Waverider Design Based on 3D Leading Edge Shapes

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Nomenclature

ICFA	=	Internal Conical Flow A
α	=	osculating plane angle to the vertical axis
β	=	effective shockwave angle
γ	=	angle between shockwave profile and upper surface profile curve
θ	=	inclination of the leading edge with respect to the horizontal plane
λ	=	effective sweep angle, relative to the freestream direction

I. Introduction

Inverse waverider design has traditionally relied on first generating or selecting a supersonic shock containing flowfield. The geometry is then 'carved' out of that flowfield by designing the leading edge on the shock surface and tracing the streamlines downstream of the shock to generate the 'waveriding' lower surface of the geometry. Essentially all inverse design methods, from the simplest planar shock based designs initially proposed by Nonweiler[1], to ones based on arbitrary shock generating bodies or the osculating cones method[2] and its extensions[3],[4], have followed this approach. The design approach presented here differs in that the three dimensional shape of the leading edge does not have to be designed on a predefined shockwave shape. Instead, the method can explore the potential of building a waverider out of any leading edge shape; although there are limitations and any arbitrary 3D curve will not necessarily be able to provide a waverider shape.

The proposed method calculates the appropriate shape of the shockwave to fit a given 3D leading edge. Due to the shape of the shockwave being unknown and not a design-driving parameter, the approach followed in the osculating cones method and its extensions has to be utilized since they allow arbitrary shaping of the shockwave shape based on simple and easy to calculate axisymmetric flowfields. The proposed method can also be viewed as a

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further extension, or a different way to parameterize the osculating cones and similar (osculating axisymmetry and osculating flowfield[3],[4]) geometry generation methods, which does not require designing a shockwave profile.

II. Method description

The starting point of the proposed design method is a 3D curve that defines the leading edge of the waverider geometry. The forward end of the curve is usually at the symmetry plane and the aft end defines where the base plane of the design method lies, as shown from different perspectives in Figure 1. In order to construct the appropriate shockwave shape we utilize the geometrical relationships for osculating cones and osculating flowfield waveriders as used by Rodi[5]. These relate the effective leading edge sweep, the effective angle of the shockwave, and the angle between the leading edge curve and shockwave profile from a base plane perspective, shown in Figure 2. The effective leading edge sweep, λ, is the sweep angle of the leading edge relative to the freestream (usually different from the sweep angle viewed from a planform perspective due to dihedral/anhedral). It is calculated as:

$$\lambda = 90 - acos\left(\frac{\vec{T} \cdot \overrightarrow{\boldsymbol{u}_{inf}}}{\left|\overrightarrow{\boldsymbol{u}_{inf}}\right|}\right),$$

With $\vec{T}$ the leading edge unit tangent vector and $\overrightarrow{\boldsymbol{u}_{inf}}$ the freestream velocity vector.

The effective angle of the shockwave, β, is equal to the shock angle for the osculating cones method, but has to be calculated when the shockwave is curved and not of equal strength along the length of the geometry for the osculating flowfield and osculating axisymmetry methods. This involves the calculation of the angle of the ray between the foremost and aft-most points of the curved shock. The angle and shape of the shock, depending on the method followed, is chosen in the beginning along with the flow conditions. Given the effective leading edge sweep, the effective shock angle and utilizing equation 1 we can calculate the angle γ, between the leading edge projection on the base plane and the shockwave. We can then calculate the angle of the osculating planes to the vertical axis, α, using equation 2.

$$\sin(\gamma) = \tan(\lambda)\tan(\beta)$$

$$\gamma = \operatorname{asin}(tan(\lambda)\, tan(\beta)) \qquad (1)$$

$$\alpha = \gamma - \theta \qquad (2)$$

Given the angles, α, for all osculating planes along the leading edge we only need to calculate the point where the shockwave is on each osculating plane. The distance between the shockwave profile and the leading edge's projection on the base plane can be calculated given the effective shock angle, β, and the local length of the geometry on each osculating plane, which is fully defined by the three-dimensional leading edge. That distance is equal to:

$$D_{local} = L_{local}\tan(\beta) \qquad (3)$$

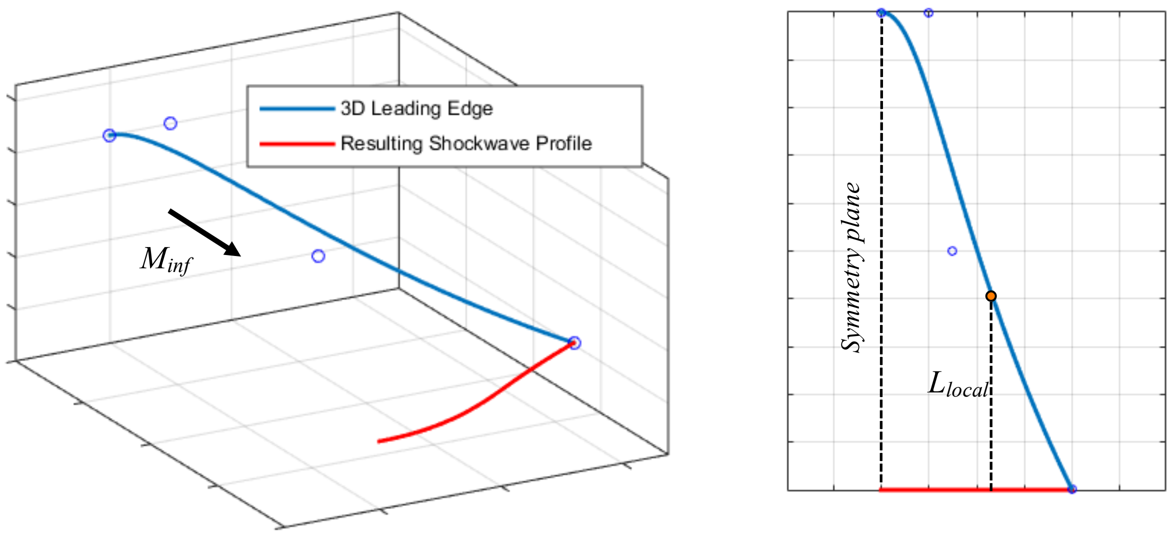


Figure 1. Perspective and planform view of the 3D leading edge curve and its resulting shockwave profile on the base plane.

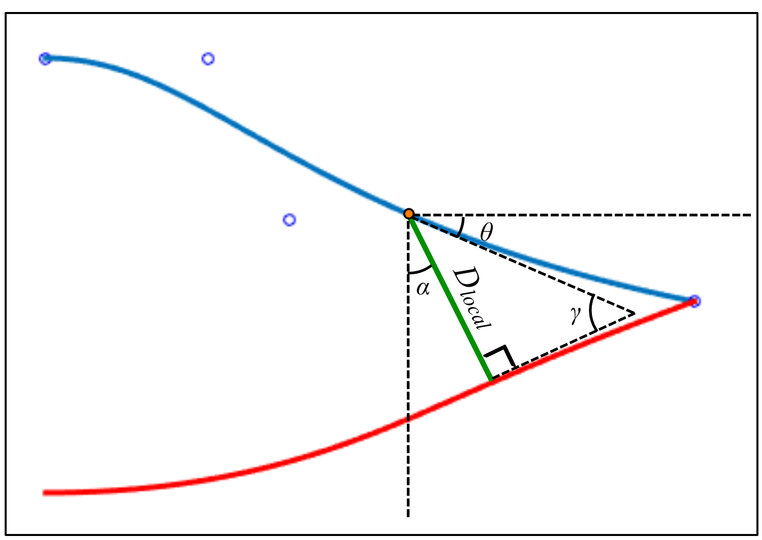
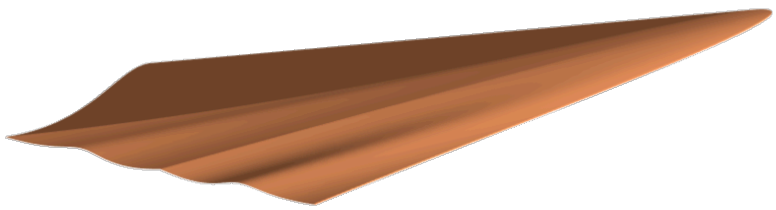


Figure 2. Base plane view of the leading edge with the geometrical features used for the calculation of the osculating plane angles and resulting shockwave profile.

Knowing the distance of the shockwave from the base plane projection of the leading edge on each plane we can now fully reconstruct the shockwave profile curve of the osculating cones method. The radius of curvature along the obtained shockwave can be calculated either numerically or through a series of complex chain rules relating it to the derivatives of the 3D leading edge curve; here we use the former. To increase the accuracy of the numerical calculation we can utilize additional 'dummy' shockwave profile curve points that are not used past this point and are placed in pairs on either side and close to the points where the radius of curvature is evaluated in order to enable a much more accurate 3 point calculation. The geometry generation process can, from this point on, be completed as described by the osculating cones method by tracing streamlines from the leading edge to the base plane to generate the lower surface of the waverider. The waverider generated using the 3D leading edge curve of Figures 1 and 2, a shock angle of 15^0, and freestream Mach number equal to 6 can be seen in Figure 3.

It is, of course, impossible to generate waverider geometries out of any arbitrary 3D shape that instantiates a parametric curve. While it is not very straightforward to formulate constraints that can be applied to the three-dimensional leading edge shape in order to always satisfy the requirements for generating a waverider geometry, the computational cost of constructing the potential shockwave profile from any 3D curve is negligible and we can instantly know if a specific curve can be matched with a viable waverider shockwave shape. Among the parametric curves tested we found that having second order continuity is preferable. Another obvious constraint is that the effective sweep should not go beyond the freestream Mach angle. Moreover, we can expand the capabilities of the osculating cones method, as far as the range of supported leading edge shapes goes, by enabling handling of concave shockwave shapes, as will be explained in the next section. Once this was applied, we found that the majority of leading edge shapes that should intuitively be able to provide a waverider shape would work with the method, rendering it much less restrictive than originally anticipated.



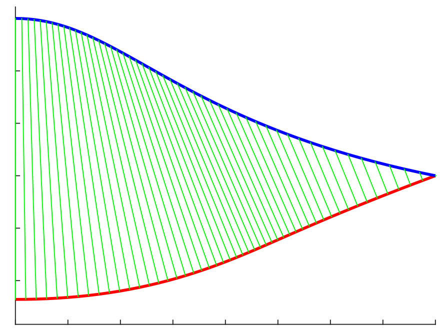


Figure 3. Resulting 3D leading edge designed waverider with converging ICFA flow used for part of the geometry generation (left). Osculating planes and profile curves on the base plane (right).

III. Additional considerations

Since we are not in direct control of the shockwave shape and it is derived in order to fit the three-dimensional leading edge, there are cases where the shockwave profile will also be concave. Normally, the osculating cones method can only handle convex shockwave profiles. Therefore, we can expand the range of leading edge geometries that meet the requirements of the method if we enable handling of shockwave shapes with concave regions. A concave shockwave shape also points to a converging flow, and the type of converging flow that best fits our purposes is the ICFA (Internal Conical Flow A) flow as described by Moelder[6], which has also been previously used for waverider design[7]. This type of flowfield enables stream tracing on osculating planes where the shockwave shape is concave by utilizing the Taylor-Maccoll equations, it does however come with its own limitation. The ICFA flowfield terminates at a singularity beyond which stream tracing is not possible. This effectively places a limitation on the minimum concave radius of curvature of the shockwave profile curve, which depends on two parameters: the angle in spherical coordinates where that singularity occurs for the given flow conditions and the local length of the waverider geometry on the osculating plane, as shown in Figure 4. Getting around this by allowing the lower surface generation to continue past the singularity following the last given direction of the ICFA streamline can be considered, since the shape of the shock will not be affected as long as we do not reach regions where the shock is getting close to the local axis. Finally, while the ICFA flowfield can work with the osculating cones method since they are both described by straight shocks, when the proposed 3D leading edge design method is used with the osculating axisymmetry or osculating flowfield approaches, other types of converging flowfields will need to be numerically constructed.

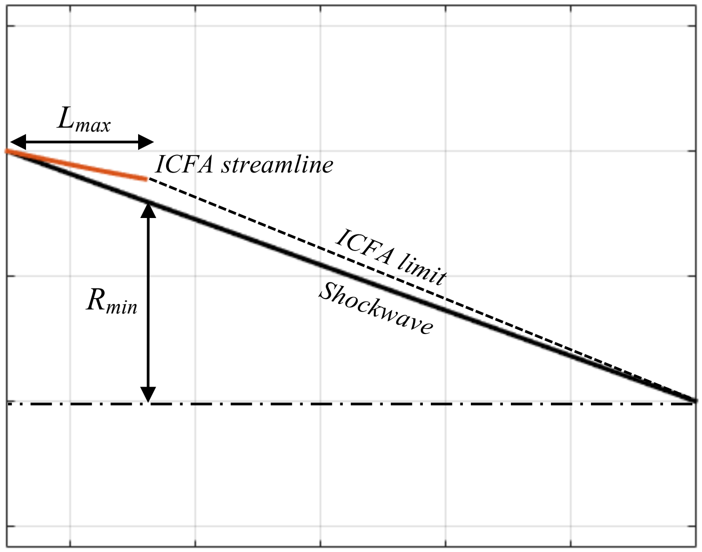


Figure 4. ICFA flowfield limit.

One advantage of previous design methods is that by directly designing the shockwave in the region of the engine inlet, its integration with the forebody becomes very straightforward. While the method presented here does not inherently provide direct control of the shockwave shape, there are two potential ways to obtain the desired flow in the inlet region. If, for example, a uniform flow and planar shock shape is required at a certain region, this can be enabled by keeping the angle of the osculating planes, α, constant. This, in turn, means that a constraint needs to be placed on the three-dimensional leading edge curve relating its effective sweep angle, λ, with its inclination from a base plane perspective, θ (*y-z* plane if *x* is the freestream direction), where:

$$\theta = \operatorname{asin}(tan(\lambda)\, tan(\beta)) - \alpha \qquad (4)$$

Depending on the type of parametric curve used, this can be complex to implement. Another way to obtain the desired shockwave shape in part of the underside of the geometry while maintaining any advantages provided by the three-dimensional definition of the leading edge curve for the rest of it, is to use a hybrid design approach as explained in [8]. We can use the osculating cones method as originally described and specify an upper surface profile or inlet capture curve and shockwave profile shape only for the region of the engine inlet for example, and use the 3D leading edge method for the rest of the geometry. Equation 4 will only need to be applied at the interface of the two methods and essentially provides a tangency condition for one end of the 3D leading edge curve; this is much easier to implement than applying the condition for a segment of the curve.

IV. Conclusions

The method presented in this paper enables generation of waverider geometries from fully defined three-dimensional leading edge shapes. Although it utilizes principles of existing waverider generation methods, it is a clear departure from the notion of having to specify a shockwave shape and then draw the leading edge on the shock surface. Apart from enabling any 3D leading edge shape of being considered for generating waverider geometries, it can also potentially increase the efficiency of parametric waverider geometry models as only one curve needs to be parameterized, albeit a three-dimensional one. This means that, given specific design requirements for the shape of the leading edge, it can be much more efficient to use this method to parameterize the geometry prior to a design optimization study, with the alternative being the use of two parametric planar curves. We also presented a method to expand the range of shapes that can provide viable shockwave shapes by enabling converging flow and concave shockwave shapes using the ICFA flowfield. Finally, we included a number of additional observations for the parameterization of the leading edge curve and how it can be used in conjunction with existing design methods that can fully define the shape of the shockwave in specific regions for engine inlet integration. As far as the aerodynamic performance of the geometries obtained goes, the method can match what the original osculating cones method is capable of. We expect that by utilizing the osculating axisymmetry and osculating flowfield extensions of the method the designs will be able to match the peak of what other studies have achieved, since the underlying design concept is the same. Another area that is worth further investigating is the potential expansion of the design space from an aircraft stability perspective, offered by enabling concave regions of the shock which can lead to greater flexibility in moving the center of aerodynamic forces.

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References

[1] Nonweiler, T. R. F., “Aerodynamic Problems of Manned Space Vehicles”, *Journal of the Royal Aeronautical Society*, Vol. 63, 1959, pp. 521–528.

[2] Sobieczky, H., Dougherty, F., and Jones, K.D., “Hypersonic Waverider Design from Given Shock Waves”, *First International Hypersonic Waverider Symposium*, University of Maryland, College Park, MD, 1990.

[3] Sobieczky, H., Zores, B., Wang, Z., and Qian, Y.J., “High Speed Flow Design Using Osculating Axisymmetric Flows”, *Proc. 3rd Pacific Int. Conf. on Aerospace Science and Technology*, 1997, pp. 182-187.

[4] Rodi, P.E., “The Osculating Flowfield Method of Waverider Geometry Generation”, AIAA Paper 2005-0511, *44th AIAA Aerospace Sciences Meeting*, Reno, NV, 2005.
doi:10.2514/6.2005-511

[5] Rodi, P. E., “Geometrical Relationships for Osculating Cones and Osculating Flowfield Waveriders”, AIAA Paper 2011-1188, 49th Aerospace Sciences Meeting, Orlando, FL, 2013.

[6] S. Molder, “Internal, axisymmetric, conical flow,” AIAA Journal, vol. 5, 1967, pp. 1252-1255.

[7] Goonko, Y.P., Mazhul, I.I., and Markelov, G.N., “Convergent-Flow-Derived Waveriders”, Journal of Aircraft, 37(4), 2000, pp.647–654.

[8] Kontogiannis, K., Sóbester, A., Taylor, N. J., “Efficient Parameterization of Waverider Geometries”, Journal of Aircraft, 2016, accessed November 28th 2016, doi: http://arc.aiaa.org/doi/abs/10.2514/1.C033902