Optimal Asset Allocation for Strategic Investors

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Abstract

This paper studies optimal asset allocation for investors over multiple investment horizons. Rather than first model the various features of the conditional return distribution and subsequently characterize the portfolio choice, we focus directly on the dependence of the portfolio weights on the predictor variables through a linear parametric portfolio policy rule. This characterization allows us to apply GMM estimation and testing methods to sample analogues of the multiperiod Euler equations that characterize our optimal portfolio choice. Our model accommodates an arbitrarily large number of assets in the portfolio and state variables in the information set. The empirical results for a portfolio of stocks, bonds and cash provide ample support to the linear specification of the portfolio weights and reveal significant differences between myopic (one-period) and strategic (long-term) optimal portfolio allocations.

Key words: dynamic hedging demand; intertemporal portfolio theory; parametric portfolio policy rules; return predictability; strategic asset allocation.

JEL Codes: E32, E52, E62.

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1 Introduction

Optimal portfolio decisions depend on the details of the economic and financial environment: the financial assets that are available, their expected returns and risks, and the preferences and circumstances of investors. These details become particularly relevant for long-term investors. Such investors must concern themselves not only with expected returns and risks today, but with the way in which expected returns and risks may change over time. It is widely understood at least since the work of Samuelson (1969) and Merton (1969, 1971, 1973) that the solution to a multiperiod portfolio choice problem can be very different from the solution to a static portfolio choice problem. In particular, if investment opportunities vary over time, then long-term investors care about shocks to investment opportunities as well as shocks to wealth itself. This can give rise to intertemporal hedging demands for financial assets and lead to strategic asset allocation as a result of the farsighted response of investors to time-varying investment opportunities.

Unfortunately, intertemporal asset allocation models are hard to solve in closed form unless strong assumptions on the investor's objective function or the statistical distribution of asset returns are imposed. A notable exception is when investors exhibit log utility with constant relative risk aversion equal to one. This case is relatively uninteresting because it implies that Merton's model reduces to the static model. Another exception within the class of utility functions describing constant relative risk aversion and represented by the family of power utility functions is when asset returns are log-normally distributed. In this case, maximizing expected utility is equal to the mean-variance analysis proposed by Markowitz (1952) in his seminal study. In this model, the investor trades off mean against variance in the portfolio return. The relevant mean return is the arithmetic mean return and the investor trades the log of this mean linearly against the variance of the log return. The coefficient of relative risk aversion acts as a penalty term adding to the variance of the return.

More generally, the lack of closed-form solutions for optimal portfolios with constant relative risk aversion has limited the applicability of the Merton model and has not displaced the Markowitz model. This situation has begun to change as a result of several developments in numerical methods and continuous time finance models. More specifically, some authors such as Barberis (2000) and Brennan, Schwartz, and Lagnado (1997, 1999), among a few others, provide discrete-state numerical algorithms to approximate the solution of the portfolio problem over infinite horizons. Closed-form solutions to the Merton model are derived in a continuous time model with a constant risk-free interest rate and a single risky asset if long-lived investors have power utility defined over terminal wealth (Kim and Omberg (1996), or if investors have power utility defined over consumption (Watcher (2002)), or if the investor has Epstein and Zin (1989, 1991) utility with intertemporal elasticity of substitution equal to one (Campbell and Viceira (1999); Schroder and Skiadas (1999). Approximate analytical solutions to the Merton model have been developed in Campbell and Viceira (1999, 2001, 2002) and Campbell, Chan, and Viceira (2003) for models exhibiting an intertemporal elasticity of substitution not too far from one. An alternative to solving the investor's optimal portfolio choice problem has been proposed by Brandt (1999), Ait-Sahalia and Brandt (2001) and Brandt and Clara (2006). Ait-Sahalia and Brandt (2001), for example, show how to select and combine variables to best predict the optimal portfolio weights, both in single-period and multiperiod contexts. Brandt and Clara (2006) solve the dynamic portfolio selection problem by expanding the asset space to include mechanically managed portfolios and compute the optimal static portfolio within this extended asset space. The intuition of this strategy is that a static choice of managed portfolios is equivalent to a dynamic strategy.

The current paper builds on the seminal articles initiated by Brandt (1999) and Ait-Sahalia and Brandt (2001). More specifically, we contribute to the literature on financial forecasting by proposing an optimal portfolio allocation for investors with constant relative risk aversion (CRRA) utility functions defined over multiple, potentially infinite, investment horizons. Rather than first model the various features of the conditional return distribution and subsequently characterize the portfolio choice, we focus directly on the dependence of the portfolio weights on the predictor variables over a multiperiod investment horizon. We do this by solving sample analogues of a set of multiperiod Euler equations that characterize our portfolio choice. This method is made operational through a linear parametric portfolio policy rule that models the dynamics of the portfolio weights, see Ait-Sahalia and Brandt (2001), over the investor's multiperiod horizon. In contrast to most of the related literature, our model accommodates an arbitrarily large number of assets in the portfolio and state variables in the information set. The main advantage of our linear portfolio policy rule is that the first order conditions of the maximization problem yield a simple system of equations that is overidentified and provides a very intuitive empirical representation. Furthermore, in our framework we avoid the implementation of time-consuming stochastic dynamic programming methods.

The sample analogues of the multiperiod Euler conditions obtained from the investor's maximization problem allow us to apply the generalized method of moments (GMM) of Hansen and Singleton (1982) for estimation and also for testing the linear parametric portfolio policy specification. We do this by developing two different but related tests. First, we adapt the specification J-test obtained from the overidentified system of Euler equations to assess whether the linear parametric portfolio policy is statistically correctly specified for long investment horizons. Second, we adapt the incremental testing approach developed by Sargan (1958, 1959) to assess the marginal statistical relevance of the state variables in the linear portfolio policy specification. We complete the econometric section by proposing a further test that gauges the effect of the number of investment horizons on the optimal allocation of assets to the portfolio. This is done by developing a Hausman type test that compares different specifications of the investor's maximization problem in terms of the investment horizon. More specifically, we contemplate a short-term and a long-term investment horizon and assess statistically the informational content of the interval spanning between the short and long-term horizons.

Our empirical application compares the optimal asset allocation of a myopic investor only concerned with maximizing one-period-ahead wealth with the allocation of a strategic investor with a long multiperiod investment horizon. The empirical application closely follows similar studies such as Brennan, Schwartz, and Lagnado (1997) and Brandt (1999). The investor is assumed to invest in a portfolio given by three assets: a one-month Treasury bill as risk-free security, a long-term bond, and an equity portfolio. The variables that predict expected returns on these assets are the detrended short-term interest rate, the U.S. credit spread, the S&P 500 trend and the one-month average of excess stock and bond returns. Our econometric specification shows that the strategic allocation to the S&P 500 and G0Q0 Bond index differs in two main aspects with respect to the myopic asset allocation. First, the absolute value of the optimal portfolio weights in the strategic case is usually larger than in the myopic case. Second, the strategic allocation to the S&P 500 index is found to be positively and significantly related to the trend variable and negatively related to the detrended short-term interest rate. In contrast, the strategic allocation to bonds is found to be negatively and significantly related to the detrended short-term interest rate with such relationship increasing with the degree of risk aversion. The analysis of the optimal stocks and bonds' hedging demands varies significantly with the state variables and highlights the importance of the dynamics of the state variables in determining the differences between the myopic and strategic optimal portfolio allocations. We also compare the performance of these two portfolios by analyzing their annualized certainty equivalent return and simulating the wealth of each investment strategy over time. The results show the outperformance of the strategic portfolio over the myopic portfolio. These differences in portfolio performance can be attributed to the presence of larger exposures to each asset for the long-term investment strategy that result in higher profits.

We also perform several robustness exercises. First, we assess the robustness of our choice of state variables to the inclusion of other variables in the investor's information set with the potential to predict the dynamics of excess returns. Interestingly, we find that the dividend yield is not statistically significant for predicting the dynamics of the optimal portfolio weights, however, other factors related to investors' risk aversion such as the Baker and Wurgler (2006) investor's sentiment index may help to determine the optimal asset allocation. However, the inclusion of this variable is rejected by the incremental Sargan specification test proposed in the paper to assess the correct specification of the state variables. As a second robustness exercise, we explore the investors' optimal asset allocation when the investment opportunity set also includes a size and a value portfolio. These portfolios present features very different from the market portfolio, as the Fama and French (1993, 1996) three factor model suggests, and hence, can be considered to offer alternative diversification and hedging possibilities to strategic investors. Our results confirm this and reveal that investors take negative exposures to the size portfolio and positive exposures to the value portfolio. Our empirical exercise also provides an out-of-sample analysis of the robustness of our results to the evaluation period.

The closest contributions to our study are Brennan, Schwartz, and Lagnado (1997) and Brandt (1999), Campbell, Chan, and Viceira (2003), and more recently, Brandt and Clara (2006). In contrast to these seminal contributions, our specification of the strategic asset allocation problem is tractable and provides a simple econometric solution to the portfolio problem over long horizons without having to rely on dynamic stochastic programming as in Brennan, Schwartz, and Lagnado (1997), parametric specifications of the joint dynamics of the state variables and the portfolio returns as in Campbell, Chan, and Viceira (2003) and Brandt and Clara (2006), or on complex methods to expand the asset space with artificially managed portfolios as in Brandt and Clara (2006). Our modeling and estimation strategy is also related to Britten-Jones (1999). This author derives the optimal portfolio weights for a myopic, mean-variance investor, as the coefficients of an OLS regression. In our case, we focus directly on the dependence of the portfolio weights on a set of state variables that forecast changes in the investment opportunity set and obtain the model parameter estimates by GMM applied to a set of multiperiod Euler equations.

The rest of the article is structured as follows. Section 2 introduces the investor's maximization problem over a multiperiod horizon and discusses the solution to the problem for constant relative risk averse investors. Section 3 discusses suitable econometric methodology to estimate the optimal portfolio weights and make inference on these parameters. The section also presents some additional specification tests to assess the suitability of the proposed linear portfolio policy for describing the variation of the optimal portfolio weights and, more specifically, the choice of state variables for forecasting changes in the investment opportunity set. Section 4 presents an empirical application that compares the investment strategy of a myopic investor maximizing one period utility against the investment strategy of an investor maximizing a multiperiod objective function. Section 5 presents a robustness exercise to assess the validity of our results to alternative specifications of the state variables and the investment opportunity set. We also extend the results to an out-of-sample setting. Section 6 concludes.

2 The Model

Consider the portfolio choice of an investor who maximizes the expected utility of wealth (w_t), defined in real terms, over K periods. Assume that the multiperiod utility function is additively time separable and exhibits constant relative risk aversion. Then, an investor maximizes

$$\sum_{j=0}^{K} \beta^j E_t \left[\frac{w_{t+j}^{1-\gamma}}{1-\gamma} \right], \tag{1}$$

with $\gamma > 0$ and $\gamma \neq 1$. Two parameters describe CRRA preferences. The discount factor β measures patience, the willingness to give up wealth today for wealth tomorrow. The coefficient γ captures risk aversion, the reluctance to trade wealth for a fair gamble over wealth today.

The standard multiperiod maximization problem is completed by defining the accumulation equation determining the buildup of real wealth by the investor over time:

$$w_{t+1} = (1 + r^p_{t+1})w_t. \tag{2}$$

The investor begins life with an exogenous endowment $w_0 \geq 0$. At the beginning of the period $t + 1$ the investor receives income from allocating the wealth accumulated at time t in an investment portfolio offering a real return r^p_{t+1}. The return on this portfolio is defined as

$$r^p_{t+1}(\alpha_t) = r_{f,t+1} + \alpha'_t r^e_{t+1}, \tag{3}$$

with $r^e_{t+1} = (r_{1,t+1} - r_{f,t+1}, \ldots, r_{m,t+1} - r_{f,t+1})'$ denoting the vector of excess returns on the m risky assets over the real risk-free rate $r_{f,t+1}$, and $\alpha_t = (\alpha_{1,t}, \ldots, \alpha_{m,t})'$.

2.1 Optimal portfolio choice under constant relative risk aversion

In this section we derive the first order conditions of the optimal portfolio choice for a risk-averse investor exhibiting a power utility function with $\gamma > 1$. These conditions are obtained for a dynamic portfolio with optimal weights determined by a linear parametric specification in terms of a vector of state variables. We note first that investor's wealth at time $t + j$ can be expressed as

$$w_{t+j} = \prod_{i=1}^{j}(1 + r^p_{t+i}(\alpha_{t+i-1}))w_t. \tag{4}$$

Hence, the investor's objective function (1) becomes

$$\max_{\{\alpha_{t+j}\}} \left\{ \sum_{j=1}^{K} \beta^j \frac{w_t^{1-\gamma}}{1-\gamma} E_t \left[\prod_{i=1}^{j} (1 + r^p_{t+j+1-i}(\alpha_{t+j-i}))^{1-\gamma} \right] \right\}.$$

In order to be able to solve a multiperiod maximization problem that accommodates in a parsimonious way arbitrarily long investment horizons we entertain the parametric portfolio policy rule introduced in the seminal contributions of Ait-Sahalia and Brandt (2001), Brandt and Clara (2006) and Brandt, Clara, and Valkanov (2009):

$$\alpha_{h,t+i} = \lambda_h' z_{t+i}, \;\; h = 1, \ldots, m, \tag{5}$$

with z_{t+i} a $n \times 1$ vector comprising a constant and a set of $n-1$ macroeconomic and financial variables reflecting all the information available to the investor at time $t+i$, and λ_h the corresponding vector of parameters. Time variation of the optimal asset allocation is introduced through the dynamics of the state variables. This specification of the portfolio weights has two main features. First, it allows us to study the marginal effects of the state variables on the optimal portfolio weights through the set of parameters λ, and second, it avoids the introduction of time consuming dynamic stochastic programming methods. A potential downside of this parametric approach is to force the individual's optimal portfolio policy rule to be linear and with the same parameter values over the long-term horizon. Nevertheless, for finite horizon ($K < \infty$) objective functions, more sophisticated models can be developed that entertain horizon-specific parametric portfolio policy rules. This approach significantly increases the computational complexity of the methodology and is beyond the scope of this paper.

Under the above parametrization of the portfolio weights the maximization problem (1) becomes

$$\max_{\{\lambda_{hs}\}} \left\{ \sum_{j=1}^{K} \beta^j \frac{w_t^{1-\gamma}}{1-\gamma} E_t \left[\prod_{i=1}^{j} (1 + r^p_{t+i}(\lambda_h' z_{t+i-1}))^{1-\gamma} \right] \right\}. \tag{6}$$

The first order conditions of this optimization problem with respect to the vector of parameters λ_{hs}, with $h = 1, \ldots, m$ and $s = 1, \ldots, n$, provide a system of mn equations characterized by the following conditions:

$$E_t \left[\sum_{j=1}^{K} \beta^j \psi_{t,j}(z_s; \lambda_h) \right] = 0, \tag{7}$$

with

$$\psi_{t,j}(z_s; \lambda_h) = \left(\sum_{i=1}^{j} \frac{z_{s,t+i-1} r^e_{h,t+i}}{1 + r^p_{t+i}(\lambda_h' z_{t+i-1})} \right) \left(\prod_{i=1}^{j} (1 + r^p_{t+i}(\lambda_h' z_{t+i-1}))^{1-\gamma} \right). \tag{8}$$

The introduction of the vector of state variables z_t allows us to incorporate forecasts of the investment opportunity set in the optimal allocation of assets to the portfolio.

$$E \left[\sum_{j=1}^{K} \beta^j \psi_{t,j}(z_s; \lambda_h) \otimes z_t \right] = 0 \tag{9}$$

where $\otimes$ denotes element by element multiplication. This system of equations provides a simple alternative to the setting introduced by Brandt and Clara (2006) to derive the optimal portfolio weights in dynamic settings. In contrast to this seminal contribution, our approach does not require expanding the asset space with managed and timing portfolios. The algebra is also considerably simpler. As mentioned earlier, our approach can also accommodate alternative formulations of the portfolio policy rule beyond the linear specification (5). However, for simplicity and consistency with the related literature we consider the linear portfolio policy.

To complete the section we also discuss the truncation of the infinite horizon model characterizing the investor's strategic behavior. To do this we note that as $j \to K = \infty$, the contribution of the functions $\psi_{t,j}(z_s; \lambda_h)$ in (7) converges to zero justifying a truncation of the infinite horizon model. To do this, condition (7) is rescaled such that the sum of the K weights is equal to 1. By doing so, each equation of the system defined in (9) can be written as a convex combination of the functions $\psi_{t,j}(z_s;\lambda_h)$, and the optimal portfolio problem is the solution to

$$E\left[\sum_{j=1}^{K} w_{j,K}^{*}\psi_{t,j}(z_s;\lambda_h)\otimes z_t\right]=0, \tag{10}$$

with $w_{j,K}^{*}=\frac{1-\beta}{1-\beta^{K+1}}\beta^{j}$. The proposed truncation is determined by K^{*} with $K^{*}=\min\{j \mid w_{j,K}^{*}\leq tol,\ j=1,\ldots,K\}$ with tol a value at the discretion of the researcher such that the objective function of interest becomes

$$E\left[\sum_{j=1}^{K^{*}} w_{j,K}^{*}\psi_{t,j}(z_s;\lambda_h)\otimes z_t\right]=0. \tag{11}$$

3 Econometric methodology

The parameters of interest of the strategic optimal portfolio allocation problem are the coefficients λ_{hs}, with $h=1,\ldots,m$ and $s=1,\ldots,n$. These parameters can be consistently estimated by using the empirical counterpart of the multiperiod Euler equation (10) stating the set of first order conditions of the investor's optimization problem. A natural technique to estimate these parameters is the generalized method of moments proposed by Hansen (1982) and Hansen and Singleton (1982). In this section, we extend this method to accommodate the convex combination of moment conditions derived in (11). This methodology leads to an overidentified system of equations that allows us not only to estimate and make statistical inference on the parameters determining the optimal portfolio policy but also to test for the correct specification of such policy and variations of it.

3.1 Estimation

A suitable empirical representation of the Euler equation (10) is

$$\widehat{\phi}_{h,s}(z_s;\lambda_h) = \frac{1}{T-K^*}\sum_{t=1}^{T-K^*}\left(\sum_{j=1}^{K^*} w^*_{j,K}\psi_{t,j}(z_s;\lambda_h)\otimes z_t\right) = 0, \tag{12}$$

where T is the sample size used for estimating the model parameters. For each pair (h,s), condition (12) yields a $n \times 1$ vector of moment conditions. Let

$$\widehat{\phi}^{(\widetilde{s})}_{h,s}(z_s;\lambda_h) = \frac{1}{T-K^*}\sum_{t=1}^{T-K^*}\left(\sum_{j=1}^{K^*} w^*_{j,K}\psi_{t,j}(z_s;\lambda_h)\; z_{\widetilde{s},t}\right) \tag{13}$$

denote each element of such vector with $\widetilde{s} = 1,\ldots,n$ where $z_{1,t} = 1$; and let $g_T(c)$ be the $mn^2 \times 1$ vector that stacks each of the sample moments $\widehat{\phi}^{(\widetilde{s})}_{h,s}$ indexed by h, s and $\widetilde{s}$, with $h = 1,\ldots,m$ and $s,\widetilde{s} = 1,\ldots,n$. The idea behind GMM is to choose $\widehat{\lambda}$ so as to make the sample moments $g_T(c)$ as close to zero as possible. More formally, the GMM estimator of the matrix of λ parameters is defined as

$$\widehat{\lambda}_T = \arg\min_{c\in\Lambda}\; g_T(c)'\widehat{V}_T^{-1}g_T(c)$$

where $\widehat{V}_T$ is an $mn^2 \times mn^2$, possibly random, non-negative definite weigth matrix, whose rank is greater than or equal to mn. This matrix admits different representations. In a first stage $\widehat{V}_T$ can be the identity matrix or some other matrix, as for example, $I_{mn}\otimes Z'Z$, with I_{mn} the identity matrix of dimension mn and Z the $(T-K)\times n$ matrix corresponding to the state variables. In a second stage, to gain efficiency, this matrix is replaced by a consistent estimator of the asymptotic covariance matrix V_0 of the random vector $g_T(c)$. A natural estimator of this matrix is discussed in (16).

Statistical inference for the portfolio weights is obtained by applying asymptotic theory results from GMM estimation to the quantity $g_T(c)$. To show this, we assume the portfolio returns and state variables to be jointly stationary, and derive the consistency and asymptotic normality of $\widehat{\phi}_{h,s}^{(\widetilde{s})}$. In particular, we note that for K^* fixed, condition

$$\sum_{t=-\infty}^{\infty}\sum_{j=1}^{K^*}\sum_{j'=1}^{K^*} w_{j,K}^* w_{j',K}^* E\left|\psi_{t,j}(z_s;\lambda_h)\psi_{t',j'}(z_s;\lambda_h)\ z_{\widetilde{s},t} z_{\widetilde{s},t'}\right| < \infty \tag{14}$$

is sufficient to guarantee that

$$\widehat{\phi}_{h,s}^{(\widetilde{s})} \xrightarrow{p} \phi_{h,s}^{(\widetilde{s})} = E\left[\sum_{j=1}^{K^*} w_{j,K}^* \psi_{t,j}(z_s;\lambda_h)\ z_{\widetilde{s},t}\right], \tag{15}$$

with $\xrightarrow{p}$ denoting convergence in probability as the sample size T increases. This condition depends on the persistence of the portfolio returns, and in particular, of the set of state variables $z_{\widetilde{s},t}$. Similarly, under suitable regularity conditions, the central limit theorem implies that

$$\sqrt{T}\left(\widehat{\phi}_{h,s}^{(\widetilde{s})} - \phi_{h,s}^{(\widetilde{s})}\right) \xrightarrow{d} N\left(0, V_{hs}^{(\widetilde{s})}\right)$$

with $N\left(0, V_{hs}^{(\widetilde{s})}\right)$ denoting a Normal distribution with $V_{hs}^{(\widetilde{s})}$ the relevant asymptotic variance.

Following similar arguments for the different elements of the vector $g_T(c)$ it is not difficult to see that under condition (11) the standardized vector $g_T(c)$ converges in distribution to a multivariate Normal distribution with covariance matrix $V_0 = E\left[g_T(c)g_T'(c)\right]$ defined by the elements $E\left[\phi_{h_1,s_1}^{(\widetilde{s}_1)}\phi_{h_2,s_2}^{(\widetilde{s}_2)}\right]$ with $s_1, \widetilde{s}_1, s_2, \widetilde{s}_2 = 1, \ldots, n$ and $h_1, h_2 = 1, \ldots, m$. This matrix can be consistently estimated by its sample counterpart $\widehat{V}_T$ defined by the elements

$$\frac{1}{(T-K^*)^2}\sum_{t=1}^{T-K^*}\sum_{j=1}^{K^*}\sum_{j'=1}^{K^*} w_{j,K}^* w_{j',K}^* \psi_{t,j}(z_{s_1},\lambda_{h_1})\psi_{t,j'}(z_{s_2},\lambda_{h_2})\ z_{\widetilde{s}_1,t} z_{\widetilde{s}_2,t}$$

$$+\frac{1}{(T-K^*)^2}\sum_{t=1}^{T-K^*}\sum_{\substack{t'=1\\t'\neq t}}^{T-K^*}\sum_{j=1}^{K^*}\sum_{j'=1}^{K^*} w^*_{j,K} w^*_{j',K}\psi_{t,j}(z_{s_1},\lambda_{h_1})\psi_{t',j'}(z_{s_2},\lambda_{h_2})\ z_{\widetilde{s}_1,t} z_{\widetilde{s}_2,t'} \tag{16}$$

with $s_1, \widetilde{s}_1, s_2, \widetilde{s}_2 = 1, \ldots, n$ and $h_1, h_2 = 1, \ldots, m$. This estimator highlights the strong persistence in the covariance matrix V_0. This persistence is due to the overlapping of periods produced by considering an strategic investment horizon ($K^* > 1$) in the investor's objective function.

The above properties also establish the asymptotic distribution of the parameter estimators defining the optimal portfolio weights, that satisfy

$$\sqrt{T}\left(\widehat{\lambda}_T - \lambda\right) \xrightarrow{d} N(0, W_0) \tag{17}$$

with $W_0 = (D_0 \Omega_0^{-1} D_0)^{-1}$, where $\Omega = E\left[g(c)g'(c)\right]$, $D_0 \equiv D(c) = \frac{\partial g(c)}{\partial c}$ and $D(c)$ is continuous at $c = \lambda$. The vector $g(c)$ stacks each of the $mn^2 \times 1$ population moments $\phi^{(\widetilde{s})}_{h,s}$ defined in (15).

3.2 Specification tests

This section discusses three statistical tests based on the overidentified system of equations introduced above. We discuss first an specification test to assess the validity of the linear policy rule (5). The second test is an incremental Sargan type test concerned with testing the relevance of the state variables within the linear specification. The third test allows us to compare different specifications of the investor's maximization problem in terms of the investment horizon. A particularly interesting application of the latter is to compare the myopic and strategic asset allocations.

The systems of equations defined in (11) entail the existence of testable restrictions implied by the econometric model. Estimation of λ sets to zero mn linear combinations of the mn^2 sample orthogonality conditions $g_T(c)$. The correct specification of the model implies that there are $mn(n-1)$ linearly independent combinations of $g_T(\widehat{\lambda}_T)$ that should be close to zero but are not exactly equal to zero. This hypothesis is tested using the Hansen test statistic (Hansen (1982)). Let $s(\widehat{\lambda}_T) = g_T(\widehat{\lambda}_T)'\widehat{V}_T^{-1}g_T(\widehat{\lambda}_T)$ with $\widehat{V}_T$ defined in (16), that under the null hypothesis of correct specification of the model, satisfies

$$s(\widehat{\lambda}_T) \xrightarrow{d} \chi^2_{mn(n-1)}. \tag{18}$$

In contrast to the standard GMM specification tests for independent and identically distributed (*iid*) observations, in our framework the components of the vector $g_T(\widehat{\lambda}_T)$ exhibit strong serial correlation due to the overlapping of investment horizons. Hence, it is not advisable to assume that $V_0 = \Omega_0/(T-K^*)$ as it would be the case in the *iid* case. Therefore, an appropriate standardization to achieve a chi-square distribution under the null hypothesis is obtained by the construction of $s(\widehat{\lambda}_T)$ presented above and not by $(T-K^*)g_T(\widehat{\lambda}_T)'\widehat{\Omega}_T^{-1}g_T(\widehat{\lambda}_T)$, with $\widehat{\Omega}_T$ a consistent estimator of Ω_0 defined as

$$\widehat{\Omega}_T = \frac{1}{T-K^*}\sum_{t=1}^{T-K^*}\sum_{j=1}^{K^*}\sum_{j'=1}^{K^*} w^*_{j,K}w^*_{j',K}\psi_{t,j}(z_{s_1},\lambda_{h_1})\psi_{t,j'}(z_{s_2},\lambda_{h_2})\ z_{\widetilde{s}_1,t}z_{\widetilde{s}_2,t}.$$

The second test, based on the incremental Sargan (1958, 1959) tests, assesses the appropriateness of different subsets of the moment conditions. In particular, we apply it to test the relevance of the state variables within the linear specification (5). Consider a set of $n-2$ state variables defining a vector z_t of dimension $n-1$, obtained from also including a constant in the portfolio policy specification, and take

$$E_t\left[\sum_{j=1}^{K} w^*_{j,K}\psi_{t,j}(z_s;\lambda_h)\right] = 0 \tag{19}$$

for $h = 1,\dots,m$ and $s = 1,\dots,n-1$ as the maintained hypothesis. We wish to test the correct specification of the linear policy rule (5) obtained from including an additional state variable $z_{n,t}$ to the vector z_t. To do this, we need to test the suitability of the above set of restrictions for $s = n$ and $h = 1, \ldots, m$. This implies a joint test of $(2n-1)m$ conditions obtained as the difference between mn^2 and $m(n-1)^2$ restrictions. The incremental Sargan test is defined in this context as

$$S_d = s(\widehat{\lambda}_T) - s_1(\widehat{\lambda}_{1T}) \tag{20}$$

with $s_1(\widehat{\lambda}_{1T}) = g_{1T}(\widehat{\lambda}_{1T})'\widehat{V}_{1T}^{-1} g_{1T}(\widehat{\lambda}_{1T})$, where $g_{1T}(\widehat{\lambda}_{1T})$ denotes the subset of $g_T(\widehat{\lambda}_T)$ determined by $m(n-1)^2$ conditions in (12), $\widehat{\lambda}_{1T}$ is the vector minimizing $s_1(\widehat{\lambda}_{1T})$ and $\widehat{V}_{1T}$ is the $m(n-1) \times m(n-1)$ version of $\widehat{V}_T$ defined in (16). This test statistic converges under the null hypothesis given by the correct specification of $z_{n,t}$ in the linear portfolio rule (5) to a chi-square distribution with $(2n-1)m$ degrees of freedom.

The third test is a Hausman type test that allows us to compare different specifications of the investor's maximization problem in terms of the investment horizon. More specifically, we contemplate a short-term and a long-term investment horizon and compare the informational content of the period spanning between the short and long-term horizons. Under the null hypothesis, the informational content of this period is null implying that the relevant terms in (7) are equal to zero. For K finite, this test can be interpreted as a Hausman type test. This is so because whereas under the null hypothesis the parameter estimates corresponding to the short-term investment horizon are consistent they are not under the alternative hypothesis.

Let $K_1 < K_2$ denote the short-term and long-term horizons, and consider the joint null hypothesis

$$H_0 : E\left[\sum_{j=K_1+1}^{K_2} w^*_{j,K_2}\psi_{t,j}(z_s;\lambda_h) \otimes z_t\right] = 0, \tag{21}$$

for $h = 1, \ldots, m$ and $s = 1, \ldots, n$. The consistency of the sample moments entails under the null hypothesis that

$$\frac{1}{T-K^*}\sum_{t=1}^{T-K^*}\left(\sum_{j=K_1+1}^{K_2} w^*_{j,K_2}\psi_{t,j}(z_s;\lambda_h)z_{\widetilde{s},t}\right)\xrightarrow{p} 0 \tag{22}$$

for all $h = 1,\ldots,m$ and $s,\widetilde{s} = 1,\ldots,n$. A suitable test statistic for this joint hypothesis is $s_r(\widehat{\lambda}^o_T)$ with $s_r(\widehat{\lambda}^o_T) = g_{rT}(\widehat{\lambda}^o_T)'\widehat{V}_T^{-1}g_{rT}(\widehat{\lambda}^o_T)$, where $g_{rT}(c)$ is the vector of moment conditions stacking the mn^2 sample moments in (22) and $\widehat{\lambda}^o_T$ denoting the set of parameters estimated under the null hypothesis (21). Under the null hypothesis H_0, the test statistic satisfies

$$s_r(\widehat{\lambda}^o_T)\xrightarrow{d}\chi^2_{mn^2}. \tag{23}$$

A natural application of this test is to compare the suitability of the myopic asset allocation problem given by $K_1 = 1$ against the strategic allocation given by $K_2 = K^*$. Under the null hypothesis, both maximization problems should yield similar optimal portfolio weights and entail, hence, the consistency of the λ parameter estimates in both scenarios. The alternative hypothesis implies differences in the optimal portfolio allocation between the myopic and strategic asset allocations. This test can also be applied to assess the suitability of truncations of the infinite horizon model determined by K^*. To do this, we consider $K_1 = K^*$ and $K_2 = K^* + \epsilon$, with $\epsilon > 0$ some large arbitrary number. Under the null hypothesis (21), the truncation of the infinite horizon model given by K^* provides consistent estimates of the optimal portfolio weights.

4 Empirical application

In this section, we analyze the one-month horizon investment strategy intended to represent the myopic investor (Brennan, Schwartz, and Lagnado (1997)) versus the long-term strategy that is intended to reflect the strategic investor. Following similar studies such as Brennan, Schwartz, and Lagnado (1997), Brandt (1999) and Campbell, Chan, and Viceira (2003), we consider an investor that can allocate wealth among stocks, bonds and the one-month real Treasury bill rate. By doing this, we implicitly take into account the role played by inflation in the formation of optimal portfolios. As in Campbell, Chan, and Viceira (2003) we do not impose short-selling restrictions.

The time-variation of the investment opportunity set is described by a set of state variables that have been identified in the empirical literature as potential predictors of the excess stock and bond returns and the short-term ex-post real interest rates. These variables are the detrended short-term interest rate (Campbell (1991)), the U.S. credit spread (Fama and French (1989)), the S&P 500 trend (Ait-Sahalia and Brandt (2001)) and the one-month average of the excess stock and bond returns (Campbell, Chan, and Viceira (2003)). The detrended short-term interest rate detrends the short-term rate by subtracting a 12-month backwards moving average. The U.S. credit spread is defined as the yield difference between Moody's Baa- and Aaa-rated corporate bonds. The S&P 500 momentum is the difference between the log of the current S&P 500 index level and the average index level over the previous 12 months. We demean and standardize all the state variables in the optimization process (Brandt, Clara, and Valkanov (2009)).

4.1 Data description

Our data covers the period January 1980 to December 2010. We collect monthly data from Bloomberg on the S&P 500 financial index and the G0Q0 Bond Index. The G0Q0 Bond Index is a Bank of America and Merrill Lynch U.S. Treasury Index that tracks the performance of U.S. dollar denominated sovereign debt publicly issued by the U.S. government in its domestic market. We collect the nominal yield on the U.S. one-month risk-free rate from the Fama and

French database. The consumer price index (CPI) time series and the yield of the Moody's Baa- and Aaa-rated corporate bonds are obtained from the U.S. Federal Reserve.

Table 1 reports the sample statistics of the annualized excess stock return, excess bond return and short-term ex-post real interest rates. The bond market outperforms the stock market during this period. In particular, the excess return on the bond index is higher than for the S&P 500 and exhibits a lower volatility entailing a Sharpe ratio almost three times higher for bonds than stocks. Additionally, the excess bond return has larger skewness and lower kurtosis. This anomalous outperformance of the G0Q0 index versus the S&P 500 is mainly explained by the last part of the sample and the consequences of the subprime crisis on the valuation of the different risky assets.

[Insert Tables 1 to 3 about here]

Table 2 shows the estimates of the seemingly unrelated regression estimation of the excess stock return, excess bond return and short-term ex-post real interest rate using as explanatory variables the detrended short-term interest rate, the U.S. credit spread, the S&P 500 trend and the one-month average of the excess stock and bond returns. The value of the intercept in those regressions is not reported for sake of space. Table 3 reports the correlation of the state variables and the excess stock return, excess bond return and short-term ex-post real interest rate innovations obtained from the SURE model. The results obtained from these tables shed some light on the dynamics of excess stock and bond returns and their variation over time linked to our set of state variables. A first conclusion that can be drawn from these estimates, and in particular from the low R^2 statistics, is the difficulty in predicting excess asset returns.

4.2 Strategic vs. myopic asset allocation

We study the optimal portfolio problem of an investor that faces a time-varying investment opportunity set that can be forecast by the set of state variables discussed above. We distinguish between a short-term investor, whose time horizon is one-month and is intended to represent the myopic strategy (Brennan, Schwartz, and Lagnado (1997)), and a long-term (strategic) investor that is infinitely long lived. The differences between the short-term and long-term optimal portfolios are explained by the role of hedging demand that the different assets offer against changes in the time-varying investment opportunity set (Merton (1973); Brennan, Schwartz, and Lagnado (1997); Campbell, Chan, and Viceira (2003)).

We compute optimal portfolio rules for $\gamma = 2, 5, 20$ and 100, and assume a value of $\beta = 0.95$. We provide an approximate solution to the infinitely long-lived investor by choosing a value of $K^* = 100$ in the system of equations given in (12). This restriction implies a tolerance level of 0.03 determined by the standardized weight function $w^*_{j,\infty} = \beta^j(1-\beta)$.[1] A two-step Gauss-Newton type algorithm using numerical derivatives is implemented to estimate the model parameters. In a first stage we initialize the covariance matrix $\widehat{V}_T$ with the matrix $I_{mn} \otimes Z'Z$, and in a second stage, after obtaining a first set of parameter estimates, we repeat the estimation replacing this matrix by (16). This matrix $\widehat{V}_T$ is also used to perform the different specification tests described below.

Our theoretical framework has several advantages over other methods proposed in the literature for solving the strategic asset allocation problem. First, under the assumption that the optimal portfolio policy rule depends on the realization of the state variables we can directly estimate the optimal strategic portfolio allocation to stocks, bonds and cash without estimating asset expected returns, volatilities and correlations. Second, we take advantage of

[1] Alternatively, we could set an ex-ante tolerance level, tol, and obtain K^* from the definition $K^* = \min\{j \mid w^*_{j,K} \leq tol,\ j = 1, \ldots, K\}$. Instead, for simplicity, we choose an ex-ante K^* and adjust the value of tol accordingly.

the overidentified system of equations to statistically test the correct specification of (5) and provide statistical evidence on the validity of the state variables. Third, the tests presented in the preceding section also allow us to statistically confirm the differences between the myopic and strategic asset allocation problems.

Table 4 reports the estimates and t-statistics of the optimal myopic and strategic asset allocation rules specified in (5) that assumes that both allocations to stocks and bonds are linearly related to our set of state variables. The table also presents the test statistic (18) to assess statistically the correct specification of our model. The analysis of the myopic case reveals the following two findings. First, the one-month average of the excess stock and bond return is the only variable that is significantly and positively related to the myopic allocation to the S&P 500. In contrast, the U.S. credit spread, the S&P 500 trend and the average of the one-month excess stock and bond returns are significantly and negatively related to the myopic allocation to the G0Q0 Bond Index. These findings are consistent across risk aversion levels. Second, the magnitude of the parameter estimates λ increases as the degree of risk aversion decreases and reveals the existence of an inverse relationship between the degree of investor's risk aversion and the responsiveness of the portfolio weights to changes in the information set.

The strategic allocation to the S&P 500 financial index and G0Q0 Bond Index differ in two main aspects with respect to the myopic asset allocation. First, the absolute value of the coefficients in the strategic case is usually larger than in the myopic case and the parameters are estimated more precisely as shown by the lower p-values. Second, the strategic allocation to the S&P 500 is found to be positively and significantly related to the trend variable and negatively related to the detrended short-term interest rate. The detrended short-term interest rate is also found to be negatively and significantly related to the dynamic optimal allocation to bonds for higher values of risk aversion.

[Insert Table 4 about here]

Figures 1 and 2 plot the optimal myopic and strategic asset allocations, respectively, to stocks and bonds for an investor with $\gamma = 5$. The myopic case shows that the optimal asset allocation responds aggressively to changes in the information set. The strategic case shows that the optimal hedging demands on stocks and bonds vary significantly with the state variables and can be positive or negative depending on the realization of the state variables.

[Insert Figures 1 and 2 about here]

To gain better understanding of the differences between the myopic and strategic asset allocations for different degrees of risk aversion, we compute in Table 5 the mean myopic allocation to each asset as well as the mean hedging portfolio demand for different values of γ. The sum of the optimal myopic and hedging components equals, by construction, the optimal strategic allocation to each asset. The optimal myopic demand consists of a long position in stocks and bonds and a short position in the one-month Treasury bill (cash). As expected, the mean optimal myopic portfolio allocation is tilted towards bonds, which have the largest Sharpe ratio among the three asset classes considered in our sample, with an optimal ratio of bonds to stocks of about 2.4. The results reported in Table 5 also suggest that the optimal allocation to risky assets, such as stocks and bonds, decreases as the relative risk aversion coefficient increases making the optimal myopic allocation to cash increase as investors become more conservative.

[Insert Table 5 about here]

The intertemporal hedging demand for bonds is positive reaching a percentage of 15% of the total strategic asset allocation and implying that the optimal mean strategic allocation to bonds is larger than the optimal mean myopic allocation to bonds. Our simple SURE model (in Table 2) reports a negative relationship between the average of the one-month excess stock and bond return shocks and expected excess bond returns. The model also uncovers the existence of a positive correlation between the unexpected excess bond return and the average of the one-month excess stock and bond return shocks. This finding suggests that poor bond returns are correlated with improvements in the future investment opportunity set. As a result, we note that bonds can be used to hedge time variation in their own returns making the strategic investor allocate a higher fraction of wealth to bonds. This effect is more important for more aggressive investors that are especially exposed to bond market fluctuations. This reasoning could also rationalize the low absolute value of the coefficient linked to the average of the one-month excess stock and bond return for the optimal strategic asset allocation to bonds compared to the absolute value of the coefficient in the myopic case.

Table 5 also shows that the mean intertemporal hedging demand for stocks is slightly negative and not very large in absolute terms. This result constitutes a difference with recent literature that finds a very positive intertemporal hedging demand for stocks, especially linked to the dividend yield variable (Brennan, Schwartz, and Lagnado (1997); Campbell, Chan, and Viceira (2003)). This finding is mainly due to the differences in the exposures of the detrended short-term interest rate (λ_{Tb}) and the one-month average of the excess stock and bond returns (λ_{ARP}) to the optimal allocation to the S&P 500 index between the myopic and strategies portfolios reported in Table 4. The large positive exposure of the portfolio weight to the ARP state variable and the positive correlation between this variable and the S&P 500 index predict a positive hedging demand to the S&P 500 index. This positive hedging demand is, however, partly offset by the negative exposure of the detrended short-term interest rate to the optimal portfolio allocation to the stock index. This negative exposure is observed for both myopic and strategic portfolios, however, its magnitude is larger for the strategic case implying a net negative effect of this state variable on the hedging demand to the S&P 500 index. Interestingly, for very large values of the risk aversion coefficient ($\gamma = 100$), the exposure of both state variables to the optimal allocation to the S&P 500 index is negligible for both myopic and strategic portfolios entailing, in turn, a null hedging demand in Table 5.

The myopic and strategic optimal portfolio allocations are also compared in terms of economic performance. To do this, we compute the certainty equivalent return (CER) corresponding to each investment strategy. This measure is defined as the guaranteed return, in annualized wealth, that would provide the investor with the same expected utility as the given optimal portfolio rule. For the multi-horizon utility function, the CER is defined as the return r^{CER} that solves the following equation:

$$\sum_{j=1}^{K^*}\beta^j \frac{w_t^{1-\gamma}}{1-\gamma}\prod_{i=1}^{j}(1+r^{CER})^{1-\gamma} = \sum_{j=1}^{K^*}\beta^j \frac{w_t^{1-\gamma}}{1-\gamma} E_t\left[\prod_{i=1}^{j}(1+r^p_{t+j+1-i}(\alpha_{t+j-i}))^{1-\gamma}\right], \tag{24}$$

where α_t denotes the strategic optimal portfolio allocation. Because the power utility function is homothetic in wealth, without loss of generality, the initial wealth can be normalized to one. Therefore, we can compute the certainty equivalent return as the annualized return on wealth earned with certainty that provides the investor with the same utility as the optimal portfolio. The above condition becomes

$$E_t\left[\sum_{j=1}^{K^*}\beta^j\left((1+r^{CER})^{j(1-\gamma)} - \prod_{i=1}^{j}(1+r^p_{t+j+1-i}(\alpha_{t+j-i}))^{1-\gamma}\right)\right] = 0. \tag{25}$$

The empirical counterpart of this expression yields the following equation:

$$\frac{1}{T-K^*}\sum_{t=1}^{T-K^*}\left[\sum_{j=1}^{K^*}\beta^j\left((1+r^{CER})^{j(1-\gamma)} - \prod_{i=1}^{j}(1+r^p_{t+j+1-i}(\alpha_{t+j-i}))^{1-\gamma}\right)\otimes z_t\right] = 0. \tag{26}$$

For simplicity, we derive r^{CER} for the unconditional version of the above expression that considers $z_t = 1$.

Table 6 reports the main summary statistics of the myopic and strategic portfolios. To evaluate the in-sample performance of these portfolios using the CER measure, we impose realistic portfolio constraints that rule out short selling positions in the risky assets and set the upper bound of portfolio weights to be equal to two (Lan (2015)). Differences in the annualized certainty equivalent return between the strategic and myopic portfolios vary from 0.80% for $\gamma = 2$ to 6.23% for $\gamma = 20$, but are positive in all cases. The summary statistics in Table 6 also reveal the outperformance of the strategic portfolio in terms of Sharpe ratio and terminal wealth.

4.3 Specification tests

The test of overidentifying restrictions in (18) shows that all the strategic asset allocation models estimated under different assumptions on the degree of risk aversion are well specified (p-values larger than 0.2) with the only exception of the model that considers $\gamma = 2$ that reports a p-value of 0. To provide further statistical evidence on the validity of our specification of the policy rule and the state variables we also carry out the tests based on the incremental Sargan test discussed above. To do this we concentrate on the case $\gamma = 5$, $\beta = 0.95$ and $K^* = 100$, and analyze the statistical significance of the state variables comprising our linear policy rule.

We analyze first the linear portfolio policy exclusively considering the detrended short-term interest rate (Tb). Note that in this case expression (5) becomes $\alpha_{h,t+j} = \lambda_{h,1} + \lambda_{h,2} Tb_{t+j}$, implying a value of $n = 2$ and $m = 2$ risky assets as parameters in the incremental Sargan test. The test statistic takes a value of 1.83 and a p-value of 0.93 obtained from a chi-square distribution with 6 degrees of freedom. This result establishes the statistical relevance of the specification, and in particular of the state variable, to solve the strategic investor asset allocation problem. We add to this specification the one month average of the excess stock and bond return. The test statistic takes a value of 12.30 obtained as the difference between 14.13 and 1.83. The p-value, obtained from a chi-square distribution with 10 degrees of freedom, is 0.26 validating the augmented specification of (5). Similarly, for the U.S. credit spread, we obtain a test statistic of 11.15 obtained as the difference between 25.28 and 14.13, yielding a p-value of 0.67 from a chi-square distribution with 14 degrees of freedom. Finally, for the S&P 500 trend we observe a test statistic of 14.23 obtained as the difference between 39.50 and 25.28 yielding a p-value of 0.71, and derived from a chi-square distribution with 18 degrees of freedom. These results shed sufficient statistical evidence to accept the marginal relevance of each of the variables comprising our policy rule.

We also test statistically the suitability of the truncation of the infinite horizon model given by different values of the truncation parameter K^* in (12) for the same parametrization of the investor's maximization problem ($\gamma = 5$ and $\beta = 0.95$). In particular, we have considered $K^* = 10, 18, 32, 60, 70, 80, 90, 95$ and 100. To do this, we compute the test (23) using these different values of K^* as K_1 and considering $K_2 = 110$; as a robustness exercise, we have repeated the tests also considering $K_2 = 150$. The null hypothesis implies that horizons further than K_1 periods ahead do not contain relevant information to the investor. This condition is equivalent to ascertaining that the investor's multiperiod maximization problem is really finite. The tests corresponding to different values of K_1 reveal that the null hypothesis (21) is only accepted for values of K^* around 100 and higher. In particular, the Wald type test reports a statistic of 1.11 for $K_2 = 110$ and 0.41 for $K_2 = 150$, respectively. These results provide sufficient statistical evidence to accept the choice of $K^* = 100$ as a valid approximation of the infinite horizon problem.

Finally, we implement a version of this test that compares statistically the myopic and strategic asset allocation problems. More precisely, we check whether the optimal solution characterized by $K = 1$ in (12) reports the same optimal portfolio weights as the strategic allocation characterized by $K = 100$ that proxies the infinite horizon problem. To do this, we simply consider $K_1 = 1$ and $K_2 = 100$ in the hypothesis test (21). The test statistic reports a value of 5.98 providing sufficient evidence to reject statistically the hypothesis that the portfolio weights obtained from these portfolio allocations are equal.

5 Robustness analysis

One of the main advantages of our methodology is that we can accommodate an arbitrarily large number of assets in the investment opportunity set and state variables in the investor's information set. We analyze separately each of these possibilities.

5.1 An augmented information set

The purpose of this exercise is twofold. First, we show that our methodology can easily accommodate a larger number of state variables in the investor's information set and, second, we explore the statistical significance of additional variables traditionally considered as well established predictors of financial returns. In particular, we extend our set of state variables by adding the following four variables: the dividend yield, the term spread, measured as the difference between the ten-year government bond yield and the three-month Treasury bill rate, the Baker and Wurgler (2006) sentiment index and the ratio of newly issued equity over the sum of newly issued debt and equity.

The rationale for choosing these variables is the following. Campbell (1991) finds that aggregate dividend yield strongly predicts excess returns, and the predictability is stronger at longer horizons. Since dividend yield hardly predicts dividend growth, most of the variation of dividend yields is related to changing forecasts of expected returns. Brennan, Schwartz, and Lagnado (1997) find that the mean reversion in stock prices induced by the dividend yield makes investment in stocks less risky for long-horizon investors, who optimally invest a larger proportion of their wealth in stocks compared to myopic investors. Fama and Bliss (1987) and Campbell and Shiller (1991) find that the slope of the yield curve predicts the expected excess returns on U.S. bonds. Baker and Wurgler (2006) find that when the investor sentiment is low (high) stocks that are more difficult to value earn high (low) subsequent returns. Laborda and Olmo (2014) also find evidence of an empirical relationship between investor sentiment and the expected bond risk premia. The inclusion of the variable capturing the ratio of newly issued equity over the sum of newly issued debt and equity responds to the findings in Baker and Wurgler (2006). These authors find that firms issue relatively more equity than debt just before periods of low market returns or high stock market valuations.

For sake of presentation, we only analyze the optimal portfolio rules for a relative risk aversion coefficient $\gamma = 5$ and assume a value of $\beta = 0.95$. Data on these state variables are freely available. In particular, dividend data are obtained from Robert Shiller's web page http://www.econ.yale.edu/$\sim$shiller/data.htm, and the investor sentiment index and the new equity share variables from Jurgen Würgler's web page http://people.stern.nyu.edu/jWürgler/. The ten-year government bond yield and the three-month Treasury bill rate are obtained from the U.S. Federal Reserve.

Table 7 reports the estimates and t-statistics of the parametric portfolio rule (5) characterizing the allocation to stocks and bonds. The results show that our model is capable of estimating the effect of additional state variables in the information set. More importantly, the main conclusion obtained from this empirical exercise is that the dividend yield does not play a significant role in predicting the optimal portfolio allocation once our proposed state variables are included in the information set. More specifically, the parameter corresponding to the dividend yield is not statistically significant for either the S&P 500 or G0Q0 Bond Indexes. This observation is consistent with studies such as Lettau and Ludvigson (2001) and Goyal and Welch (2003) that point out that predictability by the dividend yield is not robust to the inclusion of the 1990's decade. Ang and Bekaert (2007) find similar results for returns at long horizons and note that excess return predictability by the dividend yield is not statistically significant, not robust across countries, and not robust across different sample periods. In this sense, these authors claim that the predictability attributed to the dividend yield that has been the focus of most recent finance research is simply not there.

[Insert Table 7 about here]

Table 7 also shows that the term spread and the investor sentiment are significantly related to the strategic allocation to the S&P 500, but not to the G0Q0 Index Bond. These findings suggest that state variables reflecting risk aversion in financial markets are more relevant for describing the allocation to stocks than bonds. In contrast, the share of new equity over the sum of new debt and equity is only significantly related to the G0Q0 Index Bond.

For completeness, we also carry out the incremental Sargan test for the augmented set of state variables that adds separately each of the new state variables to our initial four state variable specification. In all cases, we find a p-value of the Sargan test close to zero that suggests that the addition of more state variables to our initial linear parametric portfolio policy is not statistically appropriate as the additional constraints do not conform with the data. These empirical findings indicate that the information content of these additional variables is subsumed under our initial set of state variables.

5.2 Strategic asset allocation to the size and value factors

This section analyses the performance of our methodology if the investment opportunity set considers assets beyond the stock market portfolio (S&P 500 index) and the bond market portfolio (G0Q0 index). In particular, we enlarge the investment opportunity set by considering two additional portfolios with a prominent role in determining the risk premium on risky assets: a portfolio replicating the size factor and a portfolio that replicates the value factor, see the three-factor asset pricing model in Fama and French (1993, 1996) for more information on these portfolios. The size factor allows the investor the possibility of investing in a portfolio that holds long positions in small capitalization stocks (small caps) and short positions in large capitalization stocks. The value factor replicates the return on a portfolio that holds a long position in high book-to-market stocks and a short position in low book-to-market stocks (value factor).

The profitability of these portfolios resides in their ability to capture future changes in the economic environment, such as the impoverishment in credit conditions reflected by an increase of the business failure rates, see Kapadia (2011), changes in the level of economic activity, see Liew and Vassalou (2000), or changes in financial market conditions, such as counter-cyclical dynamics of the Sharpe ratio, see Pérez-Quirós and Timmermann (2000). Therefore, by including these portfolios in the investment opportunity set, and following the rationale behind the three-factor asset pricing model, we aim to offer strategic investors the possibility of hedging exposures to factors unrelated to the market portfolio. Furthermore, the above-mentioned investment literature has shown that it is possible to establish a relationship between these portfolios replicating the size and value factors and macroeconomic and financial state variables, thus, it is reasonable to assume the parametric portfolio policy rule (5) defined by our set of state variables to predict the dynamic allocation to these additional assets in the investor's portfolio.

We compute optimal portfolio rules for $\gamma = 5$ and assume a value of $\beta = 0.95$. The solution to the optimal allocation problem of an infinitely long-lived investor is approximated by choosing a value of $K^* = 100$ in the multiperiod investment horizon problem. Table 8 reports the estimates and t-statistics of the optimal linear portfolio policy rule (5) comprised by stocks, bonds and size or value allocations. The results show that the strategic asset allocation to the size factor is significantly related to the variables that capture the overall financial conditions (see Laborda, Laborda, and Olmo (2016)). In particular, it is negatively related to the detrended short-term interest rate and the U.S. credit spread, indicating a negative exposure to the size factor as borrowing conditions tighten. A similar explanation can be found to justify the negative correlation between the size factor and the S&P trend state variable. In contrast, we only find the one-month average of the excess stock and bond returns state variable to be related to the value factor.

[Insert Table 8 about here]

Table 9 shows the mean myopic allocation to each asset as well as the mean hedging portfolio demand. Interestingly, in contrast to previous exercises, the inclusion of the size factor in the investor's optimal portfolio makes the strategic asset allocation to the S&P 500 index larger than the myopic one. The optimal strategic allocation to the size factor is negative. This result suggests that the investor uses the size portfolio as a hedging instrument in bull market episodes during which the size portfolio performs very well and in financial distress periods in which the size portfolio underperforms. We, nevertheless, find a positive hedging demand on this asset with respect to the corresponding myopic allocation. In contrast, the optimal strategic allocation to the value factor is positive. As for the size factor, the hedging demand associated to the S&P 500 index is positive, however, there is a negative hedging demand associated to the G0Q0 index.

[Insert Table 9 about here]

5.3 Out-of-sample results

This section presents several out-of-sample experiments that provide further robustness to the above results. In this out-of-sample exercise, the optimal portfolio is re-estimated every year using a rolling window of data until the end of the sample covering the period January 1980 to August 2014. The first portfolio is computed with data from January 1980 to December

2000. The second portfolio is computed with data from January 1981 to December 2001 and similarly until the end of the evaluation period. The investor uses the information available at period t, reflected in the values of the state variables, to estimate the dynamic weight function (5) defining the optimal portfolio between t and $t+1$. We compute optimal portfolio rules for $\gamma = 5$ and assume a value of $\beta = 0.95$. We provide an approximate solution to the infinitely long-lived investor by choosing a value of $K^* = 100$ in the system of equations given in (12).

Figure 3 plots the dynamics of the out-of-sample parameter estimates describing the sensitivity of the optimal allocation to the S&P 500 index with respect to each of the four state variables initially proposed. The results show patterns compatible with the in-sample exercise. Interestingly, for the U.S. credit spread, the S&P 500 trend and the one-month average of excess stock and bond returns, we observe a trend in the parameter estimates. This trend can be the result of the deterioration of the future investment prospects due to a prolonged decrease in short term interest rates over the last decade and the increase in risk aversion observed during the last part of the sample. Thus, whereas the parameter estimates associated to the U.S. credit spread and the S&P 500 trend exhibit a positive trend, the parameter associated to the average of the one-month excess stock and bond returns reveals a negative trend.

Figure 4 plots the dynamics of the out-of-sample parameter estimates describing the sensitivity of the optimal allocation to the G0Q0 Bond Index. The dynamics of the parameter estimates are very different to those exhibited for the exposures to the S&P 500 index. Thus we find negative signs for the parameter estimates associated to the four state variables for most of the out-of-sample exercise. It is interesting to note, though, the positive sign of the parameter associated to the average of the one-month excess stock and bond return state variable during the $2007-2008$ crisis period. The positive exposure to this state variable during this period is accompanied by very negative exposures to the U.S. credit spread and the S&P 500 trend.

Figure 5 plots the wealth obtained by the strategic investor and the myopic one over the out-of-sample period. To evaluate the out-of-sample performance of the myopic and strategic portfolios, we entertain the same portfolio constraints discussed in the in-sample exercise that rule out short selling positions in the risky assets and set the upper bound of portfolio weights to be equal to 2. The plot shows that the strategic investor's optimal strategy yields terminal wealth about 50% larger than the terminal wealth obtained from the myopic portfolio. In addition, the optimal strategic portfolio delivers an annualized return larger than the optimal myopic portfolio (11.36% vs 7.90%) exhibiting a higher short term volatility (15.06% vs 11.31%) that, nevertheless, leads to a higher Sharpe ratio (0.75 vs 0.69). Thus, the optimal strategic portfolio profits from the ability of the assets to hedge against changes in the time-varying investment opportunity set while the myopic investor only cares about the level of the risk premiums at each point in time.

[Insert Figures 3 to 5 about here]

We also perform an out-of-sample comparison of CER performance between the myopic and strategic optimal portfolios[2]. Figure 6 plots the out-of-sample certainty equivalent of wealth defined as the counterpart of expression (26). The construction of this out-of-sample exercise in rolling windows implies that we obtain a certainty equivalent return for each window. Figure 6 shows that the certainty equivalent wealth for the strategic investor is larger than the myopic one especially in the last part of the out-of-sample period. The average difference between the strategic certainty equivalent wealth and the myopic certainty equivalent wealth is about 10%. These differences are reflected in portfolio gains close to 1% in terms of annualized certainty equivalent returns.

[2]Alternative measures to compare the economic performance of investment portfolios out of sample such as the out-of-sample variance, Sharpe ratio, and turnover are developed in DeMiguel, Garlappi, Nogales, and Uppal (2009). Nevertheless, for consistency with the in-sample exercise and given that the main focus of the paper is not to compare performance across strategies but the suitability of our long-term investment strategy, we restrict our attention to CER measures.

[Insert Figure 6 about here]

6 Conclusion

This paper has proposed a simple framework to study the investor's optimal asset allocation problem over long, potentially infinite, horizons. The method accommodates an arbitrarily large number of assets in the portfolio and state variables in the information set. In contrast to most of the literature on dynamic asset allocation, our method does not rely on dynamic stochastic programming or numerical approximations. It is made operational through the first order conditions of the maximization problem of an strategic investor under the assumption that the optimal portfolio weights are described by a parametric linear portfolio policy. Under these conditions, we apply GMM methods to estimate the parameters driving the dynamics of the optimal portfolio weights and make statistical inference on the correct specification of the model and investment horizon.

The empirical results for a portfolio of stocks, bonds and the one-month real Treasury bill provide ample support to the suitability of a linear specification of the optimal portfolio weights determined by the detrended short-term interest rate, the U.S. credit spread, the S&P 500 trend and the one-month average of the excess stock and bond returns for investors exhibiting constant relative risk aversion. Our results are also supportive of statistically significant differences between the myopic and strategic optimal portfolio. More specifically, our application reveals that the absolute value of the optimal portfolio weights in the strategic case is usually larger than in the myopic case reflecting more aggressive responses to changes in the state variables in the strategic case.

Several robustness exercises provide additional insights. First, we show that the dividend yield does not exhibit predictive power to describe the optimal portfolio weights. Second, we acknowledge some explanatory power to the sentiment index developed by Baker and Wurgler (2006), although not sufficient to be statistically significant in the determination of the optimal portfolio weights. Third, we show that the inclusion of the size portfolio in our set of investment assets provides hedging to long-term investors in periods of financial distress. Finally, in-sample and out-of-sample comparisons confirm the outperformance of the long-term strategy compared to the myopic one.

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TABLES AND FIGURES

Table 1: Sample statistics

	Mean	Volatility	Sharpe ratio	Skewness	Kurtosis
$R^e_{S\&P500}$	0.0266	0.131	0.21	-1.12	4.88
R^e_{Bonds}	0.0290	0.0566	0.51	0.15	2.17
r_f	0.0183	0.021		0.38	3.16

This table presents the sample statistics of the excess stock return $\left(R^e_{S\&P500}\right)$, excess bond return $\left(R^e_{Bonds}\right)$ and short-term ex-post real interest rates (r_f). The sample data covers the period January 1980 to December 2010. The return horizon is one month. Mean and volatility are expressed in annualized terms.

Table 2: Seemingly unrelated regression estimation of the excess stock return, excess bond return and short-term ex-post real interest rates.

$R^e_{S\&P500}$	T_b	Def	Trend	ARP	R^2
$\beta_{R^e_{S\&P500}}$	-0.23	-0.13	-0.02	1.19	0.10
p-value	0.25	0.51	0.90	0.00	
R^e_{Bonds}	T_b	Def	Trend	ARP	R^2
β_{Bonds}	-0.02	-0.04	-0.01	-0.02	0.03
p-value	0.82	0.62	0.15	0.01	
r_f	T_b	Def	Trend	ARP	R^2
β_{r_f}	-0.05	0.25	0.19	-0.08	0.06
p-value	0.33	0.00	0.00	0.11	

This table presents the estimates of the seemingly unrelated regression estimation (SURE) of the excess stock return $\left(R^e_{S\&P500}\right)$, excess bond return $\left(R^e_{Bonds}\right)$ and short-term ex-post real interest rates $\left(r_f\right)$, using as explanatory variables the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP). We use monthly data from January 1980 to December 2010.

Table 3: Correlation matrix of the state variables with the excess stock return, excess bond return and short-term ex-post real interest rate innovations.

	$R^e_{S\&P500}$	R^e_{Bonds}	r_f	Tb	Def	Trend	ARP
$R^e_{S\&P500}$	1						
R^e_{Bonds}	-0.01	1					
r_f	-0.01	0.23	1				
Tb	-0.01	-0.14	-0.11	1			
Def	-0.05	0.07	0.07	-0.31	1		
Trend	0.21	0.00	0.00	0.21	-0.44	1	
ARP	0.87	0.40	0.14	-0.12	0.00	0.27	1

This table presents the estimated correlations of the state variables with the excess stock return $\left(R^e_{S\&P500}\right)$, excess bond return $\left(R^e_{Bonds}\right)$ and short-term ex-post real interest rate $\left(r_f\right)$ innovations obtained from the seemingly unrelated regression estimation (SURE). The state variables are the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP). We use monthly data from January 1980 to December 2010.

Table 4: Myopic and Strategic asset allocation.

	Myopic CRRA=2	Strategic CRRA=2	Myopic CRRA=5	Strategic CRRA=5	Myopic CRRA=20	Strategic CRRA=20	Myopic CRRA=100	Strategic CRRA=100
S&P 500								
λ_{Tb}	-1.10	-3.01	-0.40	-1.28	-0.09	-0.28	-0.01	-0.06
t-stat	(-1.21)	(-7.69)	(-1.11)	(-4.54)	(-1.04)	(-3.58)	(-0.69)	(-4.84)
λ_{Def}	0.23	0.09	0.08	0.28	0.02	-0.08	0.01	-0.02
t-stat	(0.26)	(0.22)	(0.23)	(-0.97)	(0.26)	(-1.18)	(0.57)	(-1.18)
λ_{Trend}	-0.63	0.84	-0.21	0.55	-0.04	0.13	-0.01	0.03
t-stat	(-0.65)	(2.91)	(-0.54)	(2.37)	(-0.45)	(1.81)	(-0.29)	(2.12)
λ_{ARP}	6.09	8.37	2.34	3.71	0.56	0.90	0.11	0.15
t-stat	(5.55)	(23.71)	(5.33)	(13.35)	(5.22)	(9.08)	(4.78)	(10.67)
Bonds								
λ_{Tb}	-1.13	0.19	-0.52	-0.48	-0.13	-0.20	-0.02	-0.04
t-stat	(-0.7)	(0.38)	(-0.8)	(-1.41)	(-0.85)	(-1.97)	(-0.88)	(-2.16)
λ_{Def}	-5.99	-8.08	-2.45	-3.36	-0.63	-0.70	-0.14	-0.10
t-stat	(-3.43)	(-9.68)	(-3.39)	(-5.58)	(-3.45)	(-5.01)	(-3.62)	(-3.12)
λ_{Trend}	-3.92	-7.00	1.61	-3.64	-0.40	-0.91	-0.07	-0.16
t-stat	(-2.01)	(-10.02)	(-2.04)	(-5.49)	(-2.01)	(-5.03)	(-1.73)	(-5.86)
λ_{ARP}	-3.68	-1.46	-1.49	-1.05	-0.37	-0.33	-0.07	-0.04
t-stat	(-1.92)	(2.32)	(-1.89)	(-2.71)	(-1.86)	(-2.50)	(-1.72)	(-1.19)
χ^2		81.73		39.49		37.3		46.69
d.f		40		40		40		40
p-value		0.00		0.49		0.59		0.22

This table shows estimates of the optimal investment strategy policy of a myopic investor, whose time horizon is one month, and a strategic investor that is infinitely long lived. We consider an investor that can allocate her wealth among stocks, bonds and the one-month Treasury bill rate. The optimal portfolio rule is specified in equation (5) and optimized for a power utility function with different CRRA coefficients (γ=2, 5, 20, 100) and a value of β=0.95, using these state variables: the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP). We use monthly data from January 1980 to December 2010. The statistics are reported with *p*-values in parentheses and associated degrees of freedom (d.f.).

Table 5: Mean asset demands.

	Myopic CRRA=2	Hedging demand CRRA=2	Myopic CRRA=5	Hedging demand CRRA=5	Myopic CRRA=20	Hedging demand CRRA=20	Myopic CRRA=100	Hedging demand CRRA=100
S&P 500	2.53	0.27	1.05	-0.07	0.26	-0.04	0.05	0.00
Bonds	6.24	0.52	2.49	0.42	0.59	0.04	0.09	-0.01
Cash	-7.78	-0.79	-2.55	-0.36	0.14	0.00	0.86	0.01

This table shows the mean optimal allocation in percentage points to stocks, bonds and cash of a myopic investor, whose time horizon is one month, and the mean hedging optimal allocation in percentage points to stocks, bonds and cash of a strategic investor that is infinitely long lived. The optimal hedging allocation to stocks, bonds and cash is defined as the difference between the strategic investor asset demand that is infinitely long lived and the myopic investor asset demand, whose time horizon is one month. The optimal portfolio rule is specified in equation (5) and optimized for a power utility function with different CRRA coefficients (γ=2, 5, 20, 100) and a value of β=0.95, using the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP) as state variables. We use monthly data from January 1980 to December 2010.

Table 6: Investment performance of the optimized strategies in-sample.

	Myopic CRRA=2	Strategic CRRA=2	Myopic CRRA=5	Strategic CRRA=5	Myopic CRRA=20	Strategic CRRA=20	Myopic CRRA=100	Strategic CRRA=100
Mean	0.020	0.021	0.021	0.023	0.014	0.018	0.004	0.005
Volatility	0.041	0.042	0.041	0.041	0.029	0.034	0.007	0.008
Sharpe ratio	0.49	0.51	0.52	0.55	0.49	0.55	0.61	0.59
Skewness	1.18	1.17	1.44	1.39	2.96	2.32	2.69	3.23
Kurtosis	4.32	4.65	4.68	4.25	14.54	8.64	14.90	18.90
Final wealth	1527	1568	1773	2771	155	760	4.41	6.04
ΔCER		0.80%		2.51%		6.23%		1.23%

This table shows the in-sample performance of the optimized investment strategies over the period 1980:01 to 2010:12. The table reports the mean, the volatility, the skewness and the kurtosis of the monthly returns. It also shows the final wealth and the difference of certainty equivalent ΔCER between the strategic portfolio and the myopic one, The optimal portfolio rule is specified in equation (5) and optimized for a power utility function with different CRRA coefficients (γ=2, 5, 20, 100) and a value of β=0.95, using these state variables: the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the average of one-month excess stock and bond returns (ARP).

Table 7: Strategic asset allocation. In-sample results adding the dividend yield, the term spread, the Baker and Wurgler (2006) investor sentiment index and the ratio of newly issued equity over total new equity and debt issues.

Panel A

	S&P 500			
β_{Tb}	-1.08	-1.53	-1.28	-1.25
t-stat	(-3.7)	(-3.21)	(-3.56)	(-3.36)
β_{Def}	0.30	-0.95	0.47	0.06
t-stat	(0.35)	(-2.03)	(0.56)	(0.23)
β_{Trend}	0.57	0.71	0.62	0.70
t-stat	(1.48)	(1.76)	(1.39)	(2.15)
β_{ARP}	3.76	4.48	4.32	3.67
t-stat	(14.34)	(10.23)	(9.89)	(8.68)
B_{dp}	-0.18			
t-stat	(-0.36)			
B_{term}		0.73		
t-stat		(2.23)		
B_{sent}			-1.60	
t-stat			(-6.28)	
B_{Eshare}				-0.17
t-stat				(-0.5)

Panel B

	Bonds			
β_{Tb}	-0.40	-0.97	-0.44	-0.71
t-stat	(-0.46)	(-1.59)	(-0.77)	(-0.89)
β_{Def}	-5.17	-2.87	-2.84	-4.63
t-stat	(-7.88)	(-3.06)	(-1.94)	(-7.39)
β_{Trend}	-5.35	-4.28	-2.06	-3.60
t-stat	(-3.87)	(-4.22)	(-2.46)	(-4.14)
β_{ARP}	-0.41	-1.00	-1.07	-1.18
t-stat	(-0.55)	(-1.54)	(-1.42)	(-2.24)
B_{dp}	1.85			
t-stat	(1.47)			
B_{term}		-0.70		
t-stat		(-1.09)		
B_{sent}			1.47	
t-stat			(1.61)	
B_{Eshare}				-2.52
t-stat				(-2.67)

This table shows estimates of the optimal investment strategy policy of a strategic investor that is infinitely long lived. We consider an investor that can allocate her wealth among stocks, bonds and the one-month Treasury bill rate. The optimal portfolio rule is specified in equation (5) and optimized for a power utility function with CRRA coefficients γ=5 and a value of β=0.95, using the state variables: the detrended short-term interest rate (Tb), the U.S. credit spread (Def), the S&P 500 trend (Trend), the average of one-month excess stock and bond returns (ARP), the dividend yield (dp), the term spread (term), the Baker and Wurgler (2006) investor sentiment index (sent) and the share of equity issues in total new equity and debt issues (Eshare). We use monthly data from January 1980 to December 2010.

Table 8: Strategic asset allocation. In-sample results adding the size or value factors.

	S&P 500	Bonds	Size factor
β_{Tb}	-0.42	-2.11	-1.57
t-stat	(-1.17)	(-4.06)	(-4.21)
β_{Def}	-0.00	-1.93	-1.17
t-stat	(0.00)	(-1.45)	(-2.67)
β_{Trend}	1.18	-3.20	-2.13
t-stat	(2.56)	(-5.46)	(-3.68)
β_{ARP}	4.70	-2.70	-0.38
t-stat	(6.02)	(-3.44)	(-1.54)
	S&P 500	**Bonds**	**Value factor**
β_{Tb}	-0.81	-0.82	0.12
t-stat	(-3.04)	(-1.57)	(0.29)
β_{Def}	-0.55	-2.88	0.00
t-stat	(-1.77)	(-3.41)	(0.03)
β_{Trend}	0.15	-2.76	-0.81
t-stat	(0.37)	(-3.28)	(-1.35)
β_{ARP}	5.70	-0.91	3.08
t-stat	(14.05)	(-0.6)	(10.83)

This table shows estimates of the optimal investment strategy policy of a strategic investor that is infinitely long lived. We consider an investor that can allocate her wealth among stocks, bonds, the size or value factor and the one-month Treasury bill rate. The optimal portfolio rule is specified in equation (5) and optimized for a power utility function with CRRA coefficients γ=5 and a value of β=0.95, using the state variables: the detrended short-term interest rate (Tb), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the average of one-month excess stock and bond returns (ARP). We use monthly data from January 1980 to December 2010.

Table 9: Mean asset demands. In-sample results adding the size or value factors.

	Myopic CRRA=5	Hedging CRRA=5	Myopic CRRA=5	Hedging CRRA=5
S&P 500	1.21	0.23	1.10	0.42
Bonds	2.44	0.39	2.51	-1.18
Size	-0.41	0.17		
Value			0.10	0.54
Cash	-2.24	-0.79	-2.71	-0.36

This table shows the mean optimal allocation in percentage points to stocks, bonds, size or value factors and cash of a myopic investor, whose time horizon is one month, and the mean optimal allocation in percentage points to stocks, , bonds, size or value factors and cash of a strategic investor that is infinitely long lived. The optimal portfolio rule is specified in equation (5) and optimized for a power utility function with CRRA coefficients γ=5 and a value of β=0.95, using these state variables: the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the average of one-month excess stock and bond returns (ARP). We use monthly data from January 1980 to December 2010.

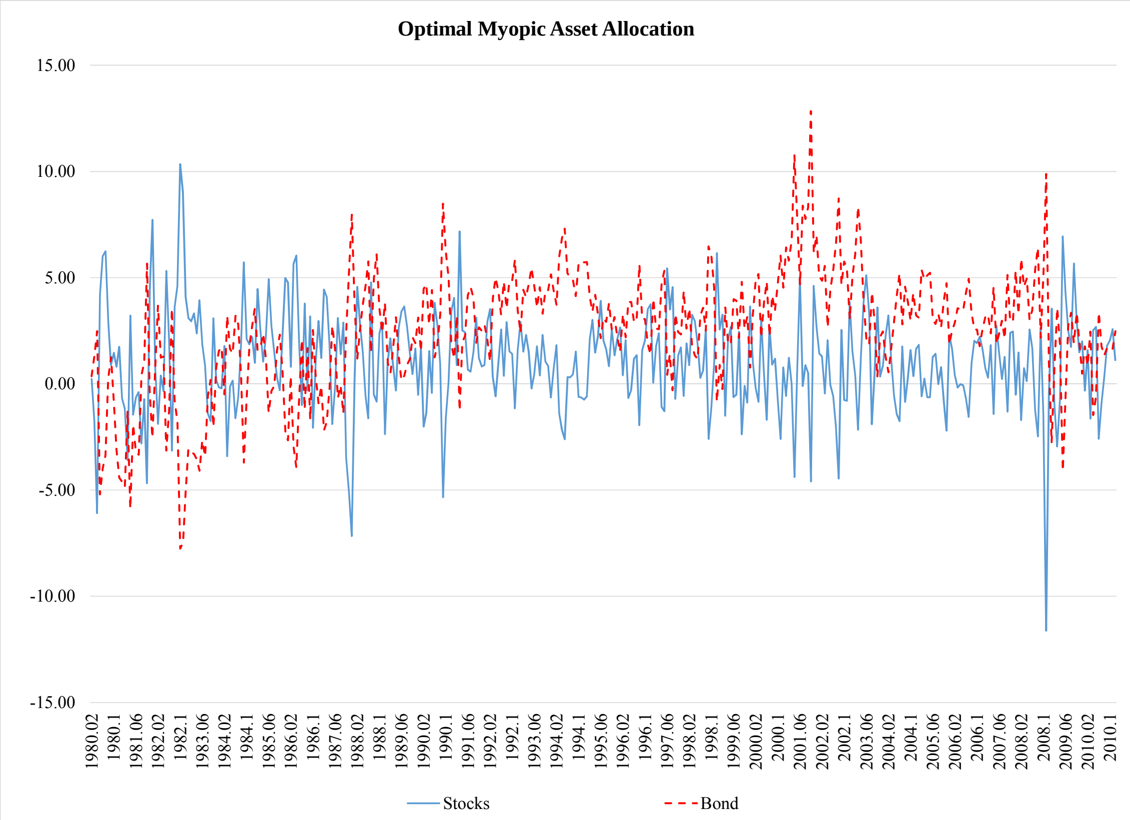

Figure 1: Optimal myopic allocation

This figure plots the optimal myopic allocation to stocks and bonds. The optimal portfolio rule is specified in equation (5) and optimized for a power utility function with a CRRA coefficient γ=5 and a value of β=0.95, using the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP) as state variables. We use monthly data from January 1980 to December 2010.

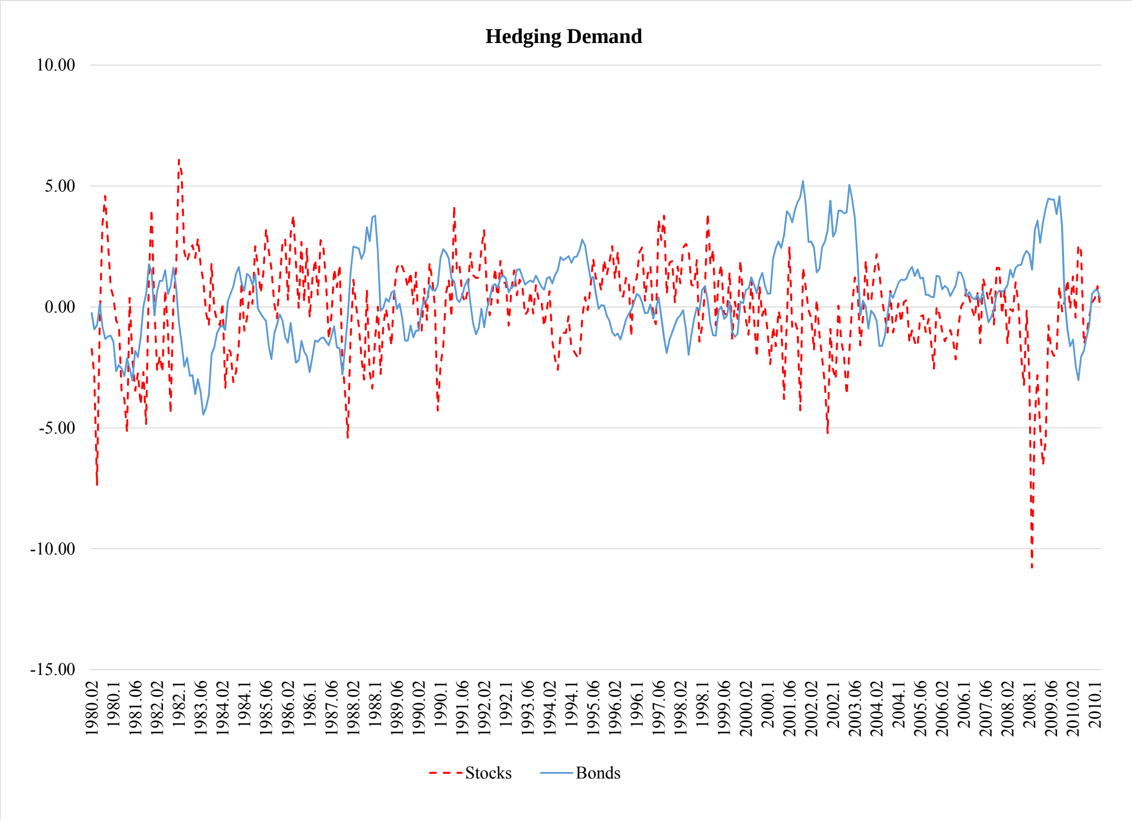

Figure 2: Optimal Hedging Demand

This figure plots the optimal hedging allocation to stocks and bonds defined as the difference between the investor strategic asset demand that is infinitely long lived and the myopic investor asset demand, whose time horizon is one month. The optimal portfolio rule is specified in equation (5) and optimized for a power utility function with a CRRA coefficient γ=5 and a value of β=0.95, using the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP) as state variables. We use monthly data from January 1980 to December 2010.

S&P 500

Figure 3: Estimated parameters for the S&P 500 index for the out-of-sample period

This figure shows the estimated parameters in the out-of-sample period from December 2000 to July 2013. The optimal portfolio is re-estimated on a yearly basis using a rolling window of data until the end of the sample. The investor uses the information available in period *t*, reflected in the values of the state variables, to estimate the dynamic weight function defining the optimal portfolio between *t* and *t*+1. The first portfolio is computed with data from January 1980 to December 2000. The second portfolio is computed with data from January 1981 to December 2001 and similarly until the end of the evaluation period. The optimal portfolio rule is specified in equation (5) and optimized for a power utility function with a CRRA coefficient γ=5 and a value of β=0.95 using the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the average of one-month excess stock and bond returns (ARP) as state variables.

G0Q0 Bond Index

Figure 4: Estimated parameters for the G0Q0 bond index for the out-of-sample period

This figure shows the estimated parameters in the out-of-sample period from December 2000 to July 2013. The optimal portfolio is re-estimated on a yearly basis using a rolling window of data until the end of the sample. The investor uses the information available in period *t*, reflected in the values of the state variables, to estimate the dynamic weight function defining the optimal portfolio between *t* and *t*+1. The first portfolio is computed with data from January 1980 to December 2000. The second portfolio is computed with data from January 1981 to December 2001 and similarly until the end of the evaluation period. The optimal portfolio rule is specified in equation (5) and optimized for a power utility function with a CRRA coefficient γ=5 and a value of β=0.95 using the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the average of one-month excess stock and bond returns (ARP) as state variables.

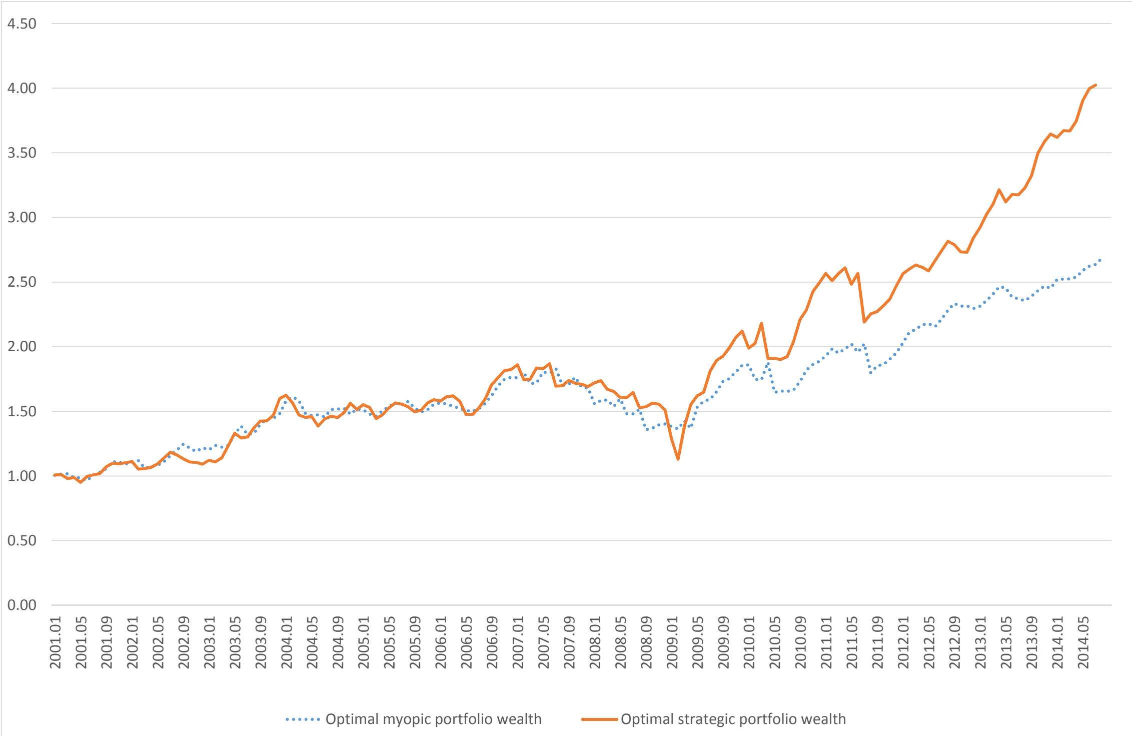

Figure 5: Out-of-sample wealth

This figure shows the out-of-sample wealth for the strategic investor and the myopic one. To evaluate the out-of-sample performance of the myopic and strategic portfolio we impose realistic portfolio constraints by ruling out short selling positions in the risky assets and setting the upper bound of portfolio weights to be 2 (Lan, 2014). The optimal portfolio is re-estimated on a yearly basis using a rolling window of data until the end of the sample. The investor uses the information available in period *t*, reflected in the values of the state variables, to estimate the dynamic weight function defining the optimal portfolio between *t* and *t+1*. The first portfolio is computed with data from January 1980 to December 2000. The optimal portfolio rule is specified in equation (5) and optimized for a power utility function with a CRRA coefficient γ=5 and a value of β=0.95 using the detrended short-term interest rate (Tb), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the average of one-month excess stock and bond returns (ARP) as state variables.

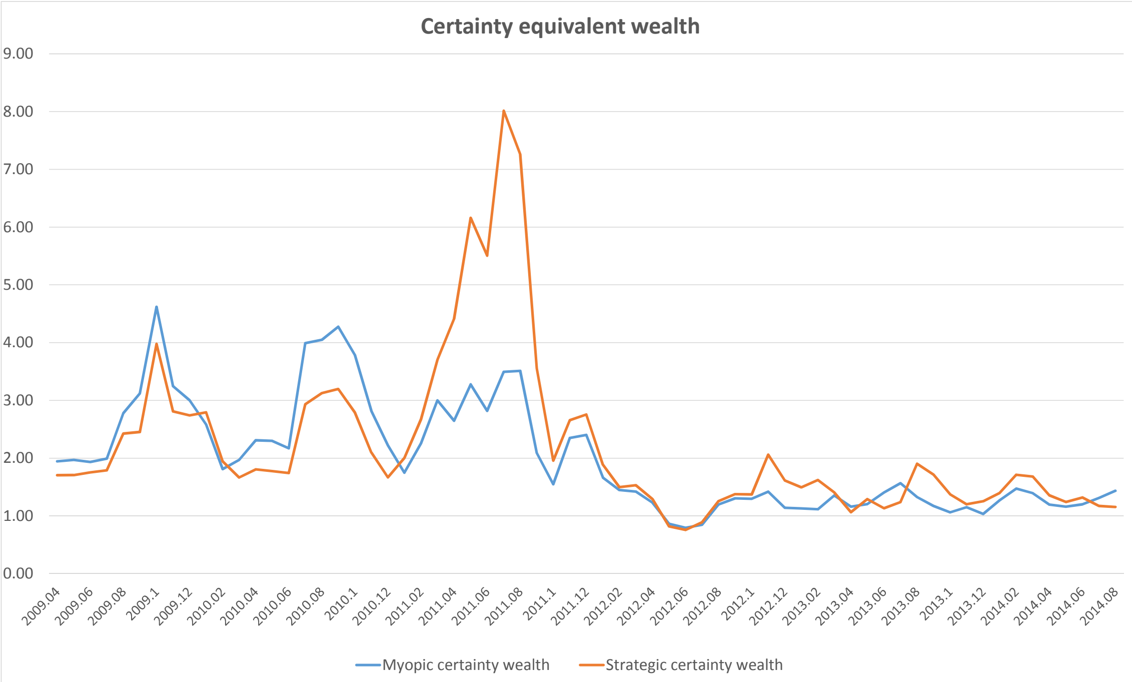

Figure 6: Out-of-sample certainty equivalent of wealth

This figure shows the out-of-sample certainty equivalent of wealth for the strategic investor and the myopic one. The out-of-sample certainty equivalent of wealth is defined as the amount of wealth such that the investor is indifferent between receiving it for sure at the horizon and having his current wealth today and invest it optimally up to the horizon.To evaluate the out-of-sample performance of the myopic and strategic portfolio we impose realistic portfolio constraints by ruling out short selling positions in the risky assets and setting the upper bound of portfolio weights to be 2 (Lan, 2014). The optimal portfolio is re-estimated on a yearly basis using a rolling window of data until the end of the sample. The investor uses the information available in period t, reflected in the values of the state variables, to estimate the dynamic weight function defining the optimal portfolio between t and $t+1$. The first portfolio is computed with data from January 1980 to December 2000. The optimal portfolio rule is specified in equation (5) and optimized for a power utility function with a CRRA coefficient γ=5 and a value of β=0.95 using the detrended short-term interest rate (Tb), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the average of one-month excess stock and bond returns (ARP) as state variables.