THEMATIC WORKING GROUP 9

TOOLS AND TECHNOLOGIES IN MATHEMATICAL DIDACTICS: RESEARCH FINDINGS AND FUTURE DIRECTIONS

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There exists a considerable amount of research on the potential of tools and resources, particularly computer-based technologies, in the teaching and learning of mathematics. Examples of such research can be found in the work of the thematic group on tools and technologies in mathematical didactics at the first and second European conferences on research in mathematics education (Gutiérrez *et al*, 1999; Jones *et al*, 2001). Such research can be categorised as focusing on one or more of three levels: the level of interactions between tool and mathematical knowledge, the level of interactions between knowledge, tool and the learner of mathematics, and the level of integration of the tool in a mathematics curriculum and in the mathematics classroom.

The potential of computer-based tools, in particular, to enhance the teaching and learning of school subjects, including mathematics, is reflected in the increase in Government spending over recent years on providing such technology for schools. For example, the British Educational Communications and Technology Agency (BECTa) estimate that the total UK spending on computers (and associated training and curriculum development work) for schools over the period 1998 to 2004 amounts to 2.65 billion Euro (BECTa, 2001a).

Given this level of spending, Governments, not just in the UK but worldwide, are beginning to ask about the impact of such spending on student learning. To date, the evidence of widespread improvement in student achievement due to using computers is not all that obvious. For example, Angrist and Victor (2001) found no evidence that increased provision of computers in Israeli schools during the mid-to-late 1990s raised pupil test scores. In the USA, analysis of data from the 1996 National Assessment of Educational Progress (NAEP) in mathematics found that high levels of general school computer use was not necessarily related to improved mathematics achievement for either fourth-grade or eighth-grade pupils (Wenglinsky, 1998). More recent UK research is suggesting a weak but positive effect on pupil learning in mathematics of providing schools with computers (BECTa, 2001a). Given this rather weak effect, some of the factors that are likely to have influenced this limited impact in the UK did emerge from the BECTa study. For example, it seems that over half (52%) of UK primary pupils never or hardly ever use computers in their mathematics lessons. This rises to 67% of lower secondary school pupils (aged 11-14). For upper secondary school pupils (aged 14-16), the proportion never, or hardly ever, using computers in their mathematics lessons is as much as 82%.

While the reasons for this lack of use of computers are varied, as Cuban observes (2001: 179), "it is premature to call the investment in computers in schools a failure because of lack of evidence for increased productivity and transformed teaching and learning. As the infrastructure matures and teachers' beliefs about teaching and learning evolve, more and more teachers will change their practices and become serious users of computers". Research would suggest that a number of factors need to be in place in order to reach a threshold above which ICT is more likely to have a significant impact and support education and learning effectively. Nevertheless, as the BECTa report observes (BECTa, 2001b), "it is unclear at this stage which factors are the most significant and how many are required in order to reach the threshold."

Such considerations underline the importance of the work of the thematic group on tools and technologies in mathematical didactics at the third European conference on research in mathematics education. In pursuing research related to the three levels of analysis noted above (the level of interactions between tool and mathematical knowledge, the level of interactions between knowledge, tool and the learner of mathematics, and the level of integration of the tool in a mathematics curriculum and in the classroom), the papers of the tools and technologies thematic group contained within this volume relate to three linked themes:

Theme 1: theoretical ideas and resources for research design and analysis

Theme 2: the design/function of technological tools/teaching activities

Theme 3: technology and the teacher/teacher development

Theme 1: theoretical ideas and resources for research design and analysis

A range of theoretical frameworks and approaches appear to hold some promise when researching the use and impact of tools and technologies. Some approaches are general mathematics education theories while others are more specific to the use of technology. The following is a list from general to specific.

Embodied cognition and metaphors Mathematical meaning is rooted in our experience of common phenomena such as movement and works as a metaphor (see, from the CERME II volume, Ainley *et al*, 1999). This approach is explicit, for instance, in research studies on the concept of function when technology helps to deepen the understanding of this phenomenon (Arzarello and Paolo, this volume).

Cognitive gaps and transitions: In research studies looking at teaching/learning as a negotiation of transitions, possibly involving gaps or obstacles, technology is seen not specially as a means to facilitate the transitions, but rather as changing the transitions themselves. For instance, the notion of function can be approached independently from algebra and not necessarily afterwards (Giraldo and Carvalho, this volume).

Situated abstraction (cognition) and systemic thinking: From the idea that cognition is necessarily situated, teaching/learning should aim to connect knowledge acquired (built) in different situations. With this perspective, learning with technology is obviously situated and therefore researchers look at the connections that can be made with other situations and also how technology can help to make connections (see, for instance, Lemut, this volume).

Semiotic mediation: From Vygotsky and others, in social contexts, significations can be created from specific uses of an instrument. Dragging, for instance, can be associated with understanding a *Cabri* construction as a figure rather than just a drawing. Talmon & Yerushalmy, this volume, provide one example. Further accounts of the mediational impact of technology can be found in the papers of, for example, Chiappini and Reggiani (for a general elaboration), Falcade (in the particular case of dynamic geometry software) and Robutti (in the case of graphing calculators).

Instrumentation: This approach distinguishes the instrument (a psychological construction) from the artefact (the material object involved in an instrumented action). The instrument is made of specific schemes in the Piagetian sense, and these schemes are related both to the artefact and to mathematics. The construction of schemes is the instrumental genesis that researchers consider especially when studying long-term uses of technology (see, for example, the papers by Haspekian and by Lagrange, this volume).

In researching the use and impact of tools and technologies in mathematical didactics, the role of such theories are twofold:

Role 1: Exploring the development of knowledge in specific mathematical concepts. For instance, as mentioned before, researchers consider varied theories when tackling the teaching/learning of the concept of function or geometric knowledge

Role 2: Exploring the impact of technology on the activity of students and teachers. It is known that new technologies have a strong impact on the delicate balance of teaching/learning. Theories can help to better appreciate this impact.

Theme 2: design/function of technological tools/teaching activities

This theme is characterised by the following questions:

> How might technology change learning sequences or bring together aspects of the curriculum that are often treated separately (and at different times)?
>
> What is the impact of using technology on didactical gaps, didactical transitions, and epistemological obstacles?
>
> What might be suitable theoretical framework(s) for designing technological tools/teaching activities?

It is widely recognised that tools change the way that activities are carried out and can shape the conceptions of the user (see, Gutiérrez *et al*, 1999; Jones *et al*, 2001). This means that learning sequences are not independent of the tools being used, and neither are didactical gaps, didactical transitions, nor epistemological obstacles. In the papers of this thematic group contained within this volume there are examples of studies where technology has altered learning sequences and/or impacted on transitions. Examples include the transition from number to algebra (see, for example, the paper by Haspekian) and the learning sequence from algebra to analysis/calculus (see, for example, the paper by Yerushalmy and Chazan). Examples of projects can be seen in the short papers in this volume (for example, those of Delgado & Santiago and Hyde & Clark-Jeavons) while implications of research can be found in the short paper by Jones.

Based on the findings of such projects, future research projects might include long-term studies of a cycle of software/technology development which might begin with a technology which provokes a (minor) change in curriculum which, in turn, provokes a change in the software/technology which leads to further curriculum change, and so on.

Theme 3: technology and the teacher/teacher development

This theme is characterised by the following questions:

> How can we understand how mathematics teachers can integrate technology into their teaching?
>
> How might we encourage more mathematics teachers to use technology?
>
> How does using technology change the ways mathematics teachers think about teaching and learning?

As the evidence from the large-scale BETCa project demonstrates (BECTA, 2001a), there is still a long way to go before students are regularly using computers in their mathematics lessons (although it should be noted that an optimum level of computer use for mathematics learning has yet to be determined). As it stands, it appears that most pupils only rarely use computers in their mathematics lessons (and even more rarely to do mathematics at home). In the papers of the thematic group within this volume there are examples of research projects aimed at understanding how mathematics teachers can integrate technology into their teaching and at understanding how to encourage more teachers to use technology. Useful approaches documented in the papers include beginning with the sorts of problems teachers have and thinking of ways that technology might help to solve the problem (such as the teaching of proof and proving, see the paper by Tapan) together with other ways of supporting teachers and/or building on or developing their practice (see, for example, the full papers by Lins; Ludwig *et al*; Miller *et al*; and by Zuccheri, and the short paper by Leder). Based on the findings of such projects, future research projects might include further experimental work linked to case studies of mathematics teachers who have changed their practice to include technology and of beginning teachers who have successfully started using technology.

Concluding Comments

While the set of papers of this thematic group provides a rich source of data and findings about the use and impact of tools and technologies in mathematical didactics, it is probably fair to say that, as yet, insufficient research results have emerged about how ICT might contribute to effective pedagogy, although plenty of speculation is available. An unresolved question, for example, is whether ICT initiatives should aim to support current pedagogy and improve attainment within the frameworks that schools already have, or whether new pedagogies will emerge replacing the old. Another challenge is to produce more high-quality research. The preface to the BECTa (2001a) report, for example, judges that research into the effects of using computers on student achievement "is often on small samples, rarely controls out the effects of things other than ICT, and is rarely rigorous enough in its methodology, or its search for explanations of findings, to support the weight that has been put on it." Murphy *et al* (2002), delineate similar major methodological problems with much research on the impact of computers on student learning.

Overall, the work of the Thematic Group, as exemplified by the set of papers included in this volume covers a variety of software technologies (DGS, spreadsheet, CAS, multimedia, distance education, calculators), school levels (from primary to university students) and methodologies (including small scale case studies, software design, surveys, and research reviews). The outcome for the participants was the development of a range of common notions and the sharing of a variety of concerns and ideas. As thematic group leaders we hope that this variety is captured in this set of papers and that these inform the future work of this thematic group within the European Society of Research in Mathematics Education.

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List of contributions

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