$$

By the same arguments as above, the terms multiplying μ sum up to a non-positive number. For $u_{FS} \geq u_{FF}$ the remaining terms add to a strictly negative number, since $\frac{\bar{p}_S^H - \sigma_H^2/(1-\bar{p}_S^H)}{\tilde{p}_S^H} > 1$ while $(1-\tilde{p}_S^H) > (1-\bar{p}_S^H + \sigma_H^2/(1-\bar{p}_S^H))$. Hence, a situation with $u_{FS} \geq u_{FF}$ does not minimize implementation costs.

Proof of Proposition 7

An independent bonus contract is a contract where $u_{12} = u_{11} \equiv u_F$, $u_{22} = u_{21} \equiv u_S$, the individual rationality constraint becomes:

$$u_F + \bar{p}_S^H(u_S - u_F) \geq u_0 + c^H + \mathcal{A}^H$$

where for $a \in H, L$,

$$\mathcal{A}^a = \frac{1}{\alpha}\log\left(\int_M \exp\left[-\alpha\hat{p}_S^H(u_S - u_F)\right]d\hat{\mu}^a\right).$$

The IC constraint becomes

$$(\bar{p}_S^H - \bar{p}_S^L)(u_S - u_F) \geq c^H - c^L + \mathcal{A}^H - \mathcal{A}^L.$$

In the absence of ambiguity aversion, $\mathcal{A}^H = \mathcal{A}^L = 0$. Since $\exp^{-\alpha u}$ is convex in u for $\alpha > 0$ and $\hat{\mu}^H$ is a mean-preserving spread of $\hat{\mu}^L$, under ambiguity aversion $\mathcal{A}^H \geq \mathcal{A}^L \geq 0$. Hence, both constraints tighten weakly, which means that the principal's profits can never increase.

Proof of Proposition 8

We show that, while implementation costs for tournaments stay bounded, they tend to infinity for independent contracts as ambiguity and ambiguity aversion grow sufficiently large. Consider first an independent bonus contract. Consider the extreme case of infinite ambiguity aversion, the limiting case of the CAAA specification of the smooth ambiguity model as α approaches infinity. Recall that such preferences can be represented by the maxmin model of ambiguity aversion[4]. Assume ambiguity is high enough so that $\min_{(1-p_S^H, p_S^H) \in M^H} p_S^H \leq \epsilon$.

A bonus contract needs to satisfy the IC constraint:

$$\min_{(1-p_S^H, p_S^H) \in M^H} u_F + p_S^H(u_S - u_F) - c^H \geq \min_{(1-p_S^L, p_S^L) \in M^L} u_F + p_S^L(u_S - u_F) - c^L.$$

Since the minimization problem on the left hand side is solved by $(1-\epsilon, \epsilon)$, it becomes necessary that $\epsilon(u_S - u_F) \geq c^H - c^L$. It is intuitive that as ϵ gets very small, the principal needs to pay a very high bonus, which leads to large costs to provide incentives for H. To see this formally note that the IR constraint binds in every optimal bonus contract,

$$u_F = u_0 + c^H - \epsilon(u_S - u_F).$$

The implementation costs for the principal satisfy

$$h(u_F) + p_S^H(h(u_S) - h(u_F)) > h(u_F + p_S^H(u_S - u_F)) \geq h\left(u_0 + c^H + p_S^H \frac{c^H - c^L}{\epsilon} - (c^H - c^L)\right).$$

Since h is convex, the right hand tends to infinity as ϵ decreases, i.e. for a sufficiently small ϵ implementation costs for a bonus contract get arbitrarily large. It might however be possible that no bonus contract meets the constraints (if e.g. $\epsilon = 0$, if the low action is not too ambiguous, or if u is bounded so that the domain of h is too small.)

Now consider a tournament. Regardless of the degree of ambiguity aversion, the IR constraint is: $u^L + \frac{1}{2}(u^W - u^L) \geq u_0 + c^H$. The IC is always satisfied if $\frac{1}{2}(\bar{p}_S^H - \bar{p}_S^L)(u^W - u^L) = c^H - c^L$. Hence, a tournament where $(u^W - u^L)$ is determined in this way, while u^L is determined to satisfy the IR provides an upper bound on the principal's implementation costs. Thus, given infinite ambiguity aversion, if ambiguity gets high, implementation costs will become unbounded for bonus contracts (or it becomes impossible to use them), while the costs for the best tournament stay bounded. It may however be the case that the principal prefers to implement L for both agents, if she finds a constant contract better than all tournaments (and hence independent contracts). Provided u is unbounded, this will not be the case if $q_S - q_F$ is large enough.

Proof of Proposition 9

Step 1 (Describing ambiguity free wage schemes.). As in the case of two outcomes, a wage scheme eliminates all ambiguity iff

$$\sum_{i,j=1}^{I} p_i^H p_j^H u_{ij} = \text{constant over } M^H. \tag{17}$$

Written in matrix-form, where p^H is interpreted as a row vector and the matrix $\mathcal{U}$ is defined by $\mathcal{U} = (u_{ij})_{i\in I, j\in I}$, it is required that $p^H \mathcal{U} (p^H)^T$ is a constant for all $p \in M^H$. This holds iff $p^H(\mathcal{U} + \mathcal{U}^T)(p^H)^T$ is constant.

Step 2 (Constant-sum schemes eliminate all ambiguity.). We normalize u_{11} to equal zero, since if equation (17) is satisfied for scheme, it is satisfied by any other scheme that differs by a constant. Then, if a scheme satisfies the constant-sum property, $u_{ij} + u_{ji} = u_{11} + u_{11} = 0$ for all $i \in Q$ and $j \in Q$. Thus, a scheme is constant-sum iff $U + U^T = 0$ in this case, so that indeed equation (17) holds.

Step 3 (If there are less then I(I+1)/2 priors in M^H, there exists a non-constant-sum contract that also eliminates all ambiguity about wages.). Fix any prior $\check{p} \in M^H$. Assume (w.l.o.g) that $\mathcal{U}$ is a diagonal matrix, and $u_{11} = 0$. Condition (17) is satisfied if $\sum_{i,j=1}^{I} p_i^H p_j^H u_{ij} = \sum_{i,j=1}^{I} \check{p}_i^H \check{p}_j^H u_{ij}$ for all $p^H \in M^H \setminus \{\check{p}^H\}$. Hence, this condition can be interpreted as a (linear homogeneous) system of $|M^H| - 1$ equations in $I(I+1)/2 - 1$ unknowns. If $|M^H|$-1 is less than $I(I+1)/2 - 1$, there must be a solution other than the trivial solution $\mathcal{U} = 0$. Any scheme where $\mathcal{U} \neq 0$ (or $\mathcal{U} + \mathcal{U}^T \neq 0$) does not have the constant sum property.

Step 4 (If there are I(I+1)/2 suitably chosen priors, only constant-sum schemes eliminate ambiguity.). We now describe, for any fixed average probability distribution $\bar{p}^H >> 0$, an example of a set M^H of exactly $I(I-1)/2$ priors which is sufficiently diverse to ensure that $\mathcal{A}^H = 0$ only for constant-sum schemes. Suppose first that $\bar{p}^H \in M^H$. Let e_i be an I-dimensional vector that equals 1 for coordinate i and 0 otherwise. Suppose that M^H consists of at least the following elements. For every pair (i, j) such that $1 \leq j < i$, M^H contains an element p^{ij} such that, for some sufficiently small ε, $p^{ij} \equiv \bar{p}^H + \varepsilon e_i - \varepsilon e_j$. Also suppose that M^H contains, for $i \geq 2$, $\tilde{p}^i = \bar{p}^H - \gamma_i e_i + \gamma_i e_1$. It is easy to check that some $\gamma_i > 0$ can be chosen such that indeed $\bar{p}^H$ remains the average probability distribution. For this set of priors, we will now show that $\mathcal{A}^H = 0$ only for constant sum schemes. Now let $\mathfrak{U}(p) \equiv p\mathcal{U}p^T$. Suppose $\mathcal{A}^H = 0$. Then $\mathfrak{U}(p)$ is constant over all elements in M^H. We now show that this implies three properties of such contracts.

Property 1. Since $\mathfrak{U}(\bar{p}^H) - \mathfrak{U}(p^{1i}) = \mathfrak{U}(\bar{p}^H) - \mathfrak{U}(\tilde{p}^i) = 0$ for all $i \geq 2$, $(\mathfrak{U}(\bar{p}^H) - \mathfrak{U}(p^{1i})/\epsilon) - (\mathfrak{U}(\bar{p}^H) - \mathfrak{U}(\tilde{p}^i)/\gamma_i) = 0$. Since omitted calculations reveal that $(\mathfrak{U}(\bar{p}^H) - \mathfrak{U}(p^{1i})/\epsilon) - (\mathfrak{U}(\bar{p}^H) - \mathfrak{U}(\tilde{p}^i)/\gamma_i) = -(\gamma_i + \varepsilon)(u_{ii} - (u_{1i} + u_{i1}))$, it must be the case that $u_{ii} = (u_{1i} + u_{i1})$ for all $i \in I$.

Property 2. Now, for arbitrary $1 < j < i$, one can show that $(\mathfrak{U}(p^{ij}) - \mathfrak{U}(\bar{p}^H)) - (\mathfrak{U}(p^{1i}) - \mathfrak{U}(\bar{p}^H)) + (\mathfrak{U}(p^{1j}) - \mathfrak{U}(\bar{p}^H)) = \epsilon\,(u_{ij} + u_{ji} - (u_{1j} + u_{j1}) - (u_{1i} + u_{i1}))$. Hence $u_{ij} + u_{ji} = (u_{1j} + u_{j1}) + (u_{1i} + u_{i1})$ for all $1 < j < i$.

Property 3. Using properties 1 and 2 one can show that, $\mathfrak{U}(p^{1i}) - \mathfrak{U}(\bar{p}^H) = \epsilon(u_{1i} + u_{i1})$ so that that $u_{i1} + u_{1i} = 0$ for arbitrary i. Together, these three properties imply that $u_{ij} + u_{ji} = 0$ for all combinations of $i, j \in Q$. Hence, for the set of priors described above, if $\mathcal{A}^H = 0$, the contract must be a constant-sum scheme.

Thus, if M^H contains e.g. these beliefs, only constant-sum schemes can implement the first best.

Step 5 (Coarse constant sum schemes satisfy IC). Now, we show that a certain coarse constant-sum tournament always provides incentives to choose the high action.

Since we assume that H is better for the principal, $\sum_i \bar{p}_i^H q_i > \sum_i \bar{p}_i^L q_i$. Thus, there must exist a $k > 1$ such that $\sum_{i \geq k} \bar{p}_i^H > \sum_{i \geq k} \bar{p}_i^L$.Otherwise $\bar{p}^L$ would first-order stochastically dominate $\bar{p}^H$, so that the principal would prefer L. Now suppose that the principal designs a coarse tournament (with random tie-braking) that considers only whether or not each agents' outputs exceed the output level q_k (i.e the function f, used in the definition of a coarse tournament, is given by $f(i) = 2$ for $i \geq k$ and 1 otherwise). Arguments parallel to the case of two outcomes establish that a high enough prize provides incentives for H, and since no ambiguity premium needs to be paid for constant-sum schemes, the first best can still be achieved.

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