Transmit Antenna Subset Selection for Single and Multiuser Spatial Modulation Systems Operating in Frequency Selective Channels

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Abstract—Transmit antenna (TA) subset selection (TAS) is a well known multiple-input multiple-output (MIMO) wireless transmission scheme that requires a single RF chain. Specifically, the SM system activates only a single transmit antenna (TA) out of several available antennas in each channel use, where the choice of the active antenna is made based on the data bits to be transmitted. Furthermore, a symbol selected from a conventional signal set such as QAM/PSK is transmitted over the selected TA. Since only one TA is activated in each channel use, the SM system completely eliminates the inter-channel interference (ICI) at the receiver, thereby enabling a very low-complexity ML detection [7]. The benefits of energy efficient transmitter and the low-complexity optimal detection at the receiver have made the SM scheme an attractive candidate for the next generation wireless systems [9], [10].

Although the SM system is known to give better bit error rate (BER) performance [1]-[2] compared to the conventional MIMO schemes at low and moderate throughputs, the SM system suffers from lack of transmit diversity gain owing to the single available RF chain at the transmitter. A significant research effort was spent on improving the transmit diversity gain by employing both open and closed loop techniques. The open loop techniques mainly constitute amalgamation of space-time block coding [12] with the SM scheme. Specifically, an Alamouti code [13] aided SM scheme was conceived in [14], while a complex interleaved orthogonal design was proposed in [15]. As a further development, a SM scheme employing Alamouti STBC having a cyclic structure was proposed in [16], while a SM scheme also relying on Alamouti STBC with phases was conceived in [17]. All the aforementioned schemes achieve a transmit diversity order of two, while requiring two transmit RF chains, except for the scheme in [15], which requires a single transmit RF chain.

The closed loop techniques [18]-[26] mainly constitute modulation order and antenna subset selection schemes. Specifically, link-adaptive modulation scheme was studied in [18], while both capacity based and Euclidean distance (ED) based antenna selection (EDAS) schemes were proposed in [19] and their performances were studied under imperfect channel conditions in [20]. Furthermore, low-complexity antenna selection algorithms were proposed in [21], [22]. The transmit diversity order of EDAS was quantified in [23], while Sun et. al. [24] proposed a cross-entropy based method for reducing the search complexity of EDAS. In [25], Yang et. al. [25] proposed an improved low-complexity implementation of EDAS by striking a beneficial performance vs. complexity trade-off. Recently, Sun et. al. [26] have proposed a reduced-dimensional EDAS-equivalent criterion, which results in the

Index Terms—Antenna subset selection, diversity gain, frequency selective channel, multiuser communication.

I. INTRODUCTION

Spatial modulation (SM) [1]-[10] is a relatively new multiple-input multiple-output (MIMO) wireless transmission scheme that requires a single RF chain at the transmitter in comparison to the conventional MIMO systems [11], which require multiple RF chains. Specifically, the SM system activates only a single transmit antenna (TA) out of several available antennas in each channel use, where the choice of the active antenna is made based on the data bits to be transmitted. Furthermore, a symbol selected from a conventional signal set such as QAM/PSK is transmitted over the selected TA. Since only one TA is activated in each channel use, the SM system completely eliminates the inter-channel interference (ICI) at the receiver, thereby enabling a very low-complexity ML detection [7]. The benefits of energy efficient transmitter and the low-complexity optimal detection at the receiver have made the SM scheme an attractive candidate for the next generation wireless systems [9], [10].

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Abstract—Transmit antenna (TA) subset selection (TAS) is a well known multiple-input multiple-output technique that exploits the channel state information (CSI) at the transmitter in order to improve the attainable bit error rate (BER) performance. The extensive study of TAS in the context of spatial modulation (SM) has recently revealed that a significant performance gains are attainable compared to SM systems without TAS. However, the existing TAS techniques conceived for SM were studied by considering a frequency-flat channel, which does not represent the practical channels which are frequency-selective. In this paper, we address this hitherto-not-addressed problem by studying the TAS schemes for zero-padded single-carrier (ZP-SC) SM systems. Specifically, we employ the partial interference cancellation receiver with SIC in order to convert the frequency-selective channel into parallel sub-channels and invoke Euclidean distance based antenna subset selection (EDAS) over each of the sub-channels. This SIC aided TAS algorithm is termed as SIC-TAS. Furthermore, we show using theoretical analysis that the parallel sub-channels thus obtained are nearly identical, which enables us to employ the majority logic to obtain a single TA subset to be used in all the sub-channels. The majority logic based TAS scheme (MAJ-TAS) reduces the feedback overhead to that of frequency-flat scenario as it requires a single TA subset to be used over all the sub-channels. Furthermore, the computational burden of MAJ-TAS is further reduced by restricting the number of sub-channels over which the EDAS is invoked. This reduced complexity TAS scheme is termed as L-MAJ-TAS scheme, where L represents the number of sub-channels over which the EDAS is invoked. Furthermore, the proposed TAS schemes are extended to the multi-user scenario. All the theoretical insights are validated using simulation results. Furthermore, it is observed through numerical simulations that the proposed TAS schemes provide a significant BER performance improvement when compared to the systems without TAS. Specifically, a signal-to-noise ratio (SNR) gain as high as 3dB is observed in single user scenario and of about 1dB in case of two-user scenario while employing TAS.
same performance as that of EDAS, albeit at a reduced complexity. In [27], Naresh et al. have studied the ED based mirror activation pattern selection schemes in the context of RF-mirror aided spatial modulation systems.

Note that while the SM schemes discussed above correspond to the diversity enhancement methods meant for coherent communication, where the channel state information (CSI) is assumed to be available at the receiver, a significant amount of research effort has been also spent on non-coherent counterpart [28]. Since this paper mainly deals with the coherent SM communication, we are not delving into the details of non-coherent diversity enhancement techniques conceived for SM [29]-[35].

Against this background, the following are the contributions of this paper:

1) Owing to the high transmit diversity order attainable by EDAS [23], it has attracted significant research interest in the recent past. However, all the existing antenna subset selection schemes [19]-[26] including the EDAS have been studied only under flat-fading channel conditions, which do not reflect the real world channel conditions where the channel is frequency selective. To the best of our knowledge, the antenna subset selection problem in the context of SM systems operating in the frequency selective channel has not been studied in the existing literature. In this paper, we attempt to address this problem by considering an SM system operating in a frequency selective channel with the aid of zero-padded single carrier (ZP-SC) transmission [5]. Specifically, we employ the partial interference cancellation receiver with successive interference cancellation (PIC-R-SIC) [5] to convert the frequency selective channel into a parallel non-interfering sub-channels and invoke EDAS over each of these sub-channels. Since $K$ can be large, the feedback overhead $K \log_2 \left( \binom{N_t}{N_{SM}} \right)$ bits would be very high, where $\lceil \cdot \rceil_{2p}$ represents ceiling to the nearest integer power of two. Considering the fact that the available bandwidth in the feedback channel is very limited, it may not be feasible to convey all the $K \log_2 \left( \binom{N_t}{N_{SM}} \right)$ bits that encode the antenna subsets to be used at the transmitter. This issue is overcome by employing a majority logic based antenna subset selection without compromising on the performance, where the antenna subset that appears in the majority of the $K$ sub-channels is chosen. This antenna selection scheme is termed as the majority logic based TA subset selection (MAJ-TAS). Thanks to the channel symmetry that arises due to PIC-R-SIC, more than 90% of the parallel sub-channels are observed to yield the same optimal antenna subset under EDAS. Furthermore, owing to the identical antenna subset in a large number of sub-channels, we can further reduce the computational complexity by invoking EDAS over only a few sub-channels instead of over all the $K$ sub-channels. The reduced complexity MAJ-TAS scheme where the EDAS is invoked over only $L \ll K$ sub-channels is termed as $L$-MAJ-TAS.

2) Secondly, we propose a space-division multiple access (SDMA) aided SM system for uplink communication in a frequency selective channel, where we generalise the PIC-R-SIC decoding for mitigating both inter-channel and inter-user interference for decoding each user’s signal. This decoding algorithm is termed as the multiuser PIC-R-SIC receiver (MU-SIC). Furthermore, with the aid of MU-SIC we propose a TAS scheme for multiuser SM communication, a problem which has hitherto not been studied in the literature. More specifically, we consider the uplink communication scenario, where the base station (BS) is assumed to have $N_r$ receive antennas with equal number of RF chains and serving $U$ users each equipped with $N_t$ TAs. Each of the users is assumed to employ only $N_{SM} \leq N_t$ TAs for SM, where the TA subset at each of the users is chosen based on the information fed back from the BS.

The remainder of the paper is organized as follows. The single and multiuser SM systems operating in the frequency selective channel are described in Section II. The proposed TAS algorithms for single user SM communication are presented in Section III. In Section IV, the proposed multiuser TAS algorithm is presented. Our simulation results and discussions are discussed in Section V, while Section VI concludes the paper.

Notations: $\mathbb{C}$ and $\mathbb{R}$ represent the field of complex and real numbers, respectively. The uppercase boldface letters represent matrices and lowercase boldface letters represent vectors. The notations of $\| \cdot \|_F$ and $\| \cdot \|$ represent the Frobenious norm of a matrix and the two-norm of a vector, respectively. The notations of $(\cdot)^H$ and $(\cdot)^T$ are the Hermitian transpose and transpose of a vector/matrix, respectively, while $| \cdot |$ represents the magnitude of a complex quantity, or the cardinality of a given set. $\mathbf{H}( [c : d] : )$ represents a matrix with rows $c, c + 1, \ldots, d$ of $\mathbf{H}$ and $\mathbf{H}( [c : d] )$ is a matrix with columns $c, c + 1, \ldots, d$ of $\mathbf{H}$. span($\mathbf{A}$) represents the space spanned by the columns of $\mathbf{A}$. $\text{Tr}( \cdot )$ represents the trace of a matrix. Given a matrix $\mathbf{A}$, the projection matrix onto its column space is denoted by Proj($\mathbf{A}$). Given a set $\mathcal{A}$, the majority element in $\mathcal{A}$ is represented by maj($\mathcal{A}$). Expected value of a random variable $Y$ is denoted by $\mathbb{E}(Y)$, while the smallest non-zero Eigenvalue of matrix $\mathbf{Y}$ is denoted by $\lambda_{sm}(\mathbf{Y})$. Furthermore, $Q(x)$ represents the tail probability of standard normal distribution given by $\frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp \left( -\frac{u^2}{2} \right) du$. A circularly symmetric complex-valued Gaussian distribution with mean $\mu$ and variance $\sigma^2$ is represented by $\mathcal{CN}(\mu, \sigma^2)$.

II. System Model

In this section, we briefly describe the single and multiuser SM system models operating with the aid of TAS in a frequency selective channel.

A. Single User SM System with TAS

Consider a MIMO system having $N_r$ receive and $N_t$ TAs, and operating in a quasi-static frequency-selective fading channel having $P$ resolvable paths between each of the transmit and receive antenna pairs. Let the number of antennas used for
SM be $N_{SM} \leq N_t$. The received signal vector corresponding to the $i^{th}$ channel use is given by
\[
y_i = \sum_{j=0}^{P-1} H_j x_{i-j} + n_i,
\]
where $y_j \in \mathbb{C}^{N_r \times 1}$ and $x_j \in \mathbb{C}^{N_t \times 1}$ are the received and transmitted vectors in the $j^{th}$ channel use, $H_k \in \mathbb{C}^{N_r \times N_t}$ is the $k^{th}$ multipath channel matrix, and $n_j \in \mathbb{C}^{N_r \times 1}$ is the noise vector in the $j^{th}$ channel use. The entries of the multipath channel matrix $H_k$ are from $\mathcal{CN}(0, \omega_0 \beta^k)$ such that $0 \leq \beta \leq 1$ and $\sum_{k=0}^{P-1} \omega_0 \beta^k = 1$, and those of noise vector are from $\mathcal{CN}(0, \sigma^2)$ where $\sigma^2$ is the noise variance per complex dimension and taken to be $\sigma^2 = \frac{1}{\rho}$ in order to ensure that the average received SNR at each receive antenna is $\rho$. Furthermore, we assume that $x_k$ has unit average energy, i.e. the input signal constellation such as PSK/QAM is normalized to have unit energy.

Assuming that each data frame is prefixed with $(P-1)$ zeros and the $K$ transmission symbols constituting a single data frame, we have $q^{th}$ zero-padded data frame given by
\[
\begin{bmatrix}
0, 0, \ldots, \hat{0} \times K \text{ vectors} \\
\hat{x}_K, \hat{x}_{K-1}, \ldots, \hat{x}_1, 0, \ldots, 0
\end{bmatrix}
\]
where $\hat{0}$ denotes an all-zero vector of size $N_t \times 1$. The $q^{th}$ received data frame is given by (3), where $K' = K + P - 1$ and $\hat{0}$ represents a $(N_r \times N_t)$-element zero matrix. Since the number of multipaths is $P$, $(P-1)$ length zero-padding ensures that the successive data frames do not suffer from inter frame interference. In the SM scheme, each transmitted vector is given by [1]
\[
\hat{x}_k = \begin{bmatrix}
0, \ldots, 0, s_k, 0, \ldots, 0
\end{bmatrix}^T \in \mathbb{C}^{N_r \times 1},
\]
where $s_k$ is a complex symbol from the signal set $\mathcal{S}$ having $|\mathcal{S}| = M$ and $l_k \in A_k \subset \mathcal{A} = \{1\}^{N_t}$. where $A_k$ represents the TA subset chosen for SM in the $k^{th}$ channel use. Thus, each $x_k$ may assume $N_{SM} M$ different values, since $|A_k| = N_{SM}$ for $0 \leq k \leq K - 1$. The corresponding set of transmit vectors is given by $\mathcal{X}_k = \{\hat{x}_k \mid l_k \in A_k, s_k \in \mathcal{S}\}$.

### B. Multisuser SM System with TAS

Consider the uplink communication scenario where the BS is assumed to have $N_r$ receive antennas with equal number of RF chains, and serving $U$ users simultaneously with aid of zero padded single carrier transmissions. Since all the $U$ users are served simultaneously by the BS, each user’s signal experiences interference from other users’ signal at the BS. Analogous to (3), the data frame received by the BS is given by
\[
\hat{y} = \sum_{j=1}^{U} \hat{H}^{(j)} \hat{x}^{(j)} + \hat{n},
\]
where $\hat{H}^{(j)}$ represents the block Toeplitz channel matrix between the BS and the $j^{th}$ user and $\hat{x}^{(j)} \in \mathbb{C}^{K N_t}$ is the data stream of the $j^{th}$ user. Analogous to the single user case, the transmit vector of the $j^{th}$ user is given by
\[
\hat{x}_k^{(j)} = \begin{bmatrix}
0, \ldots, 0, s_k^{(j)}, 0, \ldots, 0
\end{bmatrix}^T \in \mathbb{C}^{N_r \times 1},
\]
where $s_k^{(j)}$ is the $j^{th}$ user’s complex symbol chosen from the signal set $\mathcal{S}$ and $l_k^{(j)} \in A_k^{(j)} \subset \mathcal{A} = \{1\}^{N_t}$, where $A_k^{(j)}$ represents the $j^{th}$ user’s TA subset chosen for SM in the $k^{th}$ channel use. The $j^{th}$ user’s set of transmit vectors for the $k^{th}$ channel use is given by $\mathcal{X}_k^{(j)} = \{\hat{x}_k^{(j)} \mid l_k^{(j)} \in A_k, s_k^{(j)} \in \mathcal{S}\}$.

### III. PROPOSED TAS ALGORITHMS FOR SM SYSTEM IN FREQUENCY SELECTIVE CHANNEL

In this section, we present various TAS algorithms proposed for SM system operating in the frequency selective channel.

#### A. Proposed PIC-R-SIC aided TAS (SIC-TAS) Algorithm

We first convert the system in (3) into a set of non-interfering parallel channels by employing PIC-R [5], whose details are given as follows. Let
\[
I_k = \{N_t i + 1, N_t i + 2, \ldots, N_t (i+1)\},
\]
where $\cup_{i=0}^{K-1} I_i = \mathbb{I} = \{i\}^{N_t}$ and $I_k$ be the matrix having columns of $\mathcal{H}$ that are indexed by the elements of $I_k$. The system in (3) can be written as
\[
\hat{y} = \sum_{i=0}^{K-1} G_k \hat{x}_i + \hat{n},
\]
where $\hat{x}_i = \hat{x}(\{i N_t + 1 : (i+1) N_t\}) \in \mathbb{C}^{N_r}$, for $0 \leq i \leq K - 1$.

Let $G_k = [G_{k0}, G_{k1}, \ldots, G_{k i}, G_{k i+1}, \ldots, G_{k K-1}]$. The matrix projecting onto to the orthogonal complement space of $G_k^c$ is given by $P_{k} = \mathbb{I}_{N_r(K+P-1)} - Q_{k}$, where $Q_{k} = G_k^c (G_k^c)^H G_k^c)^{-1} (G_k^c)^H$. Thus, we have $P_k G_k = 0$ for $i \in \{j\} \setminus k$. Consider $z_{k} = P_k \hat{y}$ given by
\[
z_{k} = P_k \sum_{i=1}^{K-1} G_k \hat{x}_i + P_k \hat{n},
\]
\[
= P_k G_k \hat{x}_k + P_k \hat{n}.
\]
The PIC-R solution for the $k^{th}$ subsystem is given by
\[
\hat{x}_k^{PIC-R} = \arg \min_{x \in \mathcal{X}_k} \|z_k - P_k G_k x\|^2,
\]
for $0 \leq k \leq K - 1$, where $\hat{x}_k$ represents the set of transmit vectors chosen for the $k^{th}$ channel use depending on $A_k$.

**Proposition 1:** The PIC-R solution in (10) achieves a diversity gain of $N_r P (N_t - N_{SM} + 1)$ when $\hat{x}_k$, $0 \leq k \leq K - 1$ are chosen based on EDAS [19].

**Proof:** The proof directly follows from Proposition 3 [5] and Proposition 2 [23].

The EDAS [23] based TA subset is given by
\[
A_k = \arg \max_{\mathcal{A} \subset \mathcal{A}, \mathcal{A} \neq \mathcal{A}_k} \min_{\mathcal{X}_k \neq \mathcal{X}_2} \|R_k(\mathcal{A}) (x_1 - x_2)\|^2,
\]
where $R_k = P_k^c G_k^c x_1$ and $x_2$ are $N_{SM} \times 1$ SM vectors whose non-zero elements are drawn from $S$. Note that
although the Proposition 1 guarantees high transmit diversity gain, the overhead involved in computing the optimal antenna subsets (11) and conveying that information to the transmitter is enormous. Specifically, we have the following major issues when invoking EDAS over all the $K$ sub-channels:

- Since EDAS is invoked over each of the $K$ sub-channels, the $K$ antenna subsets require $K \log_2 \left( \left( \frac{N_x}{N_{s,t}} \right)^2 \right)$ bits for conveying the chosen antenna subsets over the feedback channel to the transmitter. This may impose a significant overhead in the feedback channel owing to the limited bandwidth available;
- Secondly, the computational complexity involved in employing EDAS over each of the sub-channels can be quite burdensome, especially when $K$ is very large.

The above issues can be overcome by the proposed SIC-TAS algorithm (Algorithm 1) which employs PIC-R-SIC [5].

**Algorithm 1 SIC-TAS for the ZP-SC SM system**

**Require:** $k = 0, \mathcal{A}_{SIC} = \{\cdot\}$.

while $k < K$ do

1. $G^c_k = [G_{I_k+1}, \ldots, G_{I_{K-1}}]$, $Q_{I_k} = G^c_k \left( (G^c_k)^H G^c_k \right)^{-1} (G^c_k)^H$, $P_{I_k} = I_{N_p(K+P-1)} - Q_{I_k}$, $R_k = P_{I_k} G_{I_k}$.

2. Obtain

$$ \mathcal{A}_k = \arg \max_{|A'| = N_{M,s}} \min_{x_1 \neq x_2} \| R_k (:, A') (x_1 - x_2) \|^2, $$

$\mathcal{A}_{SIC} \leftarrow \mathcal{A}_{SIC} \cup \mathcal{A}_k$, $k \leftarrow k + 1$.

end while

return $\mathcal{A}_{SIC}$

Note that the matrix computations in step 1 of Algorithm 1 can be significantly reduced by employing the results from [36]. However, the computational complexity involved in step 2 is quite high even when considering the latest low-complexity EDAS solutions of [26]. In order to reduce the computational burden involved in step 2, it is important to develop insight into the set of antenna subsets $\mathcal{A}_{SIC}$.

Note that in case of PIC-R-SIC, the transmit vectors are decoded and cancelled from the received signal in the order $x_0, x_1, \ldots, x_{K-1}$ by taking $G^c_k = [G_{I_{k+1}}, \ldots, G_{I_{K-1}}]$ for $0 \leq k \leq K - 1$. In order to gain insight into the nature of parallel sub-channels in PIC-R-SIC, let us partition $G^c_k$ into $[J_k, W_k]$, where $J_k = [G_{I_{k+1}}, \ldots, G_{I_{K-1}}]$ corresponds to the sub-channels that interfere with $G_{I_k}$ and $W_k = [G_{I_{k+1}}, \ldots, G_{I_{K-1}}]$ corresponds to the sub-channels that are orthogonal to $G_{I_k}$. The following proposition throws light on the interfering and non-interfering sub-channels in PIC-R-SIC.

**Proposition 2:** In the PIC-R-SIC detection with the decoding order mentioned above, each of the parallel sub-channels $P_{I_k} G_{I_k} \equiv [I_{N_x(K+P-1)} - \text{Proj}(P_{I_{k+P-1}}, J_k)] G_{I_k}$. (12)

**Proof:** Recall that $P_{I_k} = I_{N_x(K+P-1)} - Q_{I_k}$, where

$Q_{I_k} = G^c_k \left( (G^c_k)^H G^c_k \right)^{-1} (G^c_k)^H$. Taking $G^c_k = [J_k, W_k]$, we have

$$ Q_{I_k} = [J_k, W_k] \left[ J_k^H J_k \quad J_k^H W_k \quad W_k^H W_k \right]^{-1} \left[ J_k^H \quad W_k^H \right]. $$

By the block matrix inversion [37], we have

$$ \left[ J_k^H J_k \quad J_k^H W_k \quad W_k^H W_k \right]^{-1} = \left[ \begin{array}{ccc} A & B & C \\ W_k^H J_k & W_k^H W_k & C \\ \end{array} \right], $$

where $A$, $B$, $C$ and $D$ are given by (15)-(18). Since $G_{I_k} \perp W_k$, we have

$$ Q_{I_k} G_{I_k} = [J_k, W_k] \left[ \begin{array}{ccc} A & B & C \\ W_k^H J_k & W_k^H W_k & C \\ \end{array} \right] \left[ J_k^H G_{I_k} \quad O \right], $$

$$ = J_k A J_k^H G_{I_k} + W_k C J_k^H G_{I_k}, $$

$$ = J_k A J_k^H G_{I_k} - W_k W_k^H W_k^H W_k \left[ J_k \right], $$

$$ = \left[ I - W_k W_k^H W_k^H \right] J_k A J_k^H G_{I_k}, $$

$$ = P_{I_{k+P-1}} J_k A J_k^H G_{I_k}. $$

Furthermore, we have

$$ A = \left[ J_k^H J_k - J_k^H W_k W_k^H W_k \right], $$

$$ = \left[ J_k^H \left( I - W_k W_k^H W_k^H \right) \right], $$

$$ = \left[ J_k^H P_{I_{k+P-1}} J_k \right]. $
This concludes the proof.

A pictorial depiction of the interfering and non-interfering subspaces is given in Fig. 1. Note that when $J_k = \{1\}$, then each of the parallel sub-channels do not experience any interference. This corresponds to the case where $P = 1$. As a result, the $A_{SIC}$ will have only one distinct element. However, in the presence of interference where $J_k \neq \{1\}$, it is not clear whether $A_{SIC}$ has a single distinct element or several distinct elements. The following discussion throws light on this scenario.

Let us now study the variation in sub-channels by considering $P_{z_k}G_{T_k} = [I_{N_r}(K + p - 1) - \text{Proj}(P_{z_k+p-1}J_k)]G_{T_k}$ and $P_{z_k}G_{T_k+1} = [I_{N_r}(K + p - 1) - \text{Proj}(P_{z_k+p}J_{k+1})]G_{T_k+1}$. Before proceeding further, let us define the following matrices.

Let $E = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \in \mathbb{R}^{K+P-1 \times K+P-1}$ (31)

and $M = E \otimes I_{N_r}$. Note that the interfering sub-channels in case of $P_{z_k+1}, G_{T_k+1}$ i.e. $J_{k+1}$ is a shifted version of $J_k$. That is, $M_{J_k} = J_{k+1}$ for $k \leq K - P + 1$. A similar statement holds with regard to $P_{z_k+p-1}$ and $P_{z_k+p}$, when $K$ is large.

The following proposition makes use of these facts to show that the parallel sub-channels in PIC-R-SIC are essentially the same except for the shift, when $K$ is large.

**Proposition 3:** In the PIC-R-SIC detection with the decoding order mentioned earlier, we have

$$\lim_{K \to \infty} \begin{bmatrix} P_{z_k}G_{T_k} - M^TP_{z_k+1}G_{T_k+1} \end{bmatrix} = 0. \quad (32)$$

**Proof:** Since $M_{G_{T_k}} = G_{T_k+1}$, we have

$$\Delta_k = P_{z_k}G_{T_k} - M^TP_{z_k+1}G_{T_k+1} = P_{z_k}G_{T_k} - M^TP_{z_k+1}M_{G_{T_k}} = P_{z_k} - M^TP_{z_k+1}M_{G_{T_k}}. \quad (33)$$

Furthermore, we have

$$P_{z_k} - M^TP_{z_k+1}M = (I - M^TM) + (M^TQ_{T_k+1}M - Q_{T_k}). \quad (36)$$

Since $M^TM \to I$ and $M^TG_{T_k} \to G_{T_k}^c$ as $K \to \infty$, we have $(P_{z_k} - M^TP_{z_k+1}M) \to O$. This concludes the proof.

We can infer from Proposition 3 that the parallel sub-channels are essentially the same when $K$ is large and hence we can expect only a few distinct TA subsets in $A_{SIC}$ when employing Algorithm 1. We now analyse Algorithm 1 by considering several statistical metrics and validate the theoretical insights with the aid of numerical studies.

Let $D$ represent the number of distinct elements in $A_{SIC}$, where $1 \leq D \leq \binom{N_r}{K+P-1}$ and $P_{D}(l)$ represent the percentage of occurrence of $l$th majority elements in $A_{SIC}$. That is, $P_{D}(l)$ represents the percentage of occurrence of the majority elements in $A_{SIC}$ and $P_{D}(2)$ represents the the percentage of occurrence of the second largest majority elements, so on and so forth. We study

- the cumulative distribution function (CDF) of $D$, i.e. $F_D(d) = Pr(D \leq d)$, which enables us to quantify the number of distinct elements in the set $A_{SIC}$.
- the average $P_{D}(l)$ for $l = 1, 2, \ldots$ which enables us to understand the distribution of the size of the majority sets.
in $A_{SIC}$, both computed using Monte Carlo simulations.

![Graph](image1)

**Fig. 2.** Plot (a) depicts the CDF of $D$ for various values of channel decay factor $\beta$ in a ZP-SC SM system having $N_t = N_r = 6$, $N_{SM} = 4$, $K = 8$, $P = 4$ and employing SIC-TAS with 32-QAM, while Plot (b) corresponds to the variation of $P_D(l)$ in the aforementioned system.

![Graph](image2)

**Fig. 3.** Plot (a) depicts the CDF of $D$ for various values of data frame length $K$ in a ZP-SC SM system having $N_t = N_r = 6$, $N_{SM} = 4$, $K = 8$ and employing SIC-TAS with 32-QAM, while Plot (b) corresponds to the variation of $P_D(l)$ in the aforementioned system.

Consider a ZP-SC SM system having $N_t = N_r = 6$, $N_{SM} = 4$, $K = 8$ and employing SIC-TAS with 32-QAM signal set. Fig. 2(a) depicts the variation in $F_D(d)$ for various values of $\beta \in \{0.2, 0.4, 0.8\}$, whereas Fig. 2(b) corresponds to the variation in $P_D(l)$. The three values of $\beta$ considered above correspond to the fast, moderate and slow channel decay scenarios, respectively. It is evident from Fig. 2(a) that for any value of $\beta$, we have $F_D(d) = Pr(D \leq d) \approx 1$ for $d$ as small as four, which means no more than four distinct TA subsets occur on average in the set $A_{SIC}$. Furthermore, from Fig. 2(b) we notice that a single antenna subset constitutes at least 75% of the antenna subsets in $A_{SIC}$ independent of the channel decay factor $\beta$, thanks to the nearly same parallel sub-channels due to PIC-R-SIC. It can be observed from Fig. 2(b) that for the moderate value of $\beta = 0.4$, the majority set constitutes 84% where as the second majority set constitutes less than 15%.

Figure 3 depicts the variation in $F_D(d)$ and $P_D(l)$ with respect to $K$, where the channel decay factor is fixed to be $\beta = 0.4$. It is evident from Fig. 3(a) that the number of distinct sets remains almost the same for any value of $K$. Fig. 3(b) shows that the size of the majority set grows with $K$, as predicted by Proposition 3. Specifically, when $K = 32$ the the majority set constitutes 94% of the set $A_{SIC}$, while the second majority set constitutes less than 5%. Thus, by having a sufficiently large $K$, we can ensure that a single TA subset constitutes nearly the entire $A_{SIC}$.

The above findings motivate us to consider a majority based TA subset selection, which enables us to have a single antenna subset for all the $K$ channel uses and in turn utilise only $\log_2(C(\frac{N_r}{N_{SM}})_2\nu)$ bits in the feedback channel, which reduces the feedback overhead in the frequency selective scenario to that of the flat-fading scenario. In case of majority based TA subset selection, the antenna subset to be used in all the $K$ channel uses of the data frame is given by

$$A_{MAJ-TAS} = \text{maj}(A_{SIC}).$$

This selection scheme is referred to as MAJ-TAS. Although the MAJ-TAS scheme reduces the feedback overhead, it still requires $A_{SIC}$ to be computed, which is computationally expensive. In order to alleviate this issue, we need to understand the variation in $Pr(A_k \neq A_{MAJ-TAS})$ over $0 \leq k \leq K - 1$, so that we can avoid computing the entire set $A_{SIC}$.

Specifically, we look for a subset $K \subset \{i\}_{i=0}^{K-1}$ such that $Pr(A_k \neq A_{MAJ-TAS}) \leq \epsilon$ for $k \in K$ for a small positive $\epsilon$.

![Graph](image3)

**Fig. 4.** Plot (a)-(c) depict the $Pr(A_k \neq A_{MAJ-TAS})$ for various values of $K$ in a ZP-SC SM system having $N_t = N_r = 6$, $N_{SM} = 4$, $\beta = 0.4$, $P = 4$ and employing SIC-TAS with 32-QAM.

Figure 4 depicts the $Pr(A_k \neq A_{MAJ-TAS})$ for various values of $K$. It is evident from Fig. 4(a)-(c) that the $Pr(A_k \neq A_{MAJ-TAS})$ is very small for $k < K/2$ and slowly increases as $k$ approaches $K - 1$. Thus, by taking $K = \{K/2 - i\}_{i=1}^{L}$
we propose a low-complexity MAJ-TAS scheme termed as L-MAJ-TAS:

$$A_{L-MAJ-TAS} = \text{maj}\{A_{K/2-L}, \ldots, A_{K/2-2}, A_{K/2-1}\}.$$  

(38)

Note that (38) requires (11) to be computed only $L$ times, whereas (37) requires (11) to be computed $K$ times. Furthermore, it is evident from Fig. 4 that as $K$ becomes large the $Pr(A_k \neq A_{MAJ-TAS})$ drops to zero, which further validates Proposition 3. We show in Section IV that $L = 1$ would be sufficient to attain the same performance as that of the SIC-TAS scheme with $L = K$.

B. Proposed MU-TAS Algorithm

We first generalise the PIC-R-SIC [5] to multiuser scenario to obtain MU-SIC, which enables us to mitigate both inter-user and inter-channel interferences. The proposed MU-SIC is presented in Algorithm 2.

Algorithm 2 MU-SIC for the multiuser ZP-SC SM system

Require: $k = 0, u = 1$.

while $u \leq U$ do

T\(^{(u)}\) = $[\tilde{H}^{(u+1)}, \tilde{H}^{(u+2)}, \ldots, \tilde{H}^{(U)}]$

while $k < K$ do

1. $G_{I_k}^{(u)} = [G_{I_{k+1}}^{(u)}, \ldots, G_{I_{K-1}}^{(u)}, T^{(u)}]$, 

$Q_{I_k}^{(u)} = G_{I_k}^{(u)}(G_{I_k}^{(u)})^{-1}H$, 

$P_{I_k}^{(u)} = I_{N_t}(K+P-1) - Q_{I_k}^{(u)}$, 

2. Obtain

$z_{I_k}^{(u)} = P_{I_k}^{(u)}\hat{y}$,

$\hat{x}_k^{(u)} = \arg \min_{x \in \mathcal{A}_k^{(u)}} \|z_{I_k}^{(u)} - P_{I_k}^{(u)}G_{I_k}^{(u)}x\|^2$,

$\hat{y} \leftarrow \hat{y} - G_{I_k}^{(u)}\hat{x}_k^{(u)}$, 

$k \leftarrow k + 1$.

end while

$u \leftarrow u + 1$.

end while

Note that the Algorithm 2 is similar to PIC-R-SIC [5], except that in addition to the inter-channel interference, the inter-user interference is also taken into account, which is captured by $T^{(u)}$ in Algorithm 2. Furthermore, the MU-SIC is feasible only when the number of receive antennas at the BS is sufficiently large. That is, we need to have $UK\hat{N}_t \geq N_r(K + P - 1)$ in order to employ the MU-SIC. In other words, when $K$ is large we need to ensure that $N_r \geq U\hat{N}_t$ in order to support $U$ users. Note that this may not be an issue since the BS can be equipped with massive number of antennas in contrast to the users’ equipment, which have limited form factor.

The proposed MU-TAS is presented in Algorithm 3, which exploits the MU-SIC in order to arrive at the optimal antenna subsets $(A_{SIC})^{(u)}$ for each of the users. Note that when $U = 1$, the MU-TAS presented in Algorithm 3 reduces to SIC-TAS presented in Algorithm 1.

Let us now study the signal and interference spaces in the case of multiple users. Let $k \gg P$, $U^{(j)}_{k} = [G_{I_k}^{(j)}, \ldots, G_{I_{K-1}}^{(j)}, G_{I_{K+1}}^{(j)}, \ldots, G_{I_{K+P-1}}^{(j)}]$ represent the inter-user interference due to $j$th user in the $k$th channel use. In order to gain insight into the nature of parallel sub-channels in MU-TAS in Algorithm 3, let us partition $G_{I_k}^{(u)}$ into $[J_{k}^{(u)}, W_{k}^{(u)}]$, where

$$J_{k} = [G_{I_{k+1}}^{(u)}, \ldots, G_{I_{K+P-1}}^{(u)}, j' = u+1]U^{(j)}_{k}$$

represents the sub-channels that interfere with $G_{I_k}^{(u)}$ due to inter-channel and inter-user interference, and

$$W_{k} = [G_{I_{k+1}}^{(u)}, \ldots, G_{I_{K+P-1}}^{(u)}, j' = 1]U^{(j)}_{k}$$

corresponds to the sub-channels that are orthogonal to $G_{I_k}^{(u)}$. Proceeding along the lines of Proposition 2, it can be shown that

$$P_{I_k}^{(u)}G_{I_k}^{(u)} = [I - \text{Proj}(I - \text{Proj}(W_{k}^{(u)})))J_{k}^{(u)}]G_{I_k}^{(u)}.$$  

(39)

For any given $u$, we have $J_{k}^{(u)} = J_{k+1}^{(u)}$, $W_{k}^{(u)} = W_{k+1}^{(u)}$, and $G_{I_k}^{(u)} = G_{I_{k+1}}^{(u)}$ except for the shift when $K \rightarrow \infty$. As a result, we have $P_{I_k}^{(u)}G_{I_k}^{(u)} \rightarrow M^T P_{I_k}^{(u)}G_{I_k}^{(u)}$ when $K \rightarrow \infty$ and $k \gg P$. Thus, the parallel sub-channels are nearly the same even in case of multi-user scenario when $K$ is sufficiently large. Let us now validate the above observations by considering a MU-TAS aided ZP-SC SM system serving two users ($U = 2$), where both users’ signals are assumed to have the same received power at the BS. Let the number of receive antennas at the BS is $N_r = 6$ and the number of TAS at each of the two users be $N_l = 3$ and $N_{SM} = 2$. Let both the users be assumed to be employing 32-QAM and operating in a frequency selective channel having $\beta = 0.4$ and $P = 4$ with the aid of ZP-SC SM system with $K = 8$ and employing MU-TAS. Note that only the first user suffers from the inter-user interference, while the second user does not owing to the SIC. Thus, we explicitly study the first user’s $F_D(d) = Pr(D \leq d)$ and $P_D(l)$. Figure 5 depicts the variation in $F_D(d)$ and $P_D(l)$ with respect to $K$ in the first user’s case. It is evident from Fig. 5(a) that the number
of distinct sets remains almost the same for any value of \( K \). Fig. 5(b) shows that the size of the majority set grows with \( K \), which is in accordance with the above analysis. Specifically, when \( K = 32 \) the majority set constitutes 95\% of the set \( A_{SIC}^{(1)} \), while the second majority set constitutes less than 5\%. Thus, by having a sufficiently large \( K \), we can ensure that a single TA subset constitutes nearly the entire \( A_{SIC}^{(1)} \). Hence, analogous to the single user scenario, we may have

\[
A_{MAJ-TAS}^{(u)} = \text{maj}(A_{SIC}^{(u)}),
\]

which is termed as MAJ-MU-TAS.

![CDF of D for different K](image)

Fig. 5. Plot(a) depicts the CDF of \( D \) for various values of data frame length \( K \) of the first user in a two-user \((U=2)\) ZP-SC SM system having \( N_r = 6, N_t = 3, N_{SM} = 2 \), operating in a frequency selective channel having \( \beta = 0.4, P = 4 \) and employing MU-TAS with 32-QAM, while the Plot(b) corresponds to the variation of \( P_{D}(l) \) in the aforementioned system.

![Average P D](image)

![Average P D](image)

![Average P D](image)

Fig. 6. Plot(a)-(c) depict the \( \Pr(A_k^{(u)} \neq A_{MAJ-TAS}^{(u)}) \) for various values of \( K \) of the first user in a two-user \((U=2)\) ZP-SC SM system having \( N_r = 6, N_t = 3, N_{SM} = 2 \), operating in a frequency selective channel having \( \beta = 0.4, P = 4 \) and employing MU-TAS with 32-QAM signal set.

Figure 6 depicts the \( \Pr(A_k^{(u)} \neq A_{MAJ-TAS}^{(u)}) \) for various values of \( K \). It is evident from Fig. 6(a)-(c) that the \( \Pr(A_k^{(u)} \neq A_{MAJ-TAS}^{(u)}) \) is very small for \( k \approx K/2 \) and slowly increases as \( k \) deviates from \( K/2 \). Thus, by taking \( K = \{K/2 - i\}_{i=1}^L \), we propose a low-complexity MAJ-TAS scheme for multi-user scenario:

\[
A_{L-MAJ-TAS}^{(u)} = \text{maj}\{A_{K/2-L}^{(u)} \cdots A_{K/2-2}^{(u)} A_{K/2-1}^{(u)} \},
\]

which is termed as L-MAJ-MU-TAS. Analogous to the single user scenario, we demonstrate in Section IV that \( L = 1 \) would be sufficient to attain the same performance as that of the MU-TAS with \( L = K \). In the next section, we study the BER performance of various proposed TAS schemes.

IV. SIMULATION RESULTS AND DISCUSSIONS

**Simulation scenario:** In all our simulations, we have employed at least \( 10^{l+1} \) bits for evaluating a bit error rate (BER) of \( 10^l \). All the simulation results are obtained by considering a frequency selective channel having \( \beta = 0.4 \) and \( P = 4 \), and the ZP-SC SM system is assumed to be operating with a frame length of \( K = 8 \). The receiver is assumed to have perfect CSI in all the detection algorithms considered.

**A. Performance of TAS Schemes in Single User Scenario**

Consider a ZP-SC SM system employing SIC-TAS algorithm in conjunction with 32- and 64-QAM signal sets. Let us consider two system configurations: a) \( N_r = N_t = 4 \) and \( N_{SM} = 2 \); and b) \( N_r = N_t = 6 \) and \( N_{SM} = 4 \). Figure 7 compares the BER performance of the aforementioned system in both the configurations against their counterparts where no TAS is employed. Specifically, Fig. 7(a) and Fig. 7(b) compare the BER performance in the configurations a) and b), respectively. The observations from Fig. 7 are listed as follows:

- In both the configurations, the system employing SIC-TAS attains a better BER performance than that which does not. Specifically, at a BER of \( 10^{-4} \) there is an SNR gain of about 3dB in case of 32-QAM and of about 2dB in case of 64-QAM in configuration a). Furthermore, there is an SNR gain of about 1dB in case of both 32- and 64-QAM signal sets in configuration b).
- The gain in the SNR due to TAS diminishes as the size of the signal set is increased. This is expected since the minimum ED in receive signal space diminishes as the constellation size is increased.

Figure 8 compares the various TAS schemes proposed in the paper. Namely, the SIC-TAS, MAJ-TAS, and \( L \)-MAJ-TAS with \( L = 1 \) are compared in both the configurations considered above. It is evident from Fig. 8(a) and Fig. 8(b) that the MAJ-TAS and \( 1 \)-MAJ-TAS attain the same performance as that attained by the SIC-TAS. Thus, the low-complexity version of the SIC-TAS i.e. the \( 1 \)-MAJ-TAS is sufficient for attaining the performance gains guaranteed by TA subset selection.

Let us now compare the BER performance of \( 1 \)-MAJ-TAS scheme under various equalization algorithms. Figure 9
Fig. 7. Comparison of the BER performance in the ZP-SC SM system having $K = 8$ and employing SIC-TAS. When employing SIC-TAS, $N_r = N_t = 4$, $N_{SM} = 2$ are used in conjunction with 32- and 64-QAM signal sets which correspond to Plot(a), and the Plot(b) corresponds to $N_r = N_t = 6$, $N_{SM} = 4$ case.

Fig. 8. Comparison of the BER performance in the ZP-SC SM system having $K = 8$ and employing SIC-TAS, MAJ-TAS and L-MAJ-TAS with $L = 1$ (1-MAJ-TAS). The Plot(a) corresponds to the case where $N_t = N_r = 4$ and $N_{SM} = 2$, while the Plot(b) corresponds to the case where $N_t = N_r = 6$ and $N_{SM} = 4$.

Fig. 9. Comparison of the BER performance in the ZP-SC SM system having $K = 8$ and employing PIC-R-SIC, PIC-R, ZF and MMSE detectors. The Plot(a) corresponds to the case where $N_t = N_r = 4$ and $N_{SM} = 2$, while the Plot(b) corresponds to the case where $N_t = N_r = 6$ and $N_{SM} = 4$. Both the systems are assumed to be employing $L$-MAJ-TAS with $L = 1$ (1-MAJ-TAS).

set. Let $N_r = 6$, $N_t = 3$ or 2 and $N_{SM} = 2$. The case where $N_t = 2$ corresponds to No TAS scenario and the case where $N_t = 3$ corresponds to the MU-TAS scenario. Figure 10 compares the attainable BER performance of both the users in No TAS and MU-TAS scenarios when employing 64-QAM signal set. It is evident from Fig. 10 that the MU-TAS outperforms the NO TAS scenario in case of both the users. Specifically, an SNR gain of about 1dB is observed in case of user 2 at a BER of $10^{-5}$. Furthermore, the BER performance of user 2 is observed to be better than that of user 1 by about 4dB at a BER of $10^{-4}$ in case of MU-TAS. Furthermore, in case of user 2 an SNR gain of about 1dB is observed when employing MU-TAS compared to the No TAS scenario.

Fig. 11 compares the BER performance of various multiuser TAS schemes proposed in the paper, such as MU-TAS, MAJ-MU-TAS, and L-MAJ-MU-TAS. It is evident from Fig.11 that both MAJ-MU-TAS and 1-MAJ-MU-TAS scheme attain the nearly the same performance as that attained by the MU-TAS. Thus, the 1-MAJ-MU-TAS which requires minimal feedback overhead and computational burden is sufficient to attain the same performance as that attained by the MU-TAS.

V. CONCLUSIONS

We have proposed the TAS schemes for single and multiuser ZP-SC SM systems operating in a frequency selective channel, which has hitherto not been studied in the literature. Specifically, the frequency selective channel in case of a single user is first converted into a set of non-interfering parallel sub-channels by employing PIC-R-SIC. Then the EDAS is invoked over each of the parallel sub-channels in order to improve the attainable system performance. The parallel sub-channels owing to the PIC-R-SIC are theoretically shown to be nearly identical, which enabled us to reduce the feedback.
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